Triangulating Rebound Attack on AES-like Hashing[*]

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Abstract. The rebound attack was introduced by Mendel *et al.* at FSE 2009 to fulfill a heavy middle round of a differential path for free, utilizing the degree of freedom from states. The inbound phase was extended to 2 rounds by the Super-Sbox technique invented by Lamberger *et al.* at ASIACRYPT 2009 and Gilbert and Peyrin at FSE 2010. In ASIACRYPT 2010, Sasaki *et al.* further reduced the requirement of memory by introducing the non-full-active Super-Sbox. In this paper, we further develop this line of research by introducing *Super-Inbound*, which is able to connect multiple 1-round or 2-round (non-full-active) Super-Sbox inbound phases by utilizing fully the degrees of freedom from both states and key, yet without the use of large memory. This essentially extends the inbound phase by up to 3 rounds. We applied this technique to find classic or quantum collisions on several `AES`-like hash functions, and improved the attacked round number by 1 to 5 in targets including `AES-128` and `SKINNY` hashing modes, `Saturnin-Hash`, and `Grøstl-512`. To demonstrate the correctness of our attacks, the semi-free-start collision on 6-round `AES-128-MMO/MP` with estimated time complexity 2^{24} in classical setting was implemented and an example pair was found instantly on a standard PC.

Keywords: Triangulating Rebound · Quantum Attack · Collision Attacks · Rebound Attacks · Triangulation Algorithm · Super-Inbound

1 Introduction

Cryptographic hash function compresses a binary string of arbitrary length into a short fixed-length digest. It is one of the most fundamental cryptographic

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primitives that act as the underlying building blocks in various advanced cryptographic protocols, including digital signatures, authenticated encryption, secure multiparty computation and post-quantum public-key cryptography, etc. A cryptographic hash function must fulfill three basic security properties: collision resistance, preimage resistance, and second-preimage resistance. Due to the breakthroughs in collision attacks [76,77,74] on the hash functions MD5 and SHA-1, the academic communities advanced new hashing designs, including the best-known SHA-3 [10]. AES-like hashing, inspired by the elegant yet secure and efficient design strategies of Advanced Encryption Standard (AES) [21], is another new hashing design. It adopts block ciphers or permutations with similar features to AES as the underlying building blocks, such as Whirlpool [6], Grøstl [34], AES-MMO, Grindahl [52], ECHO [8], Haraka v2 [53], etc. These hash functions are commonly known as AES-like hashing.

Rebound Attacks [62], introduced by Mendel, Rechberger, Schläffer, and Thomsen at FSE 2009, is one of the most effective tools on the cryptanalysis of AES-like hash functions. It can be applied to both block cipher based and permutation based constructions. The idea of the rebound attack is to divide an attack into two phases, an inbound and an outbound phase. In the inbound phase, degrees of freedom are used to realize part of the differential characteristic deterministically. The remainder of the characteristic in the outbound phase is fulfilled in a probabilistic manner.

In order to penetrate more rounds, at ASIACRYPT 2009, Lamberger et al. [56] proposed the multiple inbound phases and connected them by leveraging the degrees of freedom of the key. Later, Gilbert and Peyrin [35] and Lamberger et al. [56] further extended the inbound phase by treating two consecutive AES-like rounds as the Super-Sbox [20]. At ASIACRYPT 2010, Sasaki et al. [70] reduced the memory cost by exploiting the differential property of non-full-active Super-Sboxes. At CRYPTO 2011, Naya-Plasencia [66] improved the rebound attack by introducing advanced algorithms for merging large lists. At CRYPTO 2012, Dinur et al. [26] further reduced the memory cost of the rebound attack by proposing the dissection technique. The rebound attack has since become a basic cryptanalysis tool to evaluate hash functions against collision attacks or distinguishing attacks [60,45,71,46,63,25,51,32,59]. Interestingly, the idea of the rebound attack was in turn used to improve the Demirci-Selçuk MITM attacks [24,33] and biclique attack [11].

Quantum Cryptanalysis attracts more and more attention due to Shor's quantum attacks [73] breaking the security of public-key crypto-systems RSA and ECC. For symmetric ciphers, the community has also witnessed many important quantum cryptanalysis results recently, such as quantum distinguisher on 3-round Feistel [54], key-recovery attack on Even-Mansour construction [55], key-recovery attacks or forgeries on MACs and authenticated encryption schemes [47], and more [57,13,27]. However, most attacks need to query the online quantum oracles with superposition states, which is believed to be an impractical projec-

tion for quantum physics. Thereafter, the quantum attacks [12,14,48,19,36,67,38] only using offline quantum computers are considered to be of more practical relevance.

Since hash functions can be implemented in offline quantum circuit, the attackers can freely make offline quantum superposition queries. There are several quantum generic (multi-target) preimage attacks [2,19] and multicollision attacks [42,41,58]. For generic quantum collision attacks, three quantum generic algorithms exist under different assumptions of the availability of quantum and classical memory resources:

- Condition 1: Exponentially large quantum random access memory (qRAM) is available. Brassard, Høyer, and Tapp [15] introduced the generic quantum collision attack (named as BHT algorithm) with $2^{n/3}$ quantum time complexity and $2^{n/3}$ qRAM.
- Condition 2: Neither exponentially large qRAM nor classic RAM is available. The quantum version of parallel rho's algorithm [75,9,39] achieves a time-space trade off of time $\frac{2^{n/2}}{S}$ with S computers.
- Condition 3: Exponentially large qRAM is not available but large classical RAM is. Chailloux, Naya-Plasencia, and Schrottenloher [19] introduced the CNS algorithm to find collision in time $2^{2n/5}$ with classical RAM of size $2^{n/5}$.

At EUROCRYPT 2020, Hosoyamada and Sasaki [39] introduced the first quantum dedicated collision attacks by converting the classical rebound attacks into quantum rebound attacks. This showed that, under their respective bounds of generic algorithms, quantum attacks are able to penetrate more rounds than classical attacks. At ASIACRYPT 2020, Dong et al. [30] reduced the requirement of qRAM in the quantum rebound attack by exploiting non-full-active Super-Sbox technique [70]. At CRYPTO 2021, Hosoyamada and Sasaki [40] introduced the quantum collision attacks on reduced SHA-2. At ASIACRYPT 2021, Dong et al. [31] studied the quantum free-start collision attacks.

1.1 Our Contributions

In order to extend the attacked round by the rebound attack, Lamberger et al. [56] connected two local inbound phases by consuming the degree of freedom in the key. Later, Super-Sbox [56,35] and non-full-active Super-Sbox [70] were proposed. In this paper, we further generalize this idea by bridging multiple inbound phases with available degrees of freedom both from the key and the internal data path, to build the *Super-Inbound*. With the help of Khovratovich et al.'s [50] triangulation algorithm, we identify truncated differential trails for `AES` hashing modes with sound configurations on the positions of several local inbound parts, as well as relatively low complexity to bridge the multiple local inbound parts. We name this method as *triangulating rebound attack*. With this advanced method in hand, we achieve the following results:

`AES-MMO`. The MMO-mode (shown in Figure 2) instantiated with `AES` [21] is a popular `AES`-like hashing design, which is standardized by Zigbee [1] and also

suggested by ISO [43]. Many advanced cryptographic protocols, e.g., multi-party computation [37,49], adopt `AES-MMO` due to its high efficiency when implemented with `AES-IN`. The best classical collision attacks are for 6 rounds [35,56]. At EUROCRYPT 2020, Hosoyamda and Sasaki [39] introduced two 7-round quantum collision attacks better than BHT algorithm [15] or parallel rho's algorithm [75]. At ASIACRYPT 2020, Dong et al. [30] introduced 7-round quantum collison better than CNS algorithm [19].

We extend the previous 2-round inbound phase into 3- or 4-round *Super-Inbound* for `AES-128`. Thereby, we build the first 7-round semi-free-start collision attack on `AES-128-MMO/MP` in classical setting, while the previous classical collision attack reaches 6 rounds [35,56]. In addition, a 6-round practical instance of semi-free-start collision and an 8-round quantum semi-free-start collision attack are given. Moreover, the first 8-round quantum collision attack on `AES-128-MMO/MP` is given, with time complexity better than parallel rho's algorithm [75,9,39].

`Saturnin-Hash`. It is the hash function among the post-quantum lightweight cipher family `Saturnin` designed by Canteaut et al. [17]. It is one of the round 2 candidates of the NIST lightweight cryptography competition. The designers of `Saturnin` introduced the 10-round related-key recovery attack on the `Saturnin` block cipher [16]. At ASIACRYPT 2021, Dong et al. [31] proposed several quantum and classical collision attacks on `Saturnin-Hash` and its compression function, including a 7-round semi-free-start quantum collision attack and an 8-round free-start quantum collision attack.

We extend the previous 2-round inbound phase to the 4-round *Super-Inbound*. Finally, the time complexity of the 8-round free-start quantum collision attack is significantly reduce from $2^{122.5}$ to $2^{89.65}$. Further, the first 10-round free-start quantum collision attack is derived with two more rounds than Dong et al.'s result [31].

`SKINNY-128-384-MMO`. As `SKINNY` [7] is running for ISO standard, it is natural to build lightweight hash with the ISO suggested hashing mode MMO. At ASIACRYPT 2021, Dong et al. [31] introduced the 16-round quantum free-start collision attack on `SKINNY-128-384-MMO`. In this paper, by exploring the large degrees of freedom of the key schedule for `SKINNY-128-384-MMO`, we extend the previous 2-round inbound phase into a 5-round *Super-Inbound*, and finally achieve 5 more rounds on the free-start quantum collision attack. In addition, a 19-round classical free start collision attack is given, which is even better than previous quantum attack.

`Grøstl`. `Grøstl` is one of the five finalists of SHA-3. In this paper, by bridging the differences with degrees of freedom from the states, we propose the memoryless method to solve the 3-round non-full-active inbound part for `Grøstl-512`. Thereafter, the improved semi-free-start collisions with one more round than before for both `Grøstl-512` and its predecessor are derived. The results are summarized in Table 1.

