Private Join and Compute from PIR with Default

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Abstract. The private join and compute (PJC) functionality enables secure computation over data distributed across different databases, and is applicable to a wide range of applications, many of which address settings where the input databases are of significantly different sizes.
We introduce the notion of private information retrieval (PIR) with default, which enables two-party PJC functionalities in a way that hides the size of the intersection of the two databases and incurs sublinear communication cost in the size of the bigger database. We provide two constructions for this functionality, one of which requires offline linear communication, which can be amortized across queries, and one that provides sublinear cost for each query but relies on more computationally expensive tools. We construct inner-product PJC, which has applications to ads conversion measurement and contact tracing, relying on an extension of PIR with default. We evaluate the efficiency of our constructions, which can enable $\mathbf{2^8}$ PIR with default lookups on a database of size $\mathbf{2^{25}}$ (or inner-product PJC on databases with such sizes) with the communication of **44**MB, which costs less than **0**.**17**c. for the client and **26**.**48**c. for the server.

1 Introduction

Private set intersection (PSI) enables two parties who have private input sets to identify items that they have in common without learning any other information. While PSI has proven its broad applicability, there are settings which require more refined functionality that does not reveal the whole intersection but rather enables restricted computation on the data in the intersection. We refer to this functionality as *private join and compute (PJC)* [Goo19].

An important difference in the privacy requirements relevant for the PJC and the PSI settings, is that while the intersection size is inherently revealed by the PSI output, in the PJC case this is an additional privacy leakage, which should be avoided in many scenarios. The cost of the "compute" part in a private join and compute protocol is determined by the size of the intersection, which is often much smaller than the size of the input sets, thus the dominant efficiency cost is the cost of the step computing the intersection. Similarly to the PSI setting, when the two input datasets are of the same size, the intersection computation is necessarily linear in the input size. However, when we have asymmetric inputs where one of the datasets is much larger than the other, the efficiency goal is to avoid linear dependence on the size of the larger input set. This raises the question whether it is possible, in the private join and compute setting, to address both the privacy requirement of hiding the intersection size and at the same time provide sublinear efficiency.

The PSI-Sum solution of Ion et al. [IKN+20], which was deployed in practice, does not provide either of the above properties, and they will be highly beneficial for that setting. First, that solution scales poorly for the party with the smaller input set, which also often has much more constrained resources, but needs to incur cost proportional to the larger set. Second, it inherently reveals the intersection size, which can be significant leakage especially when one of the inputs is small – their protocol mitigates the issue by allowing the party with the small input to learn the intersection size first and decide to abort if it is too small. Our construction addresses both of these issues. Additionally, we also allow revealing the intersection cardinality in a differentially private manner. Further, we extend the functionality that can be computed over the intersection, including allowing both parties to contribute associated values. While we mainly focus on a specific functionality (described below), we also discuss how to extend our work to generic functionalities.

We specifically consider the problem of private join and compute (Inner Product PJC) which allows computing an inner product between attribute values associated with the intersection IDs in each of the two input datasets. In this setting the two input sets are of the form $(X, W) = \{(x_1, w_1), \ldots, (x_t, w_t)\}$ and $(Y, V) = \{(y_1, v_1), \ldots, (y_n, v_n)\}$ and the computation evaluated by the PJC functionality is defined as follows: $f((X, W), (Y, V)) = \sum_{i \in [t], j \in [n], x_i = y_j} w_i v_j$.

1.1 Our Motivation

We motivate the above functionality with two practical applications. The first application involves privacy-preserving computation for the effectiveness of advertising campaigns, which is a generalization of the functionality supported by Ion et al. [IKN+20]. A transaction data provider (TDP) has a database of transaction values *tdp_db* which contains $(id, spending)$. Here, the customer "id" has seen an ad, and then makes a purchase with an amount "spending". The Ad tech company has a database *at_db* which contains $(id, type)$. Here, the customer "id" has seen an ad with a "type" supplied by the ad tech company. The "type" can be the time spent watching ads. Typically the number of ad impressions over a particular time period is orders of magnitude higher (millions) than the corresponding number of transactions on a fixed date (thousands), thus the sets are highly asymmetric. The TDP may want to partition based on user attributes such as new/returning customer, whether the customer is a loyalty card member, or some demographic information, and may want to learn an inner-product for each partition. The following query on the join of these two databases computes the sum of the transaction values of users who saw ads weighted according to the type (or weight) supplied by the ad tech company.

$$\texttt{SELECT}\ \ sum(tdp\_db.spending * at\_db.type)$$
$$\texttt{FROM}\ \ at\_db\ \ \texttt{INNER JOIN}\ \ tdp\_db$$
$$\texttt{ON}\ \ at\_db.id = tdp\_db.id$$

This problem can be seen as an instance of inner product PJC, where set sizes are asymmetric, and hiding the exact intersection size may be especially important, since the computation may be repeated with overlapping partitions from the TDP.

The inner product PJC functionality could also be used to enhance the privacy guarantees of exposure notification protocols in the existing decentralized contact tracing solutions [AGC20, CGH+20, TSS+20, DP320]. In such solutions, user devices broadcast BLE packets that contain pseudorandom values generated from a daily secret key. Users who test positive for COVID-19 can report their secret keys for the periods when they were infectious to a central server. Each key is accompanied with a transmission risk score based on the diagnosis and user symptoms. Anyone who downloads the server database can therefore check whether the random values that her app has received were derived from any of the reported secrets. However, this approach also allows learning information about the values transmitted in individual BLE packets. We can view the above problem as an instance of inner product PJC where the server database contains the reported pseudorandom values with their risk scores, and where the user has the pseudorandom values she has observed, and possibly with corresponding weights determined by the time elapsed since the exposure incident, the exposure duration, and other parameters. The goal is for the user to obtain the weighted sum of the transmission risks of the pseudorandom values matching all her exposures. We note that this application also has a natural input size asymmetry: the client set is much smaller than the server database.

1.2 Our Contributions

With these two applications in mind, we present two different instantiations of our approach, tailored for two distinct settings. We assume that the participants are semi-honest, they follow the protocol but attempt to obtain extra information from the execution transcript. Our first construction is in the setting allowing offline precomputation and initialization. In this setting, the server's database is fixed beforehand and can be computed on in an "offline" phase. The goal is to minimize the cost of (possibly repeated) client queries in the "online" phase when the client data becomes available. Our first construction in this setting incurs a setup time that is linear in the size of the server's (larger) dataset. The subsequent client queries are highly efficient, and have computation and communication time linear in the client's dataset and essentially independent of the size of the server dataset. This is similar to approaches taken by [KLS+17, RA18], which send an encoded server database to the client in the offline phase, allowing highly efficient "online" intersections. Our work can be seen as extending the functionality achieved by these previous works by enabling computation over the intersection but keeping the intersection itself hidden, while preserving the desirable efficiency properties for the online phase. This construction is well-suited to applications where many small PJC executions are run against a single large databases. For example, in the conversion-measurement setting, the client's dataset may arrive in small batches, or the client may want to make multiple overlapping queries based on different demographic slices. Previous works incur the costs proportional to the larger database each PJC query.

The second construction is in the fully online setting (without precomputation). In this case, we instantiate our construction using techniques derived from Private Information Retrieval (PIR). The resulting construction allows the client to incur costs that are asymptotically linear in the size of its own dataset, and logarithmic in the server's dataset size. In practice, this makes it so the bulk of the costs of executing the protocol are shifted from the client to the server. In this way, our work improves on [PSTY19] by making the costs incurred by each party more equitable in the asymmetric input size setting, This is especially beneficial when the client is a constrained device like a mobile phone, such as in the contact tracing application.

Both our constructions compose with differential privacy in a straightforward way, which allows repeated client queries on a single server database, using the differential-privacy noise to hide correlations between the outputs of the different queries. This allows our protocol to hide and/or apply differential privacy noise to the intersection size as well as the function computed over the intersection. This is an improvement over PJC [IKN+20] and related works such as [BKM+20], which require revealing the intersection size without noise.

PJC from PIR-with-Default The main building block for one of our PJC constructions provides another primitive of independent interest which we call *private information retrieval (PIR) with default*. This is a primitive which enables PIR queries over a sparse database where the client has an input index and receives either the data stored at that index, or a default value, if there is no item with this index in the database. The server does not learn anything about the query including whether the client received a database value or a default value. The client does not learn any further information about the database or the default value apart from her output. In particular, if the database values and the default value are indistinguishable, then the client does not learn whether the query index was present in the database. We also present a multi-query PIR-with-Default construction.

PIR-with-Default on its own is sufficient to compute private set intersection-sum [IKN+20]. Another application of PIR-with-Default outside the PJC setting, is a way to distribute anonymous tokens [KLOR20] as follows: the users who belong to the database stored by the server receive one type of an authentication token (which is used as the associated value for all database entries in the PIR-with-Default execution), while every other user receives a second type of an authentication token which is used for the default value. The server does not

		Construction 1 Offline	Construction 1 Online	Construction 2	Circuit-based PSI [PSTY19, SGRP19]	Labeled PSI [CHLR18]
Communication	Client	-	$O(t)$	$O(t\log(n/t))$	$O(t+n)$	$O(t\log(n/t))$ + $\lvert\mathtt{GC}(t)\rvert$
	Server	$O(n)$				
Computation	Client	-		$O(t\log(n/t))$	$t\log(t)$	$O(t\log(n/t))$ + $\lvert\mathtt{GC}(t)\rvert$
	Server	$O(n)$		$O(n)$	$n\log(n)^2$	

Table 1: Theoretical costs of PJC protocols. In Construction 2, the log factor comes from the asymptotic behavior of the underlying PIR scheme, and can be replaced with the efficiency of the specific PIR scheme. The computational complexity of [PSTY19] is slightly improved by mega-bin hashing. Poly-ROOM [SGRP19] achieves asymptotics similar to [PSTY19], thus, we group it in the circuit-based PSI. Label-PSI [CHLR18] achieves similar asymptotic efficiency as Construction-2, but has worse concrete performance (see [LPR+20]) and requires extra cost due to using a generic MPC. We denote the extra cost as $\lvert\mathtt{GC}(t)\rvert$.

learn which of the two groups the user belongs to, and if the two types of tokens are indistinguishable, the client does not learn which type it received.

We also introduce a small extension of the PIR-with-Default functionality, which we call Extended-PIR-with-Default, that enables both parties to contribute associated values. In this case, the parties will learn shares the product of the associated values, or the default value. If the parties sum the shares they receive from multiple queries, they will receive shares of the inner-product over the intersection, which then directly achieves the inner-product PJC functionality.

Table 1 shows the theoretical communication and computation complexity of our protocol compared with prior works. Note that [CHLR18] is secure against malicious adversaries, but only for the Labeled PSI functionality itself and not for PSI with computation. Table 1 lists the cost of semi-honest Labeled PSI [CHLR18].

