Fuzzy Asymmetric Password-Authenticated Key Exchange

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Abstract. Password-Authenticated Key Exchange (PAKE) lets users with passwords exchange a cryptographic key. There have been two variants of PAKE which make it more applicable to real-world scenarios:

– *Asymmetric* PAKE (aPAKE), which aims at protecting a client's password even if the authentication server is untrusted, and
– *Fuzzy* PAKE (fPAKE), which enables key agreement even if passwords of users are noisy, but "close enough".

Supporting fuzzy password matches eases the use of higher entropy passwords and enables using biometrics and environmental readings (both of which are naturally noisy).

Until now, both variants of PAKE have been considered only in separation. In this paper, we consider both of them simultaneously. We introduce the notion of *Fuzzy Asymmetric PAKE* (fuzzy aPAKE), which protects against untrusted servers *and* supports noisy passwords. We formulate our new notion in the Universal Composability framework of Canetti (FOCS'01), which is the preferred model for password-based primitives. We then show that fuzzy aPAKE can be obtained from oblivious transfer and some variant of robust secret sharing (Cramer et al, EC'15). We achieve security against malicious parties while avoiding expensive tools such as non-interactive zero-knowledge proofs. Our construction is round-optimal, with message and password file sizes that are independent of the schemes error tolerance.

1 Introduction

In a world of watches interacting with smartphones and our water kettle negotiating with the blinds in our house, communicating devices are ubiquitous. Developments in user-centric technology are rapid, and they call for authentication methods that conveniently work with, e.g., biometric scans, human-memorable passwords or fingerprints derived from environmental readings.

Password-authenticated Key Exchange (PAKE) protocols [BM92, BPR00, BMP00, KOY01, GL03, KV11, CDVW12, BBC$^+$13] are the cryptographic answer to this need. They solve the problem of establishing a secure communication channel between two users who share nothing but a low-entropy string, often simply called *password*. Two interesting variants of PAKE protocols that are known from the literature are *asymmetric* PAKE [BM93,GMR06,JKX18,BJX19] which aims at protecting the user's password even if his password file at some server is stolen, and *fuzzy* PAKE [DHP^{+}18] which can tolerate some errors in the password. The former is useful in settings where authentication servers store thousands of user accounts and the server cannot be fully trusted. The latter introduces a usability aspect to PAKE protocols used by humans trying to remember passwords exactly. Furthermore, fuzzy PAKE broadens applicability of PAKE to the fuzzy setting and thereby allows using environmental readings or biometrics as passwords.

This work is the first to consider a combination of both PAKE variants. Namely, we introduce the notion of *fuzzy asymmetric PAKE* (fuzzy aPAKE). This new primitive allows a client and an untrusted server to authenticate to each other using a password, and both parties are guaranteed to derive the same cryptographic key as long as their passwords are within some predefined distance (in some predefined metric). Consider a client authenticating to a server using his fingerprint scan. In this setting, asymmetric PAKE protocols would not work since subsequent scans do not match exactly. Fuzzy PAKE, on the other hand, would require the server to store the fingerprint (or at least some template of it that uniquely identifies the person) in the clear, which is unacceptable for sensitive and ephemeral personal data that is biometrics. Fuzzy asymmetric PAKE, as introduced in this paper, is the only known cryptographic solution that applies to this setting: it works with fuzzy authentication data *and* does not reveal this authentication data to the server.

Why is this hard? Given that there is a lot of literature about both asymmetric PAKE and fuzzy cryptography, one could ask whether existing techniques could be used to obtain fuzzy aPAKE. As explained already in [DHP^{+}18], techniques from fuzzy cryptography such as information reconciliation [BBR88] or fuzzy extractors [DRS04] cannot be used with passwords of low entropy. Essentially, these techniques lose several bits of their inputs, which is acceptable when inputs have high entropy, but devastating in case of passwords.

Looking at techniques for asymmetric PAKE, all of them require some kind of password hardening such as hashing [GMR06, HL19, PW17], applying a PRF [JKX18] or a hash proof system [BJX19]. Unfortunately, such functions destroy all notions of closeness of their inputs by design. Further, it is unclear how to define a fuzzy version of, e.g., an oblivious PRF as used in [JKX18] that is not simply a constant function. While such definitions exist for "fuzzy" cryptographic hashing (e.g., robust property-preserving hashing [BLV19]), these functions either do not provide useful error correction or already their description leaks too much information about the password of the client. Overall, there seems to be no candidate asymmetric PAKE which can be made fuzzy.

Regarding more naive approaches, it is tempting to try to apply generic techniques for multi-party computation to obtain a fuzzy PAKE such as garbled circuits [Yao86]. The circuit would be created w.r.t some function of the password $h \leftarrow H(\mathsf{pw})$. The user's input would be pw'. Now the circuit finds all passwords close enough to pw' and outputs the shared key if one of these passwords yield h. Despite the inefficiency of this approach, it is unclear how to actually write down the circuit. As shown in [Hes19], h needs to be the output of some idealized assumption such as a programmable random oracle, and thus has no representation as a circuit.

Our contributions In this paper, we give the first formal definition of fuzzy asymmetric PAKE. Our definition is in the Universal Composability framework of Canetti [Can01], which is the preferred model for PAKE protocols (cf., e.g., [JKX18] for reasons). Essentially, we take the aPAKE functionality from [GMR06] (in a revised version due to [Hes19]) and equip it with fuzzy password matching (taken from the fuzzy PAKE functionality $\mathcal{F}_{\mathsf{fPAKE}}$ from [DHP+18]). Our resulting functionality $\mathcal{F}_{\mathsf{faPAKE}}$ is flexible in two ways: it can be optionally equipped with a mutual key confirmation (often called *explicit authentication*), and, just as $\mathcal{F}_{\mathsf{fPAKE}}$, $\mathcal{F}_{\mathsf{faPAKE}}$ can be parametrized with arbitrary metrics for distance, arbitrary thresholds and arbitrary adversarial leakage. Thus, our model is suitable to analyze protocols for a wide range of applications, from tolerating only few language-specific typos in passwords [CWP+17] to usage of noisy biometric scans of few thousand bits length.

We then give two constructions for fuzzy asymmetric PAKE. Our first construction Π_{faPAKE} uses error-correcting codes (ECC)[3] and oblivious transfer (OT) as efficient building blocks. Π_{faPAKE} works for Hamming distance and can correct $\mathcal{O}(\log(n))$ errors in n-bit passwords. Let us now give more details on Π_{faPAKE}.

The idea of our protocol is to first encode a cryptographic key and store it at the server, in a file together with random values to hide the codeword. The exact position of the codeword in the file is dictated by the password. A client holding a close enough password is thus able to retrieve almost the whole codeword correctly and can thus decode the session key given the error correction capabilities of the encoding. An attacker stealing the password file, however, cannot simply decode since the file contains too much randomness. To remove this randomness, he is bound to decode subsets of the file until he finds two subsets which decode to the same session key. Since decoding can be assumed to be as expensive as hashing, the effort of an off-line dictionary attack on the password file follows from a purely combinatorial argument on the parameters of the scheme (i.e., password size and error correction threshold).

To bound the client to one password guess per run of the protocol (which is the common security requirement for PAKE), we employ an n-times 1-out-of-2 OT scheme. Each OT lets the client choose either the true or the random part of the codeword for each of the n password bits (here we assume that the codeword is from $\mathbb{F}^n$ for some large field $\mathbb{F}$). Further, we apply randomization techniques to keep a client from collecting parts of the password file over several runs of the protocol.

[3] More precisely, we use a variant of *Robust Secret Sharing*, which can be instantiated with some class of error-correcting codes. However, since most readers are presumably more familiar with the latter, we describe our constructions in terms of codes.

A plus of our protocol is that it elegantly circumvents usage of expensive techniques such as non-interactive zero-knowledge proofs to ensure security against a malicious server. Indeed, a malicious server could make the client reconstruct the session key regardless of her password by entering only the true codeword in the OT. Such attacks would be devastating in applications where the client uses the session key to encrypt her secrets and sends them to the bogus server. Thus, the client needs a means to check correct behavior of the server. We achieve this by letting the server send his transcript of the current protocol run (e.g., the full password file) to the client, symmetrically encrypted with the session key. The client decrypts and checks whether the server executed the protocol with a password close enough to his own. Crucially, a corrupted client can only decrypt (and thus learn the server's secrets) if he holds a close enough password, since otherwise he will not know the encryption key.

Our proof of security is in the UC model and thus our protocol features composability guarantees and security even in the presence of adversarially-chosen passwords. As shown in [Hes19], strong idealized assumptions are necessary in order to achieve security in the UC model in case of asymmetric PAKE protocols. The reason lies in the adaptive nature of a server compromise attack (an adversary stealing the password file), against which our fuzzy version of asymmetric PAKE should also provide some protection. And indeed, our proof is in the generic group model and additionally requires encryption to be modeled as an ideal cipher. Both assumptions provide our simulator with the power to monitor off-line password guesses (*observability*) of the environment as well as to adjust a password file to contain a specific password even after having revealed the file (*programmability*)[4]. As a technicality, usage of the generic group model requires the client to perform decoding *in the exponent*. We give an example of a code that is decodable in the exponent.

