Unidirectional Updatable Encryption and Proxy Re-encryption from DDH

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Abstract. Updatable Encryption (UE) and Proxy Re-encryption (PRE) allow *re-encrypting* a ciphertext from one key to another in the symmetric-key and public-key settings, respectively, without decryption. A long-standing open question has been the following: do *unidirectional* UE and PRE schemes (where ciphertext re-encryption is permitted in only one direction) necessarily require stronger/more structured assumptions as compared to their bidirectional counterparts? Known constructions of UE and PRE seem to exemplify this "gap" – while bidirectional schemes can be realized as relatively simple extensions of public-key encryption from standard assumptions such as DDH or LWE, unidirectional schemes typically rely on stronger assumptions such as FHE or indistinguishability obfuscation (iO), or highly structured cryptographic tools such as bilinear maps or lattice trapdoors.

In this paper, we bridge this gap by showing the first feasibility results for realizing unidirectional UE and PRE from a new generic primitive that we call Key and Plaintext Homomorphic Encryption (KPHE) – a public-key encryption scheme that supports additive homomorphisms on its plaintext and key spaces simultaneously. We show that KPHE can be instantiated from DDH. This yields the first constructions of unidirectional UE and PRE from DDH.

Our constructions achieve the strongest notions of *post-compromise security* in the standard model. Our UE schemes also achieve "backwards-leak directionality" of key updates (a notion we discuss is equivalent, from a security perspective, to that of unidirectionality with no-key updates). Our results establish (somewhat surprisingly) that unidirectional UE and PRE schemes satisfying such strong security notions *do not*, in fact, require stronger/more structured cryptographic assumptions as compared to bidirectional schemes.

1 Introduction

Cryptographic encryption is a powerful tool for ensuring data confidentiality. A common security guarantee offered by any encryption scheme (either symmetric-key or public-key) is the following: encrypted data can only be decrypted using a certain secret key. However, a limitation of traditional encryption schemes is that once data is encrypted, it is generally hard to allow a third party to transform

the ciphertext so that it can be decrypted with a different key, without sharing either the original or the new secret key with the third party.

Re-encryption schemes such as Proxy Re-encryption (PRE) [BBS98] and Updatable Encryption (UE) [BLMR13] circumvent this limitation by enabling a public transformation of ciphertexts from encryption under one key to that of another, while protecting the underlying secret keys. Classic applications for such schemes include key rotation for secure outsourced storage [BBB+12, Pay18], access control, the delegation of email access, and many more.

Proxy Re-encryption (PRE). PRE is a public-key encryption scheme which enables a party Alice, with the help of a proxy, to re-encrypt her ciphertexts for decryption by an alternate party Bob. To facilitate re-encryption, Alice and Bob, with key pairs $(\mathsf{pk}_A, \mathsf{sk}_A)$ and $(\mathsf{pk}_B, \mathsf{sk}_B)$ respectively, will together compute a re-encryption key rk_{AB} and then provide this to the proxy. Whenever the proxy needs to perform re-encryption, it can use rk_{AB} to transform a ciphertext encrypted under pk_A into a ciphertext encrypted under pk_B. Security of the PRE scheme guarantees that the proxy learns nothing about the underlying plaintext during the re-encryption process.

Updatable Encryption (UE). UE was introduced by Boneh et al. [BLMR13] to address the problem of key rotation for secure outsourced storage. UE addresses re-encryption by using similar techniques to those of PRE, with two main differences: (1) UE is a symmetric-key encryption scheme, and (2) UE typically only allows sequential updates. More specifically, in UE we divide time into a series of epochs. In the first epoch a fresh symmetric key $\mathtt{k}_0$ is chosen and used to encrypt all data. When we rotate a key from $\mathtt{k}_{\mathsf{e}-1}$ to $\mathtt{k}_{\mathsf{e}}$, we transition to the next epoch by calculating an update token Δ_{e}. All new ciphertexts are encrypted under the new key $\mathtt{k}_{\mathsf{e}}$ and all existing ciphertexts $\mathsf{ct}_{\mathsf{e}-1}$ are re-encrypted using the update token Δ_{e} so that they can be decrypted by $\mathtt{k}_{\mathsf{e}}$. The benefit of this approach is that the storage server can perform the re-encryption of data using the update token without the risk of exposing any plaintext data.

There are two variants of UE schemes, *ciphertext-dependent* schemes [EPRS17, BEKS20] and *ciphertext-independent* schemes [LT18, KLR19, BDGJ20, Jia20]. In ciphertext-dependent UE schemes, the update token $\Delta_{\mathsf{e},\mathsf{ctx}_{\mathsf{e}-1}}$ depends on the ciphertext $\mathsf{ctx}_{\mathsf{e}-1}$ to be updated, while in ciphertext-independent schemes, the update token Δ_{e} is generated independent of the updated ciphertext, hence a single token can be used to update all ciphertexts on the storage server. In the rest of the paper, when we refer to UE, we mean a ciphertext-independent scheme unless otherwise specified.

Directionality of PRE and UE. Re-encryption schemes are either *bidirectional* or *unidirectional*. A scheme is said to be bidirectional if a re-encryption key/update token can be used to re-encrypt a ciphertext to either the next party/epoch or the previous party/epoch. In contrast, the re-encryption key/update token of a unidirectional scheme can only be used to re-encrypt a ciphertext to the next party/epoch and *not* the previous. So far we have only discussed the

directionality within the context of ciphertext updates. The (uni)directionality with regards to keys differs slightly between PRE and UE, as we discuss next.

Bidirectionality vs. Unidirectionality in PRE. In bidirectional PRE schemes, the re-encryption key rk_{AB} from Alice to Bob is generated from Alice's key-pair $(\mathsf{pk}_A, \mathsf{sk}_A)$ and Bob's key-pair $(\mathsf{pk}_B, \mathsf{sk}_B)$. Given rk_{AB} along with sk_A (resp. sk_B), it is usually possible to derive sk_B (resp. sk_A). In unidirectional PRE schemes, the re-encryption key rk_{AB} is derived from $((\mathsf{pk}_A, \mathsf{sk}_A), \mathsf{pk}_B)$; Bob's secret key sk_B is not used. In fact, given the re-encryption key rk_{AB} and Alice's secret key sk_A, it should be impossible to derive any knowledge of Bob's secret key sk_B.

Unidirectionality in UE. For UE schemes, there is an extra level of subtlety regarding the *directionality of keys* in addition to ciphertexts. A recent work of Jiang [Jia20] extensively studied the question: given an update token Δ_{e} along with either $\mathtt{k}_{\mathsf{e}-1}$ or $\mathtt{k}_{\mathsf{e}}$, is it possible to derive the other key? A scheme has bidirectional key updates if Δ_{e} can be used to derive keys in both directions, and has unidirectional key updates if Δ_{e} can be used in one direction, to derive $\mathtt{k}_{\mathsf{e}}$ from $\mathtt{k}_{\mathsf{e}-1}$. Jiang [Jia20] showed that UE with bidirectional key and ciphertext updates implies UE with unidirectional key and ciphertext updates.