1.2   Novelty and Comparison with Previous Works

Both rebound attack [62] and triangulation algorithm [50] are existing techniques. However, combining these two techniques together as one cryptanalysis tool has not appeared before. When using these two techniques together, we can exploit many more degrees of freedom (DoF) from the key schedule algorithm, which greatly extends the inbound part of our rebound attack. Therefore, the number of possible truncated differential trails for the rebound attack increases significantly than before. In previous rebound attacks on `AES-MMO` (including [39,30]), the authors place a 2-round full/non-full active Super-Sbox in the inbound phase, and extend it forwards and backwards in the outbound phases to find a useful rebound-attack trail. However, in our attack, we have to use 3-/4-round truncated differentials as the multiple inbound phases, which have many more choices than a 2-round Super-Sbox before. Moreover, not all those 3-/4-round inbound can be solved efficiently by triangulation algorithm (actually, it is time consuming to compute a compatible pair for most 3-/4-round truncated differentials), and we have to pick a good trail from many choices.

Therefore, as another contribution, we introduce a heuristic automatic tool (CP-based) to identify good trails for our rebound attacks, which succeeds to find the 7-/8-round collision attacks on `AES-MMO` and gains 1-round improvement.

We note that the multiple inbound phases technique was already introduced and used many times before, see for instance the rebound attacks on the `LANE` [59], `ECHO` [44], `JH` [68], etc. The additional degrees of freedom were in these cases given by bigger states. The truncated differences affect both the number of rounds covered by the inbound phases, and the differential probabilities of the outbound phases. In the previous works, the choices of these truncated differences are made in ad hoc ways. In our paper, we introduce triangulation algorithm into the rebound attack, as well as the automated search of configuration for the rebound attack with multiple inbound phases, which is the main novelty comparing to previous ad hoc ways.

2   Preliminaries

2.1   `AES`-like Hashing

`AES` [21] operates on a 4×4 column-major order array of bytes, whose round function contains four major transformations as illustrated in Figure 1: `SubBytes` (`SB`), `ShiftRows` (`SR`), `MixColumns` (`MC`), and `AddRoundKey` (`AK`). By making changes to the numbers of rows and columns, the sizes of the cells, the order of the transformations, one can produce new designs named as `AES`-like round functions. Usually, the `MixColumns` is to multiply an MDS matrix to each column of the state. An exception is `SKINNY` [7], which uses the non-MDS matrix.

By using (keyed) permutations with `AES`-like round functions in certain hashing modes (like `MD`, `MMO`, and `MP` hashing modes [64, Section 9.4] in Figure 2), compression functions (denoted as `CF`) can be constructed. Plugging such compression functions into the Merkle-Damgård construction [65,22], one arrives at `AES`-

Table 1: A summary of the results.

AES-128-MMO/MP

Target	Attack	Rounds	Time	C-Mem	qRAM	Setting	Ref.
Hash	Collision	5/10	2^{56}	2^4	0	Classic	[62]
		6/10	2^{56}	2^{32}	0	Classic	[35,56]
		7/10	$2^{42.50}$	0	2^{48}	Quantum	[39]
		7/10	$2^{59.5}$	0	0	Quantum	[39]
		7/10	$2^{45.8}$	0	0	Quantum	[30]
		8/10	$2^{55.53}$	0	0	Quantum	Sect. 4.3
	Preimage	7/10	2^{125}	2^8	-		[69]
		7/10	2^{123}	2^8	-	Classic	[3]
		8/10	2^{125}	2^8	-		[4]
Compression function	Semi-free	6/10	2^{24}	-	-	Classic	Sect. 4.2
	Semi-free	7/10	2^{56}	2^{16}	-	Classic	Sect. 4.1
	Semi-free	8/10	2^{34}	-	-	Quantum	Sect. 4.2
	any	any	2^{64}	-	-	any	[75,39,9]
	any	any	$2^{42.7}$	-	$2^{42.7}$	Quantum	[15]
	any	any	$2^{51.2}$	$2^{25.6}$	-	Quantum	[19]

Saturnin-Hash

Target	Attack	Rounds	Time	C-Mem	qRAM	Setting	Ref.
Hash	Collision	5/16	2^{64}	2^{66}	0	Classic	[31]
		7/16	$2^{113.5}$	-	-	Quantum	[31]
	Preimage	7/16	2^{232}	2^{48}	-	Classic	[29]
Compression function	Free-start	6/16	2^{80}	2^{66}	-	Classic	[31]
	Semi-free	7/16	$2^{90.1}$	-	-	Quantum	[31]
	Semi-free	7/16	2^{86}	-	-	Quantum	Full Ver. [28]
	Free-start	8/16	$2^{122.5}$	-	-	Quantum	[31]
	Free-start	8/16	$2^{89.65}$	-	-	Quantum	Sect. 5.1
	Free-start	10/16	$2^{127.2}$	-	-	Quantum	Sect. 5.2
	any	any	2^{128}	-	-	any	[75,39,9]
	any	any	$2^{85.3}$	-	$2^{85.3}$	Quantum	[15]
	any	any	$2^{102.4}$	$2^{51.2}$	-	Quantum	[19]

Grøstl-512

Target	Attack	Rounds	Time	C-Mem	qRAM	Setting	Ref.
Compression function	Distinguisher	10/14	2^{392}	2^{64}	-	Classic	[45]
		11/14	2^{72}	2^{56}	-	Classic	[18]
	Semi-free	6/14	2^{180}	2^{64}	-	Classic	[72]
		7/14	2^{214}	-	-	Quantum	Full Ver. [28]
Compression function v0	Semi-free	7/14	2^{152}	2^{64}	-	Classic	[61]
		7/14	2^{152}	2^{56}	-	Classic	[70]
		8/14	2^{244}	-	-	Quantum	Full Ver. [28]
	any	any	2^{256}	-	-	any	[75,39,9]
	any	any	$2^{170.7}$	-	$2^{170.7}$	Quantum	[15]
	any	any	$2^{204.8}$	$2^{102.4}$	-	Quantum	[19]

SKINNY-128-384-MMO/MP

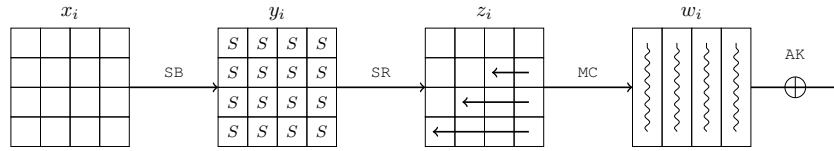| Target	Attack	Rounds	Time	C-Mem	qRAM	Setting	Ref.
Compression function	Free-start	16/56	$2^{59.8}$	-		Quantum	[31]
		19/56	$2^{51.2}$	-		Classic	Sect. 6.2
		21/56	$2^{46.2}$	-		Quantum	Sect. 6.1
	any	any	2^{64}	-	-	any	[75,39,9]
	any	any	$2^{42.7}$	-	$2^{42.7}$	Quantum	[15]
	any	any	$2^{51.2}$	$2^{25.6}$	-	Quantum	[19]

Fig. 1: The round function of `AES`

like hashings. Concrete designs include `AES-128-MMO`, `AES-128-MP`, `Saturnin-Hash` [17], `Whirlpool` [6] `Grøstl` [34].

(`DM`) Davies-Meyer   (`MMO`) Matyas-Meyer-Oseas   (`MP`) Miyaguchi-Preneel
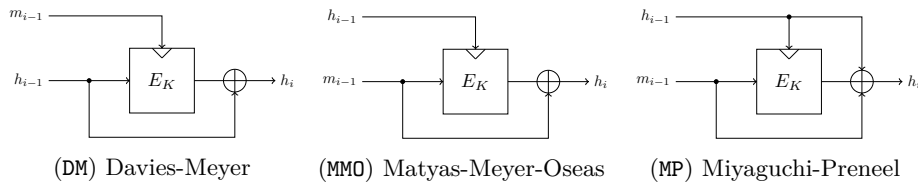
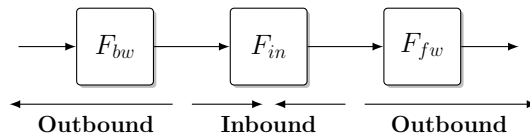Fig. 2: Common Hashing Modes

2.2   The Rebound Attack

Fig. 3: The Rebound Attack

The rebound attack was first introduced by Mendel et al. in [62]. It consists of an inbound phase and an outbound phase as shown in Figure 3, where F is an internal block cipher or permutation which is split into three subparts, then $F = F_{fw} \circ F_{in} \circ F_{bw}$.

– **Inbound phase**. In the inbound phase, the attackers efficiently fulfill the low probability part in the middle of the differential trail with a meet-in-the-middle technique. The degree of freedom is the number of matched pairs in the inbound phase, which will act as the starting point for the outbound phase.
– **Outbound phase**. In the outbound phase, the matched values of the inbound phase, i.e., starting points, are computed backward and forward through F_{bw} and F_{fw} to obtain a pair of values which satisfy the outbound differential trail in a bruteforce fashion.

Overall, the rebound attack is essentially a technique to efficiently generate a message pair fulfilling the inbound phase, which utilizes a truncated differential rather than a single differential characteristic. Suppose the probability of the outbound phase is p, then we have to prepare $1/p$ starting points in the inbound phase to expect one pair conforming to the differential trail of the outbound phase. Hence, the degree of freedom should be larger than $1/p$.

2.3   The Super-Sbox Technique

The Super-Sbox technique proposed by Gilbert and Peyrin [35] and Lamberger et al. [56] extends Mendel et al.'s [62] inbound part into 2 S-box layers by regarding them as four Super-Sboxes as shown in Figure 4 (a). In [70], Sasaki et al. further reduced the the memory complexity by considering non-full-active Super-Sboxes as shown in Figure 4 (b). In both the two techniques, AK acts as a constant addition operation and does not provide any degree of freedom.
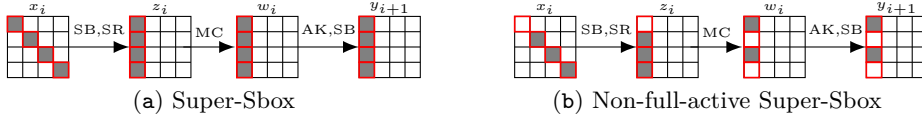

(a) Super-Sbox   (b) Non-full-active Super-Sbox

Fig. 4: The Two-Round Differential

Super-Sbox Technique. For the jth Super-Sbox $\texttt{SSB}_j$ and given input difference $\Delta x_i^{(j)}$ ($j = 0$ in Figure 4 (a)), compute $\Delta y_{i+1}^{(j)} = \texttt{SSB}_j(x \oplus \Delta x_i^{(j)}) \oplus \texttt{SSB}_j(x)$ for $x \in \mathbb{F}^{32}$. Store the pair $(x, x \oplus \Delta x_i^{(j)})$ in a table $\mathbb{L}^{(j)}[\Delta y_{i+1}^{(j)}]$. Given $\Delta y_{i+1}^{(j)}$, we find a pair conforming the two-round differential with $(\Delta x_i^{(j)}, \Delta y_{i+1}^{(j)})$ by assessing $\mathbb{L}^{(j)}[\Delta y_{i+1}^{(j)}]$. The memory cost is about 2^{32}.