Our second construction Π_{transf} is a "naive" approach of building fuzzy aPAKE from aPAKE. Namely, for a given pw, a server could simply store a list of, say, k hashes $H(\mathsf{pw}')$ for all pw' close enough to pw. Then, client and server execute k times an aPAKE protocol, with the client entering the same password every time and the server entering all hashes one by one. The fully secure protocol would need to protect against malicious behavior, e.g., by having both parties prove correct behavior. Unfortunately, this approach has two drawbacks. First, it does not scale asymptotically and has huge password files and communication overhead depending not only on the fuzziness threshold but also on the size of the password. Second, we show that Π_{transf} cannot be considered a secure fuzzy aPAKE, but has slightly weaker security guarantees.

On the plus side, Π_{transf} is already practical (and sufficiently secure) for applications where only few passwords should let the client pass. Facebook's authentication protocol, for example, is reported to correct capitalization of the

[4] We mention that already the fuzzy PAKE construction for Hamming distance from [DHP+18] relies on both the ideal cipher and random oracle model. Usage of the generic group model (together with a random oracle) has been recently shown useful in constructing strongly secure aPAKEs [BJX19].

first letter [Ale15], resulting in only two hashes to be stored in the password file. As analyzed in [CAA+16, CWP+17], correcting few common typographical mistakes as, e.g., accidental caps lock, increases usability significantly more than it decreases security. For such applications, our protocol Π_{transf} is a good choice.

1.1 Roadmap

In Section 2 we give a definition of our main building blocks, error-correcting codes which are decodable in the exponent. In Section 3, we provide the formal definition of fuzzy aPAKE and discuss the design of our functionality. Our fuzzy aPAKE protocol can be found in Section 4. Our naive approach of building faPAKE from aPAKE can be found in Section 5. Efficiency is considered in Section 6.

2 Preliminaries

2.1 Robust Secret Sharing in the exponent

An l-out-of-n secret sharing scheme allows to share a secret value s into n shares $(s_1, \cdots, s_n)$ in such a way that given at least l of these shares, the secret can be reconstructed. Simultaneously, any tuple of shares smaller than l is distributed independently of s. *Robust secret sharing* (RSS) [CDD+15] improves upon secret sharing schemes in the presence of malicious shares. Intuitively, an $(n, l-1, r)_q$-RSS is an l-out-of-n secret sharing scheme which allows the presence of up to $n-r$ corrupted shares. In detail the reconstruction of the secret is reliable for an n-tuple input $(\hat{s}_1, \cdots, \hat{s}_n)$ of r different secret shares s_i and $n-r$ random values a_i even if the positions of the correct shares are unknown.

We recall the definition of RSS as stated in [DHP+18]. For a vector $c \in \mathbb{F}_q^n$ and a set $A \subseteq [n]$, we denote with c_A the projection $\mathbb{F}_q^n \to \mathbb{F}_q^{|A|}$, i.e., the subvector $(c_i)_{i \in A}$.

Definition 1. *Let $\lambda \in \mathbb{N}$, q a λ-bit prime, $\mathbb{F}_q$ a finite field and $n, l, r \in \mathbb{N}$ with $l < r \leq n$. An $(n, l, r)_q$ robust secret sharing scheme (RSS) consists of two probabilistic algorithms $\mathsf{Share} : \mathbb{F}_q \to \mathbb{F}_q^n$ and $\mathsf{Rec} : \mathbb{F}_q^n \to \mathbb{F}_q$ with the following properties:*

- l-privacy*: for any $s, s' \in \mathbb{F}_q, A \subset [n]$ with $|A| \leq l$, the projections c_A of $c \xleftarrow{\$} \mathsf{Share}(s)$ and c'_A of $c' \xleftarrow{\$} \mathsf{Share}(s')$ are identically distributed.*
- r-robustness*: for any $s \in \mathbb{F}_q, A \subset [n]$ with $|A| \geq r$, any c output by $\mathsf{Share}(s)$, and any $\tilde{c}$ such that $c_A = \tilde{c}_A$, it holds that $\mathsf{Rec}(\tilde{c}) = s$.*

We now introduce a variant of RSS which produces shares that are hidden in the exponent of some group G, and which features a reconstruction algorithm that can handle shares in the exponent. At the same time we sacrifice absolute correctness of Rec and allow for a negligible error in the definition of robustness.

Definition 2 (Robust Secret Sharing in the Exponent). *Let $\lambda \in \mathbb{N}$, q a λ-bit prime, $\mathbb{F}_q$ a finite field and $n, l, r \in \mathbb{N}$ with $l < r \leq n$. Let $RSS = (\mathsf{Share}', \mathsf{Rec}')$ be a $(n, l, r)_q$ robust secret sharing scheme and let $G = \langle g \rangle$ be a cyclic group of prime order q. An $(n, l, r)_q$ robust secret sharing scheme in the exponent (RSSExp) with respect to G consists of two probabilistic algorithms* $\mathsf{Share} : \mathbb{F}_q \to G^n$ *and* $\mathsf{Rec} : G^n \to G$ *which are defined as follows:*

- $\mathsf{Share}(s)$: *On input a secret value $s \leftarrow \mathbb{F}_q$, obtain secret shares $(s_1, \cdots, s_n) \leftarrow$* $\mathsf{Share}'(s)$ *and output $(g^{s_1}, \cdots, g^{s_n})$.*
- $\mathsf{Rec}(g^{\hat{s}_1}, \cdots, g^{\hat{s}_n})$: *On input n group elements, this algorithm outputs $g^{\hat{s}}$, where $\hat{s} \leftarrow$* $\mathsf{Rec}'(\hat{s}_1, \cdots, \hat{s}_n)$.

Further, an (n, l, r)-RSSExp scheme fulfills the following properties:

- *l-privacy: as in Definition 1.*
- *r-robustness: for any $s \in \mathbb{F}_q, A \subset [n]$ with $|A| \geq r$, any c output by* $\mathsf{Share}(s)$, *and any $\tilde{c}$ such that $c_A = \tilde{c}_A$, it holds that* $\mathsf{Rec}(\tilde{c}) = g^s$ *with overwhelming probability in n.*

Note that any (n, l, r)-RSSExp scheme trivially fulfills the l-privacy property. In the next part of this section we show how to achieve r-robustness.

Instantiations of RSSExp In [DHP+18], it is shown how to construct an RSS scheme from any *maximum distance separable* (MDS) code. An $(n+1, k)_q$ MDS code is a linear q-ary code of length n and rank k, which can correct up to $\lfloor (n-k+1)/2 \rfloor$ errors. We refer to [Rot06] for a more in depth introduction to linear codes.

Concretely, [DHP+18] propose to use Reed-Solomon codes, which are closely related to Shamir's secret sharing scheme [MS81]. In general, we are not aware of any RSS scheme that is not also an MDS code. For this reason, we focus now on decoding algorithms of linear codes.

Which decoding alorithm works also in the exponent? In the following Lemma we show that it is possible to build an $(n, l-1, l+t, g)$-RSSExp scheme from an l-out-of-$(l+2t)$ Shamir's secret sharing scheme.

Lemma 1. *Let $n, l \in \mathbb{N}$ and* $(\mathsf{Share}', \mathsf{Rec}')$ *be an l-out-of-n Shamir's secret sharing scheme with $n = l + 2t$ for some t and $t \cdot l = \mathcal{O}(n \log n)$, $G = \langle g \rangle$ a cyclic group of order q. Further let* Share *be the algorithm that outputs $g^{\mathsf{Share}'(s)}$ on input $s \in \mathbb{F}_q$. Then there exists an algorithm* Rec *using poly$(n) \cdot \mathcal{O}(\log q)$ group operations such that* $(\mathsf{Share}, \mathsf{Rec})$ *is an $(n, l-1, l+t)$-RSSExp scheme with respect to G.*

Proof. $(l-1)$-privacy of l-out-of-n Shamir's secret sharing scheme is shown in [DHP+18], Lemma 5, and can be directly applied to the case where shares are lifted to the exponent of some group. Let Rec be the "unique decoding by randomized enumeration" algorithm defined by Canetti and Goldwasser [CG99] (essentially, the algorithm decodes random subsets of shares until it finds redundancy), but applied to shares in the exponent using, e.g., Lagrange interpolation. Peikert [Pei06] shows in his Proposition 2.1 that, if $t < (n+1-l)/2$ (i.e., the number of errors allows for unique decoding) and $t \cdot l = \mathcal{O}(n \log n)$, then Rec succeeds with overwhelming probability in n and requires $poly(n) \cdot \mathcal{O}(\log q)$ group operations. Since $n = l + 2t$, it holds that $t < (n+1-l)/2$ and hence $(l+t)$-robustness is achieved.

3 Security Model

We now present our security definition for asymmetric fuzzy password authenticated key exchange (Π_{faPAKE}). Our functionality combines the fuzzy PAKE functionality $\mathcal{F}_{\mathsf{fPAKE}}$ from [DHP⁺18] with the asymmetric PAKE functionality $\mathcal{F}_{\mathsf{apwKE}}$ [GMR06] (with revisions due to [Hes19]). In order to capture the notion of fuzziness in our model, we say that a key exchange using passwords pw and pw' is successful if $d(pw, pw') \leq \delta$, where d is an arbitrary distance function and δ a fixed threshold. $\mathcal{F}_{\mathsf{fPAKE}}$ can be parametrized with arbitrary functions $hdist()$ such as Hamming distance or edit distance.

