# Lightweight Authenticated Encryption Mode Suitable for Threshold Implementation 

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#### Abstract

This paper proposes tweakable block cipher (TBC) based modes PFB_Plus and PFB $\omega$ that are efficient in threshold implementations (TI). Let $t$ be an algebraic degree of a target function, e.g. $t=1$ (resp. $t>1$ ) for linear (resp. non-linear) function. The $d$-th order TI encodes the internal state into $d t+1$ shares. Hence, the area size increases proportionally to the number of shares. This implies that TBC based modes can be smaller than block cipher (BC) based modes in TI because TBC requires $s$-bit block to ensure $s$-bit security, e.g. PFB and Romulus, while BC requires $2 s$-bit block. However, even with those TBC based modes, the minimum we can reach is 3 shares of $s$-bit state with $t=2$ and the first-order TI $(d=1)$. Our first design PFB_Plus aims to break the barrier of the $3 s$-bit state in TI. The block size of an underlying TBC is $s / 2$ bits and the output of TBC is linearly expanded to $s$ bits. This expanded state requires only 2 shares in the first-order TI, which makes the total state size 2.5 s bits. We also provide rigorous security proof of PFB_Plus. Our second design $\operatorname{PFB} \omega$ further increases a parameter $\omega$ : a ratio of the security level $s$ to the block size of an underlying TBC. We prove security of PFB $\omega$ for any $\omega$ under some assumptions for an underlying TBC and for parameters used to update a state. Next, we show a concrete instantiation of PFB_Plus for 128 -bit security. It requires a TBC with 64 -bit block, 128 -bit key and 128 -bit tweak, while no existing TBC can support it. We design a new TBC by extending SKINNY and provide basic security evaluation. Finally, we give hardware benchmarks of PFB_Plus in the first-order TI to show that TI of PFB_Plus is smaller than that of PFB by more than one thousand gates and is the smallest within the schemes having 128-bit security.


Keywords: Authenticated encryption, threshold implementation, beyond-birthday-bound security, tweakable block cipher, lightweight.

## 1 Introduction

Data communication through IoT devices is getting more and more popular. This requires lightweight authenticated encryption (AE) schemes that can be
used comfortably in a resource-restricted environment. Since March 2019, NIST has organized a competition for determining the lightweight AE standard [38]. 56 designs were chosen as Round 1 candidates and 32 designs have been chosen as Round 2 candidates in August 2019. The design of lightweight AE schemes is one of the most actively discussed topics in the symmetric-key research filed.

Many of AE designs with provable security adopt a block cipher (BC), a cryptographic permutation, or a tweakable block cipher (TBC) as an underlying primitive. The conventional security model regards those modules as a black box and discusses the security under the black box setting. In contract, NIST's competition optionally takes into account the security in the grey box setting, where the cryptographic modules leak side-channel information. It is now important to design lightweight AE schemes such that countermeasures against side-channel attacks (SCA) can be implemented efficiently.

Masking is by far the most common countermeasure against SCA [25, 37], and thus implementing an AE scheme using a $\mathrm{BC} / \mathrm{TBC}$ primitive protected by masking is the natural way to realize an SCA-resistant AE. Threshold implementation (TI) introduced by Nikova et al. [37] is a masking particularly popular for hardware implementation. Masking, however, easily multiply the computational cost. Although hardware designers have been tackling the problem by designing serialized implementations in order to achieve an extreme of the area-speed trade-off, implementation-level optimization is reaching its limit. To push the limit further, researchers have been studying a BC optimized for TI by design, mostly focusing on TI-friendly Sboxes [13, 21]. In this paper, we follow this line of research and go one step further by introducing the TI-friendly AE mode.

TI encodes the internal state (mostly consists of the internal state to compute the underlying primitive) into multiple shares, and apply the round transformation to each of them. Hence, the area size in TI increases proportionally to the number of shares. The number of shares is $d t+1$ for the order of masking $d$ and the algebraic degree of a target function $t$, and thus it is $t+1$ for the first-order TI with $d=1$.

In lightweight AE schemes, register occupies the major circuit area. To be more precise, let $b$ and $s$ be the bit sizes of the underlying primitive and the aiming security, respectively. Then the key size needs to be at least $s$, and thus we need a $b$-bit register for the data block and an $s$-bit key for the key. We need different number of shares for the data and key because the data needs three shares for the nonlinear round function $(t>1)$, but the key needs only two shares because the key schedule function is often linear for recent algorithms. Naito and Sugawara recently proposed a TBC-based scheme which is particularly efficient with TI by exploiting this asymmetry [34].

The problem we address in this paper is to further exploit this asymmetry. More specifically, we let $\omega=s / b$ be an indicator of the asymmetry, and consider designing a scheme with higher $\omega$. Following Naito and Sugawara, we pursue TBC-based schemes because of disadvantages of other approaches as follows. The comparison is also given in Table 1.

Drawbacks of BC based schemes: To minimize a register size, i.e., the register size is (almost) equal to the BC size, the security level is compromised to the birthday-bound security regarding the block size, because birthday attacks are principally unavoidable. Hence, $2 s$-bit block and $s$-bit key are necessary to ensure $s$-bit security even without TI. SAEB [33] is an example of this case. To apply the first-order TI by assuming a linear key schedule, we need 3 shares for the data block and 2 shares for the key. Hence, we need a register of size $8 s(=3 \times 2 s+2 \times s)$ bits. Note that the key register may not be protected in the same level as the data block register because computation of the key schedule is not dependent on the value of the data block. In this strategy, the register size is $7 s(=3 \times 2 s+s)$ bits. Note that there are several beyond-the-birthday-bound (BBB) modes, but those require very unsuitable structures for TI i.e., in TI the register sizes of BBB modes are grater than those of birthday-bound ones.
Drawbacks of permutation based schemes: Let $r$ and $c$ be the number of bits for the rate and the capacity, respectively. When attackers are allowed to make decryption queries, the security of the simple duplex construction can be proven only up to the birthday bound of the capacity [12, 28]. Hence to ensure $s$-bit security, the permutation size must be at least $2 s+r$ bits. For the first-order TI, we need $3 \times(2 s+r)$ bits of the register size. Beetle [14], a recently proposed design, is provably security up to $\min (c-\log r, b / 2, r)$. To ensure $s$-bit security, we basically balance $r$ and $c$ to $s$ bits for the second term, but slightly increases $c$ to compensate ' $-\log r$ ' in the first term. Hence, the register size is $2 s+\log s$ bits without TI and $3 \times(2 s+\log s)$ for TI.
Advantages of TBC based schemes: To ensure $s$-bit security, the block size can be $s$ bits. Along with an $s$-bit key and an $s$-bit tweak, the register size without TI is $3 s$ bits, e.g. PFB [34] and Romulus [26]. To apply the first-order TI by assuming a linear key schedule, we need 3 shares for the data block and 2 shares for the key. $s$-bit tweak is a public value, and it does not need any protection. Hence, we need a register of size $6 s(=3 s+2 s+s)$ bits for TI. By the same analogy for BC, the protection of the key register may not be needed. In this case, the register size for TI becomes $5 s$ bits.

Form the above comparison, we investigate a TBC-based scheme to design a mode that is efficient for TI. In particular, we focus our attention on the property that the area size of TI mainly depends on how big $\omega(=s / b)$ is, and we aim a TBC-based mode with a large $\omega$.

Before stepping into the TI-friendly design, we first briefly introduce some knowledge that is general to the designs of AE schemes.

- To be lightweight, the use of "nonce", a value that is never repeated under the same key, offers significant advantages.
- As shown by $\Theta C B$ [29], privacy can be ensured by injecting the nonce and the block counter into the tweak for an underlying TBC.
- Authenticity can be ensured by preparing the double internal state size (the block size of an underlying TBC is a part of the internal state size) of the security level.

Table 1. Comparison of State Sizes with and without (w/o) TI. The (twea)key functions are assumed to be linear. Without TI, permutation based schemes achieve the smallest state size by using a small rate, while with TI, TBC based schemes in particular PFB_Plus outperform the others.

|  | base | BC | Permutation |  | TBC |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | example mode reference | $\begin{gathered} \text { SAEB } \\ {[33]} \end{gathered}$ | $\begin{array}{\|c\|} \hline \text { Duplex } \\ {[12,28]} \end{array}$ | Beetle <br> [14] | $\begin{array}{c\|} \hline \text { PFB,Romulus } \\ {[34,26]} \end{array}$ | $\begin{gathered} \text { PFB_Plus } \\ \text { Ours } \end{gathered}$ | PFB $\omega$ Ours |
| w/o TI | data block key tweak extra state total | $\begin{gathered} \hline \hline 2 s \\ s \\ - \\ - \\ 3 s \end{gathered}$ | $\begin{gathered} 2 s+r \\ s \\ - \\ - \\ 2 s+r \end{gathered}$ | $\begin{gathered} \hline \hline 2 s+\log s \\ s \\ - \\ - \\ 2 s+\log s \\ \hline \end{gathered}$ | $\begin{gathered} s \\ s \\ - \\ -3 s \end{gathered}$ | $\begin{gathered} \hline 0.5 s \\ s \\ s \\ 0.5 s \\ 3 s \end{gathered}$ | $s / \omega$ $s$ $s$ $s-s / \omega$ $3 s$ |
| TI | protect key not protect key | $\begin{aligned} & \hline 8 s \\ & 7 s \end{aligned}$ | $\begin{aligned} & 6 s+3 r \\ & 6 s+3 r \end{aligned}$ | $\begin{aligned} & 6 s+3 \log s \\ & 6 s+3 \log s \end{aligned}$ | $\begin{aligned} & \hline 6 s \\ & 5 s \end{aligned}$ | $\begin{aligned} & 5.5 \mathrm{~s} \\ & 4.5 \mathrm{~s} \end{aligned}$ | $\begin{aligned} & 5 s+s / \omega \\ & 4 s+s / \omega \end{aligned}$ |

- The key size must be greater than or equal to the security level.
- The maximum number of processed input blocks by all queries should be equal to the security level.

Our goal is to design a TBC-based AE mode that has a large $\omega(=s / b)$. The biggest $\omega$ among the exiting TBC modes is 1 , hence we first aim a TBC-based AE mode with $\omega=2$. To achieve the goal, we have the following obstacles.
$-b$ is a block size of TBC. For $\omega=2$, we need to ensure the security up to the double of the block size. Hence, we need to design a mode that expands an $b$-bit TBC output to a $2 b$-bit internal state. The expanded state needs to be updated only linearly, otherwise we need 3 shares for the expanded state in TI and thus does not yield any advantage compared to the case with $\omega=1$.

- To avoid using 3 shares for the key, the key schedule must be linear. To leave the tweak state unprotected (only with 1 share), the tweak and key states must be kept independent. We observe that the tweakey framework [27] is suitable for this design.
- The key size must be $2 b$ bits. To process up to $2 b$-bit block inputs, the size of the combination of the nonce and the block counter must be $2 b$ bits. Namely, we need to process $4 b$ bits for the key plus tweak, which is not easy with existing TBCs. The tweakey framework conceptually defines a way to process $4 b$-bit tweakey (tweak plus key), while exiting concrete designs only support up to $3 b$-bit tweakey. Note that Lilliput-AE [1], one of the first-round candidates at the NIST competition, specifies TBCs with $5 b-, 6 b-$, and $7 b-$ bit tweakeys. However, those ignored the rationale of the original tweakey framework to ensure the security, and were actually attacked practically [20].

Our Contributions. This paper proposes new TBC based modes that are efficient for TI. We first propose our new mode PFB_Plus (Fig. 1) that is a TIfriendly TBC-based mode for $\omega=2$ with rigorous security proof. The block size $b$ of the underlying TBC is $0.5 s$ bits for $s$-bit security. As its construction, we combine the structure of PFB with $f 9$ [43] in order to generate $2 b$-bit internal
state from $b$-bit TBC outputs and only use linear operations to update the expanded state. We then provide rigorous security proofs of PFB_Plus. The proof is advantageous in a sense that the security only depends on the number of decryption queries and independent of the length of the each query. PFB_Plus is optimized for the first-order TI, namely, 3 shares for the TBC of $0.5 s$-bit block, 2 shares for the $0.5 s$-bit extended state, 2 shares for the $s$-bit key and no protection ( 1 share) for the $s$-bit tweak. The total state size is $5.5 s$ in TI or even $4.5 s$ when the key is not protected. Those are shown in Table $1^{4}$. We also provide a tradeoff between the area size and the target security by truncating the extended $2 b$-bit internal state, which offers arbitrary security level between $b$ to $2 b$ bits. Note that such a feature cannot be achieved by PFB and Romulus: one of the second-round candidates in the NIST competition.

While PFB_Plus is optimized for the first-order TI, one may be interested in finding the theoretical limitation of our approach, i.e. how large $\omega$ can be. To answer this question, we propose an extended version called PFB $\omega$ (Fig. 2) that can handle an arbitrary $\omega$ with security proof under some assumptions for the existence of the underlying primitives (a TBC with $2 \omega b$-bit tweakey and suitable coefficients for multiplications over a finite field). When $\omega$ becomes larger, to satisfy the assumption becomes more difficult and the number of operations increases, while the area size in TI becomes smaller. The state size of PFB $\omega$ is shown in Table 1.