Non-full-active Super-Sbox. In Figure 4 (b), the Property 1 of MDS in MC is used. Look at $\Delta w_i = \texttt{MC}(\Delta z_i)$, by guessing the differences of one active byte, we can determine other differences according to Property 1. Then, for a fixed input-output differences $(\Delta x_i^{(j)}, \Delta y_{i+1}^{(j)})$ of $\texttt{SSB}_j$, we deduce all the input-output differences for the active cells of two S-box layers for each guess and then deduce their values by accessing the differential distribution table (DDT) of the S-box.

Property 1. Suppose MC is an $n \times n$ MDS matrix and $\texttt{MC} \cdot (z[1], \dots, z[n])^T = (w[1], \dots, w[n])^T$, the knowledge of any n out of $2n$ bytes of (z, w) is necessary and sufficient to determine the rest. (z, w) here can be either value or difference.

2.4   Triangulation Algorithm

At CT-RSA 2009, Khovratovich, Biryukov, and Nikolić [50] introduced the triangulation algorithm tool, an efficient Gaussian-based-algorithm to solve system

of bijective equations, to automatically detect a way to solve the nonlinear system. The algorithm models an `AES`-like block cipher as a system of key schedule and round function equations, and the state bytes and key bytes as variables. At first, the system is determined by n initial state bytes and k initial key bytes. Therefore, when m bytes are fixed, the algorithm will return $n + k - m$ "free variables" to form a basis of the system. The algorithm can output exactly the "free variables" when the system is deterministically solvable, and improve the guessing and checking the "free variables" when the system is probabilistically solvable. The idea of triangulation algorithm is roughly described following.

1. Construct a system of equations whose variables are the bytes. The predefined values are fixed to constants.
2. All variables and equations are marked as non-processed.
3. Mark the variable which is involved in only one non-processed equation as processed. Also mark this equation as processed. If no such variable exist, exit.
4. Return to Step 3 if still have non-processed equations.
5. Return all non-processed variables as "free variables".

After the "free variables" are identified, we randomly assign values for them and deduce a solution for the whole nonlinear system. For details, please refer to [50].

2.5   Collision Attacks and Its Variants

Given a hash function H, a standard collision message pair (m, m') satisfies $H(IV, m) = H(IV, m')$, where the initial vector IV is a fixed inital value. A *semi-free-start collision* is to find a pair (u, m) and (u, m'), such that $H(u, m) = H(u, m')$ $(u \neq IV)$. A *free-start collision* is to find a pair (v, m) and (v', m'), so that $H(v, m) = H(v', m')$ $(v \neq v')$. When the hash function H is built by iterating the compression function (`CF`) with Merkle-Damgård construction, we can similarly define the semi-free-start collision and free-start collision attack on the compression function. Taking the `MMO` and `MP` hashing modes in Figure 2 as examples, when considering the semi-free-start or free-start collision attack, the attackers can explore the degrees of freedom from the chaining value h_{i-1} through the key shedule algorithm, which may lead to better attacks such as [56,71]. We would like to emphasize the importance of semi-free-start and free-start collision attacks: The Merkle-Damgård security reduction assumes that any type of collision for the compression function should be intractable for the attacker, including semi-free-start and free-start collisions.

3   Triangulating Rebound Attack

3.1   Solving Non-Full-Active Super-Sbox by Key Bytes

In previous collision attacks, both Super-Sbox [56,35] and non-full-active Super-Sbox [70] techniques, the subkeys are prefixed in advance (as shown in Section

2.3). In other words, the degree of freedom of the subkeys was not be utilized. We observe that, by carefully choosing the values of subkeys, state pairs conforming the given differential can be obtained without additional computation only at a cost of memory for storing the differential distribution table (DDT). This observation enables utilization of the degrees of freedom from the corresponding subkeys and leads to collision attacks without the need of building large lists of compatible pairs for Super-Sboxes. Here we describe this new Super-Sbox technique through a concrete example in Figure 5.
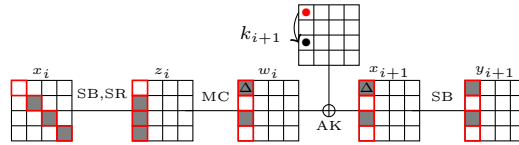

Fig. 5: An example of Super-Sbox with degrees of freedom from the subkey

Given the pair of non-full difference of $(\Delta x_i, \Delta y_{i+1})$ and zero difference in key of an Super-Sbox (marked as red for the corresponding column), we are to generate state pairs and corresponding subkey k_{i+1} conforming to that difference. First, we randomly assign a difference Δ compatible with $\Delta y_{i+1}[0]$ to $\Delta x_{i+1}[0]$ and asset to DDT to get an actual value for $x_{i+1}[0]$ conforming the difference $(\Delta, \Delta y_{i+1}[0])$. Since there is no difference in the subkey k_{i+1}, the difference $\Delta w_i[0]$ is equal to Δ as well. We use a simple property of maximum distance separable (MDS) matrix to determine all the rest bytes of Δw_i and Δz_i.

With the knowledge of 4 bytes of Δz_i and Δw_i ($\Delta z_i[0] = \Delta w_i[1,3] = 0$ and $\Delta w_i[0] = \Delta$), all bytes of Δz_i and Δw_i are uniquely determined, and so for the entire differential characteristic $(\Delta x_i, \Delta z_i, \Delta w_i, \Delta x_{i+1}, \Delta y_{i+1})$ of the Super-Sbox. By looking up DDT, the byte values before and after the active Sboxes, e.g., $x_i[5, 10, 15], z_i[1, 2, 3], x_{i+1}[0, 2]$ and $y_{i+1}[0, 2]$, are determined as well. Next, we randomly assign a value to $k_{i+1}[0]$, then $w_i[0]$ can be calculated as $w_i[0] = x_{i+1}[0] \oplus k_{i+1}[0]$. Again, Property 1 allows to determine the remaining bytes $z_i[0]$ and $w_i[1, 2, 3]$ since 4 bytes $z_i[1, 2, 3]$ and $w_i[0]$ are known. Finally, the key byte $k_{i+1}[2]$ is determined as $k_{i+1}[2] = w_i[2] \oplus x_{i+1}[2]$. In summary, after randomly assigning (compatible) values to $\Delta x_{i+1}[0]$ and $k_{i+1}[0]$, all the values and differences of the active bytes in the state as well as the subkey bytes corresponding to the active state bytes positions of AK are determined, with time complexity 1.

3.2   Connecting Multiple Inbound Phases by Key Bytes

In this section, we present a 3-round inbound phase by extending the Super-Sbox backward by one round. Since the Super-Sbox and the extended round are not solved in two inbound phases, this technique is also referred to as "multiple inbound phases". These phases are connected by free bytes of the corresponding subkeys, and so one has to ensure the value assignments to the subkeys from different rounds are not over-defined through the key schedule algorithm. This

new technique is illustrated through an example of 3-round truncated differential trail depicted in Figure 6.
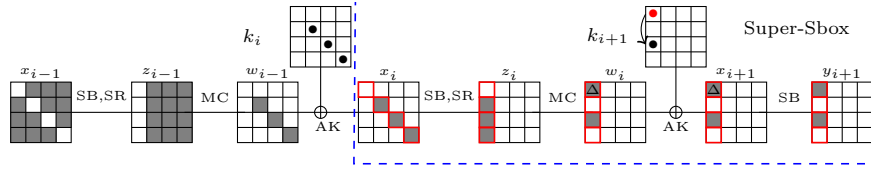

Fig. 6: An example of 3-round multiple inbound phases

Given the differences Δx_{i-1}, Δx_i, Δy_{i+1} and zero difference in key, we are to generate state and key pairs conforming the differential path with low memory requirement by utilizing the degrees of freedom from subkeys k_i and k_{i+1}. First, applying the Super-Sbox technique presented above to the given Δx_i and Δy_{i+1}, both value and difference of $\Delta x_i[5, 10, 15]$ and value of $k_{i+1}[0, 2]$ will be determined. Then, with fixed Δx_{i-1} and Δx_i, standard inbound phase of the original rebound attack can be applied, i.e., lookup `DDT` with input/output differences $(\Delta x_{i-1}, \mathtt{SR}^{-1}(\Delta z_{i-1} = \mathtt{MC}^{-1}(\Delta x_i)))$, values of the last three columns of z_{i-1} and so w_{i-1} are determined. Hence, $\Delta k_i[5, 10, 15]$ can be calculated as $\Delta w_{i-1}[5, 10, 15] \oplus \Delta x_i[5, 10, 15]$. Note, there is no direct implication between any assignment of $k_i[5, 10, 15]$ and $k_{i+1}[0, 2]$ through the AES key schedule. The whole process costs 17 `DDT` lookups and memory hosting the `DDT` lookup table.

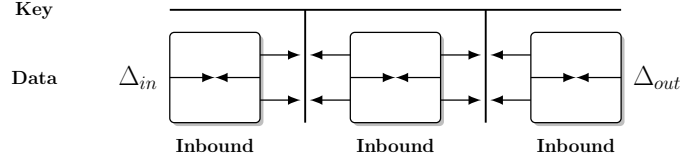### 3.3 The Triangulating Rebound Attack

Fig. 7: *Super-Inbound*: bridging multiple local inbound parts

Generally speaking, the development of the rebound attack can be viewed as a continuous process to find as many degrees of freedom (`DoF`) as possible, to either reduce the overall inbound solving complexity, or to extend for more rounds. As shown in Figure 7, the multiple local inbound parts form a *Super-Inbound*. Given the input/output differences $(\Delta_{in}, \Delta_{out})$ of the super inbound phase, bridging the multiple local inbound parts is to find the conforming pair for $(\Delta_{in}, \Delta_{out})$, while the exact differentials and values of the internal rounds are up to attacker's control. Usually, $(\Delta_{in}, \Delta_{out})$ are chosen from a truncated differential in the rebound attack. In summary, the source of `DoF` for the whole rebound attack can come from the choices of

S1 truncated differential of the internal rounds of the *Super-Inbound*,
S2 input/output differences and values of the active Sboxes of the state,
S3 values of the inactive Sboxes of the state,
S4 differences of key bytes,
S5 values of key bytes.

