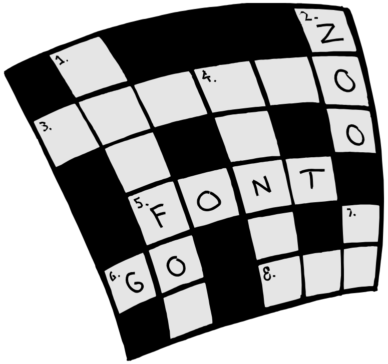
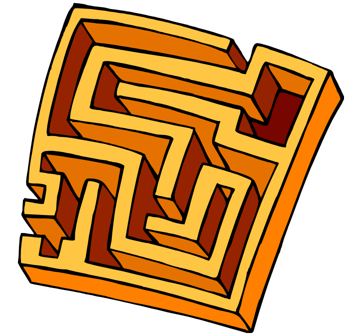


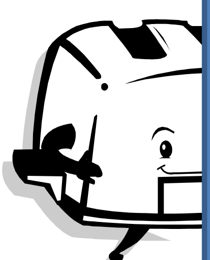


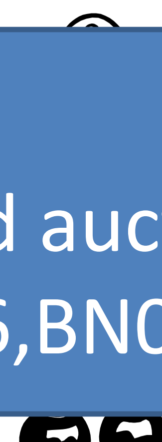
Time-Lock Puzzles
In the Random Oracle Model

Mohammad Mahmoody, **Tal Moran**, Salil Vadhan

Time-Lock Puzzles

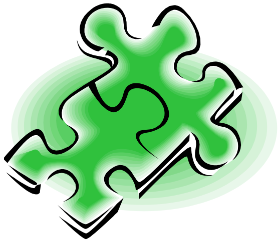
- Sending an encrypted message to the future
  - shouldn't be revealed before some future date
  - no safe storage for secrets
- Encode key as a "time-lock" puzzle
  - Bounds for computation time to solve puzzle
    - e.g., can be solved in 25 years on reasonable computer
    - Requires at least 20 on today's fastest computer
  - Puzzle generation is fast





Also useful for:
fair contract signing, sealed-bid auctions, coin flipping and more [RSW96,BN00,…]

Naïve Puzzle



- Invert a one-way function
  - Give some of the input to reduce search space
  - (Assume brute-force is the only attack)

$$y=f(x_1,x_2,\ldots,x_{100}),x_1,x_2,\ldots x_{50}$$

- Attackers might have many more computers!
  - e.g., Botnets, "cloud" servers.
  - Shouldn't gain a large advantage over legitimate solver (with one computer)

- Want a puzzle that is inherently sequential

Known Solution [RSW96]

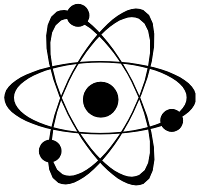
- Exponentiation (modulo N)

$$f(x)=2^{2^x} \bmod N$$

  - Fastest known method is repeated squaring
    - takes $\Omega(x)$ time
  - Can solve puzzle quickly if $\varphi(N)=(p-1)(q-1)$ is known
    - compute $x'=2^x \bmod \varphi(N)$
    - compute $2^{x'} \bmod N$