Next, we design a concrete TBC for PFB_Plus. The underlying TBC must be small in area and needs to support $4 b$-bit tweakey. In addition, to increase the efficiency in TI, the tweakey schedule should not contain any non-linear operation. We choose SKINNY with 64 -bit block as a base of our TBC because SKINNY is lightweight and indeed used in several designs submitted to the NIST competition. We extend the design of SKINNY to support TK4 so that the existing third-party security analysis remains available up to TK3. With this approach, our SKINNY-64-256 up to TK3 is secure as long as the original SKINNY is secure. We then provide the lower bounds of the number of active S-boxes in TK4 as the designers of SKINNY did the same. Moreover, we update the security analysis of SKINNY in the single key: the designers of SKINNY sometimes provided upper bounds of the number of active S-boxes both in differential and linear cryptanalysis. Alfarano et al. updated the bounds for differential cryptanalysis [4], while we update the bounds for linear cryptanalysis with the tight ones. Finally, we benchmark TI of PFB_Plus instantiated with SKINNY-64-256 in hardware by using the most practical parameters for TI. ${ }^{5}$

[^0]Finally, we give hardware performance evaluation of PFB_Plus combined with SKINNY-64-256, and compare it with the conventional PFB. As a masking scheme, we choose the first-order TI in which the TBC state and key are protected with three and two shares, respectively. Thanks to the larger $\omega$, the TI of PFB_Plus is smaller than that of PFB by more than one thousand gates $(7,439$ and 8,448 [GE], respectively), and is the smallest within the schemes having 128-bit security.

Recommendation. PFB $\omega$ is designed as a proof-of-concept of using a smaller block size, and our recommendation is PFB_Plus.

Limitations. The proposed method becomes efficient with TI, and the benefit extends to other masking schemes with $d t+1$ shares (for $t>1$ ) [25]; meanwhile, it is no longer efficient with $(d+1)$-share masking schemes [16]. We believe that $(d t+1)$-shares schemes are still important. First, the 1st-order TI is a very practical option because of its reasonable circuit area and no need for fresh randomness. Second, $(d t+1)$-share schemes can be an only option under some security requirements, e.g., when we need non-completeness to eliminate leakage by glitches without relying on registers in between gates.

PFB_Plus and PFB $\omega$ are secure if no unverified plaintext is released and no nonce is repeated, and we do not ensure the misuse security.
Previous Works. In this paper, we focus on designing TI-friendly AE schemes with respect to implementation size. Another approach to design an AE scheme with SCA resistance is leakage-resilient cryptography. The schemes $[9,10,11$, 23, 24] based on the Pereira et al.'s approach [39] assume a leak-free component, and are optimized for minimizing the number of calls to it ${ }^{6}$. However, the way how to realize the leak-free component, that determines the implementation size, is usually out of scope. Moreover, they need additional components such as hash function and pseudo-random function. Barwell et al. [7] studied another approach using pairing-based cryptography, but it is also costly.

The Sponge-based leakage resilient AE scheme ISAP [18] has a potential for lightweight implementation because it does not rely on a leak-free component. However, its implementation cost (14 [kGE]) is still larger than PFB_Plus (7.439 [kGE]). There are recent works following ISAP. The works [17, 19] gave security proofs for the Sponge-based schemes which was missing in the original paper. Degabriele et al. [17] proposed a a variant using a random function. Dobraunig and Mennink [19] gave the security proof of the duplex [12] with respect to leakage resiliency.

Another line of research is to design cryptographic primitives using minimum number of non-linear operations thereby reducing the cost for TI [3, 2]. In contrast to those studies, we approach the problem from the mode of operation by exploiting the asymmetry between non-linear round function and linear key scheduling, rather than improving the non-linear function itself. We designed SKINNY-64-256 for providing a small block length and a larger tweakey state, and not for minimizing the number of non-linear operations. We also note that

[^1]the conventional works focus on minimizing non-linear operations and thus their target primitive is BC rather than TBC (TBCs typically require a higher amount of operations than BCs in order to process a tweak), while the use of TBC is the central part of our study.

## 2 Preliminaries

Notation. Let $\varepsilon$ be an empty string and $\{0,1\}^{*}$ be the set of all bit strings. For an integer $i \geq 0$, let $\{0,1\}^{i}$ be the set of all $i$-bit strings, $\{0,1\}^{0}:=\{\varepsilon\}$, and $\{0,1\}^{\leq i}:=\{0,1\}^{1} \cup\{0,1\}^{2} \cup \cdots \cup\{0,1\}^{i}$ be the set of all bit strings of length at most $i$, except for $\varepsilon$. Let $0^{i}$ resp. $1^{i}$ be the bit string of $i$-bit zeros resp. ones. For an integer $i \geq 1$, let $[i]:=\{1,2, \ldots, i\}$ be the set of positive integers less than or equal to $i$, and $(i]:=\{0\} \cup[i]$. For a non-empty set $\mathcal{T}, T \stackrel{\$}{\leftarrow} \mathcal{T}$ means that an element is chosen uniformly at random from $\mathcal{T}$ and is assigned to $T$. The concatenation of two bit strings $X$ and $Y$ is written as $X \| Y$ or $X Y$ when no confusion is possible. For integers $0 \leq i \leq j$ and $X \in\{0,1\}^{j}$, let $\operatorname{msb}_{i}(X)$ resp. $\operatorname{lsb}_{i}(X)$ be the most resp. least significant $i$ bits of $X$, and $|X|$ be the number of bits of $X$, i.e., $|X|=j$. For integers $i$ and $j$ with $0 \leq i<2^{j}$, let $\operatorname{str}_{j}(i)$ be the $j$-bit binary representation of $i$. For an integer $b \geq 0$ and a bit string $X$, we denote the parsing into fixed-length $b$-bit strings as $\left(X_{1}, X_{2}, \ldots, X_{\ell}\right) \stackrel{b}{\leftarrow} X$, where if $X \neq \varepsilon$ then $X=X_{1}\left\|X_{2}\right\| \cdots \| X_{\ell},\left|X_{i}\right|=b$ for $i \in[\ell-1]$, and $0<\left|X_{\ell}\right| \leq b$; if $X=\varepsilon$ then $\ell=1$ and $X_{1}=\varepsilon$. For an integer $b>0$, let ozp : $\{0,1\} \leq b \rightarrow\{0,1\}^{b}$ be a one-zero padding function: for a bit string $X \in\{0,1\} \leq b, \operatorname{ozp}(X)=X$ if $|X|=b$; $\operatorname{ozp}(X)=X \| 10^{b-1-|X|}$ if $|X|<b$.
Tweakable Block Cipher. A tweakable blockcipher (TBC) is a set of permutations indexed by a key and a public input called tweak. Let $\mathcal{K}$ be the key spece, $\mathcal{T} \mathcal{W}$ be the tweak space, and $b$ be the input/output-block size. An encryption is denoted by $\widetilde{E}: \mathcal{K} \times \mathcal{T} \mathcal{W} \times\{0,1\}^{b} \rightarrow\{0,1\}^{b}, \widetilde{E}$ having a key $K \in \mathcal{K}$ is denoted by $\widetilde{E}_{K}$, and $\widetilde{E}_{K}$ having a tweak $T W \in \mathcal{T} \mathcal{W}$ is denoted by $\widetilde{E}_{K}^{T W}$.

In this paper, a keyed TBC is assumed to be a secure tweakable-pseudorandom permutation (TPRP), i.e., indistinguishable from a tweakable random permutation (TRP). A tweakable permutation (TP) $\widetilde{P}: \mathcal{T} \mathcal{W} \times\{0,1\}^{b} \rightarrow\{0,1\}^{b}$ is a set of $b$-bit permutations indexed by a tweak in $\mathcal{T} \mathcal{W}$. A TP $\widetilde{P}$ having a tweak $T W \in \mathcal{T W}$ is denoted by $\widetilde{P}^{T W}$. Let $\widetilde{\operatorname{Perm}}\left(\mathcal{T} \mathcal{W},\{0,1\}^{b}\right)$ be the set of all TPs. For a set of all TPs: $\mathcal{T} \mathcal{W} \times\{0,1\}^{b} \rightarrow\{0,1\}^{b}$ denoted by $\widetilde{\operatorname{Perm}}\left(\mathcal{T} \mathcal{W},\{0,1\}^{b}\right)$,
 adversary $\mathbf{A}$ has access to either the keyed TBC $\widetilde{E}_{K}$ or a TRP $\widetilde{P}$, where $K \stackrel{\$}{\leftarrow} \mathcal{K}$ and $\widetilde{P} \stackrel{\$}{\leftarrow} \widetilde{\operatorname{Perm}}\left(\mathcal{T} \mathcal{W},\{0,1\}^{b}\right)$, and after the interaction, A returns a decision bit $y \in\{0,1\}$. The output of $\mathbf{A}$ with access to $\mathcal{O}$ is denoted by $\mathbf{A}^{\mathcal{O}}$. The TPRPsecurity advantage function of $\mathbf{A}$ is defined as

$$
\mathbf{A d v}_{\widetilde{E}_{K}}^{\mathrm{tprp}}(\mathbf{A}):=\operatorname{Pr}\left[K \stackrel{\$}{\leftarrow} \mathcal{K} ; \mathbf{A}^{\widetilde{E}_{K}}=1\right]-\operatorname{Pr}\left[\widetilde{P} \stackrel{\$}{\leftarrow} \widetilde{\operatorname{Perm}}\left(\mathcal{T} \mathcal{W},\{0,1\}^{b}\right) ; \mathbf{A}^{\widetilde{P}}=1\right],
$$

where the probabilities are taken over $K, \widetilde{P}$ and $\mathbf{A}$.

Nonce-Based Authenticated Encryption with Associated Data. A noncebased authenticated encryption with associated data (nAEAD) scheme based on a keyed TBC $\widetilde{E}_{K}$, denoted by $\Pi\left[\widetilde{E}_{K}\right]$, is a pair of encryption and decryption algorithms $\left(\Pi . \operatorname{Enc}\left[\widetilde{E}_{K}\right], \Pi . \operatorname{Dec}\left[\widetilde{E}_{K}\right]\right) . \mathcal{K}, \mathcal{N}, \mathcal{M}, \mathcal{C}, \mathcal{A}$ and $\mathcal{T}$ are the sets of keys, nonces, plaintexts, ciphertexts, associated data $(\mathrm{AD})$ and tags of $\Pi\left[\widetilde{E}_{K}\right]$, respectively. In this paper, the key space of $\Pi\left[\widetilde{E}_{K}\right]$ is equal to that of the underlying TBC. The encryption algorithm takes a nonce $N \in \mathcal{N}, \mathrm{AD} A \in \mathcal{A}$, and a plaintext $M \in \mathcal{M}$, and returns, deterministically, a pair of a ciphertext $C \in \mathcal{C}$ and a $\operatorname{tag} T \in \mathcal{T}$. The decryption algorithm takes a tuple $(N, A, C, T) \in \mathcal{N} \times \mathcal{A} \times \mathcal{C} \times \mathcal{T}$, and returns, deterministically, either the distinguished invalid (reject) symbol reject $\notin \mathcal{M}$ or a plaintext $M \in \mathcal{M}$. We require $\mid \Pi$.Enc $\left[\widetilde{E}_{K}\right](N, A, M) \mid=$ $\left|\Pi . \operatorname{Enc}\left[\widetilde{E}_{K}\right]\left(N, A, M^{\prime}\right)\right|$ when these outputs are strings and $|M|=\left|M^{\prime}\right|$. We consider two security notions of nAEAD, privacy and authenticity. Hereafter, we call queries to the encryption resp. decryption oracle "encryption queries" resp. "decryption queries."

The privacy notion considers the indistinguishability between the encryption $\Pi$.Enc $\left[\widetilde{E}_{K}\right]$ and a random-bits oracle $\$$, in the nonce-respecting setting. $\$$ has the same interface as $\Pi$.Enc $\left[\widetilde{E}_{K}\right]$ and for a query $(N, A, M)$ returns a random bit string of length $\mid \Pi$. $\operatorname{Enc}\left[\widetilde{E}_{K}\right](N, A, M) \mid$. In the privacy game, an adversary $\mathbf{A}$ interacts with either $\Pi$.Enc $\left[\widetilde{E}_{K}\right]$ or $\$$, and then returns a decision bit $y \in\{0,1\}$. The privacy advantage function of an adversary $\mathbf{A}$ is defined as

$$
\mathbf{A d v}_{\Pi\left[\widetilde{E}_{K}\right]}^{\operatorname{priv}}(\mathbf{A}):=\operatorname{Pr}\left[K \stackrel{\$}{\leftarrow} \mathcal{K} ; \mathbf{A}^{\Pi \cdot \mathrm{Enc}\left[\widetilde{E}_{K}\right]}=1\right]-\operatorname{Pr}\left[\mathbf{A}^{\$}=1\right]
$$

where the probabilities are taken over $K, \$$ and $\mathbf{A}$. We demand that $\mathbf{A}$ is noncerespecting (all nonces in encryption queries are distinct).

The maximum over all adversaries, running in time at most $t$ and making encryption queries of $\sigma_{\mathcal{E}}$ the total number of TBC calls invoked by all encryption queries, is denoted by $\mathbf{A d v}_{\Pi\left[\widetilde{E}_{K}\right]}^{\text {priv }}\left(\sigma_{\mathcal{E}}, t\right):=\max _{\mathbf{A}} \mathbf{A d v}_{\Pi\left[\widetilde{E}_{K}\right]}^{\text {priv }}(\mathbf{A})$. When an adversary is a computationally unbounded algorithm, the time $t$ is disregarded.