Roles: In this work we consider an asymmetric setting, namely a client $\mathcal{P}_{\mathcal{C}}$ and a server $\mathcal{P}_{\mathcal{S}}$. Each party executes different code. In this setting $\mathcal{P}_{\mathcal{C}}$ uses a password pw while $\mathcal{P}_{\mathcal{S}}$ has access to some value denoted by FILE, which is generated from a password pw' but does not immediately reveal pw'. The goal of $\mathcal{P}_{\mathcal{C}}$ is convincing $\mathcal{P}_{\mathcal{S}}$ that $d(pw, pw') \leq \delta$, while $\mathcal{P}_{\mathcal{S}}$ only has access to FILE (and does not have access to pw').

Modeling Adversarial Capabilities: The standard security requirement for PAKE is that an attacker is bound to one password guessing attempt per run of the protocol. This resistance to off-line dictionary attacks is also featured by our functionality $\mathcal{F}_{\mathsf{faPAKE}}$ via the TESTPWD interface that can be called by the adversary only once per session. Since we are in the setting of asymmetric PAKE, however, the adversary can also gain access to the password file FILE by compromising the server. Such a compromise is essentially a corruption query with the effect that a part of the internal state of the server is leaked to the adversary. However, opposed to standard corruption, the adversary is not allowed to control the party or modify its internal state. $\mathcal{F}_{\mathsf{faPAKE}}$ provides an interface for server compromise named STEALPWDFILE. As a consequence of such a query (which, as natural for corruption queries, can only be asked by the adversary upon getting instructions from the environment), a dictionary attack becomes possible. Such an attack is reflected in $\mathcal{F}_{\mathsf{faPAKE}}$ by the OFFLINETESTPWD interface, which allows an unbounded number of password guesses. Accounting for protocols that allow precomputation of, e.g., hash tables of the form $H(\mathsf{pw})$, $\mathcal{F}_{\mathsf{faPAKE}}$ accepts OFFLINETESTPWD queries already *before* STEALPWDFILE was issued. $\mathcal{F}_{\mathsf{faPAKE}}$ silently stores these guesses in the form of (OFFLINE, pw) records. Upon STEALPWDFILE, $\mathcal{F}_{\mathsf{faPAKE}}$ sends the client's pw_C to the adversary in case a record (OFFLINE, pw_C) exists. This models the fact that the adversary learns the client's password from his precomputated values only upon learning the password file, i.e., compromising the server[5]. Besides offline password guesses, the adversary can use FILE of the compromised server to run a key exchange session with the user. This is captured within the IMPERSONATE interface.

All these interfaces were already present in aPAKE functionalities in the literature. The key difference of $\mathcal{F}_{\mathsf{faPAKE}}$ is now that all these interfaces apply fuzzy matching when it comes to comparing passwords. Namely, $\mathcal{F}_{\mathsf{faPAKE}}$ is parametrized with two thresholds δ and γ. δ is the "success threshold", for which it is guaranteed that passwords within distance δ enable a successful key exchange. On the other hand, γ can be seen as the "security threshold", with $\gamma \geq \delta$. Guessing a password within range γ does not enable the adversary to successfully exchange a key, but it might provide him with more information than just "wrong guess". Following [DHP+18], we enable weakenings of $\mathcal{F}_{\mathsf{faPAKE}}$ in terms of leakage from adversarial interfaces (cf. Figure 2). Here, the adversary, in addition to learning whether or not his password guess was close enough, is provided with the output of different leakage functions L_c, L_m and L_f. Essentially, he learns $L_c(\mathsf{pw}, \mathsf{pw}')$ if his guess was within range δ of the other password, L_m if it was within range $\gamma > \delta$ and L_f if it was further away than γ. $\mathcal{F}_{\mathsf{faPAKE}}$ can be instantiated with any thresholds γ, δ and arbitrary functions L_c, L_m, L_f. Looking ahead, the additional threshold γ enables us to prove security of constructions using building blocks such as error-correcting codes, which come with a "gray zone" where reliable error correction is not possible, but also the encoded secret is not information-theoretically hidden. While guessing a password in this gray zone does not enable an attacker to reliably compute the same password as the client, security is still considered to be compromised since some information about the honest party's password (and thus her key) might be leaked. To keep the notion flexible, we allow describing the amount of leakage with $L_m(\cdot, \cdot)$ and mark the record `compromised` to model partial leakage of the key.

Naturally, one would aim for δ and γ to be close, where $\delta = \gamma$ offers optimal security guarantees in terms of no special adversarial leakage if passwords are only $\delta + 1$ apart (an equivalent formulation would be to set $L_m = L_f$). $\mathcal{F}_{\mathsf{faPAKE}}$ is strongest if $L_f = L_m = L_c = \perp$. Below we provide examples of nontrivial leakage functions, verbatim taken from [DHP+18].

Since in a fuzzy aPAKE protocol the password file stored at the server needs to allow for fuzzy matching, files are required to store the password in a structured or algebraic form. An adversary stealing the file could now attempt to alter the file to contain a different (still unknown) password. This kind of attack does not seem to constitute a real threat, since the attacker basically just destroyed the file and cannot use it anymore to impersonate the server towards the corresponding client. To allow for efficient protocols, we therefore choose

[5] Recent PAKE protocols [JKX18, BJX19] have offered resistance against so-called precomputation attacks, where an attacker should not be able to pre-compute any values that can be used in the dictionary attack. Our protocols do not offer such guarantees.

to incorporate malleability of password files into our functionality $\mathcal{F}_{\mathsf{faPAKE}}$ by allowing the adversary to present a function f within an IMPERSONATE query. The impersonation attack is then carried out with $f(pw)$ instead of pw, where pw denotes the server's password.

Figure 1 depicts $\mathcal{F}_{\mathsf{faPAKE}}$ with the set of leakage functions from the second example below, namely leaking whether the password is close enough to derive a common cryptographic key.

Examples of leakage functions.

1. *No leakage.* The strongest option is to provide no feedback at all to the adversary. We define $\mathcal{F}^{N}_{\mathsf{faPAKE}}$ to be the functionality described in Figure 1, except that TESTPWD, IMPERSONATE, OFFLINETESTPWD and STEALPWDFILE use the check depicted in Figure 2 with

$$L_c^N(\mathsf{pw}, \mathsf{pw}') = L_m^N(\mathsf{pw}, \mathsf{pw}') = L_f^N(\mathsf{pw}, \mathsf{pw}') = \perp .$$

2. *Correctness of guess.* The basic functionality $\mathcal{F}_{\mathsf{faPAKE}}$, described in Figure 1, leaks the correctness of the adversary's guess. That is, in the language of Figure 2,

$$\begin{aligned} & L_c(\mathsf{pw}, \mathsf{pw}') = \text{“correct guess”}, \\ \text{and} \qquad & L_m(\mathsf{pw}, \mathsf{pw}') = L_f(\mathsf{pw}, \mathsf{pw}') = \text{“wrong guess”}. \end{aligned}$$

3. *Matching positions (“mask”).* Assume the two passwords are strings of length n over some finite alphabet, with the jth character of the string pw denoted by $\mathsf{pw}[j]$. We define $\mathcal{F}^{M}_{\mathsf{faPAKE}}$ to be the functionality described in Figure 1, except that TESTPWD, IMPERSONATE, OFFLINETESTPWD and STEALPWDFILE use the check depicted in Figure 2, with L_c and L_m that leak the indices at which the guessed password differs from the actual one when the guess is close enough (we will call this leakage the *mask* of the passwords). That is,

$$\begin{aligned} & L_c^M(\mathsf{pw}, \mathsf{pw}') = (\{j \text{ s.t. } \mathsf{pw}[j] = \mathsf{pw}'[j]\}, \text{“correct guess”}), \\ & L_m^M(\mathsf{pw}, \mathsf{pw}') = (\{j \text{ s.t. } \mathsf{pw}[j] = \mathsf{pw}'[j]\}, \text{“wrong guess”}) \\ \text{and} \qquad & L_f^M(\mathsf{pw}, \mathsf{pw}') = \text{“wrong guess”}. \end{aligned}$$

4. *Full password.* The weakest definition — or the strongest leakage — reveals the entire actual password to the adversary *if the password guess is close enough.* We define $\mathcal{F}^{P}_{\mathsf{faPAKE}}$ to be the functionality described in Figure 1, except that TESTPWD, IMPERSONATE, OFFLINETESTPWD and STEALPWDFILE use the check depicted in Figure 2, with

$$L_c^P(\mathsf{pw}, \mathsf{pw}') = L_m^P(\mathsf{pw}, \mathsf{pw}') = \mathsf{pw} \text{ and } L_f^P(\mathsf{pw}, \mathsf{pw}') = \text{“wrong guess”}.$$

The functionality $\mathcal{F}_{\mathsf{faPAKE}}$ is parameterized by a security parameter λ and tolerances $\delta \leq \gamma$. It interacts with an adversary $\mathcal{S}$ and a client and a server party $\mathcal{P} \in \{\mathcal{P}_C, \mathcal{P}_S\}$ via the following queries:

Password Registration

- On $(\textsc{StorePwdFile}, \mathsf{sid}, \mathcal{P}_C, \mathsf{pw})$ from $\mathcal{P}_S$, if this is the first $\textsc{StorePwdFile}$ message, record $(\textsc{file}, \mathcal{P}_C, \mathcal{P}_S, \mathsf{pw})$ and mark it `uncompromised`.