Implementation Evaluation We evaluate the concrete communication, computation and monetary costs of our constructions and present them together with comparisons to existing works in Section 7. For our first PJC construction, only the offline communication and computation depends (linearly) on the size of the larger dataset. The online communications and computation is determined completely by the size of the smaller set and the cost of random memory access (for datasets of size 2^8 and 2^{25}, the online computation is $\sim 2.43ms$ and the communication is 7MB). Our second construction is more computationally expensive but outperforms any existing constructions in terms of total communication when the differences of the two dataset sizes are significant, especially when the difference of input sizes is greater than a factor of 2^{10}. In terms of monetary cost, a PJC execution on sets of sizes 2^8 and 2^{25} costs ~ 0.17 c. for the client and ~ 26.48 c. for the server. Compared to the previous works, our online constructions lead to a significant reduction in client monetary costs with a small corresponding rise in server costs. For example, for $n = 2^{25}$ and $t = 2^8$, our client cost is $36.5\times$ lower than that of [PSTY19], while incurring a server cost that is only $4\times$ higher than theirs.

1.3 Improvement on Related Work

Our work is focused on privately computing a function over the intersection of two asymmetric-sized datasets, both in the setting with offline setup, and in the fully-online setting. We discuss the most important related works.

The field of private set intersection protocols is very rich, starting from the earliest PSI constructions that are based on the Diffie-Hellman assumption [Mea86]. Over the last few years, there has been a long list of works on efficient secure PSI [DCW13, CHLR18, PRTY19, PRTY20] with fast implementations, which can process millions of items in seconds. However, most of these works only allow to output the intersection itself. In our scenario we wish to compute some function of the intersection while hiding the individual elements in the intersection. There is much less related work on the more general private intersection join and compute.

In terms of works that support computing over the intersection while hiding the values, a prominent approach is *Garbled-Circuit-based PSI*. [HEK12] proposes an efficient sort-compare-shuffle circuit construction to implement PJC. [PSTY19] improves circuit-PSI using several hashing techniques. The main bottleneck in the existing circuit-based protocols is need for a large number of string comparisons, and the methods used for computing over associated values. These are done inside a generic MPC protocol, which increases the interaction round complexity, and incurs cost due to bitwise encryption of each party's dataset. Moreover, while these protocols are well-suited to symmetrically-sized input sets, they perform worse when inputs are asymmetric: both parties incur costs linear in the larger database size. Another approach in this space, which is currently used in practice by Google [Goo19], is the approach combining Diffie–Hellman and homomorphic encryption techniques [IKN+20]. While this approach has reasonable communication cost and can be extended to the PJC functionality, it also performs poorly in the asymmetric inputs setting, since both parties incur costs proportional to the other party's dataset size. In terms of work that leverages offline precomputation where one of the parties' datasets is fixed beforehand, there are several prominent works with the application of private contact discovery. Recent works [KLS+17, RA18] achieve good performance in the offline setting with asymmetric inputs. However, these works cannot be straightforwardly extended to privately compute on the intersection.

The work that achieves the closest result to ours is the protocol of [CHLR18], which uses homomorphic encryption to perform efficient PSI on sets of asymmetric sizes, with communication cost logarithmically related to the larger dataset. The authors show how to extend this construction to enable each party to retrieve labels associated to individual items in its input, with the property that the client receives "valid" labels only for the items in the intersection . They further describe how these labels can be additively masked and fed into a downstream generic MPC computation that allows privately computing a function over these labels (while hiding which specific labels were in common). This "PSI-with-Computation" extension is described mostly theoretically by [CHLR18], and is not accompanied by detailed experiments.

We see our work as improving on the approach outlined in [CHLR18] in several important ways. The first is that we use a highly tailored approach to test membership and retrieve additive shares of the labels, which greatly moderates the client cost compared to a generic approach. Secondly, the [CHLR18] protocol effectively uses a novel batched Private Information Retrieval (PIR) protocol to achieve efficiency in the asymmetric input size setting. We make the relationship to PIR explicit in our construction, which allows us to leverage techniques from the PIR literature [GR05, ACLS18, ALP+19], especially recursion and oblivious query expansion. Thirdly, our approach can be efficiently applied in the offline precomputation setting such that the client's online cost is essentially independent of the server's database size. This can provide significant gains when many queries will be made against the same database.

2 Technical Overview

Next we overview the main techniques in our constructions. We first describe the construction of PIR-with-Default, which is the core of our contributions. In particular, we show two different instantiations of PIR-with-Default: one with offline setup and one with sublinear online executions, and we describe important batching optimizations. Next, we show how to modify our constructions to achieve an extended functionality, which we call Extended-PIR-with-Default. Finally, we will describe how to build inner-product PJC from Extended-PIR-with-Default.

PIR-with-Default*:* In the PIR-with-Default functionality, we assume the server holds the larger input set $(Y, V) = \{(y_1, v_1), \ldots, (y_m, v_n)\}$ while the client holds a single input x. We want the client to receive v_j if $x = y_j$ for some j, and a server-chosen default value d otherwise. Neither party should learn anything extra, and in particular, the server should not learn which value was retrieved, and the client should not learn the other items in the server's database. The client should also not learn whether it received the default value (assuming the default value is chosen by the server to be indistinguishable from the w_j values.).

Our approach uses Bloom filters [Blo70], a data structure that allows efficient set membership tests over sparse sets. A Bloom filter (BF) is a binary vector that encodes a set. For each item x, one can check whether x is in the set or not by querying a constant number of locations in the BF. Specifically, Bloom filters have as public parameters a set of hash functions $H_1, \ldots, H_k$ and testing membership of x requires accessing only locations $H_1(x), \ldots, H_k(x)$ in the Bloom Filter and checking that they are all 1 (or alternatively, checking $k = \sum_{i \in [k]} \mathsf{BF}[H_i(x)]$). In order to allow retrieving associated values, we leverage the closely related notion of garbled Bloom filters (GBF) [DCW13], which allows to store not only a set but also a set of associated values. For value x present in the database, computing $\sum_{i \in [k]} \mathsf{GBF}[H_i(x)]$ will result in the associated value. However, if x is not present in the database, $\sum_{i \in [k]} \mathsf{GBF}[H_i(x)]$ will return a garbage value that needs to be transformed to the default value. We use a GBF in conjunction with a BF as we discuss next.

The first step is that the server creates a BF that contains the indices in Y and a GBF that contains its database (Y, V). The client and the server then execute a query protocol where the client has as input an index x and the output of the query protocol will be secret-shares of the membership bit for x in the BF and secret-shares of the value retrieved from the GBF for x (which is either a secret-share of some w_j, or a secret share of some garbage value). Next the client and the server will execute a $\mathsf{Value\text{-}Or\text{-}Default}$ protocol in which the two parties input their shares of the BF and GBF query responses and additionally the server's default value for this execution, and the client obtains either the value from the GBF query, if the BF query response was a share of 1, or the default value, otherwise.

We first describe the BF query protocol with a linear offline setup phase with a fixed server database, and client query that is available only during the online phase. We will then describe a setup-free BF query with sublinear cost in the larger database. These will constitute the difference between our two different constructions of $\mathsf{PIR\text{-}with\text{-}Default}$. After that, we will describe the $\mathsf{Value\text{-}Or\text{-}Default}$ protocol, which will be shared by both $\mathsf{PIR\text{-}with\text{-}Default}$ constructions.

BF/GBF Queries with Linear Offline Cost. In the offline phase the server sends an encryption of BF and GBF, where each entry is encrypted using an additively homomorphic encryption scheme. Now for each query x, the client can compute $H_1(x), \ldots, H_k(x)$, and can locally compute the encryption of $\mathsf{Enc}(\sum_{i\in[k]} \mathsf{BF}[H_i(x)])$. The client generates a random value $r_{\mathcal{C}}$, which it keeps as its share, and sends $\mathsf{Enc}(\sum_{i\in[k]} \mathsf{BF}[H_i(x)] - r_{\mathcal{C}})$ to the server, which the server decrypts to obtain its share $r_{\mathcal{S}}$. The client and server then transform shares $r_{\mathcal{C}}$ and $r_{\mathcal{S}}$ of $\sum_{i\in[k]} \mathsf{BF}[H_i(x)]$ into shares of the BF membership result using a single 1-out-of-$(k+1)$ oblivious transfer (OT) [Rab05] as follows. The client chooses a bit $b_{\mathcal{C}}$ and computes $B = \{b_0, \ldots, b_k\}$ where all b_i are $b_{\mathcal{C}}$, except $b_{(r_{\mathcal{C}}+k) \bmod (k+1)}$ is the client's share which is equal to $1 \oplus b_{\mathcal{C}}$. The client and the server execute 1-out-of-$(k+1)$ OT where the client is the sender with input B and the server is the receiver with input $r_{\mathcal{S}}$. The server obtains output $b_{\mathcal{S}}$ such that $b_{\mathcal{C}} \oplus b_{\mathcal{S}} = 1$ if and only if $r_{\mathcal{S}} + r_{\mathcal{C}} = k$.

In order to obtain shares of the GBF value, the client similarly locally computes $\mathsf{Enc}(\sum_{i\in[k]} \mathsf{GBF}[H_i(x)])$, and generates a random value $v_{\mathcal{C}}$, and sends the server $\mathsf{Enc}(\sum_{i\in[k]} \mathsf{BF}[H_i(x)] - v_{\mathcal{C}})$. The server decrypts this value to obtain its share $v_{\mathcal{S}}$. After these steps, the server and client have shares of the BF membership bit, and the GBF evaluation, as desired.

BF/GBF Queries with Sublinear Cost. Our second construction for the BF and the GBF queries leverages constructions for symmetric private information retrieval (PIR) [GIKM00] with sublinear communication based on homomorphic encryption (HE) [Gen09]. The general idea is that instead of transferring the entire encrypted BF and GBF to the client during a setup phase, the client instead makes PIR queries to retrieve the desired entries $H_i[x]$ of the BF and GBF. We make use of the fact that in many constructions of PIR, the client sends a homomorphic encryption of its desired index, which the server uses to obliviously compute an encryption of the query response under the same homomorphic encryption scheme, and the server can therefore sum several such responses before returning them. Specifically, our client sends PIR queries for locations $H_1(x), ..., H_k(x)$, and the server evaluates the queries to obtain $\mathsf{Enc}(\mathsf{BF}[H_i(x)])$ and $\mathsf{Enc}(\mathsf{GBF}[H_i(x)])$. The server then homomorphically sums these values, and subtracts randomly chosen masks $r_{\mathcal{S}}$ and $v_{\mathcal{S}}$ to obtain $\mathsf{Enc}(\sum_{i\in[k]} \mathsf{BF}[H_i(x)]-r_{\mathcal{S}})$ and $\mathsf{Enc}(\sum_{i\in[k]} \mathsf{GBF}[H_i(x)]-v_{\mathcal{S}})$, which it sends to the client. The client decrypts these values to get $r_{\mathcal{C}}$ and $v_{\mathcal{C}}$ respectively. The client and server engage in the 1-out-of-$(k+1)$ OT described earlier to get shares $b_{\mathcal{C}}$ and $b_{\mathcal{S}}$ of the BF membership bit. These, together with the $v_{\mathcal{C}}$ and $v_{\mathcal{S}}$ values, are the desired output of the BF/GBF Queries.