The authenticity notion considers the unforgeability in the nonce-respecting setting. In the authenticity game, an adversary $\mathbf{A}$ interacts with $\Pi\left[\widetilde{E}_{K}\right]=$ ( $\left.\Pi . \operatorname{Enc}\left[\widetilde{E}_{K}\right], \Pi . \operatorname{Dec}\left[\widetilde{E}_{K}\right]\right)$, and the goal of the adversary is to make a non-trivial decryption query whose response is not reject. The authenticity advantage of an adversary $\mathbf{A}$ is defined as

$$
\mathbf{A d v}_{\Pi\left[\widetilde{E}_{K}\right]}^{\text {auth }}(\mathbf{A}):=\operatorname{Pr}\left[K \stackrel{\$}{\leftarrow} \mathcal{K} ; \mathbf{A}^{\Pi \cdot \operatorname{Enc}\left[\widetilde{E}_{K}\right], \Pi \cdot \operatorname{Dec}\left[\widetilde{E}_{K}\right]} \text { forges }\right]
$$

where the probabilities are taken over $K$ and $\mathbf{A}$. We demand that $\mathbf{A}$ is noncerespecting (all nonces in encryption queries are distinct), that $\mathbf{A}$ never asks a trivial decryption query $(N, A, C, T)$, i.e., there is a prior encryption query $(N, A, M)$ with $(C, T)=\Pi$. $\operatorname{Enc}\left[\widetilde{E}_{K}\right](N, A, M)$, and that A never repeats a query. $\mathbf{A}^{\Pi \cdot E n c}\left[\widetilde{E}_{K}\right], \Pi \cdot \operatorname{Dec}\left[\widetilde{E}_{K}\right]$ forges means that $\mathbf{A}$ makes a decryption query whose response is not reject.

The maximum over all adversaries, running in time at most $t$ and making at most $q_{\mathcal{E}}$ encryption queries and $q_{\mathcal{D}}$ decryption queries of $\sigma$ the total number of TBC calls invoked by all queries, is denoted by $\mathbf{A d v}_{\Pi\left[\tilde{E}_{K}\right]}^{\text {auth }}\left(\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right), t\right):=$ $\max _{\mathbf{A}} \mathbf{A d v}_{\Pi!\left[\widetilde{E}_{K}\right]}^{\text {auth }}(\mathbf{A})$. When an adversary is a computationally unbounded algorithm, the time $t$ is disregarded.

## 3 PFB_Plus: Specification and Security Bounds

We design PFB_Plus, a TBC-based nAEAD mode with $b+\tau$-bit security where $0 \leq \tau \leq b$, by extending the existing TBC-based lightweight mode PFB [34]. Regarding the relation between security and internal state size, in order to achieve $s$-bit security, the internal state size must be at least $s$ bits. Thus PFB_Plus is designed so that the internal state size is minimum, i.e., $b+\tau$ bits. To do so, we extend PFB, which is a $b$-bit secure nAEAD mode and whose security level equals to the internal state size. For the extension, we need to define an additional $\tau$-bit internal state in order to have $b+\tau$-bit security. The additional internal state is designed using the idea of $f 9$ [43], which is a BC-based message authentication code.

- The first $b$-bit internal state is updated by iterating a TBC and absorbing a data block (AD/plaintext/ciphertext block), and the output of the last TBC call becomes the first $b$-bit tag. The idea comes from PFB.
- The remaining $\tau$-bit internal state is defined by XORing outputs of TBC calls. The idea comes from $\mathfrak{f} 9$, but our structure is slightly different from 99 . In PFB_Plus, a TBC is not performed after XORing all outputs of TBC calls (with $b-\tau$-bit truncation), which keeps the internal state size $b+\tau$ bits. On the other hand, in f9, a block cipher is performed after XORing all outputs of block cipher calls.

Regarding tweak elements, as shown by $\Theta C B$ [29], for the sake of perfect privacy, the nonce and the block counter are injected.

### 3.1 Specification

The specification of PFB_Plus is given in Algorithm 1 and is illustrated in Fig. 1.
Let $\ell_{\max }$ be a maximum number of $\mathrm{AD/plaintext/ciphertext} \mathrm{blocks}, \mathrm{i.e.}$, $\ell_{\max }$ and $m \leq \ell_{\max }$. The tweak space $\mathcal{T} \mathcal{W}$ consists of a nonce space $\mathcal{N}:=\{0,1\}^{n}$, a block counter space ( $\left.\ell_{\max }\right]$ and a space for tweak separations (15]. The space for tweak separations (15] is used to offer distinct permutations for handing AD, encrypting plaintexts (or decrypting ciphertexts) and generating a tag. Hence, the tweak space is defined as $\mathcal{T W}:=\{0,1\}^{n} \times\left(\ell_{\max }\right] \times(15]$.

The procedure of handing AD is given in PFB_Plus.Hash. The procedure of encrypting a plaintext is given in the steps $2-5$ of PFB_Plus.Enc, and the procedure of generating a tag is given in the steps 6-9. The procedure of decrypting a ciphertext is given in the steps $2-5$ of PFB_Plus.Dec, and the procedure of verifying a tag is given in the steps 6-9. Note that the tweaks $x$ and $y$ are defined

```
Algorithm 1 PFB_Plus
Encryption PFB_Plus.Enc \(\left[\widetilde{E}_{K}\right](N, A, M)\)
    1: \(\left(M_{0}, T_{2}\right) \leftarrow\) PFB_Plus. Hash \(\left[\widetilde{E}_{K}\right](A)\)
    2: if \(A=\varepsilon\) then \(x \leftarrow 1\); else if \(A \neq \varepsilon \wedge|A| \bmod b=0\) then \(x \leftarrow 6\); else \(x \leftarrow 11\)
    \(M_{1}, \ldots, M_{m} \stackrel{b}{\leftarrow} M\); if \(M=\varepsilon\) then \(\left\{m \leftarrow 0 ; S_{1} \leftarrow M_{0}\right.\); goto Step 7\(\}\)
    for \(i=1, \ldots, m-1\) do \(\left\{W_{i} \leftarrow \widetilde{E}_{K}^{N, i, x}\left(M_{i-1}\right) ; C_{i} \leftarrow W_{i} \oplus M_{i} ; T_{2} \leftarrow T_{2} \oplus \operatorname{lsb}_{\tau}\left(W_{i}\right)\right\}\)
    \(W_{m} \leftarrow \widetilde{E}_{K}^{N, m, x}\left(M_{m-1}\right) ; C_{m} \leftarrow \mathrm{msb}_{\left|M_{m}\right|}\left(W_{m}\right) \oplus M_{m}\)
    \(T_{2} \leftarrow T_{2} \oplus \operatorname{lsb}_{\tau}\left(W_{m}\right) ; S_{1} \leftarrow \operatorname{ozp}\left(M_{m}\right) \oplus\left(0^{\left|M_{m}\right|}| | \mid \operatorname{sb}_{b-\left|M_{m}\right|}\left(W_{m}\right)\right)\)
    if \(|M| \bmod b=0\) then \(y \leftarrow x+1\); else \(y \leftarrow x+3\)
    \(S_{2} \leftarrow \widetilde{E}_{K}^{N, m, y}\left(S_{1}\right) ; T_{2} \leftarrow \operatorname{lsb}_{\tau}\left(S_{2}\right) \oplus T_{2} ; T_{1} \leftarrow \widetilde{E}_{K}^{N, m, y+1}\left(S_{2}\right)\)
    \(C \leftarrow C_{1}\|\cdots\| C_{m} ; T \leftarrow T_{1} \| T_{2} ;\) return \((C, T)\)
```

Decryption PFB_Plus. $\operatorname{Dec}\left[\widetilde{E}_{K}\right](N, A, C, \hat{T})$
$\left(M_{0}, T_{2}\right) \leftarrow$ PFB_Plus.Hash $\left[\widetilde{E}_{K}\right](A)$
if $A=\varepsilon$ then $x \leftarrow 1$; else if $A \neq \varepsilon \wedge|A| \bmod b=0$ then $x \leftarrow 6$; else $x \leftarrow 11$
$C_{1}, \ldots, C_{m} \stackrel{b}{\leftarrow} C$; if $C=\varepsilon$ then $\left\{m \leftarrow 0 ; S_{1} \leftarrow M_{0}\right.$; goto Step 7$\}$
for $i=1, \ldots, m-1$ do $\left\{W_{i} \leftarrow \widetilde{E}_{K}^{N, i, x}\left(M_{i-1}\right) ; M_{i} \leftarrow W_{i} \oplus C_{i} ; T_{2} \leftarrow T_{2} \oplus \operatorname{lsb}_{\tau}\left(W_{i}\right)\right\}$
$W_{m} \leftarrow \widetilde{E}_{K}^{N, m, x}\left(M_{m}\right) ; M_{m} \leftarrow \operatorname{msb}_{\left|C_{m}\right|}\left(W_{m}\right) \oplus C_{m}$
$T_{2} \leftarrow T_{2} \oplus \operatorname{lsb}_{\tau}\left(W_{m}\right) ; S_{1} \leftarrow \operatorname{ozp}\left(M_{m}\right) \oplus\left(0^{\left|C_{m}\right|}| | \operatorname{lsb}_{b-\left|C_{m}\right|}\left(W_{m}\right)\right)$
if $|C| \bmod b=0$ then $y \leftarrow x+1$; else $y \leftarrow x+3$
$S_{2} \leftarrow \widetilde{E}_{K}^{N, m, y}\left(S_{1}\right) ; T_{2} \leftarrow \operatorname{lsb}_{\tau}\left(S_{2}\right) \oplus T_{2} ; T_{1} \leftarrow \widetilde{E}_{K}^{N, m, y+1}\left(S_{2}\right) ; T \leftarrow T_{1} \| T_{2}$
if $T=\hat{T}$ then return $M \leftarrow M_{1}\|\cdots\| M_{m}$; else return reject
Hash PFB_Plus.Hash $\left[\widetilde{E}_{K}\right](A)$
if $A=\varepsilon$ then return $\left(0^{b}, 0^{\tau}\right)$
$V_{0} \leftarrow 0^{b} ; H_{2} \leftarrow 0^{\tau} ; A_{1}, \ldots, A_{a} \stackrel{b}{\leftarrow} A$
for $i=1, \ldots, a-1$ do $\left\{V_{i} \leftarrow \widetilde{E}_{K}^{0^{n}, i, 0}\left(A_{i} \oplus V_{i-1}\right) ; H_{2} \leftarrow \operatorname{lsb}_{\tau}\left(V_{i}\right) \oplus H_{2}\right\}$
$V_{a} \leftarrow \widetilde{E}_{K}^{0^{n}, a, 0}\left(\operatorname{ozp}\left(A_{a}\right) \oplus V_{a-1}\right) ; H_{1} \leftarrow V_{a} ; H_{2} \leftarrow \operatorname{lsb}_{\tau}\left(V_{a}\right) \oplus H_{2} ;$ return $\left(H_{1}, H_{2}\right)$


Fig. 1. PFB_Plus. Hash and PFB_Plus.Enc. $A_{1}, \ldots, A_{a} \stackrel{b}{\leftarrow} A$ (in the hash procedure); $M_{1}, \ldots, M_{m} \stackrel{b}{\leftarrow} M$ (in the encryption procedure).
according to the lengths of $\mathrm{AD} A$ and of a plaintext $M$ (more precisely, whether AD is empty or not, whether the one-zero padding is applied to $A$ or not, and whether it is applied to $M$ or not). The concrete values are given below:

- if $A=\varepsilon \wedge|M| \bmod b=0$ then $(x, y)=(1,2)$,
- if $A=\varepsilon \wedge|M| \bmod b \neq 0$ then $(x, y)=(1,4)$,
- if $A \neq \varepsilon \wedge|A| \bmod b=0 \wedge|M| \bmod b=0$ then $(x, y)=(6,7)$,
- if $A \neq \varepsilon \wedge|A| \bmod b=0 \wedge|M| \bmod b \neq 0$ then $(x, y)=(6,9)$,
- if $A \neq \varepsilon \wedge|A| \bmod b \neq 0 \wedge|M| \bmod b=0$ then $(x, y)=(11,12)$, and
- if $A \neq \varepsilon \wedge|A| \bmod b \neq 0 \wedge|M| \bmod b \neq 0$ then $(x, y)=(11,14)$.


### 3.2 Privacy and Authenticity Bounds of PFB_Plus

## Theorem 1.

$$
\begin{gathered}
\mathbf{A d v}_{\mathrm{PFB} \_ \text {Plus }\left[\widetilde{E}_{K}\right]}^{\text {priv }}\left(\sigma_{\mathcal{E}}, t\right) \leq \mathbf{A d v}_{\widetilde{E}_{K}}^{\mathrm{tprp}}\left(\sigma_{\mathcal{E}}, t+O\left(\sigma_{\mathcal{E}}\right)\right) \\
\mathbf{A d v}_{\mathrm{PFB} \_ \text {Plus }\left[\widetilde{E}_{K}\right]}^{\text {auth }}\left(\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right), t\right) \leq \frac{q_{\mathcal{D}} \cdot 2^{b-\tau+1}}{\left(2^{b}-1\right)^{2}}+\mathbf{A d v}_{\widetilde{E}_{K}}^{\text {tprp }}(\sigma, t+O(\sigma))
\end{gathered}
$$

## 4 Proof of Theorem 1

Firstly, the keyed TBC $\widetilde{E}_{K}$ for $K \stackrel{\$}{\leftarrow} \mathcal{K}$ is replaced with a TRP $\widetilde{P} \stackrel{\$}{\leftarrow} \widetilde{\operatorname{Perm}}\left(\mathcal{T} \mathcal{W},\{0,1\}^{b}\right)$. The replacement offers the TPRP-terms $\mathbf{A d} \mathbf{v}_{\widetilde{E}_{K}}^{\text {tprp }}\left(\sigma_{\mathcal{E}}, t+O\left(\sigma_{\mathcal{E}}\right)\right)$ and $\mathbf{A d} \mathbf{v}_{\widetilde{E}_{K}}^{\mathrm{tprp}}(\sigma, t+$ $O(\sigma))$, and then the remaining works are to upper-bound the advantages $\mathbf{A d v} \mathbf{v}_{\text {PFB_Plus }[\widetilde{P}]}^{\text {priv }}\left(\sigma_{\mathcal{E}}\right)$ and $\mathbf{A d v}_{\text {PFB_Plus }[\widetilde{P}]}^{\text {auth }}\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right)$, where adversaries are computationally unbounded algorithms and the complexities are solely measured by the numbers of queries. Without loss of generality, adversaries are deterministic.