Stealing Password Data

- On $\boxed{(\textsc{StealPwdFile}, \mathsf{sid})}$ from $\mathcal{S}$, if there is no record $(\textsc{file}, \mathcal{P}_C, \mathcal{P}_S, \mathsf{pw})$, return "no password file" to $\mathcal{S}$. Otherwise, if the record is marked `uncompromised`, mark it `compromised`; regardless, for all records $(\textsc{offline}, \mathsf{pw}')$ set $d \leftarrow d(\mathsf{pw}, \mathsf{pw}')$ and do:
  - If $d \leq \delta$, send ("correct guess", pw') to $\mathcal{S}$;

  If no such pw' is recorded, return "password file stolen" to $\mathcal{S}$.
- On $(\textsc{OfflineTestPwd}, \mathsf{sid}, \mathsf{pw}')$ from $\mathcal{S}$, do:
  - If there is a record $(\textsc{file}, \mathcal{P}_C, \mathcal{P}_S, \mathsf{pw})$ marked `compromised`, then set $d \leftarrow d(\mathsf{pw}, \mathsf{pw}')$ and do:
    - If $d \leq \delta$, mark record `compromised` and send "correct guess" to $\mathcal{S}$;
    - If $d > \delta$, mark record `interrupted` and send "wrong guess" to $\mathcal{S}$.
  - Else, record $(\textsc{offline}, \mathsf{pw}')$

Password Authentication

- On $(\textsc{UsrSession}, \mathsf{sid}, \mathsf{ssid}, \mathcal{P}_S, \mathsf{pw}')$ from $\mathcal{P}_C$, send $(\textsc{UsrSession}, \mathsf{sid}, \mathsf{ssid}, \mathcal{P}_C, \mathcal{P}_S)$ to $\mathcal{S}$. Also, if this is the first $\textsc{UsrSession}$ message for ssid, record $(\mathsf{ssid}, \mathcal{P}_C, \mathcal{P}_S, \mathsf{pw}')$ and mark it `fresh`.
- On $(\textsc{SrvSession}, \mathsf{sid}, \mathsf{ssid})$ from $\mathcal{P}_S$, retrieve $(\textsc{file}, \mathcal{P}_C, \mathcal{P}_S, \mathsf{pw})$ and send $(\textsc{SrvSession}, \mathsf{sid}, \mathsf{ssid}, \mathcal{P}_C, \mathcal{P}_S)$ to $\mathcal{S}$. Also, if this is the first $\textsc{SrvSession}$ message for ssid, record $(\mathsf{ssid}, \mathcal{P}_S, \mathcal{P}_C, \mathsf{pw})$ and mark it `fresh`.

Active Session Attacks

- On $(\textsc{TestPwd}, \mathsf{sid}, \mathsf{ssid}, \mathcal{P}, \mathsf{pw}')$ from $\mathcal{S}$, if there is a record $(\mathsf{ssid}, \mathcal{P}, \mathcal{P}', \mathsf{pw})$ marked `fresh`, then set $d \leftarrow d(\mathsf{pw}, \mathsf{pw}')$ and do:
  - If $d \leq \delta$, mark record `compromised` and send "correct guess" to $\mathcal{S}$;
  - If $d > \delta$, mark record `interrupted` and send "wrong guess" to $\mathcal{S}$.
- On $(\textsc{Impersonate}, \mathsf{sid}, \mathsf{ssid}, f)$ from $\mathcal{S}$, if there is a record $(\mathsf{ssid}, \mathcal{P}_C, \mathcal{P}_S, \mathsf{pw})$ marked `fresh` and a record $(\textsc{file}, \mathcal{P}_C, \mathcal{P}_S, \mathsf{pw}')$ marked `compromised`, then set $d \leftarrow d(\mathsf{pw}, f(\mathsf{pw}'))$ and do:
  - If $d \leq \delta$, mark record `compromised` and send "correct guess" to $\mathcal{S}$;
  - If $d > \delta$, mark record `interrupted` and send "wrong guess" to $\mathcal{S}$.

Key Generation and Implicit Authentication

- On $(\textsc{NewKey}, \mathsf{sid}, \mathsf{ssid}, \mathcal{P}, \mathsf{k})$ from $\mathcal{S}$ where $|\mathsf{k}| = \lambda$ or $\mathsf{k} = \perp$, if there is a record $(\mathsf{ssid}, \mathcal{P}, \mathcal{P}', \mathsf{pw})$ not marked `completed`, do:
  - If the record is marked `compromised`, or either $\mathcal{P}$ or $\mathcal{P}'$ is corrupted, send $(\mathsf{sid}, \mathsf{ssid}, \mathsf{k})$ to $\mathcal{P}$.
  - Else if the record is marked `fresh`, $(\mathsf{sid}, \mathsf{ssid}, \mathsf{k}')$ was sent to $\mathcal{P}'$, and at that time there was a record $(\mathsf{ssid}, \mathcal{P}', \mathcal{P}, \mathsf{pw})$ with $d(\mathsf{pw}, \mathsf{pw}') \leq \delta$ marked `fresh`, send $(\mathsf{sid}, \mathsf{ssid}, \mathsf{k}')$ to $\mathcal{P}$.
  - Else if $\mathsf{k} \neq \perp$, the record is marked `fresh`, $(\mathsf{sid}, \mathsf{ssid}, \mathsf{k}')$ was sent to $\mathcal{P}'$, and at that time there was a record $(\mathsf{ssid}, \mathcal{P}', \mathcal{P}, \mathsf{pw})$ with $d(\mathsf{pw}, \mathsf{pw}') \leq \delta$ marked `fresh`, send $(\mathsf{sid}, \mathsf{ssid}, \mathsf{k}')$ to $\mathcal{P}$.
  - Else, pick $\mathsf{k}'' \overset{\$}{\leftarrow} \{0,1\}^\lambda$ and send $(\mathsf{sid}, \mathsf{ssid}, \mathsf{k}'')$ to $\mathcal{P}$.

  Finally, mark $(\mathsf{ssid}, \mathcal{P}, \mathcal{P}', \mathsf{pw})$ `completed`.

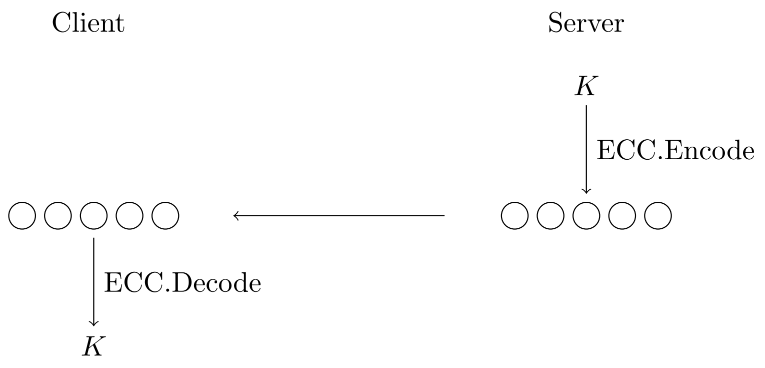
Fig. 1: Ideal functionality $\mathcal{F}_{\mathsf{faPAKE}}$. Framed queries can only be asked upon getting instructions from $\mathcal{Z}$.

– If $d \leq \delta$, mark the record `compromised` and reply to $\mathcal{S}$ with $L_c(\mathsf{pw}, \mathsf{pw}')$;
– If $\delta < d \leq \gamma$, mark the record `compromised` and reply to $\mathcal{S}$ with $L_m(\mathsf{pw}, \mathsf{pw}')$;
– If $\gamma < d$, mark the record `interrupted` and reply to $\mathcal{S}$ with $L_f(\mathsf{pw}, \mathsf{pw}')$.

Fig. 2: Modified distance checks to allow for different leakage to be used in TestPwd, OfflineTestPwd, Impersonate and StealPwdFile. In StealPwdFile, record marking is skipped.

4 Fuzzy aPAKE from Secret Sharing

We now describe our protocol for fuzzy aPAKE with Hamming distance as metric for closeness of passwords. The very basic structure of our protocol is as follows: we let the server encode a cryptographic key K using an error-correcting code[6]. The resulting codeword (different parts of codeword are depicted as white circles in the illustration below) is then transmitted to the client, who decodes to obtain the key.

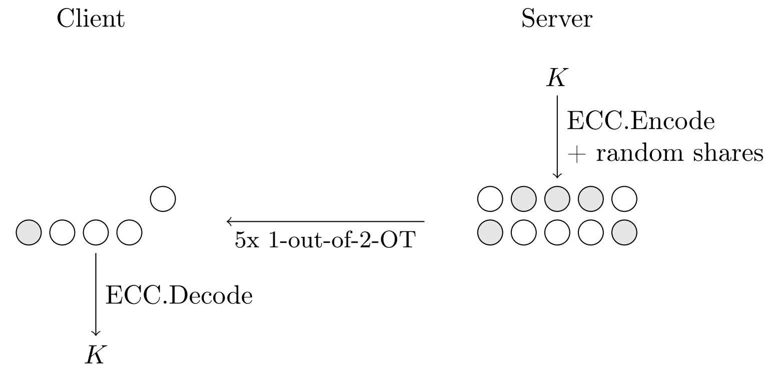


To make the retrieval of the cryptographic key password-dependent, the server stores the codeword together with randomness (depicted as grey circles below) in a password file. The position of the true codeword values in the file are dictated by the password bits. For example, in the illustration below, the server uses the password 01110. For this, we require the encoding algorithm to output codewords whose dimension matches the number of password bits. Now instead of getting the full password file, the client can choose to see only one value per column (either a part of the codeword or a random value). Technically, this is realized by employing a n-time 1-out-of-2 oblivious transfer (OT) protocol [7], where $n = 5$ is the password size of our toy example. The oblivious