Our use of PIR is heavily amenable to different kinds of optimization, which we explore in detail in Section 5.3 and the full version of this work [LPR+20]. Specifically, PIR constructions achieve sublinear communication either by using packing techniques leveraging the slots in a HE ciphertext to encrypt the entire selection vector in a single ciphertext [ACLS18, ALP+19], or using recursion where the selection vector is written as an outer product of several vectors of shorter length [GR05, ALP+19]. These two techniques are not compatible with each other, i.e. packing the entire selection vector for a query in a single HE ciphertext requires increased computation at the server and higher multiplicative degree from the HE, and does not provide efficiency benefits. However, in our setting we need to execute multiple PIR queries and we use the HE slots to pack coordinates of the selection vectors from different queries. This HE-slotting technique is also compatible with multi-query PIR approaches which use Cuckoo hashing [PR01, PSSZ15] to reduce the communication cost per query. Such hashing techniques partition both parties' inputs in a way that guarantees that the client queries are distributed evenly across the smaller server partitions and can be executed only over the partition without revealing anything about the query indices. We also instantiated this approach using two-choice hashing [CRS03, PRTY19] and compare it to Cuckoo hashing for different parameters. In both of these multi-query instantiations we can pack coordinates from queries for different partitions in the same HE ciphertext while preserving the efficiency of the server computation.

Value-Or-Default *protocol:* As we discussed above, after the BF/GBF queries, the client and server have XOR shares $b_{\mathcal{C}}$ and $b_{\mathcal{S}}$ of a bit (the output of the BF query) and additive shares $v_{\mathcal{C}}$ and $v_{\mathcal{S}}$ of a value (the output of the GBF query). In addition the server has as input a default value d. The goal of the Value-Or-Default phase is to take these shares and produce output received by the client, namely $v = v_{\mathcal{C}} + v_{\mathcal{S}}$ if $b = b_{\mathcal{C}} \oplus b_{\mathcal{S}} = 1$, and d, otherwise. We execute this phase using only two 1-out-of-2 OT executions. The first OT enables the server to learn $q = \Delta_{\mathcal{C}}+b\cdot v_{\mathcal{C}}$ where $\Delta_{\mathcal{C}}$ is a random value generated by the client. This is achieved by executing a OT where the client is the sender with messages $m_0 = \Delta_{\mathcal{C}}+b_{\mathcal{C}}\cdot v_{\mathcal{C}}$, $m_1 = \Delta_{\mathcal{C}}+(1-b_{\mathcal{C}})\cdot v_{\mathcal{C}}$ and the server is the receiver with bit $b_{\mathcal{S}}$. The second OT enables the client to obtain $\Delta_{\mathcal{C}}+b\cdot v_{\mathcal{C}}+b\cdot v_{\mathcal{S}}+(1-b)\cdot d$ from which the client can subtract $\Delta_{\mathcal{C}}$ to recover v if $b = 1$, and d, if $b = 0$, as desired. In the second OT the server is the sender with messages $m_0 = q + b_{\mathcal{S}} \cdot v_{\mathcal{S}} + (1 - b_{\mathcal{S}}) \cdot d$ and $m_1 = q + (1 - b_{\mathcal{S}}).v_{\mathcal{S}} + b_{\mathcal{S}}.d$ which the client is the receiver with input bit $b_{\mathcal{C}}$. Combining the BF/GBF queries with the Value-Or-Default phase achieves the PIR-with-Default functionality.

Extended-PIR-with-Default *from* PIR-with-Default*:* Extended-PIR-with-Default has the additions: firstly, the client holds a weight w in addition to x. Secondly, the output learned by the client should be an additively masked version of the product $w \cdot v_j - s$ if $x = v_j$ for some v_j in the server's database, and the additively masked default value $d - s$, and the server should receive the additive mask s. This extension acts as a bridge between PIR-with-Default and inner-product PJC by incorporating values from both parties, and also to more easily hide from the client whether it retrieved a "real" value or the default.

We note that the mask w can be incorporated by having the party that creates the GBF sum homomorphically multiply the GBF sum by w before proceeding with the protocol. More specifically, in the PIR-with-Default protocol with offline setup, once the client homomorphically computes the GBF sum, it can homomorphically multiply the result with the scalar w before masking it. In the protocol with sublinear costs, the client additionally sends a homomorphic encryption of w to the server along with its PIR queries. The server, after computing the PIR queries and summing the results, can multiply the GBF sum with the encryption of w before masking it.

In order to additively mask the final result, the server simply replaces the values $v_{\mathcal{S}}$ and d that it uses in the Value-Or-Default phase with the values $v_{\mathcal{S}} - s$ and $d - s$ respectively. This makes it so the final value retrieved by the client is either $w \cdot v_j - s$ or $d - s$ as desired.

Inner Product PJC from Extended-PIR-with-Default *:* In inner product PJC, the server holds larger input set $(Y, V) = \{(y_1, v_1), \ldots, (y_m, v_n)\}$ and the client holds the smaller input set $(X, W) = \{(x_1, w_1), \ldots, (x_t, w_t)\}$. In our protocol, the client and the server jointly execute t Extended-PIR-with-Default queries from the set X where the server has default value 0 for all the queries. As a result of this the client and the server have shares $\alpha_{\mathcal{C},i}$ and $\alpha_{\mathcal{S},i}$ such that $\alpha_{\mathcal{C},i} + \alpha_{\mathcal{S},i} = w_i \cdot v_j$ for all $x_i \in Y$ and $\alpha_{\mathcal{C},i} + \alpha_{\mathcal{S},i} = 0$ for all $x_i \notin Y$. Therefore, by adding their local shares $\sum_{i \in [t]} \alpha_{\mathcal{C},i}$ and $\sum_{i \in [t]} \alpha_{\mathcal{S},i}$, the client and the server obtain shares of the desired output $\sum\limits_{i \in [t], j \in [n], x_i = y_j} w_i v_j$.

3 Preliminaries

We briefly introduce notations and cryptographic primitives in this section, and refer to the full version of this work [LPR+20] for complete definitions. We denote by κ and λ the computational and statistical security parameters respectively. For $n \in \mathbb{N}$, we write $[n] = \{1, \ldots, n\}$. We define a probabilistic polynomial time (PPT) algorithm to be a randomized algorithm that runs in polynomial time in the length of its first parameter.

Oblivious Transfer (OT) [Rab05]: 1-out-of-n OT is a two-party protocol, in which a sender with n inputs $(m_1, \ldots, m_n)$ interacts with a receiver who has an input choice $b \in [n]$. The result is that the receiver learns m_i without learning anything about others $m_j, \forall j \in [n] \setminus \{i\}$, while the sender learns nothing about the receiver's choice b.

Bloom Filter (BF) [Blo70] and Garbled Bloom Filter (GBF) [DCW13]: A BF is an array $\{\mathsf{BF}[i]\}_{i\in[n]}$ of bits where each keyword x is inserted to the BF by setting $\mathsf{BF}[h_i(x)] = 1$ for all h_i in a collection of hash functions $H = \{h_1, \ldots, h_k \mid h_i : \{0,1\}^\star \to [n]\}$. A GBF is an array of integers in $\mathbb{Z}_\ell$ that implements a key-value (x, v) store, where the value v associated with key x is $v = \sum_{i=1}^{k} \mathsf{GBF}[h_i(x)]$.

Cuckoo Hashing [PR01] and 2-Choice Hashing [CRS03]: Basic Cuckoo hashing consists of m bins $B[1], \ldots, B[m]$, a stash, and k random hash functions $h_1, \ldots, h_k$ of range $[m]$. To insert an element x into a Cuckoo hash table, we place it in bin $h_i(x)$, if this bin is empty for any i. Otherwise, we choose a random $i \in [k]$ and place x in bin $h_i(x)$, evict the item currently in that bin, and recursively insert the evicted item. 2-choice hashing uses $k = 2$ random hash functions h_1, h_2 of range $[m]$, and each item x will be placed in whichever of $h_1(x), h_2(x)$ currently has fewest items.

Homomorphic Encryption (HE): HE is a form of encryption that allows to perform arbitrary computation on plaintext values while manipulating only ciphertexts. In this work, we use the BGV [BGV14] and FV [FV12] HE schemes.

Private Information Retrieval: Private information retrieval (PIR) is a cryptographic primitive that allows a client to query a database from one or multiple servers without revealing any information about the query to the database holder(s). A trivial solution suffering linear communication overhead consists in sending the whole database to the client. While the feasibility of a protocol with sublinear communication has been resolved for a long time [CKGS98], the search for concretely efficient constructions for practical applications has been an active area of research [GR05, ACLS18, ALP+19]. In this paper, we focus on the single-server setting and will use RLWE-based homomorphic encryption scheme as in [ACLS18, ALP+19].

4 Definitions

In this section, we provide the formal security definitions that we will use for our protocols. All our constructions will be proven in the semi-honest setting where the parties follow the prescribed steps in the construction.

We provide standard simulation security definitions [Gol04] for our constructions that use the following notation: $\mathsf{View}_{\mathcal{C}}^{\Pi}(1^\lambda, [X]_{\mathcal{C}}, [Y]_{\mathcal{S}})$ is the view of party $\mathcal{C}$ during the execution of protocol Π with security parameter λ between parties $\mathcal{C}$ and $\mathcal{S}$ which have inputs X and Y respectively; $\mathsf{SIM}_{\mathcal{C}}^{\Pi}(1^\lambda, O)$ is a ppt simulator algorithm, which generates the view of party $\mathcal{C}$ in the execution of a protocol Π

Parameters: Server/Sender $\mathcal{S}$ and Client/Receiver $\mathcal{R}$, agree upon
- An upper bound n on the number of key-value pairs in Server $\mathcal{S}$'s input.
- A space $\mathbb{Z}_\ell$ for the associated values and default values.
- A bound t on the number of Client $\mathcal{C}$'s queries.

Inputs:
$\mathcal{S}$: A set of key-value pairs $\mathcal{P} = \{(y_1, v_1), \ldots, (y_n, v_n)\}$ with distinct y_i, and default values $\mathcal{D} = \{d_1, \ldots, d_t\}$.
$\mathcal{R}$: A set of t queries $\{x_i\}_{i\in[t]}$.
Outputs:
$\mathcal{S}$: No output.
$\mathcal{R}$: A set $O = \{o_i\}_{i\in[t]}$ where

$$o_i = \begin{cases} v_j, & \text{if } x_i = y_j \text{ for some } j \in [n] \\ d_i, & \text{otherwise} \end{cases}$$

Fig. 1: The PIR-with-Default Functionality.