Regarding $\mathbf{A d} \mathbf{v}_{\text {PFB_Plus }[\widetilde{P}]}^{\text {priv }}\left(\sigma_{\mathcal{E}}\right)$, as tweaks of $\widetilde{P}$ are all distinct, all output blocks of $\widetilde{P}$ defined by encryption queries are chosen independently and uniformly at random from $\{0,1\}^{b}$. We thus have $\mathbf{A d v}_{\text {PFB_Plus }[\widetilde{P}]}^{\text {priv }}\left(\sigma_{\mathcal{E}}\right)=0$.

In the following, we focus on upper-bounding $\mathbf{A d v}_{\text {PFB_Plus }[\widetilde{P}]}^{\text {auth }}\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right)$.

### 4.1 Upper-Bounding $\operatorname{Adv}_{\text {PFB_Plus }[\widetilde{P}]}^{\text {auth }}\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right)$

Firstly, we fix a decryption query $\left(N^{(d)}, A^{(d)}, C^{(d)}, \hat{T}^{(d)}\right)$, and upper-bound the probability that an adversary forges at the decryption query.

In the analysis, we use the following notations. Values/variables corresponding with the decryption query are denoted by using the superscript of $(d)$ such as $N^{(d)}, M^{(d)}$, etc. Hence, this analysis upper-bounds $\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)}\right]$. The lengths $a$ and $m$ are denoted by $a_{d}$ and $m_{d}$, respectively. Similarly, for an encryption query $\left(N^{(e)}, A^{(e)}, M^{(e)}\right)$, values/variables corresponding with the encryption query are denoted by using the superscript of (e), and the lengths $a$ and $m$ are denoted by $a_{e}$ and $m_{e}$, respectively.

We next define two cases that are used to upper-bound $\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)}\right]$.

- Case1: for any previous encryption query $\left(N^{(e)}, A^{(e)}, M^{(e)}\right)$,

$$
N^{(e)} \neq N^{(d)} \vee m_{e} \neq m_{d} \vee y^{(e)} \neq y^{(d)}
$$

- Case2: for some previous encryption query $\left(N^{(e)}, A^{(e)}, M^{(e)}\right)$,

$$
N^{(e)}=N^{(d)} \wedge m_{e}=m_{d} \wedge y^{(e)}=y^{(d)}
$$

Using these cases, we have

$$
\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)}\right] \leq \max \left\{\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \text { Case1 }\right], \operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \text { Case } 2\right]\right\}
$$

These probabilities are analyzed in Subsect. 4.2 and Subsects. 4.3-4.9, respectively. The upper-bounds are given in Eqs. (1) and (4), respectively, and give

$$
\operatorname{Adv}_{\text {PFB_Plus }[\widetilde{P}]}^{\text {auth }}\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right) \leq q_{\mathcal{D}} \cdot \max \left\{\frac{1}{2^{b+\tau}}, \frac{2^{b-\tau+1}}{\left(2^{b}-1\right)^{2}}\right\}=\frac{q_{\mathcal{D}} \cdot 2^{b-\tau+1}}{\left(2^{b}-1\right)^{2}}
$$

### 4.2 Upper-Bounding $\operatorname{Pr}\left[T^{(d)}=\hat{\boldsymbol{T}}^{(d)} \mid\right.$ Case1 $]$

In Case1, the tweak tuples $\left(y^{(d)}, N^{(d)}, z^{(d)}\right)$ and $\left(y^{(d)}+1, N^{(d)}, z^{(d)}\right)$ with which the outputs of $\widetilde{P}$ define $S_{2}^{(d)}$ and $T_{1}^{(d)}$ are distinct from the tweak triples defined by the previous encryption queries. Hence, $T_{1}^{(d)}$ and $T_{2}^{(d)}$ are chosen uniformly at random from $\{0,1\}^{b}$ and independently of the previous outputs of $\widetilde{P}$. We thus have

$$
\begin{equation*}
\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \text { Case1 }\right] \leq \frac{1}{2^{b+\tau}} \tag{1}
\end{equation*}
$$

### 4.3 Upper-Bounding $\operatorname{Pr}\left[T^{(d)}=\hat{\boldsymbol{T}}^{(d)} \mid\right.$ Case2 $]$

In Case2, $S_{2}^{(d)}=S_{2}^{(e)} \Leftrightarrow S_{1}^{(d)}=S_{1}^{(e)}$ is satisfied (as $\widetilde{P}^{N^{(d)}, y^{(d)}, m_{d}}$ and $\widetilde{P}^{N^{(e)}, y^{(e)}, m_{e}}$ are the same permutation). Hence, we can focus on the cases: $S_{1}^{(d)} \neq S_{1}^{(e)} \wedge S_{2}^{(d)} \neq$ $S_{2}^{(e)} ; S_{1}^{(d)}=S_{1}^{(e)} \wedge S_{2}^{(d)}=S_{2}^{(e)}$. Using these cases, we have

$$
\begin{align*}
\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \text { Case2 }\right]= & \operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \wedge S_{1}^{(d)} \neq S_{1}^{(e)} \wedge S_{2}^{(d)} \neq S_{2}^{(e)} \mid \text { Case2 }\right] \\
& +\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \wedge S_{1}^{(d)}=S_{1}^{(e)} \wedge S_{2}^{(d)}=S_{2}^{(e)} \mid \text { Case2 }\right] \\
\leq & \underbrace{\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \operatorname{Case} 2 \wedge S_{1}^{(d)} \neq S_{1}^{(e)} \wedge S_{2}^{(d)} \neq S_{2}^{(e)}\right]}_{=: p_{1}} \tag{2}
\end{align*}
$$

The probabilities $p_{1}$ and $p_{2}$ are analyzed in Subsect. 4.4 and Subsects. 4.4-4.9, respectively. The upper-bounds are given in Eqs. (5) and (6), respectively, and give

$$
\begin{equation*}
\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \text { Case2 }\right] \leq \frac{2^{b-\tau}}{\left(2^{b}-1\right)^{2}}+\frac{2^{b-\tau}}{\left(2^{b}-1\right)^{2}}=\frac{2^{b-\tau+1}}{\left(2^{b}-1\right)^{2}} \tag{4}
\end{equation*}
$$

### 4.4 Upper-Bounding $p_{1}$ in (2)

By $S_{1}^{(d)} \neq S_{1}^{(e)} \wedge S_{2}^{(d)} \neq S_{2}^{(e)}, T_{1}^{(d)}$ is chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{T_{1}^{(e)}\right\}$, and $S_{2}^{(d)}$ is chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{S_{2}^{(e)}\right\}$, i.e., $T_{2}^{(d)}$ is chosen uniformly at random from at least $\left(2^{b}-1\right) / 2^{b-\tau}$ values. Hence, we have

$$
\begin{equation*}
p_{1}=\operatorname{Pr}\left[\hat{T}^{(d)}=T^{(d)} \mid S_{1}^{(d)} \neq S_{1}^{(e)} \wedge S_{2}^{(d)} \neq S_{2}^{(e)} \wedge \text { Case2 }\right] \leq \frac{2^{b-\tau}}{\left(2^{b}-1\right)^{2}} \tag{5}
\end{equation*}
$$

### 4.5 Upper-Bounding $p_{2}$ in (3)

Let

$$
\mathcal{I}_{V}^{\neq}=\left\{i \in\left[\max \left\{a_{e}, a_{d}\right\}\right] \mid V_{i}^{(d)} \neq V_{i}^{(e)}\right\}, \mathcal{I}_{W}^{\neq}=\left\{i \in\left[m_{d}\right] \mid W_{i}^{(d)} \neq W_{i}^{(e)}\right\}
$$

be sets of indexes with distinct blocks for $V$ and $W$, respectively, where $V_{i}^{(d)}:=\varepsilon$ for $i>a_{d}$, and $V_{i}^{(e)}:=\varepsilon$ for $i>a_{e}$.

This analysis uses the following four sub-cases of Case2.

- Case2-1: Case2 $\wedge a_{d}=a_{e} \wedge\left|\mathcal{I}_{V}^{\neq}\right|+\left|\mathcal{I}_{W}^{\neq}\right|=1$.
- Case2-2 : Case2 $\wedge a_{d}=a_{e} \wedge\left|\mathcal{I}_{V}^{\neq}\right|+\left|\mathcal{I}_{W}^{\neq}\right| \geq 2$.
- Case2-3: Case2 $\wedge a_{d} \neq a_{e} \wedge\left|\mathcal{I}_{W}^{\neq}\right|=0 \wedge A^{(d)} \neq \varepsilon \wedge A^{(e)} \neq \varepsilon$.
- Case2-4 : Case2 $\wedge a_{d} \neq a_{e} \wedge\left|\mathcal{I}_{W}^{\neq}\right| \geq 1 \wedge A^{(d)} \neq \varepsilon \wedge A^{(e)} \neq \varepsilon$.

Note that Case2 $\Rightarrow$ Case2-1 $\vee$ Case2-2 $\vee$ Case2-3 $\vee$ Case2-4 is satisfied by the following reasons. Regarding the sets $\mathcal{I}_{V}^{\neq}$and $\mathcal{I}_{W}^{\neq}$, the non-equation $\left(A^{(d)}, C^{(d)}\right) \neq$ $\left(A^{(e)}, C^{(e)}\right)$ and the condition $y^{(e)}=y^{(d)}$ (from Case2) ensure the following:

$$
\left|\mathcal{I}_{V}^{\neq}\right|+\left|\mathcal{I}_{W}^{\neq}\right| \geq 1
$$

Regarding the $\mathrm{AD} A^{(d)}$ and $A^{(e)}$, the condition $y^{(e)}=y^{(d)}$ ensures the following:

$$
\left(A^{(d)}=A^{(e)}=\varepsilon\right) \vee\left(A^{(d)} \neq \varepsilon, A^{(e)} \neq \varepsilon\right)
$$

Let Coll ${ }_{S, T}:=S_{1}^{(d)}=S_{1}^{(e)} \wedge \hat{T}_{2}^{(d)}=T_{2}^{(e)}$. Then, using the four cases, we have

$$
\left.\left.\left.\begin{array}{rl}
p_{2}=\operatorname{Pr}\left[\text { Coll }_{S, T} \mid \text { Case } 2\right] \leq \max \{ & \operatorname{Pr}\left[\text { Coll }_{S, T} \mid \text { Case2-1 }\right], \operatorname{Pr}\left[\text { Coll }_{S, T} \mid \text { Case2-2 }\right], \\
& \operatorname{Pr}\left[\text { Coll }_{S, T} \mid \text { Case2-3 }\right], \operatorname{Pr}[\text { Coll } \\
S, T
\end{array} \right\rvert\, \text { Case2-4 }\right]\right\} .
$$

These probabilities are analyzed in Subsects. 4.6, 4.7, 4.8, and 4.9, respectively. These upper-bounds are given in Eqs. (7), (8), (9), and (10), respectively, and give

$$
\begin{equation*}
p_{2} \leq \frac{2^{b-\tau}}{\left(2^{b}-1\right)^{2}} \tag{6}
\end{equation*}
$$

### 4.6 Upper-Bounding $\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)} \wedge T_{2}^{(d)}=\hat{T}_{2}^{(d)} \mid\right.$ Case2-1 $]$

In Case2-1, the number of positions with distinct output blocks is 1, and thus the output difference is propagated to $S_{1}$, i.e., $S_{1}^{(d)} \neq S_{1}^{(e)}$ is satisfied. Hence, we have

$$
\begin{equation*}
\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)} \wedge T_{2}^{(d)}=\hat{T}_{2}^{(d)} \mid \text { Case2-1 }\right]=0 \tag{7}
\end{equation*}
$$

### 4.7 Upper-Bounding $\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)} \wedge T_{2}^{(d)}=\hat{T}_{2}^{(d)} \mid\right.$ Case2-2 $]$

First, notations used in the analysis are introduced. Let $\mathcal{I}^{\neq}=\mathcal{I}_{V}^{\neq} \cup\left\{i+a_{d} \mid i \in \mathcal{I}_{W}^{\neq}\right\}$ be the set of indexes with distinct output blocks (counting from the hash function). Let $\mathcal{I}^{\neq}=\left\{i_{1}, i_{2}, \ldots, i_{\gamma}\right\}$ where $i_{1}<i_{2}<\cdots<i_{\gamma}$ and $\gamma \geq 2$. For $i \in \mathcal{I}^{\neq}$, the $i$-th output block is denoted as $Z_{i}$, where $Z_{i}:=V_{i}$ if $i \leq a_{d} ; Z_{i}:=W_{i-a_{d}}$ if $i>a_{d}$, and the data block (AD or ciphertext block) XORed with $Z_{i}$ is denoted as $D_{i}: D_{i}=A_{i+1}$ (if $i \leq a_{d}-2$ ); $D_{a_{d}-1}=\operatorname{ozp}\left(A_{a_{d}}\right) ; D_{a_{d}}=0^{b} ; D_{i}=C_{i-a_{d}}$ (if $\left.a_{d}<i<a_{d}+m_{d}\right) ; D_{a_{d}+m_{d}}=\operatorname{ozp}\left(C_{m_{d}}\right)$.