The differences and values of the active Sboxes of the state are listed together, as once differences are fixed, values can be derived from `DDT`, and vice versa. This is not the case for key bytes, as it is not necessarily for each of the key bytes to pass through Sbox depending on the key schedule algorithm. Then the Super-Sbox technique can be viewed as a way to connect two 1-round inbound parts by fully utilizing the `DoF` from S2, i.e., the output differences of first round and input differences of the second round. Our techniques presented in Section 3.1, 3.2 is from (S2,S5), (S2,S5), respectively. Under this generic view, connecting multiple inbound parts to form a *Super-Inbound* can be seen as a system of nonlinear equations, where all `DoF` serve as variables and `DoF` from subkeys are constrained by key schedule, and our goal is to find a procedure to solve this system, efficiently. There are two main considerations.

 I. The system of nonlinear equations must have solutions. In other words, the system should not be over-defined. Hence, the number `DoF` should be larger than the number of equations.
II. The system must be solvable efficiently, which for most of the time can be translated into a procedure of consuming degrees of freedom by fixing certain variables involved, and the final complexity is determined by the number variables which can not be fixed and have to be brute-forced.

We adopt the triangulation algorithm [50] as shown in Section 2.4 to solve the nonlinear system and bridge the multiple inbound parts. The algorithm can output exactly the "free variables" when the system is deterministically solvable, and improve the guessing and checking the "free variables" when the system is probabilistically solvable. We name this method as *triangulating rebound attack*.

Heuristic Method to Determine a Trail. So far, we have not mentioned the use of `DoF` from S1 and S4 yet. While S4 determines whether difference is allowed in key hence resulting in semi-free-start or free-start collision for attacks against hash functions, S1 directly determines the system of nonlinear equations, hence the number of rounds covered by the *Super-Inbound* and complexity as well. For collision attack on *n*-bit hash, the overall steps to identify a rebound attack trail are as follows:

1. Find a truncated differential trail following the *Constrained Programming* based search model from [5].
2. Choose certain consecutive rounds as the *Super-Inbound* (i.e., S1), which includes several local inbound parts. For example, in Figure 7 a *Super-Inbound* includes three local inbound parts. Usually, 2 to 4 consecutive rounds are

chosen as possible *Super-Inbound*. Following this search, all the input/output differences of the active Sboxes are fixed, and therefore corresponding values are fixed.

3. Build equation system that connects the inbound parts and through key schedule. Check if the system is solvable by triangulation algorithm.

Before checking the solvability of the system, a potential truncated trail must satisfy the condition that the number of active Sboxes in the *Super-Inbound* does not exceed the `DoF` from both key and state. This is because the system of state bytes and keys bytes are determined by the initial k key bytes and n plaintext bytes, then for any set of fixed bytes in advance, the system is potentially solvable if the number of fixed bytes is not larger than $n + k$. Our observation shows that the ideal case is when the number of active bytes in the *Super-Inbound* equals to $n + k - 1$ and only 1 free byte is left, since the scenario without free byte is likely to form an over-defined system. Therefore, this constraint is also added to our model. For classical collision attacks, after the *Super-Inbound* is fixed, the probability of the outbound phase can be computed and denoted as Pr, which should meet $Pr > 2^{-n/2}$. For quantum collision attacks, let $Pr^5 > 2^{-n}$. Similarly to previous models [39], for the attacks based on the truncated differentials, P_r is the probability for the cancellation of `MC` operator in the backward and forward trunks, as well as the cancellation for the feed-forward operator of hashing modes. After checking the potential truncated trails satisfying the above conditions for inbound and outbound phases, the triangulation algorithm is applied to detect the free bytes left followed by the step to generate sufficient data pairs for the outbound phase.

4   Improved Collision Attacks on `AES-128-MMO`

In this section, by applying the *triangulating rebound*, we introduce a 7-round classical semi-free-start collision attack and an 8-round quantum semi-free-start collision attack on `AES-MMO`. Moreover, we identify the first 8-round quantum collision attack, which is better than parallel rho's algorithm [75].

4.1   Semi-Free-Start Collision Attack on 7-Round `AES-128`

New 7-round `AES-128` Truncated Differential Trail. We introduce here our new differential trail in Figure 8. Our Super-Inbound phase covers 3 middle rounds with 2 inbound phases (marked with red and blue dashed lines), which consumes 31 bytes of degree of freedom for the fixed state bytes, i.e., 16 active bytes in x_3, 9 active bytes in x_4, and 6 active bytes in x_5. Since we have 32 bytes in total (16 bytes in key and 16 bytes in the state), only 1 free byte is left. By triangulating algorithm, the 1 free byte is identified to be $k_5[12]$. The

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[5]As shown in the introduction, we consider three quantum attack conditions. $Pr > 2^{-2n/3}$ is to be better than the BHT algorithm, $Pr > 2^{-n}$ is to be better than quantum time-space tradeoff, $Pr > 2^{-4n/5}$ is to be better than CNS algorithm. So we let $Pr > 2^{-n}$ to keep all characteristics that may lead to possible attacks.

remaining outbound phase happens with probability $p_{out} = 2^{-56}$ including the MC cancellation in round 1 and 4-byte cancellation for $\Delta P = \Delta C$.
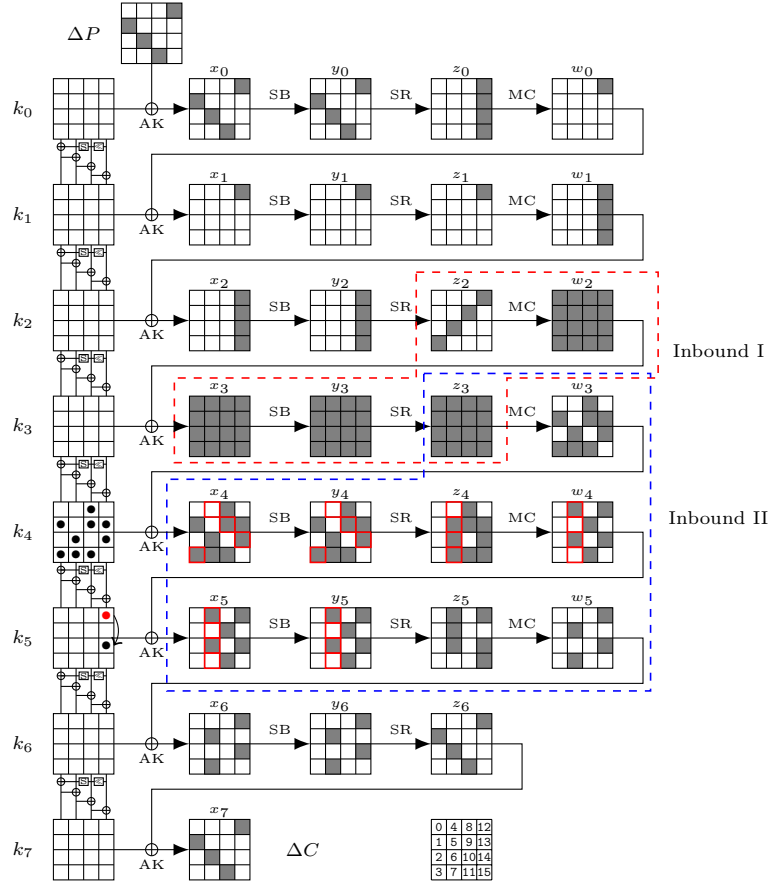

Fig. 8: Semi-free-start collision attack on 7-round AES-128

The Super-Inbound Phase. As shown in Figure 8, the Super-Inbound phase divides into 2 parts, the Inbound II marked with blue dashed line makes use of k_5 to link round 4 and round 5, and the Inbound I marked with red dashed line connects with Inbound II via k_4. To generate a data pair conforming a given difference $(\Delta z_2, \Delta w_3, \Delta w_5)$, assuming $\Delta Z_5 = \text{MC}^{-1}(\Delta w_5)$ keeps the truncated differential in Figure 8, we perform the following steps:

– Perform the Inbound II:
   1. Randomly assign a difference to $\Delta x_5[4, 9, 12]$. Then $\Delta w_4[4, 9, 12]$ are known accordingly, implies all the differences at these active cells of

z_4 and the remaining active cells of w_4 are determined as a result of the Property 1. From Δz_4, we deduce $\Delta y_4 = \mathtt{SR}^{-1}(\Delta z_4)$.