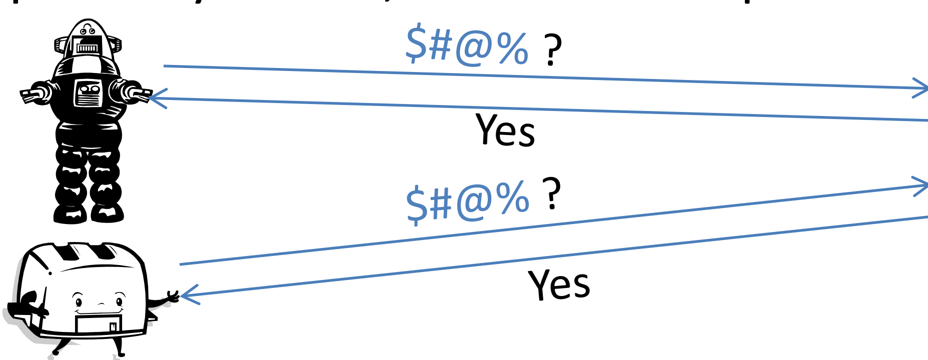
Takes time $O(\log(x)+\log(N))$

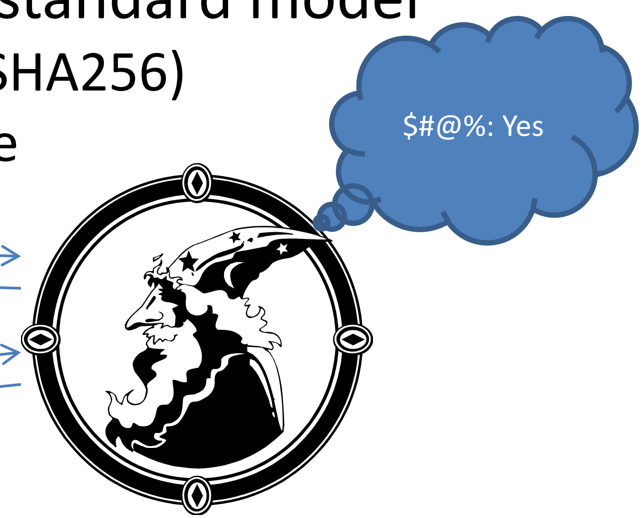
- Requires RSA assumption
  - what about quantum botnets?
  - Can we use other assumptions?



The Random Oracle Model

- Answer to each query is uniformly random (independently of other queries)
- The same query always gets the same answer
- **Complexity: count # of queries**
- Random Oracle is one-way even for computationally unbounded players
  - **Impossibility results in RO rule out black-box constructions in standard model**
- Heuristic for converting RO protocols to standard model
  - Replace RO with cryptographic hash (e.g. SHA256)
  - Not provably secure, but is used in practice



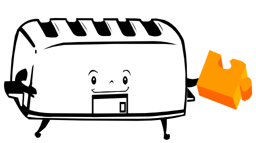


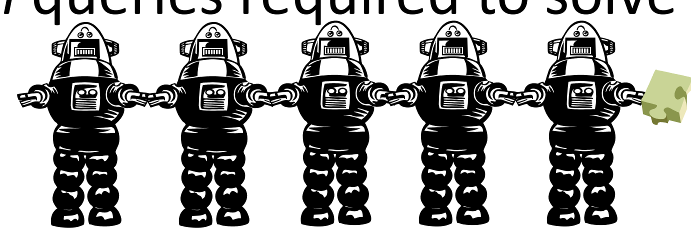
Our Results: Overview

- Main Result:
  - Time-lock puzzles that require n queries to generate can be solved in n parallel steps.
  - Rules out black-box constructions from one-way/hash functions
- Positive result:
  - Simple Time-lock puzzle satisfying
    - n *parallel* queries to construct
    - n *sequential* queries required to solve

(total # queries polynomial in honest solver)