Then, the collision $S_{1}^{(d)}=S_{1}^{(e)}$ is considered. The collision occurs if and only if $Z_{i_{\gamma}}^{(d)} \oplus D_{i_{\gamma}}^{(d)}=Z_{i_{\gamma}}^{(e)} \oplus D_{i_{\gamma}}^{(e)}$ is satisfied. In order to satisfy the equation, $D_{i_{\gamma}}^{(d)} \neq D_{i_{\gamma}}^{(e)}$ and $Z_{i_{\gamma}}^{(d)} \neq Z_{i_{\gamma}}^{(e)}$ must be satisfied. As $Z_{i_{\gamma}}^{(d)}$ is chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{Z_{i_{\gamma}}^{(e)}\right\}$, we have $\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)}\right]=\operatorname{Pr}\left[Z_{i_{\gamma}}^{(d)} \oplus D_{i_{\gamma}}^{(d)}=\right.$ $\left.Z_{i_{\gamma}}^{(e)} \oplus D_{i_{\gamma}}^{(e)}\right] \leq 1 /\left(2^{b}-1\right)$.

Next, the collision $T_{2}^{(d)}=\hat{T}_{2}^{(d)}$ is considered. The collision is of the form: $\operatorname{lsb}_{\tau}\left(Z_{i_{1}}^{(d)}\right)=\hat{T}_{2}^{(d)} \oplus \operatorname{lsb}_{\tau}\left(\bigoplus_{i \in\left[a_{d}+m_{d}\right] \backslash\left\{i_{1}\right\}} Z_{i}^{(d)} \oplus S_{2}^{(d)}\right)$. As $Z_{i_{1}}^{(d)}$ is chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{Z_{i_{1}}^{(e)}\right\}$, we have $\operatorname{Pr}\left[T_{2}^{(d)}=\hat{T}_{2}^{(d)}\right] \leq 2^{b-\tau} /\left(2^{b}-1\right)$.

These upper-bounds give

$$
\begin{equation*}
\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)} \wedge T_{2}^{(d)}=\hat{T}_{2}^{(d)} \mid \text { Case2-2 }\right] \leq \frac{2^{b-\tau}}{\left(2^{b}-1\right)^{2}} \tag{8}
\end{equation*}
$$

### 4.8 Upper-Bounding $\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)} \wedge T_{2}^{(d)}=\hat{T}_{2}^{(d)} \mid\right.$ Case2-3]

First, the collision $T_{2}^{(d)}=\hat{T}_{2}^{(d)}$ is considered. The collision is of the form $\operatorname{lsb}_{\tau}\left(V_{1}^{(d)}\right)=$ $\hat{T}_{2}^{(d)} \oplus \operatorname{lsb}_{\tau}\left(\bigoplus_{i=2}^{a_{d}} V_{i}^{(d)} \oplus \bigoplus_{i=1}^{m_{d}} W_{i}^{(d)} \oplus S_{2}^{(d)}\right)$. As $V_{1}^{(d)}$ is chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{V_{1}^{(e)}\right\}$ (if the input blocks of $V_{1}^{(d)}$ and $V_{1}^{(e)}$ are the same, $" \backslash\left\{V_{1}^{(e)}\right\} "$ is removed), we have $\operatorname{Pr}\left[T_{2}^{(d)}=\hat{T}_{2}^{(d)}\right] \leq 2^{b-\tau} /\left(2^{b}-1\right)$.

Next, the collision $S_{1}^{(d)}=S_{1}^{(e)}$ is considered. In Case2-3, $S_{1}^{(d)}=S_{1}^{(e)} \Leftrightarrow$ $H_{1}^{(d)}=H_{1}^{(e)} \Leftrightarrow V_{a_{d}}^{(d)}=V_{a_{e}}^{(e)}$ is satisfied. When $a_{d}>a_{e} \geq 1, V_{a_{d}}^{(d)}$ is chosen independently of $V_{1}^{(d)}$, and chosen uniformly at random from $\{0,1\}^{b}$. When $1 \leq a_{d}<a_{e}, V_{a_{e}}^{(e)}$ is chosen independently of $V_{1}^{(d)}$, and chosen uniformly at random from $\{0,1\}^{b}$. Hence, we have $\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)}\right] \leq 1 / 2^{b}$.

These upper-bounds give

$$
\begin{equation*}
\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)} \wedge T_{2}^{(d)}=\hat{T}_{2}^{(e)} \mid \text { Case2-3 }\right] \leq \frac{1}{2^{\tau}\left(2^{b}-1\right)} . \tag{9}
\end{equation*}
$$

### 4.9 Upper-Bounding $\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)} \wedge T_{2}^{(d)}=\hat{T}_{2}^{(d)} \mid\right.$ Case2-4 $]$

First, the collision $S_{1}^{(d)}=S_{1}^{(e)}$ is considered. Let $i=\max \mathcal{I}_{W}^{\neq}$. The collision implies $W_{i}^{(d)} \oplus C_{i}^{(d)}=W_{i}^{(e)} \oplus C_{i}^{(e)}$. As $W_{i}^{(d)}$ are chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{W_{i}^{(e)}\right\}$, we have $\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)}\right] \leq 1 /\left(2^{b}-1\right)$.

Next, the collision $T_{2}^{(d)}=\hat{T}_{2}^{(d)}$ is considered. The collision is of the form $\operatorname{lsb}_{\tau}\left(V_{1}^{(d)}\right)=\hat{T}_{2}^{(d)} \oplus \operatorname{lsb}_{\tau}\left(\bigoplus_{i=2}^{a_{d}} V_{i}^{(d)} \oplus \bigoplus_{i=1}^{m_{d}} W_{i}^{(d)} \oplus S_{2}^{(d)}\right)$. As $V_{1}^{(d)}$ is chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{V_{1}^{(e)}\right\}$ (if the input blocks of $V_{1}^{(d)}$ and $V_{1}^{(e)}$ are the same, " $\backslash\left\{V_{1}^{(e)}\right\}$ " is removed), we have $\operatorname{Pr}\left[T_{2}^{(d)}=\hat{T}_{2}^{(d)}\right] \leq 2^{b-\tau} /\left(2^{b}-1\right)$.

These upper-bounds give

$$
\begin{equation*}
\operatorname{Pr}\left[S_{1}^{(d)}=S_{1}^{(e)} \wedge T_{2}^{(d)}=\hat{T}_{2}^{(d)} \mid \text { Case2-4 }\right] \leq \frac{2^{b-\tau}}{\left(2^{b}-1\right)^{2}} . \tag{10}
\end{equation*}
$$

## 5 PFB $\omega$ : Specification and Security Bounds

We design PFB $\omega$, a TBC-based nAEAD mode with $\omega b$-bit security (under some condition), where $1 \leq \omega$. PFB $\omega$ is an extension of PFB_Plus, and the internal state size is $\omega b$ bits for achieving $\omega b$-bit security. The procedure of updating the first $b$-bit internal state of PFB $\omega$ is designed by using the PFB's idea [34]. The procedure of updating the remaining $(\omega-1) b$-bit internal sate is designed by extending the PMAC_Plus's idea $[45]^{7}$. Using these ideas, the procedure of updating the internal state of $\operatorname{PFB} \omega$ is designed as follows.

- The first $b$-bit internal state is updated by iterating a TBC and absorbing a data block ( $\mathrm{AD} /$ plaintext/ciphertext block), and the output of the last TBC call becomes the first $b$-bit tag. The idea comes from PFB.
- The $i$-th $b$-bit internal state $(2 \leq i \leq \omega)$ is updated by multiplying an output of a TBC with a constant over $G F\left(2^{b}\right)^{*}$ and then XORing the result with the current internal state. This is an extension of the PMAC_Plus's idea. In order to have $\omega b$-bit security, a condition on the constants is required, which is given in the next subsection.

Regarding tweak elements, as PFB_Plus, the nonce and the block counter are injected in order to ensure perfect privacy.

```
Algorithm 2 PFB \(\omega\)
Encryption PFB \(\omega \cdot \operatorname{Enc}\left[\widetilde{E}_{K}\right](N, A, M)\)
    \(M_{1}, \ldots, M_{m} \stackrel{b}{\leftarrow} M ;\left(M_{0}, S_{2}, \ldots, S_{\omega}, a, \ell\right) \leftarrow \operatorname{PFB} \omega \cdot \operatorname{Hash}\left[\widetilde{E}_{K}\right](A, m)\)
    if \(M=\varepsilon\) then \(\{m \leftarrow 0\); goto Step 7\(\}\)
    for \(j=1, \ldots, m\) do
        \(Z_{a+j-1} \leftarrow \widetilde{E}_{K}^{N, a, j, 0}\left(M_{j-1}\right) ; C_{j} \leftarrow Z_{a+j-1} \oplus M_{j}\)
        for \(i=2, \ldots, \omega\) do \(\left\{S_{i} \leftarrow \alpha_{i, a+j-1}^{(\ell)} \cdot Z_{a+j-1} \oplus S_{i}\right\}\)
    end for
    \(S_{1} \leftarrow M_{m} ; Z_{a+m} \leftarrow \widetilde{E}_{K}^{N, a, m, 1}\left(S_{1}\right) ; T_{1} \leftarrow Z_{a+m}\)
    for \(i=2, \ldots, \omega\) do \(\left\{S_{i} \leftarrow \alpha_{i, a+m}^{(\ell)} \cdot Z_{a+m} \oplus S_{i} ; T_{i} \leftarrow \widetilde{E}_{K}^{N, a, m, i}\left(S_{i}\right)\right\}\)
    \(C \leftarrow C_{1}\|\cdots\| C_{m} ; T \leftarrow T_{1}\|\cdots\| T_{\omega} ;\) return \((C, T)\)
```

Decryption PFB $\omega \cdot \operatorname{Dec}\left[\widetilde{E}_{K}\right](N, A, C, \hat{T})$
$\left(M_{0}, S_{2}, \ldots, S_{\omega}, a, \ell\right) \leftarrow \operatorname{PFB} \omega \cdot \operatorname{Hash}\left[\widetilde{E}_{K}\right](A, m) ; C_{1}, \ldots, C_{m} \stackrel{b}{\leftarrow} C$
if $C=\varepsilon$ then $\{m \leftarrow 0$; goto Step 7$\}$
for $j=1, \ldots, m$ do
$Z_{a+j-1} \leftarrow \widetilde{E}_{K}^{N, a, j, 0}\left(M_{j-1}\right) ; M_{j} \leftarrow Z_{a+j-1} \oplus C_{j} ;$
for $i=2, \ldots, \omega$ do $\left\{S_{i} \leftarrow \alpha_{i, a+j-1}^{(\ell)} \cdot W_{a+j-1} \oplus S_{i}\right\}$
end for
$S_{1} \leftarrow M_{m} ; Z_{a+m} \leftarrow \widetilde{E}_{K}^{N, a, m, 1}\left(S_{1}\right) ; T_{1} \leftarrow Z_{a+m}$
for $i=2, \ldots, \omega$ do $\left\{S_{i} \leftarrow \alpha_{i, a+m}^{(\ell)} \cdot Z_{a+m} \oplus S_{i} ; T_{i} \leftarrow \widetilde{E}_{K}^{N, a, m, i}\left(S_{i}\right)\right\}$
$T \leftarrow T_{1}\|\cdots\| T_{\omega} ;$ if $T=\hat{T}$ then return $M \leftarrow M_{1}\|\cdots\| M_{m} ;$ else return reject
Hash PFB $\omega$. Hash $\left[\widetilde{E}_{K}\right](A, m)$
if $A=\varepsilon$ then return $\left(0^{b}, \ldots, 0^{b}, 0, m\right)$
$Z_{0} \leftarrow 0^{b} ; A_{1}, \ldots, A_{a} \stackrel{b}{\leftarrow} A ; \ell \leftarrow a+m ;$ for $i=2, \ldots, \omega$ do $H_{i} \leftarrow 0^{b}$
for $j=1, \ldots, a-1$ do
$Z_{j} \leftarrow \widetilde{E}_{K}^{0^{n}, j, 0,0}\left(Z_{j-1} \oplus A_{j}\right)$; for $i=2, \ldots, \omega$ do $H_{i} \leftarrow \alpha_{i, j}^{(\ell)} \cdot Z_{j} \oplus H_{i}$
end for
$H_{1} \leftarrow Z_{a-1} \oplus A_{a}$ return $\left(H_{1}, \ldots, H_{\omega}, a, \ell\right)$

### 5.1 Specification

For the sake of simplifying the specification and the security proof, we consider only the case where the bit lengths of AD and plaintext/ciphertext are multiple of $b$, i.e., $|A| \bmod b=0,|M| \bmod b=0$ and $|C| \bmod b=0$. Note that arbitrary length data can be handled by introducing the one-zero padding ozp as PFB_Plus, and an extra TBC call by the padding can be avoided by adding 2 bits to the tweak space for distinguishing whether the padding is applied or not for each of AD and plaintext/ciphertext.