Parameters: Server/Sender $\mathcal{S}$ and Client/Receiver $\mathcal{R}$, agree upon
- An upper bound n on the number of key-value pairs in Server $\mathcal{S}$'s input.
- A space $\mathbb{Z}_\ell$ for the associated values and default values.
- A bound t on the number of Client $\mathcal{C}$'s queries.

Inputs:
$\mathcal{S}$: A set of key-value pairs $\mathcal{P} = \{(y_1, v_1), \ldots, (y_n, v_n)\}$ with distinct y_i, a set of default values $\mathcal{D} = \{d_1, \ldots, d_t\}$, and a set of additive masks $S = \{s_1, ..., s_t\}$. Each v_i, d_i and $s_i \in \mathbb{Z}_\ell$.
$\mathcal{R}$: A set of t pairs $\{(x_i, w_i)\}_{i\in[t]}$. Each $w_i \in \mathbb{Z}_\ell$.
Outputs:
$\mathcal{S}$: No output.
$\mathcal{R}$: A set $O = \{o_i\}_{i\in[t]}$ where

$$o_i = \begin{cases} (w_i \cdot v_j) - s_i, & \text{if } x_i = y_j \text{ for some } j \in [n] \\ d_i - s_i, & \text{otherwise} \end{cases}$$

Fig. 2: The Extended-PIR-with-Default Functionality. All arithmetic is in $\mathbb{Z}_\ell$.

(i.e. the messages received from the other participants) given input the security parameter λ and the output O that $\mathcal{C}$ receives at the end of Π.

4.1 PIR with Default

We start by defining formally our new notion of PIR-with-Default. We first recall the different existing variants of private information retrieval and their security guarantees. The notion of PIR [CGKS95] enables a client to query a public database with a private index and to obtain the corresponding entry, while the party who holds the database learns nothing about the index during the execution of the query. *Symmetric* PIR [GIKM00] adds also a privacy guarantee for the database requiring that the client learns nothing but the queried database entry. *Keyword* PIR [CGN98] addressed the setting of sparse databases where the query index is over a keyword domain, and database is index with a subset of the same domain. The query party in keyword PIR either obtains the requested value if present in the database, or learns that the query is not present in the database.

PIR-with-Default extends the notion of keyword PIR providing stronger privacy against the client hiding whether the query is present in the database. This is achieved by modifying the functionality to return either the database entry if the query is in the database, or a default value provided by the database holder, otherwise. This privacy property is stronger than symmetric keyword PIR assuming that the database entries and the default values are indistinguishable. In many real-world applications, the default value is a cryptographic object with natural pseudorandomness. As stand-alone applications of PIR-with-Default, we envision use-cases where clients retrieve cryptographic tokens from a server to utilize elsewhere. In Section 1.2, we consider the specific case of anonymous tokens [KLOR20], but this could extend to retrieving coupons (with dummy codes for non-targeted users), or a token proving allowlist-membership or blocklist-non-membership.

The precise PIR-with-Default functionality is described in Figure 1. We note that the presentation in Figure 1 allows the client to submit multiple queries, where the server specifies different default values for each client query. Single-query PIR-with-Default is equivalent to setting $t = 1$. Next we define the security properties for such a protocol.

Definition 1 (Semi-Honest Security for PIR-with-Default)**.** Let $n(\lambda)$ be an upper bound on the size of server database of (index, value) pairs $\mathcal{P}$, $t(\lambda)$ be a bound on the number of queries client's set X, and $Z_{\ell(\lambda)}$ be the domain for the database values and default values $\mathcal{D}$. Let O be a vector of length $|X|$ that contains the outputs of the PIR with default functionality executed with queries from X on database $\mathcal{P}$ and default values $\mathcal{D}$.

A PIR-with-Default protocol is $(n(\lambda), t(\lambda), \ell(\lambda))$-secure, if there exist ppt algorithms $\mathsf{SIM}_{\mathcal{C}}$ and $\mathsf{SIM}_{\mathcal{S}}$ such for any probabilistic polynomial-time adversary $\mathcal{A}$, there exists a negligible function $\mathsf{negl}(\cdot)$ such that

$$\begin{aligned}\big|\,&\Pr[\mathcal{A}(1^\lambda, \mathsf{View}_{\mathcal{S}}^{\Pi}(1^\lambda, [X]_{\mathcal{C}}, [\mathcal{P}, \mathcal{D}]_{\mathcal{S}})) = 1]\\ &- \Pr[\mathcal{A}(1^\lambda, \mathsf{SIM}_{\mathcal{S}}(1^\lambda, n, \ell, O, [\mathcal{P}, \mathcal{D}]_{\mathcal{S}})) = 1]\big| < \mathsf{negl}(\lambda)\end{aligned}$$

and

$$\begin{aligned}\Big|\,&\Pr[\mathcal{A}(1^\lambda, \mathsf{View}_{\mathcal{C}}^{\Pi}(1^\lambda, [X]_{\mathcal{C}}, [\mathcal{P}, \mathcal{D}]_{\mathcal{S}})) = 1]\\ &\qquad - \Pr[\mathcal{A}(1^\lambda, \mathsf{SIM}_{\mathcal{C}}(1^\lambda, [X]_{\mathcal{C}}, t)) = 1]\Big| < \mathsf{negl}(\lambda)\end{aligned}$$

The above security definition formalizes the intuition that the client does not learn anything more than the output of its query (the actual value if the item is present, or the default value) and the database size, and the server does not learn anything from the executions except the number of queries.

We also define the notion that extends the computation of PIR-with-Default as follows:

1. Allowing the client to also specify associated values, such that the client will learn the product of the client and server's associated values if the client identifier is in the server database.
2. Allowing the server to specify an additive mask, such that the client will receive a *masked* associated-value or default. This enables the protocol to have additively secret-shared outputs.

Extended-PIR-with-Default is formally described in Figure 2. The security definition for this primitive is the same as PIR-with-Default except the output O is computed with the extended functionality. While PIR-with-Default is a special case of Extended-PIR-with-Default, where the client's associated values are all 1, and the server's additive masks are all 0, we will be constructing both primitives in a non-blackbox way from building block components to achieve better efficiency. Note that one can use Extended-PIR-with-Default to output additive shares of items in the intersection, which can serve as input to any MPC protocol described in Section 6.2.

5 PIR with Default Construction

5.1 Construction Outline

Both of our constructions share the following three high-level steps.

The first step is a secret-shared private membership test (SS-PMT). This enables the client and server to compute a secret-share of a membership bit, i.e. the two parties obtain XOR shares of 1 or 0 if the client's query is or is not in the database.

The second step is computation of a secret-shared associated value (SS-AV). This enables the client and server to compute an additive secret share of the database value corresponding to the client's query. The outputs for the client and the server are additive shares of a value, which is the value that is in $\mathcal{S}$'s database if the query is in $\mathcal{S}$'s database. If the query is not in the database, there are no guarantees for the value underlying the secret shared output. In particular, it may be an arbitrary function of the server's database entries.

The third step is functionality called Value-Or-Default, which enables the server and the client to take their outputs from the first two steps as well as the default values on the server side, and translate them into the client's output, which is either the associated value or the default value depending on whether the output of SS-PMT was shares of 0 or of 1 .

In the following sections, we will give two constructions for PIR-with-Default. These constructions will have different implementations for SS-PMT and SS-AV, but will have the same implementation of Value-Or-Default.

5.2 Construction 1: PIR-with-Default with Offline Setup

Our first construction for PIR-with-Default involves an expensive setup phase that has communication linear in the server's database. However, the remainder

PARAMETERS:

- Security parameter λ.
- Server $\mathcal{S}$ input set size n, associated value space $\mathbb{Z}_\ell$, number of client $\mathcal{C}$ queries t .
- A 1-out-of-k OT primitive.
- Bloom Filter parameters: Bloom filter size η sufficient to hold n items, a number of hash functions k, a hash function family $\mathsf{HF} : \{0,1\}^\star \to [\eta]$.
- An additively HE scheme (HGen,HEnc,HDec) with message space $\mathbb{Z}_\ell$.

INPUT:

- Server $\mathcal{S}$: A set of key-value pairs $\mathcal{P} = \{(y_1, v_1), \ldots, (y_n, v_n)\}$ with distinct y_i, and a set of default values $D = \{d_1, ..., d_t\}$, where each $v_i, d_i \in \mathbb{Z}_\ell$. Additionally, a set of t masks $\{s_1, ..., s_t\}$ each $\in \mathbb{Z}_\ell$.
- Client $\mathcal{C}$: A set of t queries $\{x_1, ..., x_t\}$. Additionally, a set of t associated values $\{w_1, ..., w_t\}$, each $\in \mathbb{Z}_\ell$

PROTOCOL:

1. **Setup phase:**
   - $\mathcal{S}$ and $\mathcal{C}$ jointly select k hash functions $\{h_1, ..., h_k\}$ at random from HF.
   - $\mathcal{S}$ generates a HE key-pair $(pk, sk) \leftarrow \mathsf{HGen}(\lambda)$ and sends pk to $\mathcal{C}$.
   - $\mathcal{S}$ inserts a set of keys $\{y_1, \ldots, y_n\}$ into a Bloom filter BF and the set of key-value pairs $\mathcal{P}$ into a Garbled Bloom filter GBF using hash functions h_i. $\mathcal{S}$ aborts if either insertion operation fails.
   - Using pk, $\mathcal{S}$ encrypts BF and GBF as $\mathsf{EBF}[i] = \mathsf{HEnc}(pk, \mathsf{BF}[i]), \forall i \in [\eta]$ and $\mathsf{EGBF}[i] = \mathsf{HEnc}(pk, \mathsf{GBF}[i]), \forall i \in [\eta]$.
   - $\mathcal{S}$ sends EBF and EGBF to $\mathcal{C}$.
2. **Online phase:** The following steps are executed in parallel for each x_j for $j \in [t]$.
   (a) $\mathsf{SS\text{-}PMT}$ **computation:**
      - $\mathcal{C}$ chooses a random mask $r \leftarrow \mathbb{Z}_\ell$, homomorphically computes $z = \mathsf{Refresh}(-\mathsf{HEnc}(pk, r) + \sum_{i=1}^{k} \mathsf{EBF}[h_i(x_j)])$, and send the ciphertext z to $\mathcal{S}$
      - $\mathcal{S}$ decrypts the received ciphertext z using secret key sk, and obtains r'.
      - Parties invoke an instance of 1-out-of-$(k+1)$ OT:
        - $\mathcal{S}$ chooses a bit $b_\mathcal{S}$ at random.
        - $\mathcal{S}$ acts as OT's sender with input $\{b_0, \ldots, b_k\}$ where each b_i is equal to $b_\mathcal{S}$, except $b_{(k-r') \mod (k+1)}$ which is equal to $1 \oplus b_\mathcal{S}$.
        - $\mathcal{C}$ acts as OT's receiver with choice $r \mod (k+1)$.
        - $\mathcal{C}$ obtains $b_\mathcal{C}$ from the OT's functionality.