In the same work, Jiang postulated that to capture the same security level as the unidirectional PRE schemes, one requires even stronger UE schemes with *no-directional key updates*, where $\mathtt{k}_{\mathsf{e}}$ cannot be derived from $\mathtt{k}_{\mathsf{e}-1}$ and Δ_{e}. In Jiang [Jia20], the definition of no-directional key updates intuitively requires that it is *also* impossible to derive $\mathtt{k}_{\mathsf{e}-1}$ from Δ_{e} and $\mathtt{k}_{\mathsf{e}}$. The recent work of Nishimaki [Nis21] proposed a seemingly weaker notion called *backward-leak unidirectional key updates* where Δ_{e} can only be used in one direction to derive $\mathtt{k}_{\mathsf{e}-1}$ from $\mathtt{k}_{\mathsf{e}}$. However, we observe that this new notion is essentially equivalent to no-directional key updates because derivation of $\mathtt{k}_{\mathsf{e}-1}$ does not increase the adversary's advantage in breaking the scheme. In particular, if the adversary obtains a ciphertext $\mathtt{ct}_{\mathsf{e}-1}$ and corrupts Δ_{e} and $\mathtt{k}_{\mathsf{e}}$, then it can first update the ciphertext to $\mathtt{ct}_{\mathsf{e}}$ and decrypt it using $\mathtt{k}_{\mathsf{e}}$. Jiang emphasized that UE with no-directional key updates is the ideal security model, which by our argument above, extends to backwards-leak key updates. Henceforth, when we refer to unidirectional UE, we mean unidirectional UE with backwards-leak directional key updates unless otherwise specified.

Gap between Unidirectionality and Bidirectionality. In general, unidirectional UE and PRE schemes are more ideally suited to real-world applications as compared to their bidirectional counterparts due to their superior security guarantees. For example, unlike bidirectional UE schemes, unidirectional UE schemes guarantee security of data as if "freshly encrypted" in epoch e (i.e., not re-encrypted from epoch $(\mathsf{e}-1)$) even if the adversary gains access to the secret key $\mathtt{k}_{\mathsf{e}-1}$ and the update token Δ_{e}. Unidirectional PRE schemes also offer similarly superior security guarantees over their bidirectional counterparts.

Another natural point of comparison between unidirectional and bidirectional UE and PRE schemes is the nature of cryptographic assumptions from which

such schemes can be realized. Known constructions of UE and PRE seemingly exemplify an apparent "gap" in terms of the assumptions required – unidirectional schemes have historically relied on stronger/more structured cryptographic assumptions as compared to their bidirectional counterparts.

Blaze et al. [BBS98] showed how to construct bidirectional PRE schemes from the Decisional Diffie-Hellman (DDH) assumption by suitably extending the well-known ElGamal encryption scheme [Gam85]. Similarly, a long line of works [BLMR13, LT18, KLR19, BDGJ20, Jia20] have shown how to realize bidirectional UE schemes as relatively simple extensions of public-key encryption from standard assumptions such as DDH and Learning With Errors (LWE).

On the other hand, unidirectional UE and PRE schemes typically rely on a stronger set of assumptions such as FHE [Gen09] and indistinguishability obfuscation (iO) [BGI+12], or highly structured cryptographic tools such as bilinear maps [BF03] and "hard" lattice trapdoors [GPV08]. Examples of constructions of unidirectional PRE from FHE and/or structured lattice trapdoors can be found in [NX15, CCL+14, Kir14, NAL15, PWA+16, FL17, PRSV17]. Constructions of unidirectional PRE schemes have also been shown to exist from bilinear maps [AFGH06, LV11]; however, these constructions are restricted to the *single-hop* setting in the sense that they only permit a single re-encryption of a ciphertext. Known constructions of unidirectional UE include the construction in [SS21] (which relies on bilinear maps), and two constructions in [Nis21] (one which achieves backward-leak key updates from lattice-specific techniques, and one which achieves no-directional key updates from iO). Sehrawat and Desmedt show a construction of UE from bi-homomorphic lattice-based pseudorandom functions [SD19]; however, their construction only achieves unidirectional ciphertext updates while still incurring bidirectional key updates (and is hence effectively bidirectional as per the recent findings in [Jia20]). To date, there exist no constructions of unidirectional PRE or UE from the plain DDH assumption (to our knowledge).

In this paper, we are motivated by the following longstanding open question in the study of UE and PRE:

Do unidirectional UE and PRE schemes necessarily require stronger/more structured assumptions as compared to their bidirectional counterparts?

More concretely, we ask the following question:

Can we construct unidirectional UE/PRE schemes from DDH?

1.1 Our Results

In this paper, we bridge this gap between the assumptions for unidirectional and bidirectional UE/PRE. We establish (somewhat surprisingly) that unidirectional UE and PRE schemes *do not*, in fact, require stronger/more structured cryptographic assumptions as compared to their bidirectional counterparts.

More concretely, we present generic constructions of unidirectional UE and PRE from a new primitive that we call Key and Plaintext Homomorphic Encryption (KPHE). We also show that such a KPHE scheme can be instantiated

from the BHHO encryption scheme [BHHO08] based on the DDH assumption. This yields the first constructions of unidirectional UE and PRE from the plain DDH assumption.

Our main result is summarized by the following (informal) theorem:

Theorem 1 (Informal). *Assuming the existence of a Key and Plaintext Homomorphic Encryption (KPHE) scheme that satisfies certain special properties, there exist post-compromise secure unidirectional UE and PRE schemes.*

On KPHE. The KPHE scheme with special properties required in our constructions can be viewed as a generalization of the BHHO public-key encryption scheme due to Boneh et al. [BHHO08]. It is a public key encryption scheme where the secret key is a bit-string $\mathsf{sk} \in \{0,1\}^{\ell}$ and the plaintext is also a bit-string $\mathsf{m} \in \{0,1\}^{\ell'}$ (in our constructions we use $\ell = \ell' = 2n$). The specialized KPHE scheme satisfies the following three properties:

- **Distributional Semantic Security:** We require a KPHE scheme to achieve semantic security even when the secret keys are sampled from a specific distribution. In particular, we use KPHE schemes with $2n$-bit secret keys where the secret key is uniformly random subject to the constraint that it has equally many 0 and 1 bits (i,e., n bits of 0 and n bits of 1).

- **Additive Key and Plaintext Homomorphisms:** We require a KPHE scheme to satisfy the following property: let T, T' be two arbitrary affine transformations that map 0-1 vectors to 0-1 vectors of the same length (in our constructions we use permutation maps over the bits of a $2n$-bit string). Then, given a public key pk corresponding to some secret key sk and a ciphertext $\mathsf{ct} \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}, \mathsf{m})$, one can generate a public key pk' corresponding to the secret key $T(\mathsf{sk})$ and a ciphertext $\mathsf{ct}' \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}', T'(\mathsf{m}))$, without the knowledge of the original secret key sk or the original message m.