2. Compute the difference $\Delta y_5 = \mathtt{SR}^{-1}(\mathtt{MC}^{-1}(\Delta w_5))$. Next, we deduce the values of the active cells at x_4 and x_5 by assessing to the $\mathtt{DDT}$ for each cell of $(\Delta x_4, \Delta y_4)$ and $(\Delta x_5, \Delta y_5)$. The active byte values of y_4 and y_5 are determined respectively.
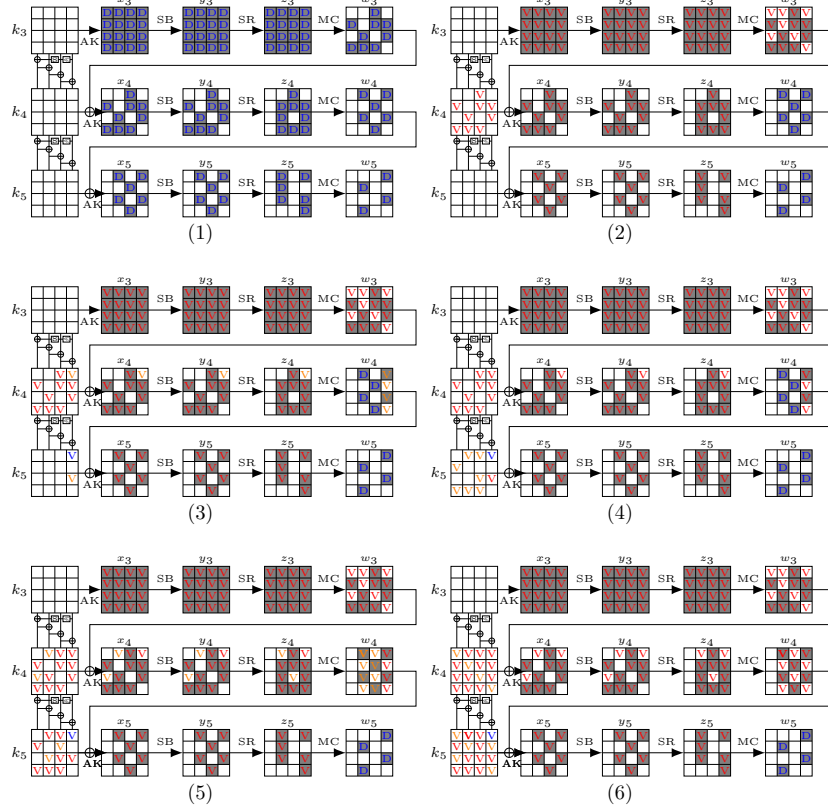

Fig. 9: Steps to recover k_5. The bytes marked by 'D' mean that the differences are known. The bytes marked by 'V' mean that the values are known. In subfigures, the bytes marked by 'V' are deduced from those marked by 'V'.

- Perform the Inbound I:
    1. Compute $\Delta x_3 = \mathtt{MC}(\Delta z_2)$ and $\Delta y_3 = \mathtt{SR}^{-1}(\mathtt{MC}^{-1}(\Delta w_3))$.
    2. Deduce full state values x_3 and y_3 by assessing the $\mathtt{DDT}$. We compute the full state values of w_3 by the action $w_3 = \mathtt{MC}(\mathtt{SR}(y_3))$.

- Connect the phases by the subkeys:

1. Since the full state of w_3 and the active bytes of x_4 are known, we obtain $k_4[1,3,6,7,8,9,11,13,14] = x_4[1,3,6,7,8,9,11,13,14] \oplus w_3[1,3,6,7,8,9,11,13,14]$ (marked as black dots).
2. Add constrains on the related cells of k_5 to the system of equations of the subkeys. We solve the constrains on k_4 and k_5 by triangulation algorithm and obtain a free byte $k_5[12]$.
3. The steps (shown in Figure 9) to recover k_5 and compute a starting point are shown as follows (equations are also given in Table 2) :
   (a) As shown in Figure 9(1), all the differences marked by 'D' are known. In Figure 9(2), by assessing the DDT, we know the values of $x_3[V]$, $x_4[V]$, and $x_5[V]$. Then deduce $k_4[V] = w_3[V] \oplus x_4[V]$.
   (b) In Figure 9(3), randomly assign a value for $k_5[12]$ marked by 'V', compute $w_4[12] = k_5[12] \oplus x_5[12]$. According to Property 1, deduce all the bytes marked by 'V' in z_4 and w_4. Then, $k_5[14] = x_5[14] \oplus w_4[14]$. From $z_4[V]$, we get $x_4[V]$. Then $k_4[V]$ is deduced.
   (c) In Figure 9(4), following the key schedule of AES-128, compute the $k_5[V]$: $k_5[8] = k_4[12] \oplus k_5[12]$, $k_5[10] = k_4[14] \oplus k_5[14]$, $k_5[11] = k_4[11] \oplus k_5[7]$, $k_5[4] = k_4[8] \oplus k_5[8]$, $k_5[7] = k_5[3] \oplus k_4[7]$, $k_5[1] = k_4[1] \oplus \mathtt{SB}(k_4[14]) \oplus r_c$, $k_5[3] = k_4[3] \oplus \mathtt{SB}(k_4[12]) \oplus r_c$.
   (d) In Figure 9(5), deduce $w_4[4] = k_5[4] \oplus x_5[4]$ and $w_4[11] = k_5[11] \oplus x_5[11]$, then acorrding to Property 1, all the bytes in w_4 and z_4 marked by 'V' are deduced. Then, two bytes marked by 'V' in k_5 are deduced, i.e., $k_5[6] = w_4[6] \oplus x_5[6]$ and $k_5[9] = w_4[9] \oplus x_5[9]$. Moreover, compute two bytes marked by 'V' in k_4, i.e., $k_4[2] = x_4[2] \oplus w_3[2]$ and $k_4[4] = x_4[4] \oplus w_3[4]$.
   (e) In Figure 9(6), following the key schedule, deduce the bytes marked by 'V' in k_5, i.e., $k_5[0] = k_4[4] \oplus k_5[4]$, $k_5[2] = k_4[6] \oplus k_5[6]$, $k_5[5] = k_4[9] \oplus k_5[9]$, $k_5[13] = k_4[13] \oplus k_5[9]$, $k_4[15] = \mathtt{SB}^{-1}(k_4[2] \oplus k_5[2])$, $k_5[15] = k_4[15] \oplus k_5[11]$.
4. Finally, full state w_3 and full subkey k_5 are obtained by using 34 DDT assesses.

Due to the probability of the outbound phase is 2^{-56}, we need to check 2^{56} starting points to find the collision. The final complexity is 2^{56} and memory needed is 2^{16} for DDT storing.

Remark. In a rebound attack [62], we typically have a probability $\frac{1}{2}$ to have a solution for each active SBox, so that we have to repeat the attack with new differences, but when the differences are compatible we obtain many solutions and the amortized cost is close to 1. In our attack, when $(\Delta x_3, \Delta y_3)$, $(\Delta x_4, \Delta y_4)$ and $(\Delta x_5, \Delta y_5)$ are fixed, we have 31 active Sboxes. Therefore, one compatible differential characteristic is found in roughly 2^{31} time, which leads to 2^{31} solutions for the active bytes of (x_3, x_4, x_5). Moreover, since we can randomly assign $k_5[12]$ (Step 3 (b)), we totally have about $2^{31+8} = 2^{39}$ starting points, which is lower than the final time 2^{56}. Briefly, for a given $(\Delta x_3, \Delta y_3)$, $(\Delta x_4, \Delta y_4)$, $(\Delta x_5, \Delta y_5)$ and $k_5[12]$, we find one starting point with time complexity of 1 on average.

1. $w_4[12] = x_5[12] \oplus k_5[12]$	15. $k_5[7] = k_5[3] \oplus k_4[7]$
2. $z_4[12], w_4[13,14,15] = \text{MC}(w_4[12], z_4[13,14,15])$	16. $k_5[11] = k_4[11] \oplus k_5[7]$
3. $k_5[14] = w_4[14] \oplus x_5[14]$	17. $w_4[11] = k_5[11] \oplus x_5[11]$
4. $x_4[12] = \text{SB}^{-1}(z_4[12])$	18. $z_4[10], w_4[8,9,10] = \text{MC}(z_4[8,9,11], w_4[11])$
5. $k_4[12] = w_3[12] \oplus x_4[12]$	19. $k_5[9] = w_4[9] \oplus x_5[9]$
6. $k_5[8] = k_4[12] \oplus k_5[12]$	20. $k_5[13] = k_5[9] \oplus k_4[13]$
7. $k_5[1] = k_4[1] \oplus \text{SB}(k_4[14]) \oplus r_c$	21. $k_5[5] = k_4[9] \oplus k_5[9]$
8. $k_5[10] = k_4[14] \oplus k_5[14]$	22. $k_4[5] = k_5[1] \oplus k_5[5]$
9. $k_5[4] = k_4[8] \oplus k_5[8]$	23. $k_4[4] = w_3[4] \oplus \text{SB}^{-1}(z_4[4])$
10. $w_4[4] = k_5[4] \oplus x_5[4]$	24. $k_5[0] = k_4[4] \oplus k_5[4]$
11. $z_4[4], w_4[5,6,7] = \text{MC}^{-1}(w_4[4], z_4[5,6,7])$	25. $x_4[2] = \text{SB}^{-1}(z_4[10])$
12. $k_5[6] = w_4[6] \oplus x_5[6]$	26. $k_4[2] = w_3[2] \oplus x_4[2]$
13. $k_5[2] = k_4[6] \oplus k_5[6]$	27. $k_4[15] = \text{SB}^{-1}(k_4[2] \oplus k_5[2])$
14. $k_5[3] = k_4[3] \oplus \text{SB}(k_4[12]) \oplus r_c$	28. $k_5[15] = k_5[11] \oplus k_4[15]$

Table 2: Steps to recover the subkey k_5 from known bytes

4.2 Semi-Free-Start Collision Attack on 8-Round `AES-128-MMO`

As shown in Figure 10, there are three inbound parts in the Super-Inbound phase. For given fixed differences of Δz_1, Δz_2, Δw_4, and Δw_5, assuming $\Delta w_1 = \text{MC}(\Delta z_1)$ and $\Delta z_5 = \text{MC}^{-1}(\Delta w_5)$ keep the truncated differential of Figure 10, we perform inbound part I, II, and III without the key information. Then, we connect the three inbound part by determining certain key values and get the starting point for the outbound phase. The probability of the outbound phase is 2^{-64}, including the condition $\Delta P = \Delta C$.