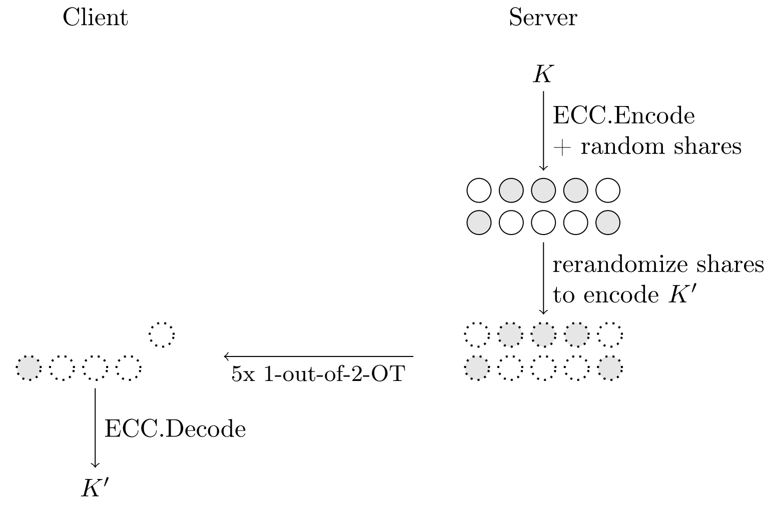
[6] Formally, we will define our scheme using the more general concept of robust secret sharing. However, for this overview it will be convenient to use the terminology of error-correcting codes.

[7] The protocol is not restricted by 1-out-of-2 OT, but can use 1-out-of-n OT for any $n \in \mathbb{N}$. In this work we consider $n = 2$, but in practice $n > 2$ might be useful to reduce the number of wrong shares (e.g. $n = 2^7$ in case of ASCII encoding).

part is crucial to keep the server from learning the client's password. With this approach, passwords within the error correction threshold of the password used by the server are sufficient to let the client decode the cryptographic key. In the illustration below, the client uses password 11110, letting him obtain 4/5 of the codeword correctly. Furthermore, an adversary stealing the password file is now faced with the computationally expensive task of finding the codeword within the file. Generalized to an $(n-2t)$-out-of-n RSS, the naive approach of finding $n-2t$ shares of the codeword by taking random subsets succeeds with probability $1/2^{n-2t}$ (as there are $\binom{n}{2t}$ "good" choices containing shares only, and $\binom{n}{2t} \cdot 2^{n-2t}$ choices overall). Here, n is the password size and t the number of errors that the fuzzy aPAKE protocol allows in passwords.



The above protocol can only be used to derive a single cryptographic key. Further, it is prone to a malicious client who could send pw and $\mathsf{pw} \oplus 1^n$ in two subsequent runs and obtain the full password file. The solution is randomization of the password file in each run of the protocol. This is straightforward for linear secret sharing.



Unfortunately, the above protocol cannot be proven UC secure. As already mentioned before, UC-secure asymmetric PAKE protocols require an idealized assumption to reveal password guesses against the file to the adversary [Hes19]. Furthermore, we need to require that a password file does not fix the password that is contained in it, in order to prove security in the presence of adaptive server compromise attacks. To remedy the situation, we let the server store the password file in the exponent of a publicly known large group and prove security of our construction in the generic group model [Sho97]. As a consequence, the client now needs to perform decoding in the exponent. We summarize in Section 2 which known decoding techniqes work also in the exponent, and detail in Section 6 how this affects the parameter choices of our scheme.

To complete our high-level protocol description, we now consider malicious behavior of client and server in the above protocol. Firstly, we observe that the client cannot cheat apart from using a different password in the OT (which does not constitute an attack) or outputting a wrong cryptographic key (which also does not constitute an attack). Things look differently when we consider a malicious server. The server could, e.g., deviate from the protocol by entering only correct codeword parts in the OT, making the key exchange succeed regardless of the password the client is using. To prevent such attacks, we let the server prove correct behavior by encrypting his view of the protocol run under the symmetric key K'. The view consists of the randomized password file as well as g^{pw}. A client being able to derive K' can now check whether the server indeed holds a password pw close enough to his own, and whether the transmitted password file parts match the password file created with pw. The formal description of our protocol can be found in Figure 3.

It is worth noting the similarity of our protocol to the fuzzy PAKE from RSS/ECC of [DHP$^+$18]. Namely, the overall idea is the same (server choosing and encoding K, sending it to the client who can decode if and only if his password is close enough). Essentially, both protocols transmit the codeword *encrypted* with the password, using a symmetric cipher that tolerates errors in the password - let us call this a *fuzzy symmetric cipher*. [DHP$^+$18] uses the following fuzzy symmetric cipher: XOR the codeword (the message) with cryptographic keys derived from the individual password bits. These cryptographic keys are exchanged using PAKE on individual password bits. Unfortunately, this approach does not work in the asymmetric setting, since the server would have to store the password in the clear to access its individual bits. For the asymmetric case, one has to come up with a fuzzy cipher that works with a key that is some function of the password. This function needs to have two properties: hide the password sufficiently, and still allow to evaluate distance of its input.

4.1 Security

Theorem 1. *Let $n, l, t \in \mathbb{N}$ with $n = l+2t$ and $(\mathsf{Share}, \mathsf{Rec})$ be an $(n, l-1, l+t)$-RSSExp scheme with respect to a generic group G. Then the protocol depicted in Figure 3 UC-emulates $\mathcal{F}^{P}_{\mathsf{faPAKE}}$ in the $\mathcal{F}_{\mathsf{IC}}, \mathcal{F}^{n}_{\mathsf{OT}}$-hybrid model, with $\gamma = 2t$,*