- **Blinding:** We also require the KPHE scheme to satisfy an associated security property called "blinding", that (informally) argues that the public key and ciphertext generated via the aforementioned homomorphic transformations are indistinguishable from freshly generated public keys and ciphertexts (we make this more formal in Section 2).

For our PRE constructions, we also require that the KPHE scheme satisfies a notion of distributional circular security (i.e., circular security when the secret keys are sampled from a specific distribution). This is not required for our UE constructions.

Instantiating KPHE. We show how to concretely instantiate a KPHE scheme satisfying all of the aforementioned properties from DDH (based on the BHHO scheme [BHHO08]).

Lemma 1 (Informal). *Assuming DDH, there exists a secure construction of KPHE that satisfies the aforementioned properties.*

Table 1: Summary of bi/unidirectional UE and PRE schemes. We focus on ciphertext-independent UE and multi-hop PRE. In this table, "bi, uni, bwd-uni, no" stand for bidirectional, unidirectional, backward-leak unidirectional, no-directional, respectively. We note that the notion of *key-directionality* differs for UE and PRE; in the case of UE, unidirectionality of key updates implies that, given the source (secret) key and the update token, the destination (secret) key can be computed. This is not the case for PRE, where unidirectional key update simply denotes that the re-key generation algorithm takes as input the source secret key and the destination public key (as opposed to bidirectional key update, where the re-key generation algorithm takes as input both secret keys).

	scheme	**dir. (ctx)**	**dir. (key)**	**security**	**assumption**
UE	[BLMR13]	bi	bi	IND-ENC	DDH/LWE
UE	[LT18, KLR19, BDGJ20]	bi	bi	IND-UE	DDH
UE	[Jia20]	bi	bi	IND-UE	DLWE
UE	[SD19]	uni	bi	IND-UE	LWE
UE	[Nis21]	uni	bwd-uni	IND-UE	LWE
UE	[Nis21]	uni	no	IND-UE	iO
UE	[SS21]	uni	no	IND-UE	SXDH
UE	Ours	uni	bwd-uni	IND-UE	DDH
PRE	[BBS98]	bi	bi	IND-CPA	DDH
PRE	[CCL+14]	uni	uni	IND-CPA	DLWE
PRE	[PWA+16]	uni	uni	IND-CCA	LWE
PRE	[PRSV17]	uni	uni	IND-CPA	RLWE
PRE	Ours	uni	uni	IND-HRA	DDH

Corollary 1 (Informal). *Assuming DDH, there exist post-compromise secure unidirectional UE and PRE schemes.*

Security of Our Constructions. Our constructions of unidirectional UE and PRE achieve the strongest notions of *post-compromise security* in the standard model. Our construction of unidirectional UE achieves the state-of-the-art post-compromise security definition due to Boyd et al. [BDGJ20], while also ensuring backward-leak unidirectional key updates [Nis21]. Our unidirectional PRE construction achieves the post-compromise security definition recently proposed by Davidson et al. [DDLM19], which is, to our knowledge, the only notion of post-compromise PRE security to be proposed to date. We present a more detailed discussion on post-compromise security (and other related security notions) of UE and PRE in the next subsection. Table 1 presents a comparison of our results with those in the existing literature.

1.2 Background and Related Work

There has been extensive research on both UE and PRE, including various settings, definitions, and constructions. Below we only mention works that are the most directly relevant. For both UE and PRE, we focus on the CPA-type definitions, which are by far the most well-studied notions.

Security Notions for UE. Since the introduction of UE in [BLMR13], several works have explored its security notions [EPRS17, LT18, AMP19, KLR19, BDGJ20, Jia20]. Most notable is the work of Lehmann and Tackmann [LT18], which improved the model and studied the notion of post-compromise security for UE. Their Indistinguishability of Update notion (IND-UPD) returns a challenge ciphertext ct^* which is either the re-encryption of a ciphertext ct_0 or ct_1. A scheme is IND-UPD secure if an adversary is unable to determine which of the ciphertexts was re-encrypted.

In subsequent works a stronger combined notion of IND-UE security has been used, first defined by Boyd et al. [BDGJ20]. The IND-UE notion requires an adversary be unable to distinguish between a fresh encryption of a plaintext m and the re-encryption of a ciphertext ct. As a result this notion captures both CPA (specifically IND-ENC) and IND-UPD security.

Security Notions for PRE. In the context of PRE, the traditional notion of IND-CPA security [ID03, AFGH06] have been shown to be insufficient in practice. To address this, Cohen [Coh19] introduced the notion of Honest Re-Encryption Attack (HRA) security where an adversary is additionally permitted to re-encrypt (from honest to corrupt users) ciphertexts previously output by the encryption oracle. While only recently considered in the analysis of PRE, the essence of this notion is also fundamental in formalizing security for UE.

More recently, Davidson et al. [DDLM19] have investigated achieving post-compromise secure PRE schemes. They introduced a notion of IND-PCS security for PRE, which can be viewed as the analogue of IND-UPD security of UE in the context of PRE, albeit for more complex re-encryption graphs. To date this is the only paper that studies the PCS security of PRE schemes. Their work again demonstrates the challenges in constructing such schemes in the unidirectional setting. They discuss two PCS-secure constructions which are based on a prior unidirectional PRE scheme, Construction 7b of Fuchsbauer et al. [FKKP19] and an extension of BV-PRE [PRSV17].

Updatable Public Key Encryption. In order to achieve forward security in public key encryption (PKE), a notion called *updatable PKE (UPKE)* has recently been proposed and studied [JMM19, ACDT20, DKW21], where any sender (encryptor) can initiate a key update by sending a special update ciphertext to the receiver (decryptor). This ciphertext updates the public key and also, once processed by the receiver, will update its secret key. These are PKE schemes that encrypt messages under different public keys and aim to achieve forward security. In contrast, UE and PRE schemes studied in this paper aim to update ciphertexts encrypted under an old key to a new key without leaking the message content. The notions of UE/PRE as well as our techniques are very different from UPKE despite the partial naming collision.

Comparison with Umbral. There exists a practically deployed construction of unidirectional PRE, namely Umbral [Nun18], from the DDH Assumption, albeit in the random oracle model. It turns out that the Umbral construction is only

single-hop, and focuses on achieving threshold PRE rather than multi-hop PRE. In particular, the Umbral construction crucially relies on the Diffie-Hellman key change, and it is unclear how to extend the construction to multiple hops. On the other hand, our primary aim is to achieve multi-hop unidirectional PRE in the traditional non-threshold setting. We note additionally that Umbral would not achieve post-compromise security, which is an important property provided by our constructions. Fundamentally, this is due to the fact that Umbral adopts a KEM-DEM style approach where only the KEM is re-encrypted.

Concurrent Work. A concurrent work by Galteland and Pan [GP23] constructs unidirectional UE with backward-leak unidirectional key update from public key encryption (PKE) schemes with certain properties, which can be realized from the DDH or LWE assumption. Their techniques are significantly different from ours and do not trivially extend to the PRE setting. The authors of [GP23] also demonstrate a formal proof that the security definition for unidirectional UE with backward-leak unidirectional key updates is equivalent to the one with no-directional key updates, which confirms our observation discussed earlier.