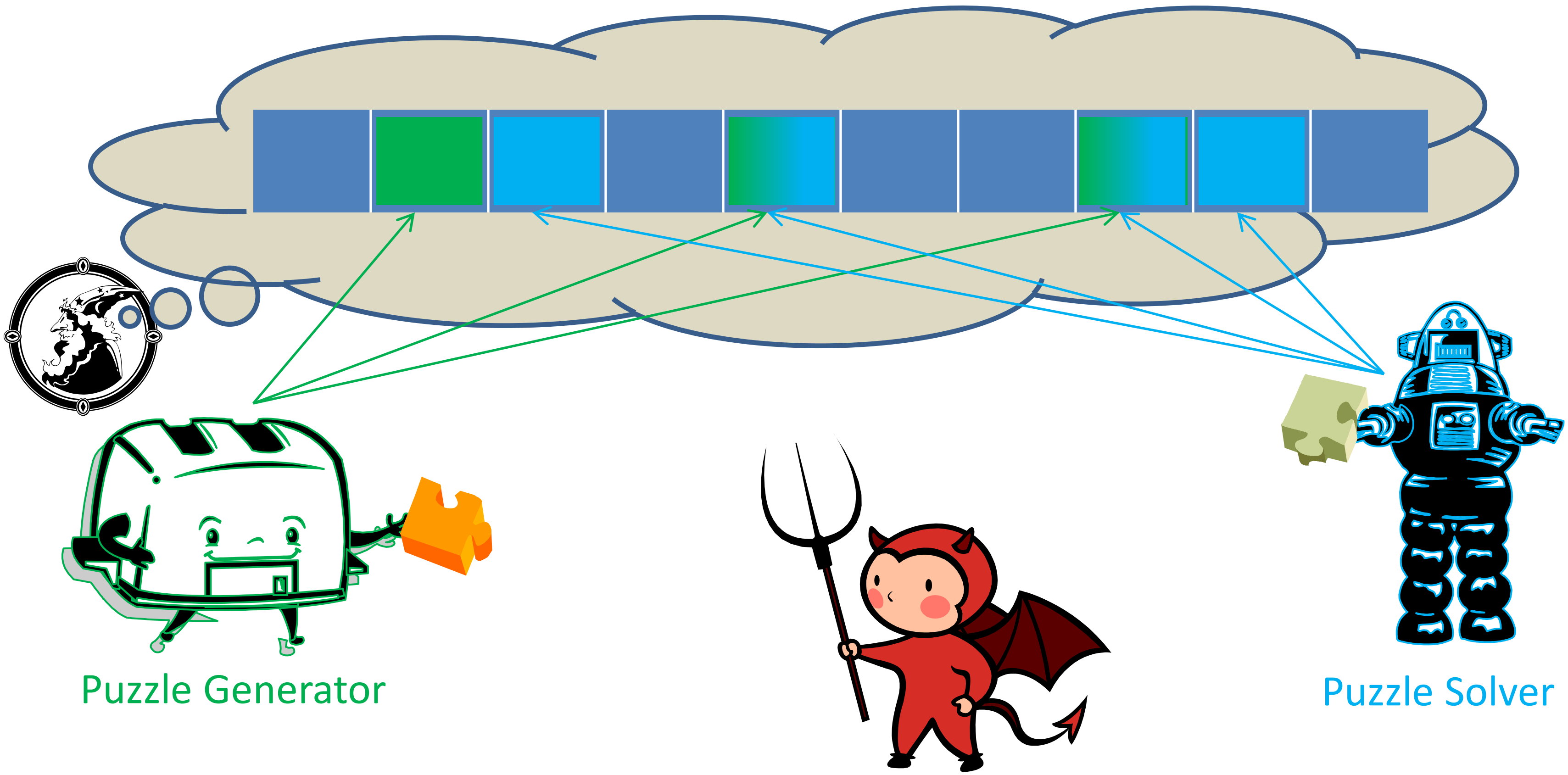
Generator with n parallel CPUs - n times faster than solver