   (b) $\mathsf{SS\text{-}AV}$ **computation:**
      - $\mathcal{C}$ chooses a random mask $v_\mathcal{C} \leftarrow \mathbb{Z}_\ell$, homomorphically computes $z' = \mathsf{Refresh}(-\mathsf{HEnc}(pk, v_\mathcal{C}) + w_j \cdot \sum_{i=1}^{k} \mathsf{EGBF}[h_i(x)])$, and sends the ciphertext to $\mathcal{S}$
      - $\mathcal{S}$ decrypts the received ciphertext z' using its secret key sk, and obtains $v_\mathcal{S}$.

   (c) $\mathsf{Value\text{-}Or\text{-}Default}$ **computation:**
      - $\mathcal{S}$ and $\mathcal{C}$ engage in a $\mathsf{Value\text{-}Or\text{-}Default}$ protocol execution described in Figure 4.
      - $\mathcal{S}$ uses inputs $b_\mathcal{S}$, $v_\mathcal{S} - s_j$ and $d_j - s_j$.
      - $\mathcal{C}$ uses inputs $b_\mathcal{C}$ and $v_\mathcal{C}$.
      - Let o_j be the output received by $\mathcal{C}$ from the $\mathsf{Value\text{-}Or\text{-}Default}$ protocol execution
3. **Output:** $\mathcal{C}$ outputs the set $O = \{o_j\}_{j \in [t]}$.

Fig. 3: Construction 1: PIR-with-Default construction with Setup. The highlighted parts are only needed for Extended-PIR-with-Default construction.

of the protocol is independent of the number of entries in the server's dataset. Therefore this protocol is well suited to scenarios where the server's database is fixed and the setup phase can be performed offline, and requires an efficient online phase once the client's input is available. Moreover, the setup phase can be run once and reused for multiple protocol executions, and for different clients.

The construction presented in Figure 3 works as follows. The server inserts its database into a Bloom filter BF and a Garbled Bloom filter GBF. The server generates a public/private key pair (pk, sk) for additively homomorphic encryption, and encrypts the entries of both BF and GBF using the public key. It sends the encrypted results to the client in the setup phase. Whenever the client wants to run a $\mathsf{PIR\text{-}with\text{-}Default}$ query x, the client invokes the online phase of computation with the server to compute $\mathsf{SS\text{-}PMT}$, $\mathsf{SS\text{-}AV}$ and $\mathsf{Value\text{-}Or\text{-}Default}$. We describe each of these computations as follows:

SS-PMT Functionality We instantiate $\mathsf{SS\text{-}PMT}$ as follows. The client first computes a sum of the encrypted entries $b = \sum_{i=1}^{k} \mathsf{EBF}[h_i(x)]$ using the homomorphic property of the encryption scheme. It is easy to see that b is an encryption of a value p which is smaller than $k+1$. Moreover, if the query x is in the server dataset Y, p is exactly equal to k. The client now needs to turn this into secret shares of the membership bit. A straw man solution is to homomorphically convert b to an encryption of a bit (0/1) so that each party can have a secret share of the bit indicating whether $x \in Y$. The conversion can be done by homomorphically evaluating the equality circuit that has multiplicative depth $\lceil \log(k) \rceil$. However this approach is relatively inefficient.

Instead, we use a simple solution that relies on oblivious transfer. More precisely, the client randomly chooses a value $r \leftarrow \mathbb{Z}_\ell$, which will be its output share, and masks b by computing $c \leftarrow b - \mathsf{Enc}(pk, r)$. The client sends the resulting value to the server, who decrypts it to obtain its output share $r' = \mathsf{Dec}(sk, c) = p - r$. The parties use their output shares of p as inputs in the next OT functionality that translates these shares into shares of a single membership bit.

The client chooses a random bit $b_{\mathcal{C}}$ and acts as OT's sender with $(k+1)$ OT messages $B = \{b_0, \dots, b_k\}$ where all b_i are $b_{\mathcal{C}}$, except $b_{(k-r') \bmod (k+1)}$ which is equal to $1 \oplus b_{\mathcal{C}}$. The server acts as OT's receiver with $r \mod (k+1)$. The OT functionality gives the server $b_{\mathcal{S}}$ such that $b_{\mathcal{C}} \oplus b_{\mathcal{S}} = 1$ if $r + r' = k$ (i.e. the client's keyword is in the server's database), otherwise $b_{\mathcal{C}} \oplus b_{\mathcal{S}} = 0$. The described process exactly implements the $\mathsf{SS\text{-}PMT}$ functionality.

Instantiating 1*-out-of-*N *OT* A trivial implementation of the 1-out-of-$(k+1)$ OT used above is via $\log(k+1)$ 1-out-of-2 OT instances. Recently, several works [KK13, KKRT16, PSZ18] have proposed efficient protocols to generalize 1-out-of-2 OT extension to 1-out-of-N OT. Each protocol has a different underlying encoding function to support an upper-bound number of N messages in OT. Kolesnikov and Kumaresan [KK13] employ 256-bit Walsh-Hadamard error-correcting code and achieve 1-out-of-N OT on random strings, for N up to approximately 256. For arbitrarily large N, the best 1-out-of-N OT protocol [KKRT16] uses 424-448 bits codeword length, which requires 424-448 bits of communication per OT and N hash evaluations. For smaller N, the best protocols [PSZ18, OOS17] use linear BCH code, in which codeword length depends on N. Our instantiation for the BF parameters yields $N = 2^5$ to achieve a BF false-positive rate of $2^{-\lambda}$. In this case, the required codeword length and the best underlying encoding are 248 bits, which are chosen according to [min] to achieve Hamming distance of two codewords at least κ security parameter.

SS-AV Functionality The SS-AV protocol uses similar but simplified approach as the one in SS-PMT. The client first computes a sum of all encrypted $\mathsf{EGBF}[h_i(x)], \forall i \in [k]$, using the additive HE property $z = \sum_{i=1}^{k} \mathsf{EBF}[h_i(x)]$. Due to the GBF property, z is an encryption of the associated value v if $(x, v) \in \mathcal{P}$, and some unrelated value otherwise. To output SS-AV, the client chooses a random $v_{\mathcal{C}}$ and sends $z - \mathsf{Enc}(pk, v_{\mathcal{C}})$ to the server who can decrypt it and obtain $v_{\mathcal{S}}$.

The work [DCW13] observed that the GBF procedure aborts when processing item x if and only if x is a false positive for a BF containing the previous items. Therefore, to bound the probability by $2^{-\lambda}$, one can use a table with $58n$ entries to store n items. In that case, the optimal number of hash functions is $k = 31$.

In the setting of Extended-PIR-with-Default, the client homomorphically multiplies its value w_i with the sum of the encrypted GBF values before masking and sending it to the server. Then if x_i is in the server database, $v_{\mathcal{S}}$, the decryption of z, will be a share of $(v_j \cdot w_i)$. Then the server can simply add $(-s_i)$ to $v_{\mathcal{S}}$ before proceeding to the next phase: now $v_{\mathcal{C}}$ and $v_{\mathcal{S}} - s_i$ are additive shares of $(v_j \cdot w_i) - s_i$.

Finally, the server and client engage in a Value-Or-Default protocol to translate the outputs of the previous two steps into the associated value or the default value. We describe this subprotocol in the next section.

Value-Or-Default Functionality We describe our Value-Or-Default protocol for the PIR-with-Default construction and note that the only change required for the Extended-PIR-with-Default construction is that the server has to modify its inputs to Value-Or-Default, but there are no changes to the Value-Or-Default protocol itself.

After SS-AV, parties hold secret shares of the associated value v if $(x, v) \in \mathcal{P}$. To complete the PIR-with-Default functionality, the client has to either reconstruct v or obtain a default value d from the server. We translate the shares into the required output using 2 OT invocations (forward and backward) as follows.

In the "forward" OT, the client chooses a random value $\Delta_{\mathcal{C}} \leftarrow \{0,1\}^{\ell}$, and acts as OT's sender with OT's messages $\{\Delta_{\mathcal{C}} + b_{\mathcal{C}} \cdot v_{\mathcal{C}}, \Delta_{\mathcal{C}} + (1 - b_{\mathcal{C}}) \cdot v_{\mathcal{C}}\}$, while the server acts as OT's receiver with a choice bit $b_{\mathcal{S}}$, and obtains q. Clearly, $q = \Delta_{\mathcal{C}} + (b_{\mathcal{C}} \oplus b_{\mathcal{S}}) \cdot v_{\mathcal{C}} = \Delta_{\mathcal{C}} + b \cdot v_{\mathcal{C}}$.

INPUT:
- Server $\mathcal{S}$: A bit $b_{\mathcal{S}}$ and two strings $v_{\mathcal{S}}$ and d each $\in Z_\ell$.
- Client $\mathcal{C}$: A bit $b_{\mathcal{C}}$ and a string $v_{\mathcal{C}} \in Z_\ell$.

DESIRED OUTPUT:
- Server $\mathcal{S}$: No output.
- Client $\mathcal{C}$: $v = v_{\mathcal{S}} + v_{\mathcal{C}}$ if $b = 1$, or d if $b = 0$, where $b = b_{\mathcal{S}} \oplus b_{\mathcal{C}}$

PROTOCOL:

1. $\mathcal{C}$ chooses $\Delta_{\mathcal{C}} \leftarrow Z_\ell$ at random.
2. Parties invoke an OT instance:
   - $\mathcal{C}$ acts as OT's sender with OT's messages $m_0 = \Delta_{\mathcal{C}} + b_{\mathcal{C}} \cdot v_{\mathcal{C}}$ and $m_1 = \Delta_{\mathcal{C}} + (1 - b_{\mathcal{C}}) \cdot v_{\mathcal{C}}$.
   - $\mathcal{S}$ acts as OT's receiver with a choice bit $b_{\mathcal{S}}$, and obtains q. Note that $q = \Delta_{\mathcal{C}} + b \cdot v_{\mathcal{C}}$ where $b = b_{\mathcal{S}} \oplus b_{\mathcal{C}}$
3. Parties invoke another OT instance:
   - $\mathcal{S}$ acts as OT's sender with inputs $m_0 = q + (b_{\mathcal{S}} \cdot v_{\mathcal{S}}) + ((1 - b_{\mathcal{S}}) \cdot d)$ and $m_1 = q + ((1 - b_{\mathcal{S}}) \cdot v_{\mathcal{S}}) + (b_{\mathcal{S}} \cdot d)$.
   - $\mathcal{C}$ acts as OT's receiver with a choice bit $b_{\mathcal{C}}$, and receives q'. Note that $q' = q + (b \cdot v_{\mathcal{S}}) + ((1 - b) \cdot d)$ where $b = b_{\mathcal{S}} \oplus b_{\mathcal{C}}$
4. $\mathcal{C}$ outputs $q' - \Delta_{\mathcal{C}}$. Note that the output is exactly $v_{\mathcal{S}} + v_{\mathcal{C}}$ if $b = 1$, or d if $b = 0$, where $b = b_{\mathcal{S}} \oplus b_{\mathcal{C}}$

Fig. 4: Our Value-Or-Default Construction. All arithmetic is implicitly in $\mathbb{Z}_\ell$.