1.3 Technical Overview

In this section, we provide a high-level overview of our techniques for constructing unidirectional UE and PRE from any generic KPHE scheme satisfying the special properties described earlier.

IND-ENC Secure UE. Our first attempt is to build a unidirectional IND-ENC secure UE scheme, and we start with a naïve idea. Take an arbitrary symmetric-key encryption scheme and each epoch key is a freshly generated key of this encryption scheme. The update token Δ_{e} from $\mathtt{k}_{\mathsf{e}-1}$ to $\mathtt{k}_{\mathsf{e}}$ is an encryption of $\mathtt{k}_{\mathsf{e}-1}$ under $\mathtt{k}_{\mathsf{e}}$, namely $\Delta_{\mathsf{e}} = \mathsf{Enc}_{\mathtt{k}_{\mathsf{e}}}(\mathtt{k}_{\mathsf{e}-1})$. When we update a ciphertext from epoch $(\mathsf{e}-1)$ to epoch e, we just attach the update token Δ_{e} to the end of the ciphertext. For a message m first encrypted in epoch e and then updated through epoch e', the resulting ciphertext is of the form:

$$\mathsf{ct}_{\mathsf{e}'} = \left(\mathsf{Enc}_{\mathtt{k}_{\mathsf{e}}}(\mathsf{m}), \mathsf{Enc}_{\mathtt{k}_{\mathsf{e}+1}}(\mathtt{k}_{\mathsf{e}}), \ldots, \mathsf{Enc}_{\mathtt{k}_{\mathsf{e}'}}(\mathtt{k}_{\mathsf{e}'-1})\right).$$

Given $\mathtt{k}_{\mathsf{e}'}$, one can easily decrypt $\mathsf{ct}_{\mathsf{e}'}$ layer by layer to recover m.

This naïve approach does not achieve IND-ENC security. We show a concrete attack in the following. Let e^* be the challenge epoch and m^* be the challenge message queried by the adversary. Let $\mathsf{ct}_{\mathsf{e}^*} = \mathsf{Enc}_{\mathtt{k}_{\mathsf{e}^*}}(\mathsf{m}^*)$ be the challenge ciphertext. To extract the secret key $\mathtt{k}_{\mathsf{e}^*}$, the adversary proceeds as follows. It first queries for an encryption of an arbitrary message m in epoch 0 and then updates it to epoch e (for some $\mathsf{e} > \mathsf{e}^*$) via a sequence of update queries. This way the adversary obtains a ciphertext of m of the form:

$$\mathsf{ct}_{\mathsf{e}} = \left(\mathsf{Enc}_{\mathtt{k}_0}(\mathsf{m}), \mathsf{Enc}_{\mathtt{k}_1}(\mathtt{k}_0), \ldots, \mathsf{Enc}_{\mathtt{k}_{\mathsf{e}}}(\mathtt{k}_{\mathsf{e}-1})\right).$$

Now the adversary corrupts the secret key $\mathtt{k}_\mathsf{e}$. Then it can recover all the previous keys from $\mathtt{k}_0$ to $\mathtt{k}_{\mathsf{e}-1}$ (including $\mathtt{k}_{\mathsf{e}^*}$) during decryption of the ciphertext ct_e.

Nonetheless, this simple approach demonstrates some nice properties of unidirectionality. For key updates, it is impossible to derive $\mathtt{k}_\mathsf{e}$ from $\mathtt{k}_{\mathsf{e}-1}$ and Δ_e. For ciphertext updates, given a *fresh* ciphertext ct_e in epoch e and the previous update token Δ_e (from epoch $(\mathsf{e}-1)$ to e), it is impossible to transition the ciphertext ct_e to the previous epoch $\mathsf{ct}_{\mathsf{e}-1}$ (i.e. the epoch prior to its existence). In fact, Cohen [Coh19] applied this idea to PRE and showed a CPA-secure but not HRA-secure PRE scheme (HRA security is inherently required in IND-ENC UE schemes).

Re-randomizing the Secret Keys. The problem with these chained ciphertexts is that during decryption of a single ciphertext, all the previous secret keys are also leaked. To resolve this problem, our hope is to somehow re-randomize all the previous secret keys in the chain, in a consistent and homomorphic manner. In particular, we want the ciphertext to be of the form

$$\mathsf{ct}_\mathsf{e} = \left(\mathsf{Enc}_{\overline{\mathtt{k}}_0}(\mathsf{m}), \mathsf{Enc}_{\overline{\mathtt{k}}_1}(\overline{\mathtt{k}}_0), \ldots, \mathsf{Enc}_{\overline{\mathtt{k}}_{\mathsf{e}-1}}(\overline{\mathtt{k}}_{\mathsf{e}-2}), \mathsf{Enc}_{\mathtt{k}_\mathsf{e}}(\overline{\mathtt{k}}_{\mathsf{e}-1})\right),$$

where $\overline{\mathtt{k}}_0, \overline{\mathtt{k}}_1, \ldots, \overline{\mathtt{k}}_{\mathsf{e}-1}$ are all re-randomized secret keys that are different for each ciphertext. During the decryption of ct_e, only these re-randomized secret keys are leaked, which does not affect the security of other ciphertexts.

To enable such re-randomization, our idea is inspired by the re-randomizable Yao's garbled circuits [GHV10]. We propose a new primitive called Key and Plaintext Homomorphic Encryption (KPHE), which can be seen as a generalization of the circular secure encryption scheme of Boneh et al. [BHHO08]. Instead of using an arbitrary symmetric-key encryption scheme, we use the KPHE scheme for encryption, where the UE secret key $\mathtt{k}_\mathsf{e}$ is a key pair $(\mathsf{pk}_\mathsf{e}, \mathsf{sk}_\mathsf{e})$ of the KPHE scheme. The update token is a KPHE encryption of the previous epoch's secret key under the current epoch's public key, namely $\Delta_\mathsf{e} = \mathsf{KPHE.Enc}_{\mathsf{pk}_\mathsf{e}}(\mathsf{sk}_{\mathsf{e}-1})$.

To update a ciphertext we exploit the two homomorphism properties of the KPHE scheme, in both the message space and the key space. Given an update token Δ_e and a ciphertext of the form

$$\mathsf{ct}_{\mathsf{e}-1} = \left(\mathsf{KPHE.Enc}_{\overline{\mathsf{pk}}_0}(\mathsf{m}), \mathsf{KPHE.Enc}_{\overline{\mathsf{pk}}_1}(\overline{\mathsf{sk}}_0), \ldots, \mathsf{KPHE.Enc}_{\mathsf{pk}_{\mathsf{e}-1}}(\overline{\mathsf{sk}}_{\mathsf{e}-2})\right),$$

we focus on the last component $\mathsf{ctx} = \mathsf{KPHE.Enc}_{\mathsf{pk}_{\mathsf{e}-1}}(\overline{\mathsf{sk}}_{\mathsf{e}-2})$ and the update token $\Delta_\mathsf{e} = \mathsf{KPHE.Enc}_{\mathsf{pk}_\mathsf{e}}(\mathsf{sk}_{\mathsf{e}-1})$, In our update operation we first generate a random permutation π and then perform two important steps:

- Use the KPHE key-space homomorphism to transform ctx from an encryption under $\mathsf{sk}_{\mathsf{e}-1}$ to an encryption under $\pi(\mathsf{sk}_{\mathsf{e}-1})$.
- Use the KPHE message-space homomorphism to transform Δ_e from an encryption of $\mathsf{sk}_{\mathsf{e}-1}$ to an encryption of $\pi(\mathsf{sk}_{\mathsf{e}-1})$.