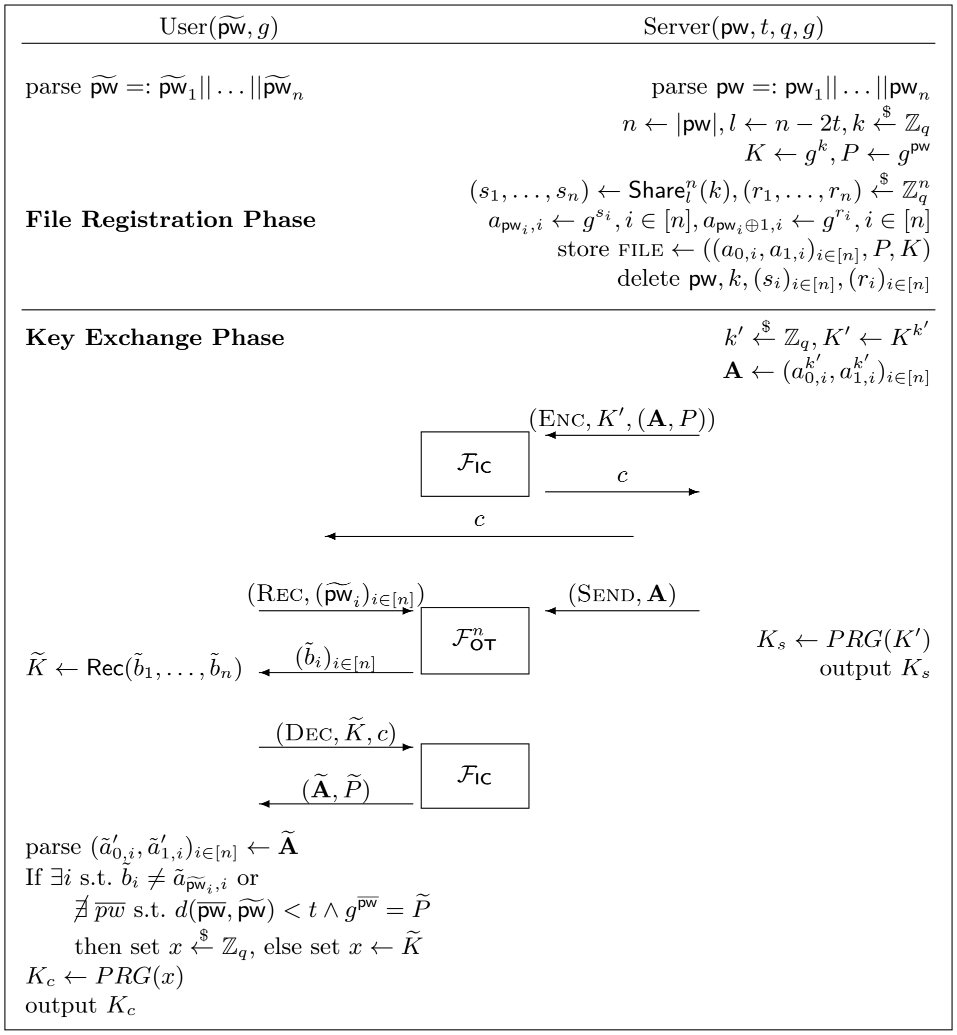

User$(\widetilde{\mathsf{pw}}, g)$
Server(pw, t, q, g)
parse $\widetilde{\mathsf{pw}} =: \widetilde{\mathsf{pw}}_1 || \dots || \widetilde{\mathsf{pw}}_n$
parse $\mathsf{pw} =: \mathsf{pw}_1 || \dots || \mathsf{pw}_n$
$n \leftarrow |\mathsf{pw}|, l \leftarrow n - 2t, k \xleftarrow{\$} \mathbb{Z}_q$
$K \leftarrow g^k, P \leftarrow g^{\mathsf{pw}}$
$(s_1, \dots, s_n) \leftarrow \mathsf{Share}_l^n(k), (r_1, \dots, r_n) \xleftarrow{\$} \mathbb{Z}_q^n$
File Registration Phase
$a_{\mathsf{pw}_i, i} \leftarrow g^{s_i}, i \in [n], a_{\mathsf{pw}_i \oplus 1, i} \leftarrow g^{r_i}, i \in [n]$
store $\textsc{file} \leftarrow ((a_{0,i}, a_{1,i})_{i \in [n]}, P, K)$
delete $\mathsf{pw}, k, (s_i)_{i \in [n]}, (r_i)_{i \in [n]}$
Key Exchange Phase
$k' \xleftarrow{\$} \mathbb{Z}_q, K' \leftarrow K^{k'}$
$\mathbf{A} \leftarrow (a_{0,i}^{k'}, a_{1,i}^{k'})_{i \in [n]}$
$(\textsc{Enc}, K', (\mathbf{A}, P))$
$\mathcal{F}_{\mathsf{IC}}$
c
c
$(\textsc{Rec}, (\widetilde{\mathsf{pw}}_i)_{i \in [n]})$
$(\textsc{Send}, \mathbf{A})$
$\mathcal{F}_{\mathsf{OT}}^n$
$\widetilde{K} \leftarrow \mathsf{Rec}(\tilde{b}_1, \dots, \tilde{b}_n)$
$(\tilde{b}_i)_{i \in [n]}$
$K_s \leftarrow PRG(K')$
output K_s
$(\textsc{Dec}, \widetilde{K}, c)$
$\mathcal{F}_{\mathsf{IC}}$
$(\widetilde{\mathbf{A}}, \widetilde{P})$
parse $(\tilde{a}'_{0,i}, \tilde{a}'_{1,i})_{i \in [n]} \leftarrow \widetilde{\mathbf{A}}$
If $\exists i$ s.t. $\tilde{b}_i \neq \tilde{a}_{\widetilde{\mathsf{pw}}_i, i}$ or
$\nexists\, \overline{pw}$ s.t. $d(\overline{\mathsf{pw}}, \widetilde{\mathsf{pw}}) < t \wedge g^{\overline{\mathsf{pw}}} = \widetilde{P}$
then set $x \xleftarrow{\$} \mathbb{Z}_q$, else set $x \leftarrow \widetilde{K}$
$K_c \leftarrow PRG(x)$
output K_c

Fig. 3: Protocol Π_{faPAKE} for asymmetric fuzzy PAKE using an n times 1-out-of-2 Oblivious Transfer.

$\delta = t$, Hamming distance $d()$ and with respect to static byzantine corruptions and adaptive server compromise.

We now provide a proof sketch for Theorem 1. The detailed proof can be found in the full version of this paper [EHOR20].

Proof sketch: The overall proof strategy is to give a simulated transcript and output of the protocol that is indistinguishable from a real protocol execution and runs independently of the parties' passwords. The simulator is allowed to make one password guess per execution (in case of compromised server the simulator can run several offline password guesses). In the following, we describe the different cases of corruption that have to be considered.

- **Honest session:** Apart from the interaction between client and server through the UC-secure OT, the only message that needs to be simulated is one ideal cipher output which is sent from the server to the client and serves as a commitment to the servers values. Since the ideal cipher generates a uniformly random ciphertext from the ciphertext space, the simulator can replace the $\mathcal{F}_{\mathsf{IC}}$ output by a random value as long as the key is unknown. Hence, the simulator runs independently from the passwords of the parties.
- **Corrupted client:** In case of corrupted client, it is crucial to bind the client to submitting all n password bits at once such that the client is not able to adaptively change the password bits based on previous OT outputs. We achieve this by using non-adaptive n times 1-out-of-2 OT executions. Hence, $\mathcal{S}$ is able to query TestPwd on the submitted password bits before it needs to simulate the OT outputs for the client. In case TestPwd returns the server's password, $\mathcal{S}$ can simulate valid OT outputs. Otherwise, $\mathcal{S}$ chooses random outputs which is indistinguishable from the real execution due to the privacy property of the RSSExp scheme.
- **Corrupted server:** Whenever the corrupted server sends the ciphertext that contains the OT inputs and g^{pw}, $\mathcal{S}$ reconstructs pw from the inputs to the ideal cipher and the generic group operations requestes by the environment. $\mathcal{S}$ then checks whether pw is close to the client's password using the TestPwd interface. If so the simulator gets the client's password and can simulate the client. Otherwise the client's behavior is independent of its password. Hence, $\mathcal{S}$ can simulate the client with an arbitrary password that is not close to the server's.
- **Server compromise:** (1) Simulating the password file. $\mathcal{S}$ assembles a table with random group element handles as password file, and a random handle corresponding to g^{k}. As soon as $\mathcal{Z}$ starts decoding with some subset of these elements by querying the GGM, $\mathcal{S}$ learns these queries. As soon as this subset of elements corresponds to a password, the simulator submits this password to OfflineTestPwd. If the answer includes the server's password, then $\mathcal{S}$ programs the GGM such that the decoding results in the handle of g^{k}.
  (2) Impersonation attacks. The environment could use a file (e.g., the one obtained from $\mathcal{S}$ or a randomized variant of it) to impersonate the server. For

this, the environment has to modify the ciphertext c to encrypt the file. Upon the environment sending an encryption query to $\mathcal{F}_{\mathsf{IC}}$ including an element P at the end of the message to be encrypted, the simulator checks if the GGM contains a tuple (pw, P). If so, $\mathcal{S}$ runs a TestPwd query on pw and learns the client's password $\widetilde{\mathsf{pw}}$ in case pw and $\widetilde{\mathsf{pw}}$ are close[8]. If there is no tuple (pw, P) in the GGM, $\mathcal{S}$ checks whether P was computed from the file (A', P') by the environment sending $f(P')$ to the GGM (and the simulator replying with P). If such a query happened, $\mathcal{S}$ issues an Impersonate query using the same function f.

– **MITM attack on honest session:** Apart from the interaction between client and server through the UC-secure OT, the only message that is sent is one ideal cipher output from the server to the client. Any attempt by $\mathcal{Z}$ to tamper with this message can be detected and hence $\mathcal{S}$ can simulate accordingly.

Password Salting. In the UC modeling each protocol session has access to a fresh instantiation of the ideal functionalities. Consequently each protocol session invokes a fresh instantiation of RO or GGM, which return different values when queried on the same input in different sessions. Therefore the password files generated for two users with the same password are different. In practice however the passwords must be salted, i.e. instead of storing the g^{pw}, the server stores $g^{(\mathsf{sid}||\mathsf{pw})}$ where sid is the respective session identifier. By applying this standard technique of salting in practice, the password files for two clients who use the same password would be different.

Use Cases for Hamming Distance metric. Although hamming distance is not the most optimal way to measure the distance of two passwords, it is quite suitable for biometric applications. As an example, a server can derive the password file from a client's iris scan or fingerprint such that the client can use this biometric data for authentication. Another example would be wearable or IoT devices. Such devices can measure unique characteristics of the user or environment, such as heart beat patterns and use these measurements for authentication. Our next construction is more suitable for password matching applications where users authenticate themselves with a human memorable password, but might input some characters of the password incorrectly.

5 Fuzzy aPAKE from standard aPAKE

We now show how to construct a fuzzy aPAKE from asymmetric PAKE. Essentially, the idea is to let the server run an aPAKE protocol with the client multiple times, entering all the passwords that are close to the password he originally registered. For formally defining the protocol, it will be convenient to assume a (possibly probabilistic) function $\mathsf{close}(pw) := \{pw_i | d(\mathsf{pw}, \mathsf{pw}_i) < \delta\}$

[8] We could alternatively let $\mathcal{S}$ issue an Impersonate query, but since the password is known issueing TestPwd works just as well.

that produces a set of all authenticating passwords. For example, for $d(), \delta$ accepting passwords where the first letter's case should be ignored, we would get $\mathsf{close}(\text{holy–moly!}) = \{\text{Holy–moly!}, \text{holy–moly!}\}$. When asking to register a password file containing pw, the server stores $\textsc{file} := \{H(\mathsf{pw}_i) | \mathsf{pw}_i \in \mathsf{close}(\mathsf{pw})\ \forall i = 1, ..., |\mathsf{close}(\mathsf{pw})|\}$ as arbitrarily ordered list of hash values of all authenticating passwords. Let $k := |\textsc{file}|$ be the number of such passwords. Now client and server execute the aPAKE protocol k times, where the client *always* enters his password, and the server enters all values from the password file (in an order determined by a random permutation τ). Then, similar to our protocol Π_{faPAKE}, the server proves honest behavior by encrypting the (permuted) password file under all k keys generated by the aPAKE protocol. The client decrypts and looks for a password file that was generated from a password that is close to his own password. If he finds such a file, he uses the corresponding decryption key (generated from aPAKE) to perform an explicit authentication step with the server. Note that this extra round of explicit authentication cannot be skipped, since otherwise the server would not know which key to output. While the computation on the client side sounds heavy at first sight, if both parties follow the protocol, all but one decryption attempts on the client side will fail. The client can efficiently recognized a failed decryption attempt by searching the decrypted message for the hash of his own password. The protocol is depicted in Figure 4.