Given fixed differences of Δz_1, Δz_2, Δw_4, and Δw_5, the procedures to get one starting point are as follows:

1. Deduce values $(x_5[0,5,10], x_5'[0,5,10])$ with Δw_4 and $\Delta z_5 = \text{MC}^{-1}(\Delta w_5)$ by accessing the `DDT`.
2. Deduce values $(x_2[0,1,2], x_2'[0,1,2])$ with $\Delta w_1 = \text{MC}(\Delta z_1)$ and Δz_2 by accessing the `DDT`. Then $z_2[0,10,13]$ are known as well.
3. For $\text{SSB}^{(0)}$ in Inbound III (marked in red), we compute $\Delta x_3 = \text{MC}(\Delta z_2)$ and $\Delta y_4 = \text{SR}^{-1}(\text{MC}^{-1}(\Delta w_4))$.
4. Randomly choose $\Delta w_3[0,2] \in \mathbb{F}_2^8 \times \mathbb{F}_2^8$,
   (a) Compute other active differences $\Delta z_3[0,2,3]$ and $\Delta w_3[3]$ by Property 1
   (b) Deduce $(z_3[0,2,3], z_3'[0,2,3])$ and $(x_4[0,2,3], x_4'[0,2,3])$ by assessing `DDT`
5. Repeat Step 4 for random differences $\Delta w_3[4,5], \Delta w_3[8,9], \Delta w_3[13,14]$ to compute all active bytes of x_3 and x_4. Next, we compute $w_4[0,5,10] = \text{MC}(\text{SR}(\text{SB}(x_4)))[0,5,10]$ and then $k_5[0,5,10] = w_4[0,5,10] \oplus x_5[0,5,10]$.
6. Assign random values to $k_3[6]$, $k_4[0]$, and $k_4[4]$. We recover the full key k_5 by the following steps in Table 3. There exists a filter of 2^{-8} in the step to recover $k_3[3]$.

Since Step 6 has a filter of 2^{-8} to meet the condition "$k_3[3] = w_2[3] \oplus x_3[3] \stackrel{?}{=} \text{SB}(k_3[12]) \oplus k_4[3] \oplus r_c$" in Table 3, the time complexity to find a starting point
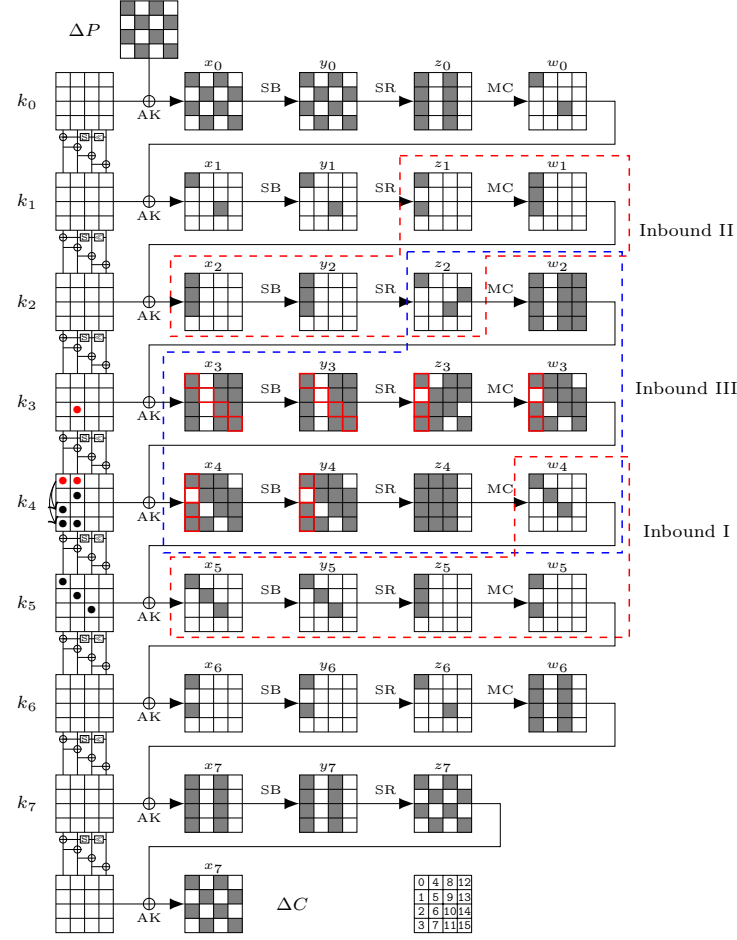
18   Xiaoyang Dong, Jian Guo, Shun Li, Phuong Pham

Fig. 10: Quantum semi-free-start collision attack on 8-round `AES-128`

(including a data pair and a key value conforming the inbound parts from Δz_1 to Δw_5) is 2^8. The probablity of the outbound phase is 2^{-64}, we need 2^{64} starting points to find one collision. In other words, given 21 bytes of Δz_1, Δz_2, Δw_4, Δw_5, $\Delta w_3[0,2]$, $\Delta w_3[4,5]$, $\Delta w_3[8,9]$, $\Delta w_3[13,14]$ and $k_3[6]$ $k_4[0]$, $k_4[4]$, we get one collision with probability of 2^{-72}. Since $21 \times 8 = 168 > 72$, we have enough degrees of freedom to find one collision.

The Quantum Attack. Given input-output differences of the active Sbox, it is expected to find one pair on average, e.g. (x, x'), satisfying the differences. In inbound phase of the quantum rebound attacks, given a valid input-output differences of l active Sboxes, we obtain about $2^l/2$ choices for starting points. To indicate which starting point to choose among $2^l/2$ choices, Hosoyamda and

1. $w_3[0] = k_4[0] \oplus x_4[0]$	23. $k_3[10] = k_4[6] \oplus k_4[10]$
2. $w_3[2,3] = \texttt{MC}^{-1}(w_3[0], z_3[0,2,3])$	24. $k_5[4] = k_5[0] \oplus k_4[4]$
3. $k_4[2,3] = w_3[2,3] \oplus x_4[2,3]$	25. $w_3[10] = x_4[10] \oplus k_4[10]$
4. $k_4[13] = \texttt{SB}^{-1}(k_4[0] \oplus k_5[0] \oplus r_c)$	26. $w_3[8,9] = \texttt{MC}^{-1}(w_3[10], z_3[8,9,10])$
5. $w_3[13] = x_4[13] \oplus k_4[13]$	27. $k_4[8,9] = w_3[8,9] \oplus x_4[8,9]$
6. $w_3[14,15] = \texttt{MC}^{-1}(w_3[13], z_3[12,13,15])$	28. $k_3[9] = k_4[5] \oplus k_4[9]$
7. $k_4[14,15] = w_3[14,15] \oplus x_4[14,15]$	29. $k_3[8] = k_4[4] \oplus k_4[8]$
8. $k_5[14] = k_5[10] \oplus k_4[14]$	30. $w_2[8,9,10] = x_3[8,9,10] \oplus k_3[8,9,10]$
9. $k_4[6] = k_4[2] \oplus k_3[6]$	31. $w_2[11] = \texttt{MC}^{-1}(w_2[8,9,10], z_2[10])$
10. $w_3[4] = x_4[4] \oplus k_4[4]$	32. $k_3[11] = w_2[11] \oplus x_3[11]$
11. $w_3[5,7] = \texttt{MC}^{-1}(w_3[4], z_3[5,6,7])$	33. $k_4[11] = k_4[7] \oplus k_3[11]$
12. $k_4[5,7] = w_3[5,7] \oplus x_4[5,7]$	34. $k_3[13] = k_4[9] \oplus k_4[13]$
13. $k_3[4] = k_4[0] \oplus k_4[4]$	35. $k_3[15] = k_4[11] \oplus k_4[15]$
14. $k_3[7] = k_4[3] \oplus k_4[7]$	36. $w_2[13,14,15] = k_3[13,14,15] \oplus x_3[13,14,15]$
15. $k_5[1] = k_4[5] \oplus k_5[5]$	37. $w_2[12] = \texttt{MC}^{-1}(w_2[13,14,15], z_2[13])$
16. $k_4[1] = k_5[1] \oplus \texttt{SB}(k_4[14]) \oplus r_c$	38. $k_3[12] = x_3[12] \oplus w_2[12]$
17. $k_3[5] = k_4[5] \oplus k_4[1]$	39. $k_3[0] = \texttt{SB}(k_3[13]) \oplus k_4[0] \oplus r_c$
18. $k_5[2] = k_4[2] \oplus \texttt{SB}(k_4[15]) \oplus r_c$	40. $k_3[2] = \texttt{SB}(k_3[15]) \oplus k_4[2] \oplus r_c$
19. $k_5[6] = k_5[2] \oplus k_4[6]$	41. $w_2[0,1,2] = x_3[0,1,2] \oplus k_3[0,1,2]$
20. $k_4[10] = k_5[6] \oplus k_5[10]$	42. $w_2[3] = \texttt{MC}^{-1}(w_2[0,1,2], z_2[0])$
21. $k_3[14] = k_4[10] \oplus k_4[14]$	43. $k_3[3] = w_2[3] \oplus x_3[3] \overset{?}{=} \texttt{SB}(k_3[12]) \oplus k_4[3] \oplus r_c$
22. $k_3[1] = \texttt{SB}(k_3[14]) \oplus k_4[1] \oplus r_c$	

Table 3: Steps to recover the subkey k_3 from known bytes

Sasaki [39, Sect. 6.2, Page 22] introduce $l-1$ auxiliary qubits. In Hosoyamda and Sasaki's attack, l is usually small, e.g. $l = 4$, then the increased time complexity due to the $l - 1$ auxiliary qubits is marginal. However, in our 8-round attack of Figure 10, we have to compute pairs for $l = 3+3+12+12 = 30$ active Sboxes with given input-output differences. If we follow Hosoyamda and Sasaki's idea [39], we have to introduce $l-1 = 29$ auxiliary qubits, which can not be ignored anymore. To deal with the problem, we introduce a trick in Supplementary Material A in our full version [28], which introduces the auxiliary qubits nearly for free. By our new trick, we perform the 8-round quantum semi-free-start collision attack. According to Equation 5 in [28], we have $l = 3 + 3 + 12 + 12 = 30$, and choose 9 bytes out of the 21 bytes to act as "$|\Delta_{in}| + |\Delta_{out}|$" to find the collision with time complexity roughly $30 \cdot 2^{72/2}/128 \approx 2^{34}$ 8-round $\texttt{AES}$, which is roughly the square root of 2^{72}.

Practical Semi-Free-Start Collision Attack on 6-Round $\texttt{AES-128}$. To prove the correctness of the 8-round semi-free-start collision, we give a 6-round practical semi-free-start collision by cutting the differential characteristic depicted in Figure 10 to 6 rounds (round 1 to 6). The total complexity to find the collision is 2^{24}, including 2^8 time for solving the Super-Inbound phase and 2^{16} for the condition $\Delta x_1 = \Delta z_6$. We have verified the validity and complexity of the attack by implementing the 6-round attack. One such example collision is presented in Table 4 in Supplementary Material of the full version [28].