The updated ciphertext becomes

$$\mathsf{ct}_{\mathsf{e}} = \left(\mathsf{KPHE.Enc}_{\overline{\mathsf{pk}}_0}(\mathsf{m}), \mathsf{KPHE.Enc}_{\overline{\mathsf{pk}}_1}(\overline{\mathsf{sk}}_0), \ldots, \mathsf{KPHE.Enc}_{\overline{\mathsf{pk}}_{\mathsf{e}-1}}(\overline{\mathsf{sk}}_{\mathsf{e}-2}), \mathsf{KPHE.Enc}_{\mathsf{pk}_{\mathsf{e}}}(\overline{\mathsf{sk}}_{\mathsf{e}-1})\right),$$

where $\overline{\mathsf{sk}}_{\mathsf{e}-1} = \pi(\mathsf{sk}_{\mathsf{e}-1})$ (with corresponding public key $\overline{\mathsf{pk}}_{\mathsf{e}-1}$). In our construction, the KPHE secret key is a $2n$-bit string, which is randomly sampled with exactly n bits of 0 and n bits of 1. The affine transformation π is a random permutation on the $2n$ bits of the string. By transforming from $\mathsf{sk}_{\mathsf{e}-1}$ to $\overline{\mathsf{sk}}_{\mathsf{e}-1}$ we ensure that a fresh secret key is used for each update operation and hence there is appropriate isolation between all ciphertexts updated in a given epoch. The blinding property of KPHE ensures that re-randomization can be done without knowledge of the underlying secret keys, and that the re-randomized ciphertexts are computationally indistinguishable from freshly generated ciphertexts.

Use of Balanced KPHE Keys. The astute reader might have noticed that we use "balanced" secret keys for our KPHE scheme, wherein each secret key is a randomly sampled $2n$-bit string with exactly n bits of 0 and n bits of 1. The restriction is required to offset some leakage that our scheme incurs during the honest re-encryption query phase in the security proofs. Informally, the adversary can use a sequence of honest re-encryption queries to learn some information about the intermediate (re-randomized) secret keys; in particular, it learns the number of 0 and 1 bits in each secret key. Intuitively, we offset this leakage by specifying at setup that all secret keys have an equal number of 0 and 1 entries. As a result, the adversary learns no additional information about these intermediate keys, irrespective of the number of honest re-encryption queries that it issues. We defer a formal treatment to the detailed proofs of security for our constructions.

Achieving Post-Compromise Secure UE. We can extend the IND-ENC secure UE construction to achieve post-compromise security. To achieve IND-UPD security, we can modify the update operation to ensure that all the chained ciphertexts are updated (rather than just the last one). In effect what our enhanced construction does is again exploit properties of the KPHE scheme to re-randomize each of the ciphertext components. This ensures that two updated ciphertexts of the same length are computationally indistinguishable. To further achieve the combined IND-UE security, we need to additionally guarantee that a freshly generated ciphertext has the same length as an updated ciphertext in a certain epoch. More details on our UE constructions are given in Section 3.

Achieving Unidirectional PRE. We can use the same high-level approach to construct a unidirectional PRE scheme, where a ciphertext consists of a chain of KPHE ciphertexts, and re-encryption exploits the two KPHE homomorphisms to transform each new KPHE ciphertext to a fresh secret key. The crucial subtlety in the PRE case, which makes proving security slightly more involved, is that we no longer consider sequential ciphertext updates but must consider re-encryption between all possible key pairs. As a result we need to further exploit the circular

security properties of the KPHE scheme to prove security. This is further detailed in Section 4.

Connections between UE and PRE. Generally speaking, unidirectional PRE can be viewed as a stronger primitive than unidirectional UE because UE only allows for sequential updates while PRE allows for re-encryption between every pair of keys. In fact, we observe that if we treat the public-secret key pair of PRE as a secret key for UE, and the PRE re-encryption key as an update token for UE, then IND-HRA secure PRE implies IND-ENC secure UE, and IND-PCS secure PRE implies IND-UPD secure UE. This is also why our constructions for unidirectional UE and PRE follow a very similar framework. On the other hand, since PRE supports re-encryption between (potentially) every pair of keys, our constructions of PRE require stronger security guarantees (in particular, circular security) from the underlying KPHE scheme.

Efficiency and Feasibility. We acknowledge that the ciphertext length in our UE/PRE constructions grows linearly with the number of epochs/re-encryption hops, unlike certain existing constructions (e.g. in [Nis21, PWA+16, PRSV17]) where the ciphertext size remains the same. In this context, we emphasize that our paper is the first to achieve backward-leak unidirectional UE and unidirectional PRE from standard assumptions, specifically DDH. It has been a long-standing open problem for over a decade whether obfuscation/FHE is necessary for unidirectional UE/(multi-hop) PRE, and our work closes this assumption gap. As a result, we believe that our results should be viewed with emphasis on the new theoretical insights/understanding into unidirectional-UE/PRE that they enable as opposed to concrete efficiency. Our work opens up the discussion of whether obfuscation/FHE is necessary for achieving unidirectional UE/PRE with "succinctness" in the ciphertext length.

We note that in the UE setting, key rotation may only happen a small number of times in practice. For example, once a year for the lifetime of the ciphertext (say 10years). Thus, taking a similar approach to [BEKS20] (from the ciphertext-dependent setting) we could bound the number of updates and have fixed-length ciphertexts through some form of padding. We also point out that while the size of ciphertexts in our general constructions grow linearly, the secret keys and update tokens/re-encryption keys remain constant-sized. We also note that for the basic versions of our UE/PRE construction (IND-CPA unidirectional UE and IND-HRA secure unidirectional PRE), the work done per update/re-encryption operation is also constant (independent of the number of epochs/update hops).

We note here that a naïve approach to achieving unidirectional UE is the so called "download–decrypt–re-encrypt–upload" approach, where the client downloads the encrypted data (e.g. from the server storing the encrypted data), locally decrypts it, re-encrypts it using the new key, and re-uploads the newly encrypted data to the server. Our UE constructions are non-trivial in the sense that we achieve significantly better properties as compared to this naïve approach. In particular, for applications of UE (e.g. key rotation) where the client outsources encrypted data to the server, this entails constant computational/

communication/storage overheads at the client during key rotation (the client simply generates and sends the update token to the server); the corresponding client-overheads are linear (in database size) in the naïve solution.