In the "backward" OT, the server acts as OT's sender with input $\{q + b_{\mathcal{S}} \cdot v + (1 - b_{\mathcal{S}}) \cdot d, q + (1 - b_{\mathcal{S}}) \cdot v + b_{\mathcal{S}} \cdot d\}$ while the client acts as OT's receiver with a choice bit $b_{\mathcal{C}}$, and receives q'. It is easy to see that $q' = q + b \cdot v_{\mathcal{S}} + (1 - b) \cdot d$.

Finally the client reconstructs it output $o = q' - \Delta_{\mathcal{C}}$.

5.3 Construction 2: PIR-with-Default with Sublinear communication.

Our second construction aims to remove the expensive offline setup phase from our first construction, replacing it by (standard) Private Information Retrieval queries.

Recall that the offline Phase in Construction 1 consists of $\mathcal{S}$ sending encrypted BF and GBF to the client. For each query x_j $\mathcal{C}$ homomorphically sums the entries corresponding to $h_i(x_j)$ for each of the k hash functions h_i, additively masks the encrypted result, and sends it to the server.

In Construction 2, $\mathcal{C}$ will instead obliviously query the server at the locations $h_i(x_j)$, and will receive the masked sum of the corresponding values at those locations in BF and GBF. Note that if $\mathcal{C}$ only needed to retrieve the entries at locations $h_i(x_j)$ (without summing or masking), then $\mathcal{C}$ could have used standard (symmetric) PIR. In order to execute the query results summed and masked, we have $\mathcal{C}$ instead use a modified version of PIR, which we call Sum-PIR.

Sum-PIR Functionality The Sum-PIR primitive allows a receiver $\mathcal{C}$, holding a set of indices $p_1, \ldots, p_k$, to interact with a server holding a database $\mathcal{P}$ and receive $\sum_{i=1}^{k} \mathcal{P}[p_i] - r$, for some additive mask r held by the server. The server should not learn the entries queried by the client.

Our construction for Sum-PIR builds on standard constructions of PIR from additively homomorphic encryption, for example [ACLS18, ALP+19]. In the basic version of these constructions, the receiver $\mathcal{C}$ sends η ciphertexts $c_1, \ldots, c_\eta$ to the server, where $\eta = |\mathcal{P}|$ is the number of items held by the server. These ciphertexts all encrypt 0, except the c_i (where i is the index $\mathcal{C}$ wishes to retrieve), which encrypts 1. The server $\mathcal{S}$ receives these ciphertexts and performs a homomorphic dot-product between these ciphertexts and its database $\mathcal{P}$. This results in a ciphertext $c^* = \sum_{j=1}^{\eta} \mathcal{P}[j] \cdot c_j$, which is an encryption of exactly $\mathcal{P}[i]$. $\mathcal{S}$ then sends c^* to the client who decrypts to receive its desired value.

We observe that if the client wishes to instead receive the sum of k entries, then it can send k PIR queries simultaneously to the server, who executes the computation described above, and *homomorphically sums* the resulting ciphertexts before returning the result to the client. The result will then contain exactly the sum of the k queried items. If we additionally want the result to be masked, then the server can homomorphically add a chosen mask r to the result before returning it to the client.

While the basic construction described has high client communication costs, we can perform several optimizations to reduce the communication and computation costs, which we describe in Section 7. We also note that the description above only requires additively homomorphic encryption. However, some of our optimizations will additionally require homomorphic multiplications. Therefore, our construction will be from RLWE-based somewhat-HE [BGV14].

We present our Sum-PIR functionality and its construction in Figure 5. The security of our Sum-PIR construction follows in a straightforward way from the security of its building block (e.g. PIR).

Building PIR-with-Default from Sum-PIR Our second PIR-with-Default construction is presented in Figure 6 and works as follows. In the setup phase, the server inserts its database into a Bloom filter BF and a Garbled Bloom filter GBF. The online execution starts the SS-PMT phase, which now consists of a Sum-PIR execution. For each item $x_{i \in [t]}$, the client inputs a set of indices $\{h_1(x_i), \ldots, h_k(x_i)\}$ while the server inputs BF and a random mask r. Similar to Construction 1, the parties obtain secret share of the value p which is smaller than $k+1$. The parties then use their obtained values as inputs to the 1-out-of-$(k+1)$ OT that translates these shares into output of SS-PMT functionality.

For SS-AV computation, the parties also invoke Sum-PIR. We observe that the client can *reuse* the queries from the SS-PMT phase in the SS-AV phase, since it is querying the same indices (i.e, $\mathcal{C}$ does not need to send PIR.Query to the server in Step (2,a) of Figure 5) while the server inputs GBF and a random mask $v_{\mathcal{S}}$. Sum-PIR directly gives parties SS-AV's outputs as desired.

In the setting of Extended-PIR-with-Default , for each $j \in [t]$, the client additionally sends encryption of w_j to the server who homomorphically multiplies

INPUT:

- Server $\mathcal{S}$: A database D of size η and an additive mask r
- Client $\mathcal{C}$: a set of indices $p_1, \ldots, p_k$.

DESIRED OUTPUT:

- Server $\mathcal{S}$: no output
- Client $\mathcal{C}$: $v = \sum_{i=1}^{k} D[p_i] - r$

PROTOCOL:

1. $\mathcal{C}$ generates a public-secret key pair (pk, sk) with $\mathsf{PIR.Gen}$, and sends pk to $\mathcal{S}$
2. $\mathcal{S}$ and $\mathcal{C}$ invoke multi-query PIR. For each $i \in [k]$,
   (a) $\mathcal{C}$ uses $\mathsf{PIR.Query}(pk, p_i)$ to generate a query q_i and sends it to $\mathcal{S}$
   (b) $\mathcal{S}$ uses $\mathsf{PIR.Answer}(pk, q_i, D)$ to generate the answer d_i.
   (c) $\mathcal{S}$ homomorphically computes $c = \sum_{i=1}^{k} d_i$
   (d) $\mathcal{S}$ homomorphically masks $c^\star$ with r as $c^\star \leftarrow c - \mathsf{HEnc}(pk, r)$.
   (e) $\mathcal{S}$ sends $c^\star$ to $\mathcal{C}$.
3. $\mathcal{C}$ outputs $\mathsf{PIR.Extract}(sk, c^\star)$

Fig. 5: Our Sum-PIR Construction.

it with the PIR results in Step (2,c) of Figure 5 before masking the result with the additive mask $v_{\mathcal{S}}$ in Step (2,d) of Figure 5.

Finally, the server and client engage in the Value-Or-Default protocol as before to translate the outputs of the previous two steps into the associated value or the default value. This protocol is the same as in Construction 1.

Hashing Based Multi-Query PIR-with-Default Construction Construction 2 based on Sum-PIR relies heavily on several PIR queries (see Step 2 of Figure 5), with one query for each client input, which is executed against the server's data at the same time. However, standard PIR techniques require the server to touch each item in its dataset for each client query, which quickly becomes expensive. In this section, we describe an optimization based on hashing to bins that enables large cost savings when executing multiple parallel PIR executions. Variants of this idea have appeared in previous work: [ACLS18, ALP+19] proposed a new PIR construction for sparse databases based on Cuckoo hashing to amortize CPU cost when making multiple PIR queries. We also show how to leverage a hashing technique [KMP+17, PRTY19] to speed up the computational cost of Construction 2.

Our main idea is that the parties use hashing to partition its items into m bins. Each bin contains a smaller fraction of inputs, which allows the parties to evaluate PIR-with-Default or Extended-PIR-with-Default bin-by-bin. The amount of data the server has to touch per query is now only the items that were mapped to the same bin as the client query, which is much more efficient computationally.

Our hashing based PIR-with-Default construction is presented in the full version of this work [LPR+20]. In this construction, parties hash their items to bins using one of the hashing schemes described above, and execute PIR-with-Default bin-by-bin. We note that when we use this hashing technique, we are able to

Parameters:

- Security parameter λ.
- Server $\mathcal{S}$ input set size n, associated value space $\mathbb{Z}_\ell$, number of client $\mathcal{C}$ queries t .
- A 1-out-of-k OT primitive and a $\mathsf{Sum\text{-}PIR}$ primitive.
- Bloom Filter parameters: Bloom filter size η sufficient to hold n items, a number of hash functions k, a hash function family $\mathsf{HF} : \{0,1\}^\star \rightarrow [\eta]$.

Input:

- Server $\mathcal{S}$: A set of key-value pairs $\mathcal{P} = \{(y_1, v_1), \ldots, (y_n, v_n)\}$ with distinct y_i, and a set of default values $D = \{d_1, ..., d_t\}$, where each $v_i, d_i \in \mathbb{Z}_\ell$. Additionally a set of t masks $\{s_1, ..., s_t\}$ each $\in \mathbb{Z}_\ell$.
- Client $\mathcal{C}$: A set of t queries $\{x_1, ..., x_t\}$. Additionally a set of t associated values $\{w_1, ..., w_t\}$, each $\in \mathbb{Z}_\ell$.

Protocol:

1. **Setup phase:**
   - $\mathcal{S}$ and $\mathcal{C}$ jointly select k hash functions $\{h_1, ..., h_k\}$ at random from HF.
   - $\mathcal{S}$ inserts a set of keys $\{y_1, \ldots, y_n\}$ into a Bloom filter BF and the set of key-value pairs $\mathcal{P}$ into a Garbled Bloom filter GBF using hash functions h_i. $\mathcal{S}$ aborts if either insertion operation fails.
2. **Online phase:** The following steps are executed in parallel for each x_j for $j \in [t]$.
   (a) $\mathsf{SS\text{-}PMT}$ **computation:**
      - $\mathcal{S}$ selects a mask $r \leftarrow \mathbb{Z}_\ell$.
      - $\mathcal{C}$ and $\mathcal{S}$ execute a $\mathsf{Sum\text{-}PIR}$ query. $\mathcal{C}$ uses inputs $h_1 x_j, ..., h_k(x_j)$. $\mathcal{S}$ uses BF and r as input.
      - $\mathcal{C}$ receives $r' = -r + \sum_{i=1}^{k} \mathsf{BF}[h_i(x_j)]$ as output.
      - Parties invoke an instance of 1-out-of-$(k+1)$ OT:
        - $\mathcal{S}$ chooses a bit $b_\mathcal{S}$ at random.
        - $\mathcal{S}$ acts as OT's sender with input $\{b_0, \ldots, b_k\}$ where each b_i is equal to $b_\mathcal{S}$, except $b_{(-r+k) \mod (k+1)}$ which is equal to $1 \oplus b_\mathcal{S}$.
        - $\mathcal{C}$ acts as OT's receiver with choice $r' \mod (k+1)$.
        - $\mathcal{C}$ obtains $b_\mathcal{C}$ from the OT's functionality.