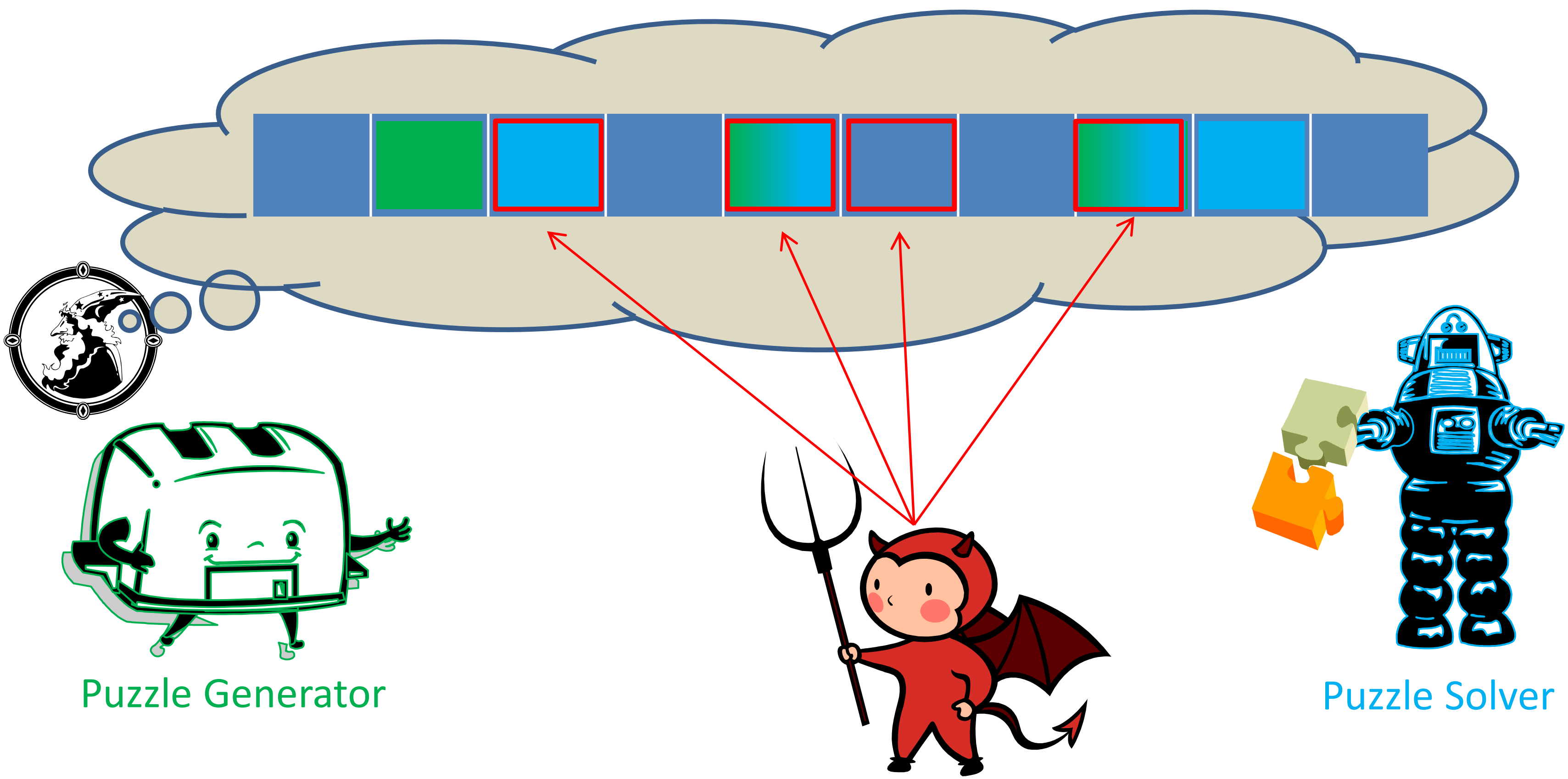
Main Result

Based on ideas from attacks on key-exchange protocols in the random oracle model [IR89,BM09]



Main Result

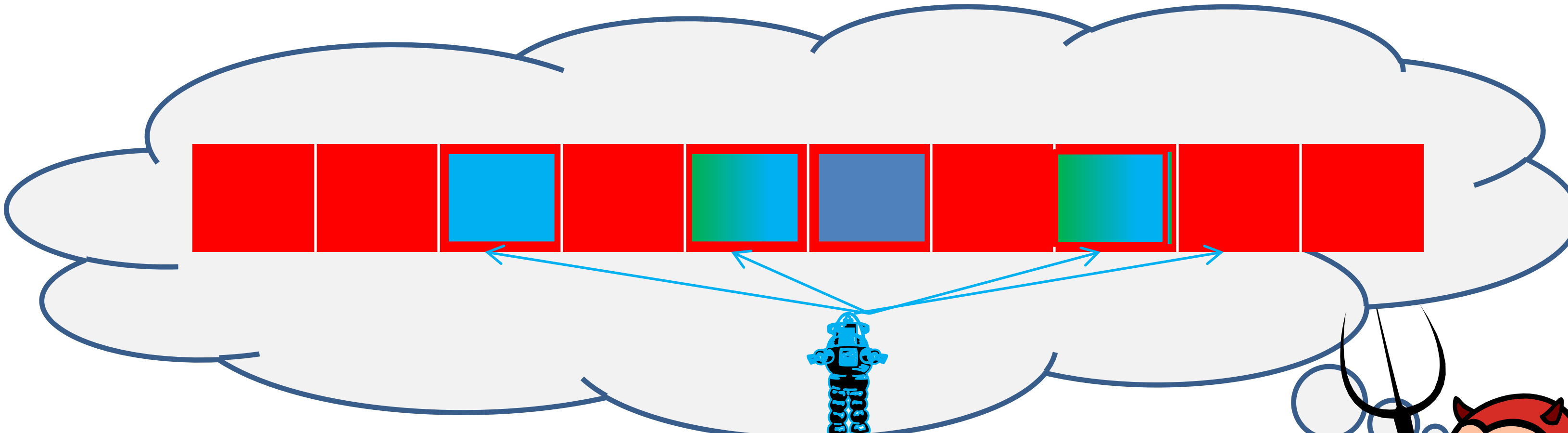
- High-level Sketch:
  - Construct adversary that finds *intersection queries*