Using Random Oracles. A possible approach towards achieving practical efficiency is to use random oracles (such as in the single-hop unidirectional threshold PRE scheme Umbral [Nun18]). Our focus is primarily on feasibility results for unidirectional UE/PRE in the standard model, and we consciously avoid the use of random oracles. We also point out that a previous result [AMP19] showed that, even in the symmetric-key setting, unidirectional UE/PRE implies public-key encryption, and so a construction from just a random oracle is unlikely. However, it might be possible to achieve efficiency gains using a random oracle. We leave investigating such a random oracle-based construction of unidirectional UE/PRE as an interesting direction of future research.

1.4 Paper Outline

The rest of the paper is organized as follows. Section 2 formally defines a KPHE scheme and its associated security properties. Section 3 presents our constructions of IND-CPA and IND-UPD secure UE from any KPHE scheme. Section 4 presents our construction of IND-HRA secure PRE from any KPHE scheme. We defer detailed proofs of security for these schemes, our constructions of IND-UE secure UE and IND-PCS secure PRE from any KPHE scheme, and the instantiation of KPHE from DDH to the full version of our paper [MPW22].

For readers not familiar with the formal definitions of UE and PRE, we present relatively self-contained background material on UE and PRE in Sections 3.1 and 4.1, respectively. Due to lack of space, the formal security notions of PRE are deferred to the full version of our paper [MPW22].

1.5 Notations

We summarize here the notations used in the rest of the paper. We write $x \xleftarrow{\$} \chi$ to represent that an element x is sampled randomly from a set/distribution $\mathcal{X}$. The output x of a deterministic (resp. randomized) algorithm $\mathcal{A}$ is denoted by $x = \mathcal{A}$ (resp. $x \xleftarrow{\$} \mathcal{A}$). For $a \in \mathbb{N}$ such that $a \geq 1$, we denote by $[a]$ the set of integers lying between 1 and a (both inclusive). We refer to $\lambda \in \mathbb{N}$ as the security parameter, and denote by $\mathrm{poly}(\lambda)$ and $\mathrm{negl}(\lambda)$ any generic (unspecified) polynomial function and negligible function in λ, respectively.[4]

2 Key and Plaintext Homomorphic Encryption

In this section, we present the definitions for the core building block for our constructions, namely key and plaintext homomorphic encryption (KPHE). Informally, a KPHE scheme has the following features:

[4] Note that a function $f : \mathbb{N} \to \mathbb{N}$ is said to be negligible in λ if for every positive polynomial p, $f(\lambda) < 1/p(\lambda)$ when λ is sufficiently large.

– **Keys and Plaintexts:** Each secret key sk is an ℓ-bit string for some $\ell = \text{poly}(\lambda)$ (λ being the security parameter). Additionally, each plaintext message m is an ℓ'-bit string for some $\ell' = \text{poly}(\lambda)$.

– **Key Distribution:** Each secret key is sampled according to some distribution $\mathcal{D}$ over $\{0,1\}^\ell$. In particular, for our applications, we assume KPHE schemes where each secret key sk is a $2n$-bit string with equally many 0 and 1 entries.

– **Key Homomorphism:** Let T be any linear transformation that maps ℓ-bit strings to ℓ-bit strings. Then, it is possible to efficiently evaluate the following:

  - Given a public key pk corresponding to some secret key $\mathsf{sk} \in \{0,1\}^\ell$, it is possible to efficiently compute a valid public key pk' corresponding to the transformed secret key $\mathsf{sk}' = T(\mathsf{sk})$, without the knowledge of sk.

  - Given a ciphertext ct encrypting a message m under some secret key $\mathsf{sk} \in \{0,1\}^\ell$, it is possible to efficiently compute a ciphertext ct' encrypting the same message m under the transformed secret key $\mathsf{sk}' = T(\mathsf{sk})$, without the knowledge of sk.

– **Plaintext Homomorphism:** Let T' be any linear transformation that maps ℓ'-bit strings to ℓ'-bit strings. Then, given a ciphertext ct encrypting a message $\mathsf{m} \in \{0,1\}^{\ell'}$ under some secret key sk, it is possible to efficiently compute a ciphertext ct'' encrypting the transformed message $\mathsf{m}' = T'(\mathsf{m})$ under the same secret key sk.

We now summarize these features of KPHE formally below.

Definition 1 (KPHE). *A KPHE scheme is a tuple of PPT algorithms of the form* $\mathsf{KPHE} = (\mathsf{Setup}, \mathsf{SKGen}, \mathsf{PKGen}, \mathsf{Enc}, \mathsf{Dec}, \mathsf{Eval})$ *that are defined as follows:*

– $\mathsf{pp} \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$*: On input the security parameter* λ*, the setup algorithm outputs a public parameter* pp*.*

– $\mathsf{sk} \xleftarrow{\$} \mathsf{SKGen}(\mathsf{pp}, \mathcal{D})$*: On input the public parameter* pp *and a distribution* $\mathcal{D}$ *over* $\{0,1\}^\ell$ *(for* $\ell = \text{poly}(\lambda)$*), the secret key generation algorithm outputs a secret key* $\mathsf{sk} \xleftarrow{\$} \mathcal{D}$*.*

– $\mathsf{pk} \xleftarrow{\$} \mathsf{PKGen}(\mathsf{pp}, \mathsf{sk})$*: On input the public parameter* pp *and a secret key* $\mathsf{sk} \in \{0,1\}^\ell$*, the public key generation algorithm outputs a public key* pk*.*

– $\mathsf{ct} \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}, \mathsf{m})$*: On input a public key* pk *and a message* $\mathsf{m} \in \{0,1\}^{\ell'}$ *(for* $\ell' = \text{poly}(\lambda)$*), the encryption algorithm outputs a ciphertext* ct*.*

– $\mathsf{m}/\perp \xleftarrow{\$} \mathsf{Dec}(\mathsf{sk}, \mathsf{ct})$*: On input a secret key* $\mathsf{sk} \in \{0,1\}^\ell$ *and a ciphertext* ct*, the decryption algorithm outputs a plaintext message string* m *or an error symbol* $\perp$*.*

– $(\mathsf{pk}', \mathsf{ct}') \xleftarrow{\$} \mathsf{Eval}(\mathsf{pk}, \mathsf{ct}, T, T')$*: On input a public key* pk*, a ciphertext* ct*, and a pair of (linear) transformations* $T : \{0,1\}^{\ell} \to \{0,1\}^{\ell}$ *and* $T' : \{0,1\}^{\ell'} \to \{0,1\}^{\ell'}$*, the homomorphic evaluation algorithm outputs a tuple consisting of a transformed public key and a transformed ciphertext* $(\mathsf{pk}', \mathsf{ct}')$*.*