Π_{transf} does not scale asymptotically, neither in the size of the password nor the number of errors. As an example, for correcting only one arbitrary error in an n-bit password, the password file size is already $k = n + 1$. For correcting up to t errors, we get $k := 1 + \sum_{i=1}^{t} \binom{n}{i}$. Note that k determines not only the size of the password file but also the number of aPAKE executions. On the plus side, the construction works with arbitrary metric and distances, does not have a "security gap" between δ and γ and has reasonable computational complexity on both the client and server side.

Unfortunately Π_{transf} cannot be proven secure given the original ideal functionality $\mathcal{F}_{\mathsf{faPAKE}}$, or rather its variant with explicit authentication (see the full version of this paper [EHOR20] for more details). In a nutshell, an attacker tampering with the single aPAKE executions can issue k password guesses using arbitrary passwords from the dictionary. A fuzzy aPAKE as defined within $\mathcal{F}_{\mathsf{faPAKE}}$, however, needs to bound the attacker to use k *close* passwords. To remedy the situation we modify the TestPwd interface of our $\mathcal{F}_{\mathsf{faPAKE}}$ functionality such that it allows n single password guesses. By single guess we mean that, instead of comparing a guess to all passwords within some threshold of the password of the attacked party (as it is done by $\mathcal{F}_{\mathsf{faPAKE}}$), it is compared to just one password. In case the client is attacked, the functionality compares with the client's password (and allows k such comparisons). In case the server is attacked, comparison is against a randomly chosen password close to the server's password[9]. Overall, the amount of information that the attacker obtains from

[9] Programming this randomized behavior into the functionality greatly simplifies proving security of Π_{transf} and does not seem to weaken the functionality compared to one using non-randomized equality checks.

both TestPwd interfaces is comparable: they both allow the attacker to exclude k passwords from being "close enough" to authenticate towards an honest party. Stated differently, to go through the whole dictionary D of passwords, with both TestPwd interfaces an attacker would need to tamper with $|D|/k$ key exchange sessions. We refer the reader to the full version of this paper [EHOR20] for more details regarding the modified functionalities.

We let $\mathcal{F}'_{\mathsf{faPAKE}}$ denote the ideal functionality $\mathcal{F}^{P}_{\mathsf{faPAKE}}$ with interfaces TestPwd and NewKey.

Theorem 2. *Protocol Π_{transf} UC-emulates $\mathcal{F}'_{\mathsf{faPAKE}}$ with arbitrary distance function $d()$ and arbitrary threshold $\delta = \gamma$ in the $(\mathcal{F}_{\mathsf{aPAKE}}, \mathcal{F}_{\mathsf{RO}}, \mathcal{F}_{\mathsf{IC}})$-hybrid model w.r.t static corruptions and adaptive server compromise and $H()$ denoting calls to $\mathcal{F}_{\mathsf{RO}}$.*

We now provide a proof sketch for Theorem 2. The detailed proof can be found in the full version of this paper [EHOR20].

Proof sketch. We need to consider the following attack scenarios:

– *Passive attacks*: The environment $\mathcal{Z}$ tries to distinguish uncorrupted real and ideal execution by merely observing transcript and outputs of the protocol, while providing the inputs of both honest parties. Since the outputs of the protocol are random oracle outputs and the transcript consists of a random ciphertext vector $\overrightarrow{e}$ output by the ideal cipher, $\mathcal{Z}$ cannot distinguish real outputs from simulated random values unless it queries either the ideal cipher functionality $\mathcal{F}_{\mathsf{IC}}$ or the random oracle $\mathcal{F}_{\mathsf{RO}}$ with the corresponding inputs. This can be excluded with overwhelming probability since these inputs are uniformly random values of high entropy chosen by honest parties.
– *Active message tampering*: We consider $\mathcal{Z}$ injecting a message into a protocol execution between two honest parties. The only messages being sent in unauthenticated channels are the encryption vector $\overrightarrow{e}$ and the explicit authentication message h. Replacing the message h would simply result in two different keys as output for the parties, simulatable by sending $\perp$ via NewKey. Tampering with $\overrightarrow{e}$ is a bit more tricky. Namely, we have to consider $\mathcal{Z}$ modifying only single components of $\overrightarrow{e}$. Tampering with each element of the vector $\overrightarrow{e}$ lowers the probability for the parties to output the same key. Hence, the simulator needs to adjust the probability for the parties to output the same key by forcing the functionality to only output the same session key with this exact probability, i.e., the simulator sends $\perp$ via NewKey with the inverse probability.
– *(Static) Byzantine corruption*: We consider the case where $\mathcal{Z}$ corrupts one of the parties.
  - In case of corrupted server, given an adversarially computed $\overrightarrow{e}$, the simulator extracts all k passwords used by $\mathcal{Z}$ from the server's inputs to $\mathcal{F}_{\mathsf{IC}}$ and $\mathcal{F}_{\mathsf{RO}}$ and submits them as password guess to $\mathcal{F}'_{\mathsf{faPAKE}}$ (via TestPwd). $\mathcal{S}$ then uses the answers (either "wrong guess" or the client's true password) to continue the simulation faithfully. In case the corrupted

server deviates from the protocol (e.g., $\overrightarrow{e}$ does not encrypt a set of passwords generated by $\mathsf{close}()$, or sends garbage to the $\mathcal{F}_{\mathsf{aPAKE}}$ instance in which the server uses the client's password), the simulator sends $\perp$ via the NewKey interface to simulate failure of the key exchange.
    - The case of a corrupted client is handled similarly using the freedom of k individual TestPwd queries.
- *Server compromise*: The password file is simulated without knowledge of the password by sampling random hash values. The simulator now exploits observability and programmability of the random oracle (that models the hash function) as follows: as soon as $\mathcal{Z}$ wants to compute $H(\mathsf{pw})$, $\mathcal{S}$ submits pw to its OfflineTestPwd interface. Upon learning the server's true password, $\mathcal{S}$ programs the random oracle such that the password file contains hash values of all passwords close to pw.
- *Attacking $\mathcal{F}_{aPAKE}$*: While using $\mathcal{F}_{\mathsf{aPAKE}}$ as hybrid functionality helps the parties to exchange the key, it gives us a hard time when simulating. Essentially, the simulator has to simulate answers to all adversarial interfaces of each instance of $\mathcal{F}_{\mathsf{aPAKE}}$ since $\mathcal{Z}$ is allowed to query them. And $\mathcal{F}_{\mathsf{aPAKE}}$ has a lot of them: StealPwdFile, TestPwd, OfflineTestPwd and Impersonate. In a nutshell, OfflineTestPwd queries can be answered by querying the corresponding interface at $\mathcal{F}'_{\mathsf{faPAKE}}$. The same holds for StealPwdFile and Impersonate, only that they can be queried only once in $\mathcal{F}'_{\mathsf{faPAKE}}$. Our proof thus needs to argue that the one answer provided by $\mathcal{F}_{\mathsf{faPAKE}}$ includes already enough information to simulate answers to all k. The most annoying interface, namely TestPwd is handled by forwarding each individual TestPwd guess to $\mathcal{F}'_{\mathsf{faPAKE}}$. This explains why $\mathcal{F}'_{\mathsf{faPAKE}}$ needs to allow k individual password guesses instead of one fuzzy one (as provided by $\mathcal{F}_{\mathsf{faPAKE}}$).

6 Efficiency

Efficiency of Π_{faPAKE}. When instantiated with the statically secure OT from [BDD$^+$17], Π_{faPAKE} is round-optimal and requires each party to send only one message. While 2 consecutive messages are in any case required for the OT, we can conveniently merge the ciphertext sent by the server with his message sent within the OT. In order to compute the total message size, let us first give more details on the OT instantiations that are compatible with Π_{faPAKE} and their communication complexity. Π_{faPAKE} can use any UC-secure protocol for 1-out-of-2 OT with the slight modification that the sender only continues the protocol after having received n input-dependent messages of the client (in UC-secure protocol, the client is usually committed to his input when sending his first message). E.g., one could modify the round-efficient statically secure OT protocol from [BDD$^+$17], Figure 3, to let the sender Alice wait for receiver Bob to complete the first step of the protocol n times. The protocol requires one round of communication. In total, 3 strings, 1 public key and 2 ciphertexts are send around per 1-out-of-2 OT. For sender inputs from $\mathbb{F}_q^2$ and security