The specification of PFB $\omega$ is given in Algorithm 2 and is illustrated in Fig. 2.
Let $a_{\max }$ be a maximum number of AD blocks, i.e., $a \leq a_{\max }$, and $m_{\max }$ be a maximum number of plaintext/ciphertext blocks, i.e., $m \leq m_{\max }$. The

[^2]

Fig. 2. PFB $\omega$.Enc and PFB $\omega$. Hash.
tweak space $\mathcal{T} \mathcal{W}$ consists of a nonce space $\mathcal{N}:=\{0,1\}^{n}$, a counter space for AD blocks ( $a_{\text {max }}$ ], a counter space for plaintext/ciphertext blocks ( $m_{\text {max }}$ ], and a space for tweak separations $(\omega]$. Hence, the tweak space is defined as $\mathcal{T} \mathcal{W}:=$ $\mathcal{N} \times\left(a_{\max }\right] \times\left(m_{\max }\right] \times(\omega]$. Let $\alpha_{i, j}^{(\ell)}$ be a $b$-bit constant in $G F\left(2^{b}\right)^{*}$ with the following condition.

- Cond: for any $1 \leq \ell \leq a_{\text {max }}+m_{\text {max }}$, a $\omega-1 \times \ell$ matrix with an $i$-th row and $j$-th column element $\alpha_{i, j}^{(\ell)}$ is MDS, i.e., for any $1 \leq \mu \leq \min \{\ell, \omega-1\}$, $2 \leq i_{1}<i_{2}<\cdots<i_{\mu} \leq \omega$, and $1 \leq j_{1}<j_{2}<\cdots<j_{\mu} \leq \ell$, the rank of the $\mu \times \mu$ sub-matrix where for each $u, v \in[\mu]$, the $u$-th row and $v$-th column element is $\alpha_{i_{u}, j_{v}}^{(\ell)}$ is $\mu$.
Examples of constants for $\omega=2,3$ are given below.
$-\omega=2: \alpha_{2, j}^{(\ell)}:=1$ for all $\ell, j$. The second $b$-bit internal state is updated by XORing all outputs of TBC calls. This is the same as the PFB_Plus's internal state updating (without truncations).
$-\omega=3: \alpha_{2, j}^{(\ell)}:=1$ and $\alpha_{3, j}^{(\ell)}:=2^{\ell-j}$ for all $\ell, j$. This is the same as the PMAC_Plus's internal state updating.


### 5.2 Privacy and Authenticity Bounds of PFB $\omega$

Theorem 2.

$$
\begin{array}{rl}
\mathbf{A d v} & \mathrm{PFB} \omega\left[\widetilde{E}_{K}\right] \\
\text { priv } \left._{\mathcal{E}}, t\right) & \leq \mathbf{A d v}_{\widetilde{E}_{K}}^{\mathrm{tprp}}\left(\sigma_{\mathcal{E}}, t+O\left(\sigma_{\mathcal{E}}\right)\right) \\
\mathbf{A d v}_{\mathrm{PFB} \omega\left[\widetilde{E}_{K}\right]}^{\text {auth }}\left(\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right), t\right) & \leq \frac{2^{\omega} \cdot q_{\mathcal{D}}}{\left(2^{b}-1\right)^{\omega}}+\mathbf{A d v}_{\widetilde{E}_{K}}^{\mathrm{tprp}}(\sigma, t+O(\sigma))
\end{array}
$$

## 6 Proof of Theorem 2

 The replacement offers the TPRP-terms $\mathbf{A} \mathbf{d v}_{\widetilde{E}_{K}}^{\text {tprp }}\left(\sigma_{\mathcal{E}}, t+O\left(\sigma_{\mathcal{E}}\right)\right)$ and $\mathbf{A} \mathbf{d v}_{\widetilde{E}_{K}}^{\text {tprp }}(\sigma, t+$ $O(\sigma))$ in the upper-bounds, and then the remaining works are to upper-bound the advantages $\mathbf{A d} \mathbf{v}_{\mathrm{PFB} \omega[\widetilde{P}]}^{\text {priv }}\left(\sigma_{\mathcal{E}}\right)$ and $\mathbf{A d v} \underset{\mathrm{PFB} \omega[\widetilde{P}]}{\text { auth }}\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right)$, where adversaries are computationally unbounded algorithms and the complexities are solely measured by the numbers of queries. Without loss of generality, adversaries are deterministic.

Regarding $\mathbf{A d v}_{\operatorname{PFB} \omega[\widetilde{P}]}^{\text {priv }}\left(\sigma_{\mathcal{E}}\right)$, as tweaks of $\widetilde{P}$ are all distinct, all output blocks of $\widetilde{P}$ defined by encryption queries are chosen independently and uniformly at random from $\{0,1\}^{b}$. We thus have $\operatorname{Adv}_{\operatorname{PFB} \omega[\widetilde{P}]}^{\text {priv }}\left(\sigma_{\mathcal{E}}\right)=0$.

Hereafter, we focus on upper-bounding $\operatorname{Adv}_{\operatorname{PFB} \omega[\widetilde{P}]}^{\text {auth }}\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right)$.

### 6.1 Upper-Bonding $\operatorname{Adv}_{\operatorname{PFB} \omega[\widetilde{P}]}^{\text {auth }}\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right)$

We first fix a decryption query $\left(N^{(d)}, A^{(d)}, C^{(d)}, \hat{T}^{(d)}\right)$ and upper-bound the probability that A forges at the decryption query. Values/variables corresponding with the decryption query are denoted by using the superscript of $(d)$ such as $N^{(d)}, M^{(d)}$, etc. The lengths $a, m$ and $\ell$ are denoted by $a_{d}, m_{d}$ and $\ell_{d}$, respectively. Thus $\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)}\right]$ is upper-bounded in the analysis. Similarly, for an encryption query $\left(N^{(e)}, A^{(e)}, M^{(e)}\right)$, values/variables corresponding with the decryption query are denoted by using the superscript of (e), and the lengths $a, m$ and $\ell$ are denoted by $a_{e}, m_{e}$ and $\ell_{e}$, respectively.

Then, $\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)}\right]$ is upper-bounded using the following two cases.

- Case1: $\forall$ enc. query $\left(N^{(e)}, A^{(e)}, M^{(e)}\right): N^{(e)} \neq N^{(d)} \vee a_{e} \neq a_{d} \vee m_{e} \neq m_{d}$.
- Case2: ヨenc. query $\left(N^{(e)}, A^{(e)}, M^{(e)}\right)$ s.t. $N^{(e)}=N^{(d)} \wedge a_{e}=a_{d} \wedge m_{e}=m_{d}$.

Using these cases, we have

$$
\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)}\right] \leq \max \left\{\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \text { Case1 }\right], \operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \text { Case2 }\right]\right\}
$$

These probabilities are analyzed in Subsects. 6.2 and 6.3, respectively. The upper-bounds are given in Eqs. (12) and (13), respectively, and give

$$
\begin{equation*}
\mathbf{A d v}_{\mathrm{PFB} \omega[\widetilde{P}]}^{\mathrm{auth}}\left(q_{\mathcal{E}}, q_{\mathcal{D}}, \sigma\right) \leq \frac{2^{\omega} \cdot q_{\mathcal{D}}}{\left(2^{b}-1\right)^{\omega}} \tag{11}
\end{equation*}
$$

### 6.2 Upper-Bounding $\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid\right.$ Case1 $]$

In Case1, tag blocks $T_{1}^{(d)}, T_{2}^{(d)}, \ldots, T_{\omega}^{(d)}$ are chosen independently and uniformly at random from $\{0,1\}^{b}$. Hence, we have

$$
\begin{equation*}
\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \text { Case1 }\right] \leq \frac{1}{2^{\omega b}} . \tag{12}
\end{equation*}
$$

### 6.3 Upper-Bounding $\operatorname{Pr}\left[\hat{\boldsymbol{T}}^{(d)}=\boldsymbol{T}^{(d)} \mid\right.$ Case2 $]$

Let $\left(N^{(e)}, A^{(e)}, M^{(e)}\right)$ be an encryption query with $N^{(e)}=N^{(d)} \wedge a_{e}=a_{d} \wedge m_{e}=$ $m_{d}$. The analysis considers the following sub-cases where $0 \leq \mu \leq \omega$.

Case2- $\mu: \exists \mu$ indexes $i_{1}<\cdots<i_{\mu}$ s.t. $\left(\forall i \in\left[i_{1}, \ldots, i_{\mu}\right]: S_{i}^{(d)}=S_{i}^{(e)}\right) \wedge$

$$
\left(\forall i \in[\omega] \backslash\left\{i_{1}, \ldots, i_{\mu}\right\}: S_{i}^{(d)} \neq S_{i}^{(e)}\right) .
$$

Using the sub-cases, we have

$$
\begin{align*}
& \operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \mid \text { Case2 }\right] \leq \sum_{\mu=0}^{\omega} \operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \wedge \text { Case2- } \mu \mid \text { Case2 }\right] \\
& \leq \frac{1}{\left(2^{b}-1\right)^{\omega}}+\sum_{\mu=1}^{\omega} 2 \cdot\binom{\omega-1}{\mu-1} \cdot \frac{1}{\left(2^{b}-1\right)^{\omega}} \leq \frac{2^{\omega}}{\left(2^{b}-1\right)^{\omega}} \tag{13}
\end{align*}
$$

The probabilities $\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \wedge\right.$ Case2- $\mu \mid$ Case2 $]$ for $0 \leq \mu \leq \omega$ are upperbounded below. In the analyses, the following set is used: $\mathcal{I}^{\neq}=\left\{j \mid Z_{j}^{(e)} \neq Z_{j}^{(d)}\right\}$.

- $\mu=0$. In this case, for all $i, S_{i}^{(d)} \neq S_{i}^{(e)}$ is satisfied, and thus $T_{i}^{(d)}$ is chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{T_{i}^{(e)}\right\}$ (as both $T_{i}^{(e)}$ and $T_{i}^{(d)}$ are defined by the same permutation $\left.\widetilde{P}^{N^{(d)}, a_{d}, m_{d}, i}\right)$. Hence, we have

$$
\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \wedge \text { Case2-0 } \mid \text { Case2 }\right] \leq \frac{1}{\left(2^{b}-1\right)^{\omega}}
$$

- $1 \leq \mu \leq \omega-1 \wedge S_{1}^{(d)}=S_{1}^{(e)}$. Note that one has $i_{1}=1$. First, $\mu-1$ indexes $1<i_{2}<\cdots<i_{\mu}$ are fixed, and the following case is considered:
$-\forall i \in\left\{1, i_{2}, \ldots, i_{\mu}\right\}: S_{i}^{(d)}=S_{i}^{(e)}$ is satisfied, and
$-\forall i \in[\omega] \backslash\left\{1, i_{2}, \ldots, i_{\mu}\right\}: S_{i}^{(d)} \neq S_{i}^{(e)}$ is satisfied.
For each $i \in[\omega] \backslash\left\{1, i_{2}, \ldots, i_{\mu}\right\}, T_{i}^{(d)}$ is chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{T_{i}^{(e)}\right\}$, we have $\operatorname{Pr}\left[\forall i \in[\omega] \backslash\left\{1, i_{2}, \ldots, i_{\mu}\right\}: T_{i}^{(d)}=\hat{T}_{i}^{(d)}\right] \leq 1 /\left(2^{b}-1\right)^{\omega-\mu}$.

Next, the collisions $S_{i}^{(d)}=S_{i}^{(e)}$ where $i \in\left\{1, i_{2}, \ldots, i_{\mu}\right\}$ are considered. Let $\mathcal{I}^{\neq}=\left\{j_{1}, \ldots, j_{\gamma}\right\}$ such that $j_{1}<\cdots<j_{\gamma}\left(\right.$ note that $\left.\forall j \in \mathcal{I}^{\neq}: Z_{j}^{(d)} \neq Z_{j}^{(e)}\right)$. The collisions are of the following forms:

$$
S_{1}^{(d)}=S_{1}^{(e)} \Leftrightarrow \underbrace{Z_{j_{\gamma}}^{(d)} \oplus Z_{j_{\gamma}}^{(e)}}_{=: Z_{j_{\gamma}}}=D_{j_{\gamma}+1}^{(d)} \oplus D_{j_{\gamma}+1}^{(e)}
$$

where $D_{j_{\gamma}+1} \in\left\{A_{j_{\gamma}+1}, C_{j_{\gamma}-a+1}\right\}$, and for $i \in\left\{i_{2}, \ldots, i_{\mu}\right\}$,

$$
S_{i}^{(d)}=S_{i}^{(e)} \Leftrightarrow \alpha_{i, j_{1}}^{\left(\ell_{d}\right)} \cdot \underbrace{\left(Z_{j_{1}}^{(e)} \oplus Z_{j_{1}}^{(d)}\right)}_{=: Z_{j_{1}}} \oplus \cdots \oplus \alpha_{i, j_{\gamma}}^{\left(\ell_{d}\right)} \cdot \underbrace{\left(Z_{j_{\gamma}}^{(e)} \oplus Z_{j_{\gamma}}^{(d)}\right)}_{=: Z_{j_{\gamma}}}=0^{b}
$$

If $\gamma \leq \mu-1$, by Cond, the collisions $S_{i}^{(d)}=S_{i}^{(e)}$ where $i \in\left\{i_{2}, \ldots, i_{\mu}\right\}$ offer a unique solution $\left(Z_{j_{1}}, \ldots, Z_{j_{\gamma}}\right)=\left(0^{b}, \cdots, 0^{b}\right)$. Hence, the collisions do not occur. If $\gamma \geq \mu$, then the collision $S_{1}^{(d)}=S_{1}^{(e)}$ offers a solution $Z_{j_{\gamma}}=D_{j_{\gamma}+1}^{(d)} \oplus D_{j_{\gamma}+1}^{(e)}$. The collisions $S_{i_{2}}^{(d)}=S_{i_{2}}^{(e)}, \ldots, S_{i_{\mu}}^{(d)}=S_{i_{\mu}}^{(e)}$, fixing $Z_{j_{\omega}}, \ldots, Z_{j_{\gamma-1}}$, offer a unique solution for $\left(Z_{j_{1}}, \ldots, Z_{j_{\omega}-1}\right)$ by Cond. Since for each $j \in\left\{j_{1}, \ldots, j_{\omega-1}, j_{\gamma}\right\}, Z_{j}^{(d)}$ is chosen uniformly at random from $\{0,1\}^{b} \backslash\left\{Z_{j}^{(e)}\right\}$, we have $\operatorname{Pr}\left[\forall i \in\left\{1, i_{2}, \ldots, i_{\mu}\right\}\right.$ : $\left.S_{i}^{(d)}=S_{i}^{(e)}\right] \leq 1 /\left(2^{b}-1\right)^{\mu}$.