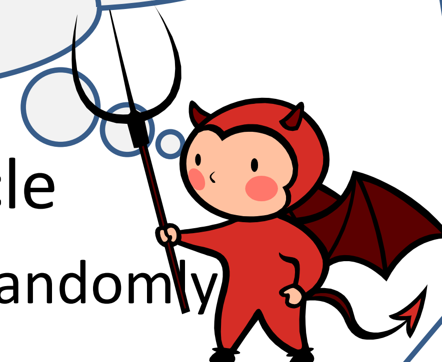


Main Result

From generator's point of view, "real" answers are identical to "fake" on unqueried indices

- High-level Sketch:
  - Construct adversary that finds *intersection queries*





  - Run honest solver with simulated oracle
    - Answer known queries correctly, others randomly
  - Success prob. identical to honest solver
  - Main hurdle: find intersections with ***low adaptivity***

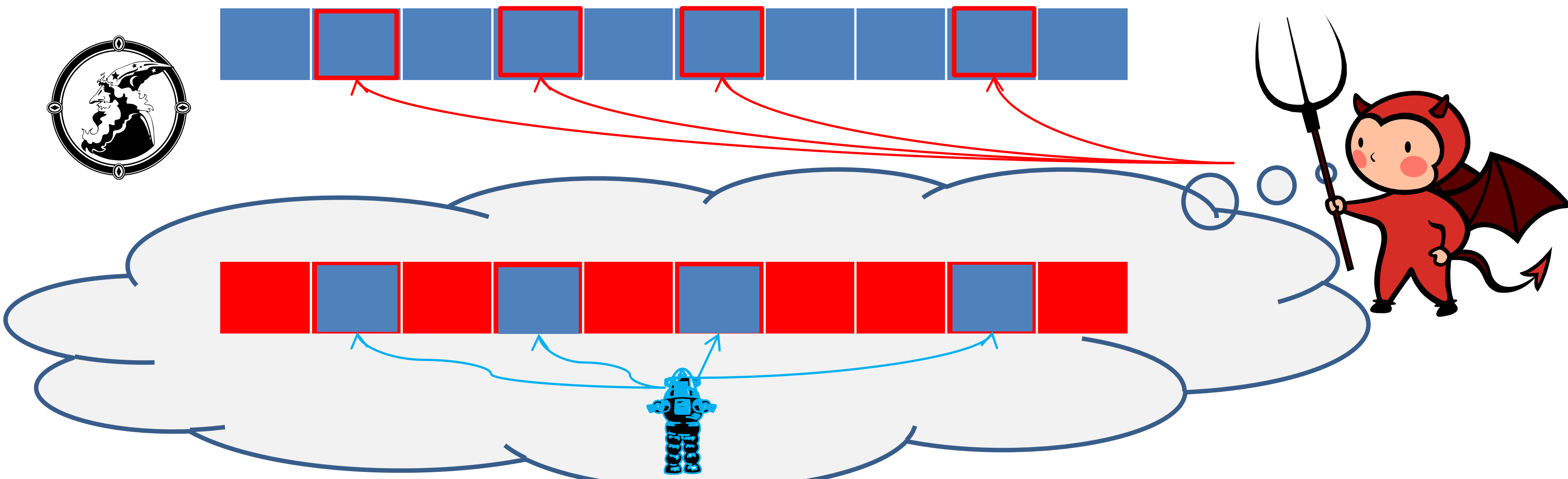
Finding Intersection Queries

(efficient adversary with non-optimal adaptivity)

- For all ε, adversary uses n/ε *rounds* of queries
  - Queries in each round can be done in parallel
- In each round:
  - Simulate honest solver
  - Answer known queries correctly, others randomly
  - Ask all queries to real oracle in parallel after every round
- Output results of randomly chosen round

queries used by generator

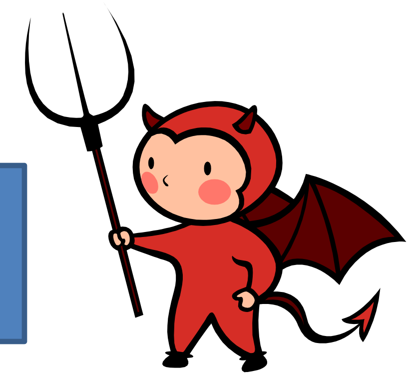
Adversary's error prob.