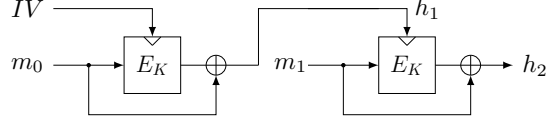### 4.3 Quantum Collision Attack on 8-Round `AES`-128

Fig. 11: Matyas-Meyer-Oseas (MMO) hashing mode with two blocks

Remove the Inbound I/II from Figure 10, we get the trail for 8-round collision attack (also shown in Figure 15 in [28]). The inbound phase covers states z_2 to w_4. The probability of the outbound phase is 2^{-96}. For a random given Δz_2, Δw_4 and the master key, the steps to get one starting point are as follows:

1. Deduce Δx_3 and Δy_4 from Δz_2 and Δw_4.
2. For $\texttt{SSB}^{(0)}$ marked in red, randomly choose $\Delta w_3[0, 2] \in \mathbb{F}_2^{16}$,
   (a) Compute other active differences $\Delta z_3[0, 2, 3]$ and $\Delta w_3[3]$ by Property 1
   (b) Deduce the values for active bytes by accessing $\texttt{DDT}$ to get $(z_3[0, 2, 3], z_3'[0, 2, 3])$ and $(x_4[0, 2, 3], x_4'[0, 2, 3])$
   (c) Deduce $(w_4[0, 2, 3], w_4'[0, 2, 3])$ by XORing $(x_4[0, 2, 3], x_4'[0, 2, 3])$ with k_4
   (d) Check if $\texttt{MC}(z_3[0, 2, 3]) \overset{?}{=} w_4[0, 2, 3]$, which is a 16-bit filter by Pro. 1
3. Repeat Step 2 in parallel for $\texttt{SSB}^{(1)}$, $\texttt{SSB}^{(2)}$, and $\texttt{SSB}^{(3)}$ to get one starting point on average.

Hence, we need 2^{16} time complexity to get one starting point for a random given a random given Δz_2, Δw_4 and the master key. Since the probability of the outbound phase is 2^{-96}, we need 2^{96} starting points to get one collision. Since $|\Delta z_2| \times |\Delta w_4| = 2^{48} < 2^{96}$, we need two blocks as shown in Figure 11 to get additional degrees of freedom from the first block. The overall time complexity of the classical time is 2^{112}. According to the quantum analysis by [39,30], we apply Grover's algorithm and get a quantum collision attack with roughly 2^{56} quantum time complexity. Full paper [28] gives a very detailed quantum collision attack on 8-round `AES`, whose time complexity is $2^{55.53}$ 8-round `AES`, which is very close to the estimated 2^{56}.

5 Improved Quantum Attacks on `Saturnin-Hash`

`Saturnin` is a block cipher with a 256-bit state and 256-bit key that was designed as the derivative of `AES` with efficient implementation by Canteaut et al. [17]. It is among the round 2 candidates of the NIST lightweight cryptography competition. The composition of two consecutive rounds starting from even round is called super-round, which is very similar to an `AES` round operating on 16-bit words except that the `SR` is replaced by a transposition. `Saturnin-Hash` is built on 16-super-round `Saturnin` block cipher with the MMO hashing mode.

5.1   Improved 8-Round Quantum Free-Start Collision

The differential trail we use here was found in [31]. As shown in Figure 12, our new method extends the inbound phase by covering round 0 to round 4 in inbound phase, and the probability of the outbound phase becomes $2^{-135.8}$.
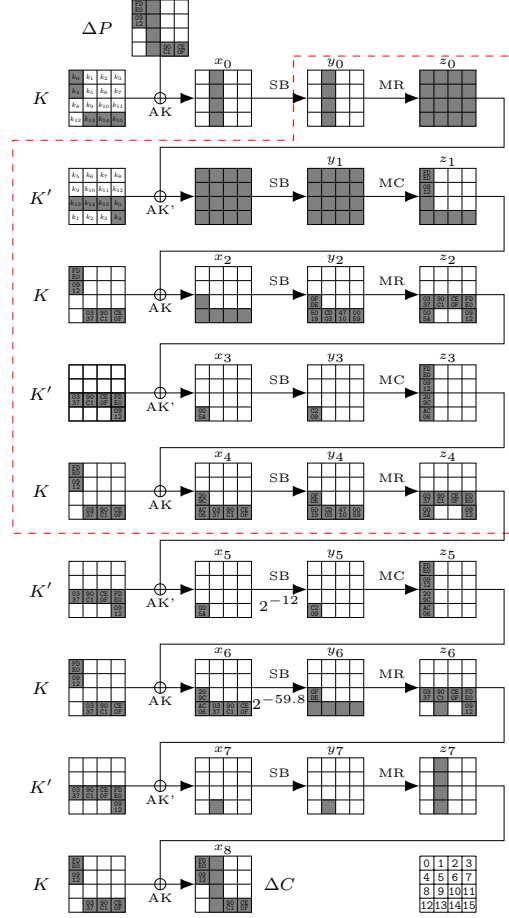

Fig. 12: Free-start collision attack on 8-round `Saturnin-Hash`

Super-Inbound Phase. In Dong et al.'s [31] attack, the inbound phase covers states from y_0 to x_3. However, our Super-Inbound covers two more rounds from state y_0 to x_5. Hence, the probabilities $Pr(\Delta x_3 \mapsto \Delta y_3) = 2^{-12}$ and $Pr(\Delta x_4 \mapsto \Delta z_4) = 2^{-59.8}$ in Dong et al.'s attack are removed in ours.

To perform the 4-round Super-Inbound phase, we first randomly assign arbitrary differences for all unfixed differences in $\Delta z_0 = \texttt{MR}(\Delta y_0)$, Δz_1 and Δz_4.

Together with the prefixed differences of Δz_2 and Δz_3, we do the following steps to find a pair to confirm the given differences in the Super-Inbound phase.
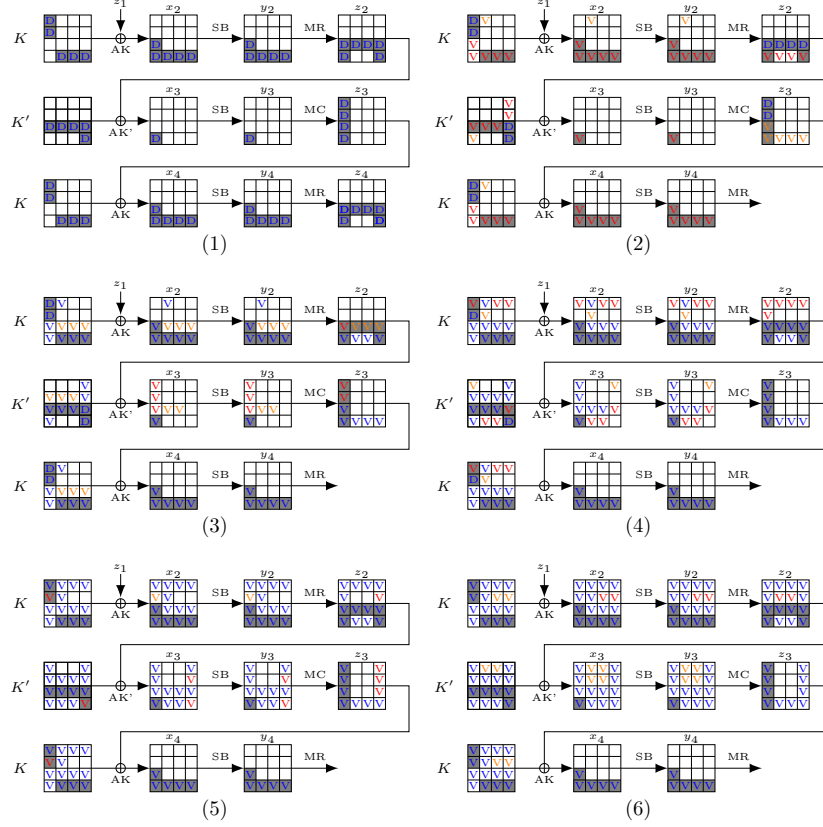

Fig. 13: Solving the inbound part for 8-round `Saturnin-Hash`

1. Deduce the differences of Δy_1, Δy_4. Compute all the active byte values of $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) by assessing the DDT. Then z_1 is known. As shown in Figure 13, we pick out round 2 to round 4 to clarify the steps to compute a conforming pair for the inbound part. As shown Figure 13 (2), compute $K[V] = z_1 \oplus x_2[V]$. Compute $K'[V] = x_3[V] \oplus z_2[V]$. Similarly, compute all other cells marked by "V" and "V".

2. In Figure 13 (3), deduce "V" and "V" cells by the "V" cells. **Guess** $z_3[0]$ to compute $y_3[V]$ and $z_3[V]$ by Property 1. Compute $z_2[V] = K'[V] \oplus x_3[V]$. **Guess** $z_2[9, 10]$ to deduce $z_2[V]$ and $y_2[V]$ by Property 1. Then, compute $K[V] = x_2[V] \oplus z_1$ and $x_3[V] = z_2[V] \oplus K'[V]$.

3. In Figure 13 (4), **guess** $z_2[0, 1, 2]$ to deduce $y_2[V]$ and $z_2[V]$. Deduce $K[V]$ and $K'[V]$. Compute $K'[V] = x_3[V] \oplus z_2[V]$.

4. In Figure 13 (5), **guess** $z_3[3]$ to deduce $z_3[V]$ and $y_2[V]$. Compute $K'[V] = x_3[V] \oplus z_2[V]$ and $z_2[V] = K'[V] \oplus x_3[V]$. Compute $x_2[V] = K[V] \oplus z_1$.
5. In Figure 13 (6), compute $z_2[V]$ and $y_2[V]$. Compute $K[V] = x_2[V] \oplus z_1$. Compute $y_3[V]$, so that the 2nd column and 3rd column of y_3 and z_3 form a filter of 2^{-32}.