These upper-bounds give

$$
\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \wedge \text { Case2- } \mu \mid \text { Case2 }\right] \leq\binom{\omega-1}{\mu-1} \cdot \frac{1}{\left(2^{b}-1\right)^{\omega}} .
$$

- $1 \leq \mu \leq \omega-1 \wedge S_{1}^{(d)} \neq S_{1}^{(e)}$ : This analysis is the same as that of the case: $1 \leq \mu \leq \omega-1 \wedge S_{1}^{(d)}=S_{1}^{(e)}$. $\mu$ indexes $1<i_{1}<i_{2}<\cdots<i_{\mu}$ are fixed, and the following case is considered:
$-\forall i \in\left\{i_{1}, i_{2}, \ldots, i_{\mu}\right\}: S_{i}^{(d)}=S_{i}^{(e)}$ is satisfied, and
$-\forall i \in[\omega] \backslash\left\{i_{1}, i_{2}, \ldots, i_{\mu}\right\}: S_{i}^{(d)} \neq S_{i}^{(e)}$ is satisfied.
Using the same analysis, we have $\operatorname{Pr}\left[\forall i \in\left\{i_{1}, i_{2}, \ldots, i_{\mu}\right\}: S_{i}^{(d)}=S_{i}^{(e)}\right] \leq 1 /\left(2^{b}-\right.$ $1)^{\mu}$, and $\operatorname{Pr}\left[\forall i \in[\omega] \backslash\left\{i_{1}, i_{2}, \ldots, i_{\mu}\right\}: T_{i}^{(d)}=\hat{T}_{i}^{(d)}\right] \leq 1 /\left(2^{b}-1\right)^{\omega-\mu}$. These upperbounds give

$$
\operatorname{Pr}\left[T^{(d)}=\hat{T}^{(d)} \wedge \text { Case2- } \mu \mid \text { Case2 }\right] \leq\binom{\omega-1}{\mu-1} \cdot \frac{1}{\left(2^{b}-1\right)^{\omega}} .
$$

## 7 SKINNY-64-256

SKINNY [8] is a tweakable block cipher adopting the tweakey framework [27] that treats the key input and the tweak input in the same way. The combined state is called tweakey which does not make a particular distinction about which part is used as a key and the tweak. For the 64-bit block, SKINNY supports the tweakey sizes up to 192 bits, (i.e. SKINNY-64-192) while what we need is SKINNY-64-256. In Sect 7.1, we show how to extend the design of SKINNY to support a 256 -bit tweakey. The rationale of our design choices are explained in Sect. 7.2. Security evaluation of SKINNY-64-256 is given in Sect. 7.3.

### 7.1 Specification

Round Transformation. We only briefly recall the round transformation of SKINNY-64-256 because SKINNY-64-256 does not modify the round transformation. Refer to the original SKINNY document [8] for the details of each operation.

The 64 -bit internal state is viewed as a $4 \times 4$ square array of nibbles. SKINNY-64-256 consists of 44 rounds, in which one round transformation is defined as an application of the following 5 operations: SubCells, AddRoundConstant, AddRoundTweakey, ShiftRows and MixColumns.

SubCells. A 4-bit S-box is applied for each nibble.
AddRoundConstant. A 6-bit constant generated by an LFSR and a single fixed bit are XORed to the top three rows of the first column.
AddRoundTweakey. The top two rows of all tweakey arrays are extracted and XORed to the top two rows of the state.
ShiftRows. Each nibble in row $i$ is rotated by $i$ positions to the right.
MixColumns. Each column is multiplied by a $4 \times 4$ binary matrix.
New Tweakey Schedule. The 256 -bit tweakey state consists of four $4 \times 4$ square arrays of nibbles. Each of them are called TK1, TK2, TK3 and TK4.

The tweakey states are updated as follows. First, a permutation $P_{T}$ is applied on the nibble positions of all tweakey arrays $T K 1, T K 2, T K 3$, and $T K 4$, where $P_{T}$ is defined as $(0, \ldots, 15) \stackrel{P_{T}}{\longrightarrow}(9,15,8,13,10,14,12,11,0,1,2,3,4,5,6,7)$.

Finally, every nibble of the first and second rows of TK2,TK3, and TK4 are individually updated with the following LFSRs.

$$
\begin{aligned}
& \text { TK2 }:\left(x_{3}\left\|x_{2}\right\| x_{1} \| x_{0}\right) \rightarrow\left(x_{2}\left\|x_{1}\right\| x_{0} \| x_{3} \oplus x_{2}\right) \\
& \text { TK3 }:\left(x_{3}\left\|x_{2}\right\| x_{1} \| x_{0}\right) \rightarrow\left(x_{0} \oplus x_{3}\left\|x_{3}\right\| x_{2} \| x_{1}\right) \\
& \text { TK4: }\left(x_{3}\left\|x_{2}\right\| x_{1} \| x_{0}\right) \rightarrow\left(x_{2}\left\|x_{1}\right\| x_{2} \oplus x_{0} \| x_{3} \oplus x_{2} \oplus x_{1}\right)
\end{aligned}
$$

### 7.2 Rationale for Newly Designed Parts

Design from Scratch vs Extension of the Original. The designers of SKINNY first searched for good parameters of ShiftRows and MixColumns to maximize the security in the single-key setting, and then later searched for the tweakey schedule to maximize the security in the TK2 and TK3 settings. Later Nikolić searched for better parameters to achieve higher number of active S-boxes [36]. The first choice we made is whether we should search for good parameters for TK4 from scratch as Nikolić did or we should extend the original SKINNY that was optimized for TK1, TK2, and TK3. In the end, we determined to design SKINNY-64-256 as a natural extension of the original SKINNY-64, i.e. not modify any components to realize $T K 1, T K 2$, and $T K 3$, though we do not have any application to use smaller tweakey sizes. This is all for higher reliability. The original SKINNY has received a lot of cryptanalytic effort by third-party and seems to generate a consensus that the design choice of SKINNY is conservative, and thus secure. We would like to design SKINNY-64-256 so that those existing results contribute to the reliability of the security of SKINNY-64-256.
Number of Rounds. Once the above strategy was established, the only components we need to design are an LFSR to update the $T K 4$ state and the number of rounds. In SKINNY, the number of rounds for TK1, TK2 and TK3 are defined to be 32,36 , and 40 , respectively. As mentioned above, those choices look quite conservative. Indeed, the maximum number of attacked rounds so far is 19 for TK1 by related-tweaky impossible differential attacks [30, 41], 23 for TK2 by related-tweaky impossible differential attacks [5, 30, 41], and 27 for TK3 by a related-tweakey rectangle attack [30]. This made us think about not increasing
the number of rounds from TK3. In the end, to be consistent with the first decision, i.e. to make it a natural extension of the original SKINNY, we determined to keep the same rate for increasing the number of rounds, namely 44 for $T K 4$.
LFSR for $\boldsymbol{T K}$. To be a secure instantiation of the tweakey framework [27], the LFSR must have a cycle of 15. The original LFSRs in SKINNY for TK2 and TK3 are quite efficient: they only require a single XOR to the LFSR. By the exhaustive search, We found that there is no more LFSR achieving cycle 15 only with a single XOR. Moreover, we found that

- there is no LFSR having cycle 15 even with two XORs.
- it is impossible to achieve cycle 15 only by updating one output bit

In the end, we picked up the LFSR that updates 2 output bits with 3 XORs.

### 7.3 Bounds of the Number of Active S-boxes

Bounds for SKINNY-64-256 (TK4). The designers of SKINNY evaluated the tight bounds of the number of active S-boxes by using Mixed Integer Linear Programming (MILP) by describing how to model the problem in details. We extended their MILP model to derive the number of active S-boxes of SKINNY-64-256 (in TK4). The lower bounds of the number of active S-boxes for SK, TK1, TK2, TK3 and TK4 are compared in Table 2. Note that according to the designers, MILP sometimes took too long, and the designers only could give upper bounds of the number of active S-boxes in such cases. The upper bounds are denoted with the upper bar in Table 2.

Table 2 shows that TK4 is a natural extension of TK3 also for the increase of the bounds. In particular, the comparison is clear in the following part.

- The bounds for 21 to 24 rounds for TK2 are 59, 64, 67, and 72, respectively.
- The bounds for 24 to 27 rounds for TK3 are 58, 60, 65, and 72, respectively.
- The bounds for 27 to 30 rounds for TK4 are 58, 62, 66, and 72, respectively.

The bounds for $r$ rounds in TK2, $r+3$ rounds in TK3, and $r+6$ rounds in TK4 are almost the same. This also implies that our choice of the total number of rounds ( 44 rounds for TK4, while 40 rounds for TK3 and 36 rounds for TK2) is quite reasonable.

To be more precise, the designers of SKINNY need to ensure at least 64 active S-boxes because their 8 -bit S-box for 128 -bit block versions also allows differential propagation with probability $2^{-2}$. For SKINNY-64, to ensure at least 32 active S-boxes is sufficient to resist a single differential characteristic, which is ensured only by 20 rounds even in TK4. Hence, our choice of 44 rounds is more conservative than the original SKINNY supporting the 128 -bit block.

Deriving Tight Bounds for Linear Cryptanalysis. As mentioned above, the designers sometimes could not derive the tight bounds. Alfarano et al. [4] later identified the tight bound for differential cryptanalysis in SK, but did not

Table 2. Lower bounds on the number of active Sboxes. The numbers for SK, TK1, TK2, TK3 and Lin are from the evaluation by the designers [8], where numbers with upper line are the upper bounds. SK[4] shows the updated tight bounds by Alfarano et al. [4]. Numbers for Lin' and TK4 were derived by us.

|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 4}$ | $\mathbf{1 5}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| SK | 1 | 2 | 5 | 8 | 12 | 16 | 26 | 36 | 41 | 46 | 51 | 55 | 58 | 61 | 66 |
| TK1 | 0 | 0 | 1 | 2 | 3 | 6 | 10 | 13 | 16 | 23 | 32 | 38 | 41 | 45 | 49 |
| TK2 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 9 | 12 | 16 | 21 | 25 | 31 | 35 |
| TK3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 10 | 13 | 16 | 19 | 24 |
| TK4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 3 | 6 | 9 | 12 | 16 |
| Lin | 1 | 2 | 5 | 8 | 13 | 19 | 25 | 32 | 38 | 43 | 48 | 52 | 55 | 58 | 64 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $\mathbf{1 6}$ | $\mathbf{1 7}$ | $\mathbf{1 8}$ | $\mathbf{1 9}$ | $\mathbf{2 0}$ | $\mathbf{2 1}$ | $\mathbf{2 2}$ | $\mathbf{2 3}$ | $\mathbf{2 4}$ | $\mathbf{2 5}$ | $\mathbf{2 6}$ | $\mathbf{2 7}$ | $\mathbf{2 8}$ | $\mathbf{2 9}$ | $\mathbf{3 0}$ |
| SK | 75 | 82 | 88 | 92 | 96 | 102 | 108 | $\overline{114}$ | $\overline{116}$ | $\overline{124}$ | $\overline{132}$ | $\overline{138}$ | $\overline{136}$ | $\overline{148}$ | $\overline{158}$ |
| SK[4] 75 | 82 | 88 | 92 | 96 | 102 | 108 | 112 | 116 | 124 | 128 | 132 | 136 | 142 | 148 |  |
| TK1 | 54 | 59 | 62 | 66 | 70 | 75 | 79 | 83 | 85 | 88 | 95 | 102 | $\overline{108}$ | $\overline{112}$ | $\overline{120}$ |
| TK2 | 40 | 43 | 47 | 52 | 57 | 59 | 64 | 67 | 72 | 75 | 82 | 85 | 88 | 92 | 96 |
| TK3 | 27 | 31 | 35 | 43 | 45 | 48 | 51 | 55 | 58 | 60 | 65 | 72 | 77 | 81 | 85 |
| TK4 | 19 | 21 | 24 | 30 | 35 | 39 | 41 | 43 | 46 | 50 | 54 | 58 | 62 | 66 | 72 |
| Lin | 70 | 76 | 80 | 85 | 90 | 96 | 102 | 107 | $\overline{110}$ | $\overline{118}$ | $\overline{122}$ | $\overline{128}$ | $\overline{136}$ | $\overline{141}$ | $\overline{143}$ |
| Lin | 70 | 76 | 80 | 85 | 90 | 96 | 102 | 107 | 110 | 115 | 121 | 127 | 130 | 135 | 141 |

show the bound for linear cryptanalysis. To present a better picture, we tried to derive the tight bounds.