   (b) $\mathsf{SS\text{-}AV}$ **computation:**
      - $\mathcal{C}$ sends $\mathsf{HEnc}(pk, w_j)$ to $\mathcal{S}$.
      - $\mathcal{S}$ selects a mask $v_\mathcal{S} \leftarrow \mathbb{Z}_\ell$.
      - $\mathcal{C}$ and $\mathcal{S}$ execute a $\mathsf{Sum\text{-}PIR}$ query. $\mathcal{C}$ uses inputs $h_1 x_j, ..., h_k(x_j)$. $\mathcal{S}$ uses GBF and $v_\mathcal{S}$ as input.
      - Prior to additively masking the $\mathsf{Sum\text{-}PIR}$ result c with $v_\mathcal{S}$ to compute $c^\star$, $\mathcal{S}$ homomorphically multiplies c with $\mathsf{HEnc}(pk, w_j)$
      - $\mathcal{S}$ receives mask $v_\mathcal{S}$ as output. $\mathcal{C}$ receives $v_\mathcal{C} = -v_\mathcal{S} + w_j \cdot \sum_{i=1}^{k} \mathsf{GBF}[h_i(x_j)]$ as output.

   (c) $\mathsf{Value\text{-}Or\text{-}Default}$ **computation:**
      - $\mathcal{S}$ and $\mathcal{C}$ engage in a $\mathsf{Value\text{-}Or\text{-}Default}$ protocol execution described in Figure 4.
      - $\mathcal{S}$ uses inputs $b_\mathcal{S}$, $v_\mathcal{S} - s_j$ and $d_j - s_j$.
      - $\mathcal{C}$ uses inputs $b_\mathcal{C}$ and $v_\mathcal{C}$.
      - Let o_j be the output received by $\mathcal{C}$ from the $\mathsf{Value\text{-}Or\text{-}Default}$ protocol execution
3. **Output:** $\mathcal{C}$ outputs the set $O = \{o_j\}_{j \in [t]}$.

Fig. 6: Construction 2: PIR-with-default construction with sublinear communication. Portions with changes highlighted are needed for achieving $\mathsf{Extended\text{-}PIR\text{-}with\text{-}Default}$

achieve a weakened version of PIR-with-Default. Specifically, the server cannot assign a particular default value specifically to the ith client query since it does not know which bin this query got assigned to. Rather, the server must assign defaults to the ith client query *per-bin*. That is, default values must be assigned bin-wise. The same holds true for masks in the case of Extended-PIR-with-Default . We observe that this does not impact any of our applications, since they have $\mathcal{S}$ choose all default values (and masks) the same way (as a random share of 0), independent of which specific client query is being responded to. Therefore these applications lose nothing from assigning default values and masks by bin.

An additional difference is that the hashing-based modification needs both the client and server to pad their inputs with dummy values so that each bin is of the same size. These dummy values need to be chosen carefully so that they are distinct for the client and server, and never occur in either party's input set. Our formulation [LPR+20] makes it so whenever $\mathcal{C}$ uses a dummy value, it always receives the default value. $\mathcal{S}$ therefore has to provide additional default values to allow for the increased number of client queries due to padding. We also note that in the case of PIR-with-Default, the client can just discard the values received for dummy items. However, for Extended-PIR-with-Default , the client must preserve these values, since the server has received a mask-share for them, and may use it in downstream computation. This implies another caveat for using hashing: the downstream computation for Extended-PIR-with-Default must also be able to smoothly handle additional default values corresponding to dummy client inputs. We observe that our applications are all able to smoothly do so, since their defaults and masks all correspond to random shares of 0, and computation that follows can accommodate additional shares of 0 while remaining correct.

We now discuss concrete hashing schemes and parameters. If there are m bins, each with maximum load γ items on the client's side, then the number of default values the server must provide is $m\gamma$. In the setting of Extended-PIR-with-Default, the number of additive masks the server must provide is also $m\gamma$.

Concretely, the client uses Cuckoo hashing or 2-choice hashing with k hash functions, and inserts her items into m bins. The server maps his points into m bins using the same set of k hash functions (i.e., each of the server's items appears k times across all over bins). Using a standard ball-and-bin analysis based on k, m, and the input size of client $|X|$, one can deduce an upper bound β such that no server bin contains more than β items with high probability $(1 - 2^{-\lambda})$.

In our protocol, we use Cuckoo hashing, the client can place its set into a Cuckoo table of size $m = 1.27t$ using $k = 3$ hash functions. There are only 3% dummy items [PSTY19] required per bin on the server's side. Therefore, the client and server maximum bin size are $\gamma = 1$ and $\beta = 1.03\lceil 3n/m \rceil$, respectively.

5.4 Correctness and Security Proofs

We observe that our constructions are correct by observation, except with the negligible probability of Bloom Filter failure. In particular, our constructions fail to be correct if the server is unable to hash its items into a BF or GBF, or if the

Bloom filter returns a false positive on a client query. However, we note that we can set parameters so that the probability of such failures is negligible.

The security proof of the following theorem is given in Appendix 5.4.

Theorem 1. *The* PIR-with-Default *constructions 1 and 2 described in Figure 3 and Figure 6 securely implement the* PIR-with-Default *functionality defined in Figure 1 in the semi-honest setting, given the OT, HE, and* Sum-PIR *functionalities described in Section 5.3.*

Because the client's associated values w_j are either masked with random or encrypted before sending to the server, the security of our Extended-PIR-with-Default constructions follows straightforwardly from the security of PIR-with-Default and the encryption scheme. Thus, we omit the proof of the following theorem.

Theorem 2. *The* Extended-PIR-with-Default *constructions 1 and 2 described in Figure 3 and Figure 6 securely implement the* Extended-PIR-with-Default *functionality defined in Figure 2 in the semi-honest setting, given the OT, HE, and* Sum-PIR *functionalities described in Section 5.3.*

6 Two Party PJC

6.1 Inner-Product Private Join and Compute

The functionality of Extended-PIR-with-Default provides directly a protocol for inner-product private join and compute. In particular, a client with input $(X, W) = \{(x_1, w_1), \ldots, (x_t, w_t)\}$ and a server with input $(Y, V) = \{(y_1, v_1), \ldots, (y_n, v_n)\}$ execute the Extended-PIR-with-Default protocol where the server uses 0 as the default value for all queries. The two parties receives as outputs additive shares of $w_i \cdot v_i$ is $x_i \in Y$, or shares of 0 otherwise. Now each of the parties sums locally all the shares they have obtained, and in doing so they obtain shares of the value $\sum\limits_{i\in[t], j\in[n], x_i=y_j} w_i v_j$, which is the desired output.

Private set intersection-SUM is a special case of inner-product PJC can also be obtained in the same way as above except that the client uses weight equal to 1 in the execution of the Extended-PIR-with-Default protocol. For a slightly more efficient implementation the parties can use a *plain* PIR-with-Default execution, where for the i-th client query, the server additively masks all values with the same mask s_i, and sets s_i to be the default value, and uses these values as input to the protocol. The client then receives effectively an additive share of the associated value or of 0, with the server's share being $-s_i$. Parties can sum their shares locally to get additive shares of the intersection-sum.

If the server sets $v_i = 1$ for all $i \in [n]$, this protocol computes the cardinality of the intersection for the two input sets. Since the two parties obtain shares of the cardinality they can further execute a two-party protocol that checks whether the cardinality is above a threshold.

6.2 General PJC

The Extended-PIR-with-Default functionality enables the two parties to obtain shares of the associated values for the server's records included in the intersection, or shares of zero for the records with identifiers in $Y \setminus X$. We note we can obtain such shares for multiple attributes values associated with record.

We can also enable the two parties to obtain shares of the client's attribute values (or vectors of attribute values) for the intersection records (and shares of 0 for the records in $Y \setminus X$) as follows: The client executes PIR-with-Default with an input x_i to receive a share of the server's associated attribute(s). The client and the server execute a 1-out-of-2 OT similar to Step 2 of the Value-Or-Default protocol, using the shares of membership bit $b_{\mathcal{C}}$ and $b_{\mathcal{S}}$ from the SS-PMT phase of the preceding PIR-with-Default, where the client uses inputs $m_0 = r_i + b_{\mathcal{C}} \cdot w_i$, $m_1 = r_i + (1 - b_{\mathcal{C}}) \cdot w_i$ for a random mask r_i, and the server uses $b_{\mathcal{S}}$ as its choice bit. The result will be an additive sharing of either w_i or 0.

At this point the two parties can run any general two-party computation protocol which takes as input the shares of the attribute values for the records in $X \cap Y$ and shares of 0 for records in $Y \setminus X$, and evaluates a function on these attribute values.

6.3 Supporting Differentially Private Outputs

The above approach to compute general functions on the inner-join data can also be extended easily to support differential privacy (DP) [DMNS06] for the output by having the two parties compute jointly DP noise that will be added to the output. Since we are constructing semi-honest protocols each party can locally compute noise with the magnitude required for the resulting output. This means that the noise will be double the standard amount of noise, but this is needed in order to prevent either of the parties from subtracting its noise contribution from the output. The ability to add noise is important when the records in the input data sets are records of individuals and the PJC output is aggregate statistics over the users in the inner-join database, which should not reveal information about individuals.

7 Implementation

In the full version of this work [LPR+20], we revisit the state of the art constructions and optimizations of single-server PIR based on RLWE-based homomorphic encryption: SealPIR [ACLS18] and MulPIR [ALP+19]. Then, we explain how to apply the optimizations of the latter works to the application setting of our new PIR-with-Default construction. In particular, note that we achieve sublinear communication using recursion and multiplicative homomorphism, and use oblivious expansion to compress the upload as in [ALP+19]. Finally, we explain how to embed weights in PIR queries for the Extended-PIR-with-Default construction. The communication cost of all protocols is calculated according to Section 7.2.