Correctness. A KPHE scheme $(\mathsf{Setup}, \mathsf{SKGen}, \mathsf{PKGen}, \mathsf{Enc}, \mathsf{Dec}, \mathsf{Eval})$ is said to be correct with respect to a distribution $\mathcal{D}$ over $\{0,1\}^{\ell}$ if for any $\mathsf{pp} \xleftarrow{\$} \mathsf{Setup}(1^{\lambda})$, any $\mathsf{sk} \xleftarrow{\$} \mathsf{SKGen}(\mathsf{pp}, \mathcal{D})$, any $\mathsf{pk} \xleftarrow{\$} \mathsf{PKGen}(\mathsf{pp}, \mathsf{sk})$, any $\mathsf{m} \in \{0,1\}^{\ell'}$, and any pair of (linear) transformations $T : \{0,1\}^{\ell} \to \{0,1\}^{\ell}$ and $T' : \{0,1\}^{\ell'} \to \{0,1\}^{\ell'}$, letting $\mathsf{sk}' = T(\mathsf{sk})$, $\mathsf{m}' = T'(\mathsf{m})$ and

$$\mathsf{ct} \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}, \mathsf{m}), \quad (\mathsf{pk}', \mathsf{ct}') \xleftarrow{\$} \mathsf{Eval}(\mathsf{pk}, \mathsf{ct}, T, T'),$$

both of the following hold with overwhelmingly large probability:

– pk' is a valid public key with respect to $\mathsf{sk}' = T(\mathsf{sk})$, i.e., for any $\bar{\mathsf{m}} \in \{0,1\}^{\ell'}$, it holds that $\mathsf{Dec}(\mathsf{sk}', \mathsf{Enc}(\mathsf{pk}', \bar{\mathsf{m}})) = \bar{\mathsf{m}}$.
– ct' is a valid encryption of m' under $(\mathsf{pk}', \mathsf{sk}')$, i.e., $\mathsf{Dec}(\mathsf{sk}', \mathsf{ct}') = \mathsf{m}'$.

Distributional Semantic Security. We (informally) say that a KPHE satisfies distributional semantic security with respect to some distribution $\mathcal{D}$ over $\{0,1\}^{\ell}$ if it remains semantically secure even when the secret key sk is sampled according to the distribution $\mathcal{D}$. Formally, this is modeled using a semantic security game where the secret key is sampled by the challenger as per the distribution $\mathcal{D}$.

Definition 2 ($\mathcal{D}$-Semantic Security). *A KPHE scheme with ℓ-bit secret keys is said to be $\mathcal{D}$-semantically secure with respect to a distribution $\mathcal{D}$ over $\{0,1\}^{\ell}$ if for any security parameter $\lambda \in \mathbb{N}$ and any PPT adversary $\mathcal{A}$, the following holds with overwhelmingly large probability:*

$$|\Pr[\mathsf{Expt}_{\mathcal{D}}^{\mathsf{DSS-KPHE}}(\lambda, \mathcal{A}) = 1] - 1/2| < \mathrm{negl}(\lambda),$$

where the experiment $\mathsf{Expt}_{\mathcal{D}}^{\mathsf{DSS-KPHE}}(\lambda, \mathcal{A})$ is as defined in Figure 1.

Distributional Circular Security. We (informally) say that a KPHE satisfies distributional circular security with respect to some distribution $\mathcal{D}$ over $\{0,1\}^{\ell}$ if it satisfies the standard notion of circular security [CL01, BRS02, BHHO08, ACPS09] even when each secret key is sampled from the distribution $\mathcal{D}$. Formally, this is modeled using a circular security game where the secret keys are sampled by the challenger as per the distribution $\mathcal{D}$.

Definition 3 ($\mathcal{D}$-Circular Security). *A KPHE scheme with ℓ-bit secret keys and ℓ-bit messages is said to be $\mathcal{D}$-circular secure with respect to a distribution $\mathcal{D}$ over $\{0,1\}^{\ell}$ if for any security parameter $\lambda \in \mathbb{N}$ and any PPT adversary $\mathcal{A}$, the following holds with overwhelmingly large probability:*

$$|\Pr[\mathsf{Expt}_{\mathcal{D}}^{\mathsf{DCC-KPHE}}(\lambda, \mathcal{A}) = 1] - 1/2| < \mathrm{negl}(\lambda),$$

where the experiment $\mathsf{Expt}_{\mathcal{D}}^{\mathsf{DCC-KPHE}}(\lambda, \mathcal{A})$ is as defined in Figure 2.

Experiment $\mathsf{Expt}_{\mathcal{D}}^{\mathsf{DSS-KPHE}}(\lambda, \mathcal{A})$:

1. The challenger generates $\mathsf{pp} \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$, $\mathsf{sk} \xleftarrow{\$} \mathsf{SKGen}(\mathsf{pp}, \mathcal{D})$, and $\mathsf{pk} \xleftarrow{\$} \mathsf{PKGen}(\mathsf{pp}, \mathsf{sk})$, and provides the adversary $\mathcal{A}$ with $(\mathsf{pp}, \mathsf{pk})$.
2. The adversary $\mathcal{A}$ issues a challenge encryption query for a pair of messages $(\mathsf{m}_0, \mathsf{m}_1)$.
3. The challenger samples $b \xleftarrow{\$} \{0,1\}$, creates the challenge ciphertext

$$\mathsf{ct}^* \xleftarrow{\$} \mathsf{Enc}\left(\mathsf{pk}, \mathsf{m}_b\right),$$

   and sends ct^* to the adversary $\mathcal{A}$.
4. The adversary $\mathcal{A}$ outputs a bit $b' \in \{0,1\}$.
5. Output 1 if $b = b'$ and 0 otherwise.

Fig. 1: The $\mathcal{D}$-Semantic Security Experiment for KPHE

Experiment $\mathsf{Expt}_{\mathcal{D}}^{\mathsf{DCC-KPHE}}(\lambda, \mathcal{A})$:

1. The challenger generates $\mathsf{pp} \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$ and provides it to the adversary.
2. The adversary $\mathcal{A}$ outputs $n = \mathrm{poly}(\lambda)$.
3. The challenger samples $\mathsf{sk}_1, \ldots, \mathsf{sk}_n \xleftarrow{\$} \mathsf{SKGen}(\mathsf{pp}, \mathcal{D})$, sets

$$\mathsf{pk}_1 \xleftarrow{\$} \mathsf{PKGen}(\mathsf{pp}, \mathsf{sk}_1), \ldots, \mathsf{pk}_n \xleftarrow{\$} \mathsf{PKGen}(\mathsf{pp}, \mathsf{sk}_n),$$

   and provides $(\mathsf{pk}_1, \ldots, \mathsf{pk}_n)$ to the adversary $\mathcal{A}$.
4. The challenger also sets the following for each $i, j \in [n]$:

$$\mathsf{ct}_{i,j,0} \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}_i, \mathsf{sk}_j), \quad \mathsf{ct}_{i,j,1} \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}_i, 0^{|\mathsf{sk}_j|}).$$

5. The challenger finally samples a bit $b \xleftarrow{\$} \{0,1\}$ and provides the adversary $\mathcal{A}$ with the ensemble $\{\mathsf{ct}_{i,j,b}\}_{i,j \in [n]}$.
6. The adversary $\mathcal{A}$ outputs a bit $b' \in \{0,1\}$.
7. Output 1 if $b = b'$ and 0 otherwise.