Our approach is to apply the combination of Matsui's search strategy [31] with MILP proposed by Zhang et al. [46]. In short, this considers the bound derived for $r-1$ rounds to efficiently search for the bounds for $r$ rounds. In more precise, it restricts the sum of the number of active S-boxes from round 1 to round $r-1$ and from round 2 to round $r$. This small changes actually allowed us to derive the tight bounds for $\operatorname{Lin}$ up to 30 rounds.

## 8 Hardware Performance Evaluation

We evaluate the hardware performance of PFB_Plus combined with SKINNY-64256 and compare it with a conventional BBB scheme, namely PFB.
Choice of Competitor. We choose PFB as a competitor in hardware performance evaluation because (i) it is the scheme PFB_Plus based on, and (ii) it shows the best performance in TI at the time of writing [34]. To achieve the same security level, we use a 128-bit variant of the SKINNY family, namely SKINNY-128-256 as an underlying cipher.
Design Policy. We follow the design policy for the previous PFB implementation [34]. The design defines a set of commands for processing block-aligned data, and an external microcontroller is supposed to dispatches the commands

Table 3. Comparison of the number of registers with and without TI. We implement and evaluate the ones with 128-bit security (PFB_Plus[SKINNY-64-256] and PFB[SKINNY-128-256]) in this section. The table also shows the conventional ones with 64-bit security (PFB[SKINNY-64-192] and SAEB[GIFT-128-128]) [34] for comparison.

| Name | TI | Sec. | Total | State | Key | Tweak | Tag | Ref. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| PFB_Plus[SKINNY-64-256] | - | 128 | 386 | 64 | 128 | 128 | 64 | Ours |
| PFB[SKINNY-128-256] | - | 128 | 386 | 128 | 128 | 128 | 0 | Ours |
| PFB_Plus[SKINNY-64-256] | $\checkmark$ | 128 | 704 | 192 | 256 | 128 | 128 | Ours |
| PFB[SKINNY-128-256] | $\checkmark$ | 128 | 768 | 384 | 256 | 128 | 0 | Ours |
| PFB[SKINNY-64-192] | - | 64 | 256 | 64 | 128 | 64 | 0 | $[34]$ |
| SAEB[GIFT-128-128] | - | 64 | 256 | 128 | 128 | 0 | 0 | $[34]$ |
| PFB[SKINNY-64-192] | $\checkmark$ | 64 | 512 | 192 | 256 | 64 | 0 | $[34]$ |
| SAEB[GIFT128-128] | $\checkmark$ | 64 | 640 | 384 | 256 | 0 | 0 | $[34]$ |

in an appropriate order to realize AD processing, encryption, and decryption of AEAD. The design aims to accelerate the main processing part, while the microcontroller is responsible for preparing the block-aligned data by padding and choosing an appropriate ID. The designs store a key, nonce, and tweak in its internal registers, and can process multiple data blocks without feeding the data redundantly. For the purpose, the tweak is updated in place by integrated nonce-updating circuitry.

Side-Channel Attack Countermeasure. We implement unprotected and protected designs for each of the algorithms. For protected implementation, we implement 3 -share TI secure up to the first-order attacks. For protected implementations, we also protect the on-the-fly tweakey schedule considering a profiling attack ${ }^{8}$.

Register Cost. We first compare the register costs of PFB_Plus[SKINNY-64256] and PFB[SKINNY-128-256] with and without TI in Table 3. The table also shows PFB[SKINNY-64-192] and SAEB[GIFT-128-128] in the previous work [34] for comparison. Without TI, the security level determines the register cost: the ones with 128- and 64 -bit security need 386 and 256 bits of registers, respectively. With TI, on the other hand, PFB_Plus[SKINNY-64-256] uses a smaller number of registers than PFB[SKINNY-128-256]. The difference comes from the different number of shares for each component: the state needs three shares, while the key and tag need only two shares because the operation is linear. There are 2 -share masking schemes that can protect the state with two shares [15], but we do not consider them because they need fresh randomness during the execution and the cost for random number generation is overwhelming [42].

[^3]

Fig. 3. Hardware architecture of PFB_Plus[SKINNY-64-256]. $f$ and $g$ functions are the decomposed 4-bit S-box [8].

### 8.1 PFB_Plus with SKINNY-64-256

Tweakey Configuration. We use the tweakey TK1 and TK2 for storing a 128-bit secret key, and TK3 and TK4 for a tweak. The tweak comprises the 4 -bit ID $x, 96$-bit nonce $N$, and a 28 -bit counter ctr. TK3 and TK4 combined store these values as:

$$
\begin{equation*}
\mathbf{T K 3}\left\|\mathbf{T K 4}=\operatorname{str}_{4}(x)\right\| \operatorname{str}_{96}(N) \| \operatorname{str}_{28}(c t r) \tag{14}
\end{equation*}
$$

Circuit Architecture for SKINNY-64-256. Following the conventional SKINNY implementations, we use the nibble-serial architecture based on 2dimensional arrays of scan flip-flops [32, 8, 34] with the decomposed 4-bit Sbox ( $f$ and $g$ functions) integrated. The design uses in-place on-the-fly tweakey schedule capable of reverting it to the original state after the final round [34]. Moreover, the TK4 array has an integrated 28-bit adder for incrementing ctr in place.
Circuit Architecture for Mode of Operation. PFB_Plus is a thin wrapper on top of the SKINNY-64-256 circuit similar to the conventional PFB implementation. The shift register ( $4 \times 16$ bits) with a feedback XOR realizes the tag accumulator.
Latency. The design finishes the round function in 16 cycles, and the entire SKINNY-64-256 in $704(=16 \times 44)$ cycles. With one more cycle for updating the tweak, the circuit consumes a single-block message with 705 cycles.
Sharing. Fig. 3 shows the number of shares in the protected implementation. As mentioned in the previous section, the implementation is heterogeneous in terms of the number of shares: (I) there is no sharing on TK3 and TK4 storing the public tweak, (II) TK1, TK2, and the tag accumulator use 2-share representation as they use linear operations only, and (III) the state array that goes through the non-linear S-box operation has three shares.


Fig. 4. Hardware architecture of PFB[SKINNY-128-256]. $f, g, h$, and $i$ functions are the decomposed 8-bit S-box [8].

### 8.2 PFB with SKINNY-128-256

Tweakey Configuration. The first tweakey array TK1 stores a 128-bit secret key, and another tweakey TK2 stores a tweak comprising the 3-bit ID $x$, 96 -bit nonce $N$, and a 29 -bit counter ctr:

$$
\begin{equation*}
\mathbf{T K 2}=\operatorname{str}_{3}(x)\left\|\operatorname{str}_{96}(N)\right\| \operatorname{str}_{29}(c t r) \tag{15}
\end{equation*}
$$

Circuit Architecture for SKINNY-128-256. Fig. 4 shows the circuit architecture of PFB_Plus with SKINNY-128-256. The circuit architecture of SKINNY-128-256 follows the previous implementation [8]: the byte-serial architecture with the decomposed 8-bit S-box ( $f, g, h$, and $i$ functions) integrated into the state array. The TK1 and TK2 arrays have the same structure as the SKINNY-64256 circuit (see Fig. 3), and support in-pace tweak updating and reverting after on-the-fly tweakey schedule.
Circuit Architecture for Mode of Operation. The circuit architecture for PFB is similar to the previous PFB_Plus and also the conventional implementation [34].
Latency. The design finishes the SKINNY-64-256 encryption in $768(=16 \times$ $44+1)$ cycles.
Sharing. This circuit also has a heterogeneous sharing, as shown in Fig. 4: (I) there is no sharing on TK2 storing the public tweak, (II) the secret key in TK1 represented by two shares, and (III) the state array in three shares.

### 8.3 Performance Evaluation and Comparison

Implementation and Evaluation Procedure. We implemented the designs in the register-transfer level with a single exception: explicit instantiation of scan flip-flops following the previous works [32]. We synthesized the design using Synopsys Design Compiler with the NanGate 45-nm standard cell library [35] while preserving the structure of major components, as shown in Table 4.

Table 4. Circuit area breakdown of PFB_Plus[SKINNY-64-256] and PFB[SKINNY-128256]

| Target | Component | Circuit area [GE] |  |
| :--- | :--- | ---: | ---: |
|  |  | Normal | TI |
| PFB_Plus with | Total | $\mathbf{4 , 3 5 1}$ | $\mathbf{7 , 4 3 9}$ |
| SKINNY-64-256 | Total/Shift register | 444 | 888 |
|  | Total/TBC | 3,713 | 6,292 |
|  | Total/TBC/State | 532 | 1,646 |
|  | Total/TBC/Key | 1,268 | 2,620 |
|  | Total/TBC/Tweak | 1,551 | 1,551 |
| PFB with | Total | $\mathbf{4 , 4 0 0}$ | $\mathbf{8 , 4 4 8}$ |
| SKINNY-128-256 | Total/TBC | 4,218 | 8,159 |
|  | Total/TBC/State | 1,098 | 3,517 |
|  | Total/TBC/Key | 1,224 | 2,546 |
|  | Total/TBC/Tweak | 1,421 | 1,470 |

Performance without TI. PFB_Plus[SKINNY-64-256] and PFB[SKINNY-128256] have similar circuit areas without TI: 4,351 and 4,400 [GE], respectively. As consistent with the register counts in Table 3, PFB_Plus[SKINNY-64-256] has the smaller state array ( 532 compared to 1,098 [GE], but needs the additional shift register.

Performance with TI. With TI, on the other hand, PFB_Plus[SKINNY-64-256] is smaller than PFB[SKINNY-128-256] by 1,009 [GE] (7,439 and 8,448 [GE]). That is also consistent with Table 3 as PFB_Plus[SKINNY-64-256] has 64bit fewer registers. A smaller S-box circuit of PFB_Plus[SKINNY-64-256] (nibblewise and two stages) compared to that of PFB[SKINNY-128-256] (byte-wise and four stages) also contributes to this advantage of over one thousand gates.
Comparison with other AEAD. Table 5 compares the proposed method with conventional implementations of AEADs protected with TI. PFB[SKINNY-64-192] is a predecessor with a lower security level and is smaller than PFB_Plus[SKINNY-64-256] by 1,581 [GE] because it has fewer registers as summarized in Table 3. In comparison with Ascon having the same 128-bit security level, PFB_Plus[SKINNY-64-256] has a smaller circuit area even compared with the one having no interface ${ }^{9}$. The advantage of PFB_Plus[SKINNY-64-256] comes from heterogeneous sharing: PFB_Plus[SKINNY-64-256] can use fewer shares for the tweak, key, and tag meanwhile Ascon needs three shares for the entire 320-bit state. We also note that the Ascon implementation has longer latency and needs fresh random bits during the execution. Based on the comparison, we can conclude that PFB_Plus[SKINNY-64-256] has the smallest circuit area in TI among the schemes having 128-bit security.

[^4]Table 5. Comparison with previous AEAD implementations with TI: the latency shows that of a single primitive call.

| Target | Sec. <br> [bits] | Area <br> [GE] | Latency <br> [cycles] | Standard-cell <br> library | Ref. |
| :--- | ---: | ---: | ---: | :---: | :---: |
| PFB_Plus[SKINNY-64-256] | 128 | 7,439 | 705 | NanGate 45-nm | Ours |
| PFB[SKINNY-128-256] | 128 | 8,448 | 768 | NanGate 45-nm | Ours |
| PFB[SKINNY-64-192] | 64 | 5,858 | 641 | NanGate 45-nm | $[34]$ |
| SAEB[GIFT128-128] | 64 | 6,229 | 1,320 | NanGate 45-nm | $[34]$ |
| Ascon w/o IF | 128 | 7,970 | 3,072 | UMC 90-nm | $[22]$ |
| Ascon w/ IF | 128 | 9,190 | 3,072 | UMC 90-nm | $[22]$ |
| Ketje-JR | 96 | 18,335 | 16 | NanGate 45-nm | $[6]$ |

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[^0]:    ${ }^{4}$ In the table, the (twea)key functions are assumed to be linear. If the functions are non-linear, 3 shares of the functions are required, and the state sizes of the TBCbased modes are grater than those of the permutation-based ones.
    ${ }^{5}$ With respect to the reliability, it can be disadvantageous that our modes cannot be instantiated with existing well-known TBCs. However, from a different viewpoint, PFB_Plus is the first use case where $2 n$-bit tweak and $2 n$-bit key sizes are useful. This can give new insight to TBC designers considering that there is no consensus about the adequate tweak size to support.

[^1]:    ${ }^{6}$ Note that some works even have misuse resistance that our research does not.

[^2]:    ${ }^{7}$ PMAC_Plus is a block-cipher-based message authentication code and has $2 b$-bit internal state, which is updated by using outputs of BC calls, XOR operations and constant field multiplications.

[^3]:    ${ }^{8}$ The designs in this paper has a room for more aggressive optimization by skipping portection of (twea)key-scheduling [8, 40, 44].

[^4]:    ${ }^{9}$ The Ascon implementation excludes a 128-bit key register (640 [GE] for 5 [GE/bit]) needed to run another encryption/decryption with the same key.