Finding Intersection Queries: Analysis

- Success probability: 1-ε
  - If simulation in output round did not hit any new intersection queries:
    simulated output is identically distributed to honest output (success probability is 1)
  - Generator asks at most n queries
    - Adv. asks a *new* intersection query in at most n rounds
  - Random round hits all intersection queries with prob. 1-ε
- Query complexity: nm/ε

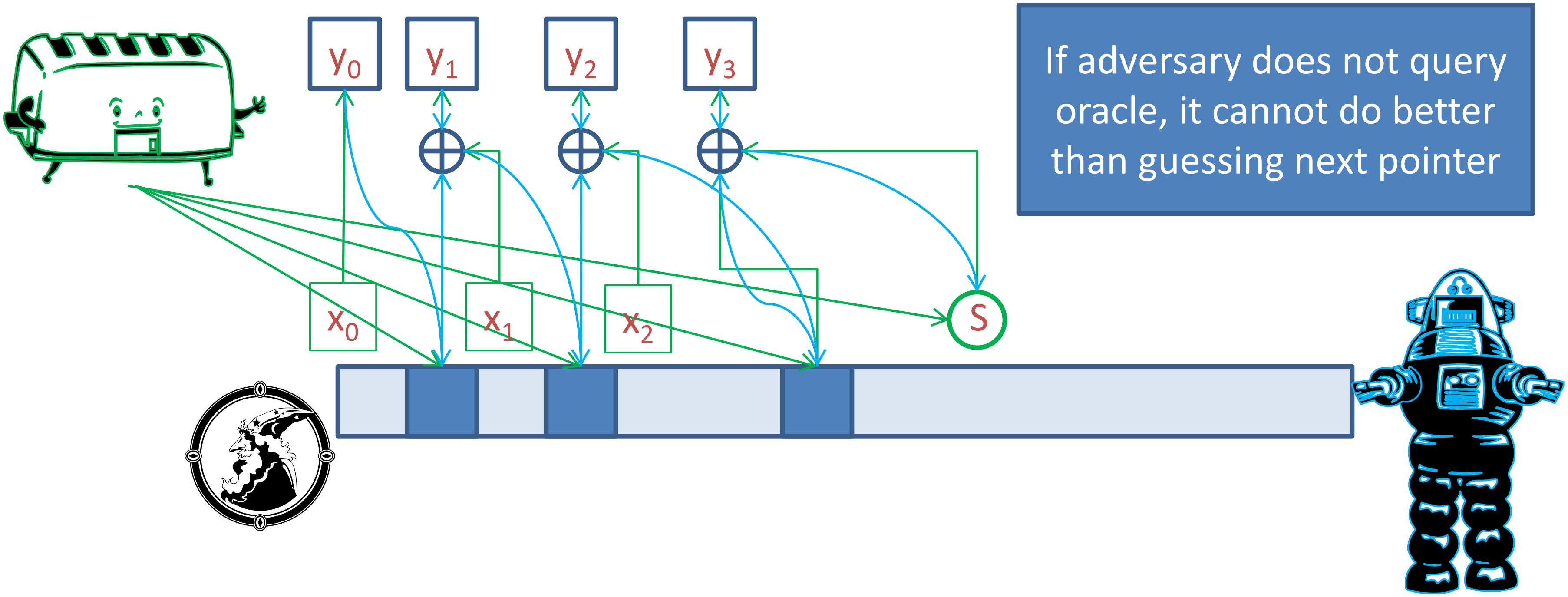
queries for honest solver



- Computational complexity:
  - polynomial in honest solver complexity

Positive Construction

- Time-lock puzzle encodes “pointer chain”
  - Generator queries in parallel
  - Solver must serially follow pointers



If adversary does not query oracle, it cannot do better than guessing next pointer

Discussion and Open Questions

- Optimally Adaptive (but inefficient) adversary
  - Uses n rather than n/ε adaptive rounds
  - Based on new learning algorithm for intersection queries.
- Corollary:
  - "Merkle puzzles" can be solved in linear parallel time
- Our negative result does not rule out "proofs of work"
  - In a proof-of-work, puzzle generator can verify solution quickly but not solve.
  - Positive solutions exist (work in progress)

- Still open:
  - Other time-lock puzzles in standard model?
  - Time-lock puzzles for quantum computers?
    - Related to [BHKKLS11] (*coming soon to a lecture hall near you!)*