Totally, we guess 7-cell $(z_3[0,3], z_2[9,10,0,1,2])$ to deduce the conforming pair. In Step 5, there is a filter of 2^{-32}, together with the outbound probability $2^{-135.8}$, we get a collision with total probability of $2^{-32-135.8} = 2^{167.8}$. We have $2^{9 \times 16} = 2^{144}$ choices for $\Delta z_0 = \text{MR}(\Delta y_0)$, Δz_1 and Δz_4. Together with the 7-cell $(z_3[0,3], z_2[9,10,0,1,2])$, the total degree of freedom is $2^{144+16 \times 7} = 2^{256} > 2^{167.8}$. Hence, we have enough degree of freedom to find the collision. Since we do not have the quantum circuit for the `DDT` of `Saturnin` like `AES`, we have to use Equation 4 in [28] to estimate the time complexity. With $l = 27$, the time is roughly $27 \cdot 2^{167.8/2+16/2}/(16 \times 8) = 2^{89.65}$ 8-round `Saturnin-Hash`.

5.2 Extend the Attack to 10-round Free-Start Collision

The 10-round differential trail is easily obtained by repeating the path of Figure 12 round 5 and round 6 one more time before entering round 7. The figure for 10-round `Saturnin-Hash` is given in Figure 17 in our full version [28]. By this way, the probability of the outbound phase decreases by $2^{-16-59.8} = 2^{-75.8}$ while the Super-Inbound phase maintains the same probability 2^{-32}. Finally, we obtain the free-start collision attack with probability $2^{167.8-75.8} = 2^{-243.6}$. Since the number of degrees of freedom 2^{256} is still larger than $2^{243.6}$, so that we can find a collision. According to Equation 4 in [28], $l = 27$, the time is roughly $27 \cdot 2^{243.6/2+16/2}/(16 \times 10) = 2^{127.2}$ 10-round `Saturnin-Hash`. Additionally, we present an improved 7-round quantum semi-free-start collision in Supplementary Material D in [28].

6 Quantum Collision Attack on `SKINNY-128-384-MMO`

`SKINNY` is a family of lightweight block ciphers designed by Beierle et al. [7]. In this section, we focus on the hashing mode of `SKINNY-128-384`. Please find the structure of `SKINNY`-n-$3n$ in [7]. The `MC` operation is non-MDS:

$$\text{MC} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \oplus c \oplus d \\ a \\ b \oplus c \\ a \oplus c \end{pmatrix} \quad \text{and} \quad \text{MC}^{-1} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} \beta \\ \beta \oplus \gamma \oplus \delta \\ \beta \oplus \delta \\ \alpha \oplus \delta \end{pmatrix}. \tag{1}$$

6.1 21-Round Quantum Free-Start Collision Attack

In quantum setting, we derive the free-start collision attack on hashing modes (MMO/MP) with 21-round `SKINNY-128-384` using the differential characteristic shown in Figure 18 in Supplementary Material D in [28]. The new differential

trail is found by using the automatic tool in [23]. The 5-round Super-Inbound covers from state w_9 to x_{14}. The outbound phase happens with probability $2^{-103.4}$.

We pick out the 5-round Super-Inbound phase from the whole rebound attack trail of Figure 18 in [28] to get Figure 14 (a). To launch the rebound attack, we first precompute a pair satisfying the 5-round inbound trail as shown in Figure 14 (b). Then thanks to the large degrees of freedom from the tweakey, we get enough starting points by changing the 384-bit tweakey.
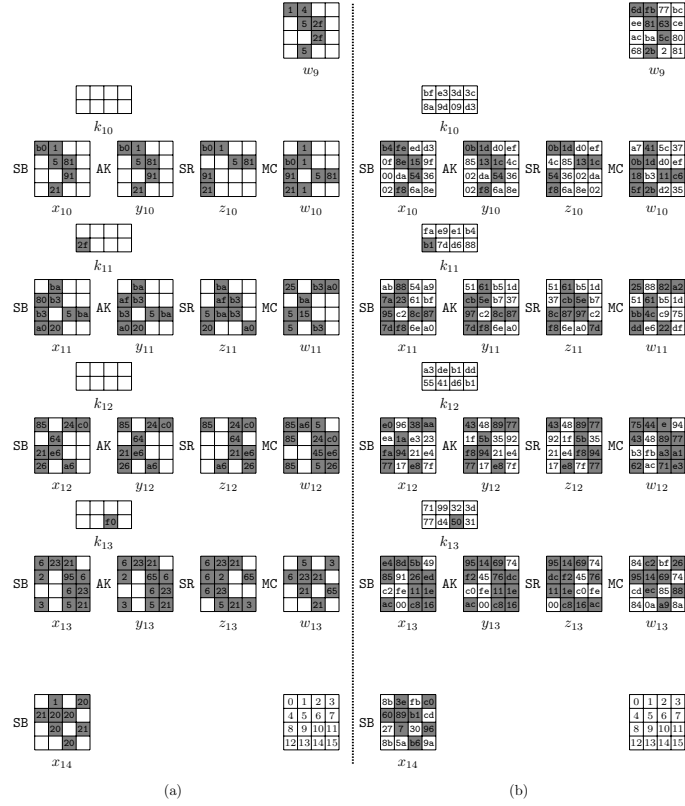

Fig. 14: 5-round Super-Inbound of `SKINNY-128-384`: (a) Differences, (b) Values. The values of k_i are the `XOR` of subkeys and constants of `AC` operator.

Precomputation in the Super-Inbound Phase. We build several steps to compute the conforming data pairs for the Super-Inbound phase.

1. In Figure 14(a), focusing on states z_{10} to x_{11}, the values in $z_{10}[8, 12] = $ $\texttt{SR}(x_{10}[10, 13])$ and all active bytes of w_{10} are deduced by assessing `DDT` with fixed differences in Figure 14(a). In the 1st column z_{10} and w_{10}, i.e., $z_{10}^{(0)}$ and $w_{10}^{(0)}$, we have a condition "$w_{10}[4] \oplus w_{10}[12] = z_{10}[8]$" due to Equation 1, which is a filter of 2^{-8}. In our trail, the differences are dedicatedly

chosen, so that we have enough pairs to verify the filters for the given differences. For example, in the condition "$w_{10}[4] \oplus w_{10}[12] = z_{10}[8]$", we choose $(w_{10}[4], w_{10}[12], z_{10}[8]) \in (\mathtt{DDT[b0_x][80_x]} \times \mathtt{DDT[21_x][a0_x]} \times \mathtt{DDT[2f_x][91_x]})$, where $\mathtt{DDT[b0_x][80_x]}$ is the subset of $\mathtt{DDT}$ with input-output differences $(\mathtt{b0_x, 80_x})$. The size $|\mathtt{DDT[b0_x][80_x]} \times \mathtt{DDT[21_x][a0_x]} \times \mathtt{DDT[2f_x][91_x]}| = 48 \cdot 2^5 \cdot 2^3 > 2^8$.

2. With the choice of $w_{10}[12], z_{11}[15]$ is known as well. $z_{11}[8, 9, 10, 12]$ and all active bytes of w_{11} are deduced by $\mathtt{DDT}$. We have similar conditions to fulfill for the 1st column of z_{11} and w_{11} with $w_{11}[0] \oplus w_{11}[12] = z_{11}[12]$, which act as a filter of 2^{-8}.

3. Do similar steps from z_{10} to w_{13}, we get a data and key pair as shown in Figure 14(b) conforming the whole 5-round inbound trail.

An additional trick is used in our inbound phase. As only the first two rows of x_9 are XORed with k_9, the last two rows of x_9 and w_8 are fully determined by w_9. Hence, in the inbound phase, we directly find the pair with $\Delta w_8[8] = \mathtt{04_x}, \Delta w_8[13] = \mathtt{54_x}$, and $\Delta w_8[14] = \mathtt{54_x}$, which increases the probability of the outbound phase from $2^{-103.4}$ to $2^{-92.4}$. The conforming pair for the Super-Inbound is given in Figure 14(b), which is generated by the source code at https://www.dropbox.com/s/2n4d9zwufkpigqt/Find_inbound.7z

Generating Starting Points for Outbound Phase. Now we have the values of subkeys $k_{10}, k_{11}, k_{12}, k_{13}$. Since the key schedule is linear and by solving a linear system on the 384-bit key with fixed values of subkeys $k_{10}, k_{11}, k_{12}, k_{13}$ in Figure 14 (b), we can derive at most 2^{128} starting points for the outbound phase. We need only $2^{92.4}$ starting points to get a collision. In quantum setting, the $2^{92.4}$ starting points are traversed by Grover's algorithm, which need roughly $2^{46.2}$ time complexity to find the collision.

6.2 Classic Free-Start Collision Attack on 19-Round

The first classical free-start collision attack on 19-round $\mathtt{SKINNY\text{-}128\text{-}384}$ hash mode is also given with the trail in Figure 19 in Supplementary Material F in [28]. The Super-Inbound covers 5 rounds from round 9 to round 13, and only one data pair, shown in Figure 20 in our full paper [28], is needed to confirm the inbound trail in the precomputation phase. Similar to the 21-round attack, the pre-computed pair of the inbound phase also satisfies the differential of the last two rows of x_8 to w_7. Since the outbound excluding the Sboxes in the last two rows of x_8 happens with probability $2^{-51.2}$, the final time complexity is $2^{51.2}$.

7 Discussion and Conclusion

7.1 Possible Generalization of Triangulating Rebound

In Section 3, we have shown that the multiple inbound phases can be connected by the key bytes. We may describe the idea in a more generalized way. In fact, for active Sbox or Super-Sbox with fixed input/output differences, the values are fixed. For active Sbox without fixed input/output differences, or inactive Sbox, the values are not fixed and up to attacker's control, which are the source of the

degree of freedom (`DoF`). In order to identify a data pair conforming the truncated differential, we may consume the degree of freedom from the unfixed Sbox to bridge several fixed values due to active Sboxes or Super-Sboxes with fixed input/output differences. This intuitive idea implies that not only the key bytes are useful to connect the differences, but also the internal states. In Supplementary Material G in [28], we give an example to connect the differences with DoF from internal states and gain some improved collision attacks on `Grøstl`.

7.2   Conclusion

In this paper, we extended the number of attacked rounds by the rebound attack by introducing *triangulating rebound attack*. The core idea is to take full advantage of the available degrees of freedom from both the subkeys and the state to greatly extend the number of rounds covered by the inbound phase, which is named as Super-Inbound. As a consequence, multiple improved classic or quantum collision attacks on AES-like hashing were presented. Possible future works are to investigate more ciphers with our method, as well as to find more applications such as distinguishing attacks based the improved rebound attacks.

Acknowledgments

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