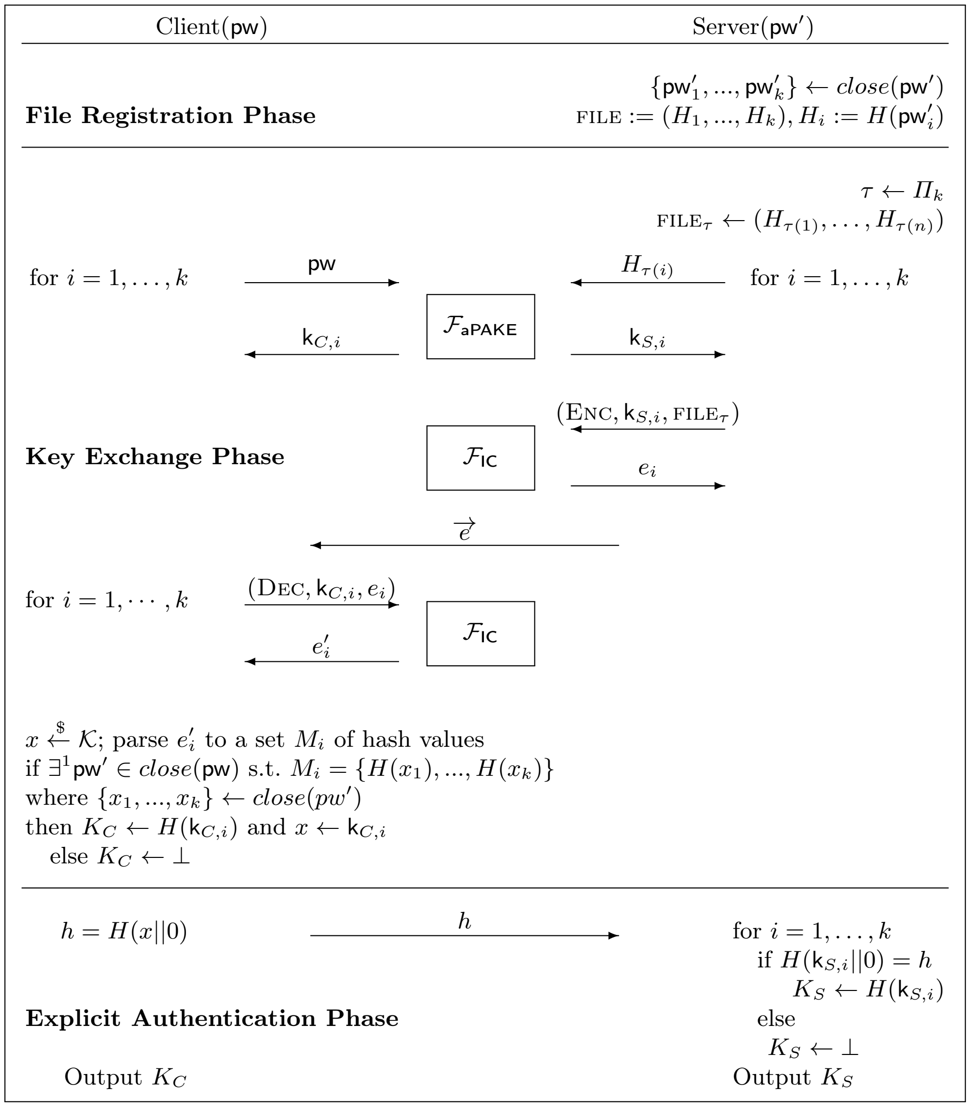

Client(pw)
Server(pw')
$\{\mathsf{pw}'_1, ..., \mathsf{pw}'_k\} \leftarrow close(\mathsf{pw}')$
File Registration Phase
$\text{FILE} := (H_1, ..., H_k), H_i := H(\mathsf{pw}'_i)$
$\tau \leftarrow \Pi_k$
$\text{FILE}_\tau \leftarrow (H_{\tau(1)}, \ldots, H_{\tau(n)})$
for $i = 1, \ldots, k$
pw
$H_{\tau(i)}$
for $i = 1, \ldots, k$
$\mathcal{F}_{\mathsf{aPAKE}}$
$\mathsf{k}_{C,i}$
$\mathsf{k}_{S,i}$
$(\text{Enc}, \mathsf{k}_{S,i}, \text{FILE}_\tau)$
Key Exchange Phase
$\mathcal{F}_{\mathsf{IC}}$
e_i
$\vec{e}$
for $i = 1, \cdots, k$
$(\text{Dec}, \mathsf{k}_{C,i}, e_i)$
$\mathcal{F}_{\mathsf{IC}}$
e'_i
$x \xleftarrow{\$} \mathcal{K}$; parse e'_i to a set M_i of hash values
if $\exists^1 \mathsf{pw}' \in close(\mathsf{pw})$ s.t. $M_i = \{H(x_1), ..., H(x_k)\}$
where $\{x_1, ..., x_k\} \leftarrow close(pw')$
then $K_C \leftarrow H(\mathsf{k}_{C,i})$ and $x \leftarrow \mathsf{k}_{C,i}$
else $K_C \leftarrow \perp$
$h = H(x||0)$
h
for $i = 1, \ldots, k$
if $H(\mathsf{k}_{S,i}||0) = h$
$K_S \leftarrow H(\mathsf{k}_{S,i})$
Explicit Authentication Phase
else
$K_S \leftarrow \perp$
Output K_C
Output K_S

Fig. 4: Protocol Π_{transf} for fuzzy asymmetric PAKE. The parties participate in k executions of the aPAKE protocol. Afterwards they verify if at least one of the produced k keys match and agree on it. We denote $\Pi_n := perm(1, ..., k)$ the set of permutations $[k] \rightarrow [k]$. $close(\mathsf{pw})$ is a function outputting a list of all authenticating passwords (see text for a formal description).

parameter λ with $q = 2^{\lambda}$, the communication complexity of the n-fold 1-out-of-2 OT is then $8\lambda n$ bits. This results in a total message size of $8\lambda n + |c| = 8\lambda n + (2n+1)\lambda \approx 10\lambda n$ bits. For each login attempt of a client, the server needs to perform $2n + 1$ group exponentiations in order to refresh the values in the password file, as well as an encryption of $2n + 1$ group elements. Finally, the server has to perform one PRG execution. Note that the server has to do some additional computations during the initial setup phase of the protocol, however since this phase is only run once, we do not consider its complexity in this section. The client's computation is where our protocol lacks efficiency. Namely, with the naive decoding technique from [CG99], client's computation is only polynomial in $|\mathsf{pw}|$ if the error correction capability δ is not larger than $\log |\mathsf{pw}|$. And still for such δ, going beyond password sizes of, say, 40 bits does not seem feasible.

Efficiency of Π_{transf}. In order to achieve the fuzzy password matching in Π_{transf}, the server is required to store one hash value for each password that lies within distance δ of the original password. As a consequence, the password file size is highly dependent on these threshold parameters. If we consider Hamming distance as done in our first construction, for $\delta = 1$ the password file is of size $\mathcal{O}(n)$. However for $\delta = 2$ it grows to $\mathcal{O}(n^2)$ and for $\delta = 3$ to $\mathcal{O}(n^3)$. Hence, such error tolerance can only be achieved in Π_{transf} at the cost of huge password files. The same correlation to the error tolerance holds for the amount of aPAKE executions in Π_{transf}.

In order to determine the computational complexity of Π_{transf} in terms of required group operations, we chose an instantiation of an aPAKE protocol, OPAQUE [JKX18], that requires a constant number of group exponentiations. As previously discussed, Π_{transf} requires k aPAKE executions with k being the size of the password file.

Despite its shortcomings when used with Hamming distance, Π_{transf} serves as a good illustration for how to construct a general purpose faPAKE protocol that already has practical relevance. Instantiated with distance and threshold suitable to correct, e.g., capitalization of first letters or transposition of certain digits, we obtain an efficient "almost secure" fuzzy aPAKE scheme.

We present a comparison of the two schemes in Table 1. Π_{transf} is listed twice. First it is compared to Π_{faPAKE} when using Hamming distance. The last row indicates its efficiency for parameters resulting in k authenticating passwords, where k can be as small as 2.

	File size	Message size	Thresholds	Metric	Client	Server	Assumption
Π_{faPAKE}	$(2n+2)\lambda$	$10\lambda n$	$2\delta = \gamma$	Hamming	$poly(n) \cdot \mathcal{O}(\log q)$	$\mathcal{O}(n \log q)$	IC, GGM
Π_{transf}	$\mathcal{O}(n^{\delta})$	$\mathcal{O}(n^{\delta})$	$\delta = \gamma$	Hamming	$\mathcal{O}(n^{\delta} \log q)$	$\mathcal{O}(n^{\delta} \log q)$	IC, ROM
Π_{transf}	λk	$\mathcal{O}(k)$	$\delta = \gamma$	arbitrary	$\mathcal{O}(k)$	$\mathcal{O}(k)$	IC, ROM

Table 1: Comparison of Π_{faPAKE} and Π_{transf}. We assume n-bit passwords in case of Hamming distance. File size and communication complexity are in bits. The Client and Server column indicate the number of group operations.

7 Conclusion

In this paper, we initiated the study of *fuzzy asymmetric PAKE*. Our security notion in the UC framework results from a natural combination of existing functionalities. Protocols fulfilling our definition enjoy strong security guarantees common to all UC-secure PAKE protocols such as protection against off-line attacks and simulatability even when run with adversarially-chosen passwords.

We demonstrate that UC-secure fuzzy aPAKE can be build from OT and Error-Correcting Codes, where fuzziness of passwords is measured in terms of their Hamming distance. Our protocol is inspired by the ideas of [DHP$^+$18] for building a fuzzy *symmetric* PAKE. We also show how to build a (mildly less secure) fuzzy aPAKE from (non-fuzzy) aPAKE. Our construction allows for arbitrary notions of fuzziness and yields efficient, strongly secure and practical protocols for use cases such as, e.g., correction of typical orthographic errors in typed passwords.

Our two constructions nicely show the trade-offs that one can have for fuzzy aPAKE. The "naive" construction from aPAKE has large password file size when used with Hamming distance, but also works for arbitrary closeness notions possibly leading to small password files and practical efficiency. The construction using Error-Correcting Codes is restricted to Hamming distance and $\log(|\mathsf{pw}|)$ error correction threshold. I comes with a computational overhead on the client side, but has only little communication and small password file size. It is worth noting that, for this construction, all efficiency drawbacks could be remedied by finding a more efficient decoding method that works in the exponent. We leave this as well as finding more fuzzy aPAKE constructions as future work. Specifically, no fuzzy aPAKE scheme with *strong* compromise security (as defined in [JKX18]) is known.

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