Parameters		Construction 1				Construction 2		Circuit PSI [PSTY19]		Poly-ROOM [SGRP19]		PJC+RLWE [IKN+20]	
		Setup		Online		Online		Online		Online		Online	
n	t	Comm. *(MB)*	Time *(/query)*	Comm. *(MB)*	Time *(/query)*	Comm. *(MB)*	Time *(/query)*	Comm. *(MB)*	Time *(/query)*	Comm. *(MB)*	Time *(/query)*	Comm. *(MB)*	Time *(/query)*
2^{16}	2^8	29	35ms	7	2.43ms	27	673ms	5	11.79ms	55	59ms*	3†	44.8ms†
	2^{12}	29	2.19ms	112	1.03ms	120	34ms	30	0.93ms	863	3.5ms*	3†	2.97ms†
	2^{16}	29	0.14ms	1794	0.72ms	801	2ms	472	0.13ms	13788	2.2ms*	6†	0.36ms†
2^{20}	2^8	465	539ms	7	2.43ms	29	11821ms	51	178ms	71	–	40†	713ms†
	2^{12}	465	34ms	112	1.03ms	213	521ms	76	11.31ms	878	–	40†	44.7ms†
	2^{16}	465	2.11ms	1794	0.72ms	1821	34ms	522	0.78ms	13837	–	44†	2.97ms†
2^{25}	2^8	14885	17252ms	7	2.43ms	44	370s	1582	5668ms	591	–	1272†	22838ms†
	2^{12}	14885	1078ms	112	1.03ms	379	15.8s	1607	354ms	1401	–	1272†	1427ms†
	2^{16}	14885	67ms	1794	0.72ms	3704	1.1s	2180	22.22ms	14391	–	1276†	89ms†

Machine: single core of Intel(R) Xeon(R) CPU E5-2696 v3 @ 2.30GHz. For all constructions and $n = 2^{25}$, times have been estimated from microbenchmarks of the core operations, and fixed cost for a random access was assumed.

* The times for Poly-ROOM are taken from [SGRP19, Fig. 17], initially provided for a database $n = 50,000$ and a number of queries $t = 5,000$ and $50,000$. Unknown machine.

† Although PJC+RLWE does not achieve the PIR-with-Default functionality, we report it for comparison purpose. Timings are estimated from microbenchmarks of NIST-P256, and RLWE-encryption with degree 2048 and 62 bit modulus.

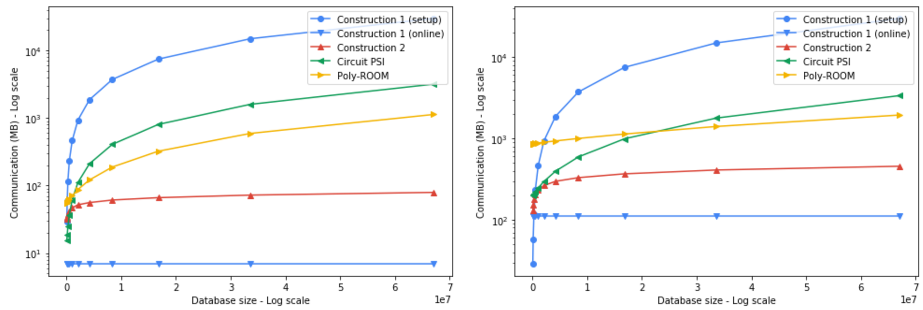
Table 2: Communication and computation costs of PIR-with-Default with elements of 32 bits. Running time is amortized over the number of client queries.

7.1 Communication and Computation

Asymptotically, Construction 2 (Figure 6) achieves sublinear communication per client query with respect to the server database size. In our benchmarking, we will make use of the hashing-based multi-query PIR-with-Default Construction described in 5.3 to reduce server costs. For both our constructions (and related work), we report the communication cost of t queries against a database of key-value pairs of size n with 32-bit values, for $2^8 \leq t \leq 2^{16}$ and $2^{16} \leq n \leq 2^{25}$, and the computational cost amortized over the number of queries t, in Table 2.

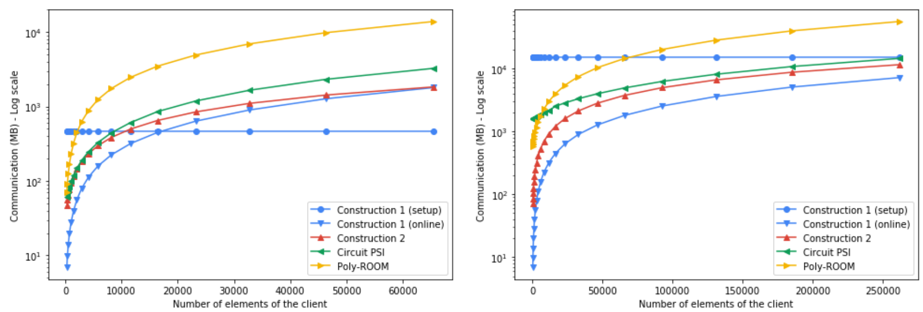
For Construction 1 (Figure 3), we report the cost of encrypting a Bloom Filter and Garbled Bloom Filter of dimension $58n$ with an homomorphic encryption scheme. We use the Shell homomorphic encryption library [she20] with HE parameters $d = 1024$, $\log_2(q) = 15$ for the encryption of the Bloom filter, and $d = 2048$, $\log_2(q) = 46$ for the encryption of the Garbled Bloom filter, both ensuring more than 128 bits of security [APS15] and allowing $k = 31$ homomorphic additions. Each coefficient of the polynomials embed a cell of the (Garbled) Bloom Filter, and rotations are performed by multiplications with x^i. As expected, the setup communication grows linearly with n and becomes larger than 15GB when $n > 2^{25}$. On the computation side, it is important to note that, assuming fixed cost for a random access, the online time *only depends on* t (and not on n).

For Construction 2 (Figure 6), we try different combinations of the optimizations and for each input size, we report the cost for the combination with smallest communication cost. In particular, we use Cuckoo hashing with three hash functions, as described in Section 5.3, and loop over 5 recursions levels (1 to 5 homomorphic multiplications). Concretely, for $n = 2^{20}$ and $t = 2^8 = 256$, we obtain 326 buckets for the Cuckoo hashing, each of size $576,461$. We perform $k = 31$ queries over each bucket, and use 4 homomorphic multiplications for recursion. The total number of elements transmitted is therefore approximately $5 \cdot B^{1/5} \cdot 31 \cdot 326 = 717,305$, which fits in 88 ciphertexts using oblivious ex-



(a) $t = 2^8$ (b) $t = 2^{12}$

Fig. 7: Communication cost of t PIR-with-Default queries, for increasing database sizes n and fixed number t.



(a) $n = 2^{20}$ (b) $n = 2^{25}$

Fig. 8: Communication cost of t PIR-with-Default queries, for increasing number t and fixed database sizes n.

pansion [ACLS18, ALP+19]. The HE parameters are $d = 8192$, $\log_2(q) = 255$, and each ciphertext is about 250kB. The key information is about 6MB, and the upload ciphertexts account for about 29MB of communication. Finally, the amortized time per query is 11.8s.

As illustrated in Figure 7, for a fixed number t of elements, Construction 2 has the smallest communication footprint as the database size n increases. For database of moderate sizes $n \leq 2^{25}$ and very few elements t, our solutions use less communication than alternatives. We note that Construction 2 becomes more communication efficient relative to other solutions as the gap between n and t grows larger, with the advantage appearing when the server size is a factor of 2^{10} larger than the client dataset. Finally, we note that the computation cost is relatively higher for Construction 2 than related works, but the bulk of the cost is incurred by the server instead of shared between client and server.

7.2 Comparison to Previous Work

We compare the resulting communication of our protocols to those of the best Circuit PSI protocol [PSTY19] and ROOM [SGRP19]. The run-time comparison of the protocols is illustrated in Table 2.

We first compute the communication complexity of [PSTY19]. The communication is composed of (a) the OPRF evaluations for each of the m bins which has an amortized communication of at most 450 bits; (b) the communication of $1.03kn$ coefficients of size $\tau + \ell$ bits each where τ is approximately equal to $\lambda + \log(knt) - \log(m)$; (c) the weighted sum garbled circuit which contains m comparison of two τ-bit elements, m multiplications of two ℓ-bit associated values, and $m - 1$ additions of two ℓ-bit associated values. Using circuit compiler [MGC+16], the weighted sum garbled circuit has $m(\tau + \ell - 1) + 993m\ell + (m - 1)(\ell - 1)$ AND gates in total. Note that each AND requires 256 bits. The communication cost of garbled circuit also requires $m(\tau + \ell)$ OT instances, each requiring 256 bits of communication.

In ROOM [SGRP19], the communication is composed of (a) the communication of n coefficients of size 128 bits each; (b) m garbled AES executions, each requiring 6400 AND gates; (c) and the same weighted sum garbled circuit as that of [PSTY19], which has $t(\tau + \ell - 1) + 993t\ell + (t - 1)(\ell - 1)$ AND gates and $m(\tau + \ell)$ OT instances.

For PJC [IKN+20], we use the NIST-P256 elliptic curve, which requires 32B to represent an element. We also use RLWE-based encryption for the associated values, with degree $d = 2048$ and $\log_2(q) = 62$-bit modulus. We use their packing technique to pack 2048 associated values into a single RLWE ciphertext, together with homomorphic rotation and addition.

Note that the labeled PSI protocol proposed in [CHLR18] can be extended to perform PJC. However, the extended protocol either reveals the intersection size, or incurs extra cost due to using a generic MPC. [CHLR18] only provide experimental numbers for their implementation of a similar functionality of keyword PIR (e.g. retrieve the associated value of an intersection item) for the server's dataset of size $n = 2^{20}$ records and the 256 queries. Their protocol takes 340ms and 20.9ms per query in the offline and online phases, respectively. To extend their functionality to PIR-with-Default, the sender needs to mask labeled (associated) message with random value using HE, and send it to the receiver. When the receiver decrypts the ciphertext, parties hold a secret share of a correct associated value, or of a random value. These shares are forwarded to a secondary MPC protocol to perform the functionality of PIR-with-Default. Indeed, the last extended step is similar to the garbled circuit phase of [PSTY19], which takes about 40% of [PSTY19]'s total cost. Therefore, we estimate that [CHLR18] requires an extra 71.2ms in the online phase to implement a PIR-with-Default query. In contrast, Construction 1 only takes 2.43ms online time to perform an instance of PIR-with-Default, a 37× improvement. However, Construction 1 is 1.5× slower than [CHLR18] in the offline phase. In terms of communication/storage cost, [CHLR18] requires at least 66.56MB transmitted without storage from the offline phase while ours needs only 7MB transmitted but 465MB storage from the offline phase. We present the performance comparison of ours and [CHLR18] for $n = 2^{20}$ and $t = 2^8$ in the full version of this work [LPR+20].

7.3 Monetary Costs

We estimate the monetary costs of our protocol compared to other works [IKN+20, PSTY19] using the same cost model. The cost is charged by Google Platform for pre-emptible virtual machines (including CPU and RAM). More details of the estimation of the monetary costs are shown in the full version of this work [LPR+20].

We observe that Construction 2 enables much lower client monetary costs compared to other protocols. However, due to the expensive server computation, we notice that the server monetary cost is higher than that of alternative protocols. However, the relative changes in cost make the comparison attractive. For example for $n = 2^{25}$ and $t = 2^8$, our client cost is $36.5\times$ lower than that of [PSTY19], while incurring a server cost that is only $4\times$ higher than theirs.

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