Fig. 2: The $\mathcal{D}$-Circular Security Experiment for KPHE

Blinding. We (informally) say that a KPHE scheme satisfies public key and ciphertext blinding if the homomorphic evaluation algorithm outputs a public key-ciphertext pair $(\mathsf{pk}', \mathsf{ct}')$ corresponding to the transformed secret key sk' and the transformed message m' such that:

- The transformed public key pk' is computationally indistinguishable from a public key sampled uniformly at random from the set of all valid public keys corresponding to the secret key sk'.
- The transformed ciphertext ct' is computationally indistinguishable from a ciphertext sampled uniformly at random from the set of all valid ciphertexts corresponding to the transformed message m' under pk'.

Experiment $\mathsf{Expt}_{\mathcal{D}}^{\mathsf{Blind-KPHE}}(\lambda, \mathcal{A})$:

1. The challenger generates $\mathsf{pp} \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$, $\mathsf{sk} \xleftarrow{\$} \mathcal{D}$, and $\mathsf{pk} \xleftarrow{\$} \mathsf{PKGen}(\mathsf{pp}, \mathsf{sk})$, and provides the adversary $\mathcal{A}$ with $(\mathsf{pp}, \mathsf{sk}, \mathsf{pk})$.
2. The adversary $\mathcal{A}$ sends a message $\mathsf{m} \in \{0,1\}^{\ell'}$ to the challenger.
3. The challenger responds to $\mathcal{A}$ with a ciphertext $\mathsf{ct} \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}, \mathsf{m})$.
4. The adversary $\mathcal{A}$ then sends a pair of (linear) transformations
$$T : \{0,1\}^{\ell} \to \{0,1\}^{\ell}, \quad T' : \{0,1\}^{\ell'} \to \{0,1\}^{\ell'}.$$
5. The challenger sets
$$\mathsf{sk}' = T(\mathsf{sk}), \quad \mathsf{m}' = T'(\mathsf{m}),$$
and computes the following:
$$(\mathsf{pk}_0, \mathsf{ct}_0) \xleftarrow{\$} \mathsf{Eval}(\mathsf{pk}, \mathsf{ct}, T, T'), \quad \mathsf{pk}_1 \xleftarrow{\$} \mathsf{PKGen}(\mathsf{pp}, \mathsf{sk}'), \quad \mathsf{ct}_1 \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}_1, \mathsf{m}'),$$
6. The challenger finally samples a bit $b \xleftarrow{\$} \{0,1\}$ and provides the adversary $\mathcal{A}$ with $(\mathsf{pk}_b, \mathsf{ct}_b)$.
7. The adversary $\mathcal{A}$ outputs a bit $b' \in \{0,1\}$.
8. Output 1 if $b = b'$ and 0 otherwise.

Fig. 3: The Blinding Experiment for KPHE

More formally, we define this blinding property as follows.

Definition 4 (Blinding). *A KPHE scheme with ℓ-bit secret keys and ℓ'-bit messages is said to satisfy blinding security with respect to a distribution $\mathcal{D}$ over $\{0,1\}^{\ell}$ if for any security parameter $\lambda \in \mathbb{N}$ and any PPT adversary $\mathcal{A}$, the following holds with overwhelmingly large probability:*

$$|\Pr[\mathsf{Expt}_{\mathcal{D}}^{\mathsf{Blind-KPHE}}(\lambda, \mathcal{A}) = 1] - 1/2| < \mathrm{negl}(\lambda),$$

where the experiment $\mathsf{Expt}_{\mathcal{D}}^{\mathsf{Blind-KPHE}}(\lambda, \mathcal{A})$ is as defined in Figure 3.

KPHE from DDH. In full version of our paper [MPW22], we prove the following (informal) theorem:

Theorem 2 (Informal). *Assuming DDH, there exists a KPHE scheme with $2n$-bit secret keys that satisfies distributional semantic security with respect to the distribution $\mathcal{U}_n$, distributional circular security with respect to the distribution $\mathcal{U}_n$, and blinding, as defined above.*

In particular, we rely on known results from [BHHO08, NS12, GHV10] for the DDH-based instantiation of KPHE. See [MPW22] for details.

KPHE from LWE. In this paper, we do not explicitly describe a construction of KPHE from LWE since there already exist constructions of unidirectional

UE/PRE from LWE [CCL+14, PWA+16, PRSV17, Nis21]. Our aim in this work is to close the gap between bidirectional and unidirectional constructions of UE/PRE in terms of assumptions, and so we choose to focus on the feasibility results from the DDH assumption.

We note, however, that constructing KPHE from LWE is a very interesting direction of future work. In particular, one needs to be careful during re-encryption, which potentially increases the level of noise in the ciphertext of the LWE-based encryption scheme and could leak extra information. For example, due to increase in the noise level during re-encryption, it is not straightforward to prove the ciphertext blinding property, which requires that a freshly created ciphertext and a re-encrypted ciphertext are distributed in an indistinguishable manner. This issue can be handled using noise flooding techniques, albeit at the cost of a larger ciphertext size.

KPHE from Other Assumptions. We also leave it as an interesting open question to construct KPHE from concrete hardness assumptions other than DDH or LWE (e.g., factorization-based assumptions or LPN). Given our results on achieving unidirectional UE/PRE from KPHE, such realizations of KPHE would immediately yield new constructions of unidirectional UE/PRE from these assumptions.

3 Unidirectional UE from KPHE

In this section, we show how to construct unidirectional UE satisfying various security notions (IND-ENC, IND-UPD and IND-UE) from any KPHE scheme.

3.1 Definition

Definition 5. *An updatable encryption (UE) scheme for message space $\mathcal{M}$ is a tuple of PPT algorithms* $\mathsf{UE} = (\mathsf{UE.setup}, \mathsf{UE.next}, \mathsf{UE.enc}, \mathsf{UE.upd}, \mathsf{UE.dec})$ *with the following syntax:*

- $\mathtt{k}_0 \xleftarrow{\$} \mathsf{UE.setup}(1^\lambda)$*: On input a security parameter 1^λ, it returns a secret key $\mathtt{k}_\mathsf{e}$ for epoch $\mathsf{e} = 0$.*
- $(\mathtt{k}_{\mathsf{e}+1}, \Delta_{\mathsf{e}+1}) \xleftarrow{\$} \mathsf{UE.next}(\mathtt{k}_\mathsf{e})$*: On input a secret key $\mathtt{k}_\mathsf{e}$ for epoch e, it outputs a new secret key $\mathtt{k}_{\mathsf{e}+1}$ and an update token $\Delta_{\mathsf{e}+1}$ for epoch $\mathsf{e}+1$.*
- $\mathsf{ct}_\mathsf{e} \xleftarrow{\$} \mathsf{UE.enc}(\mathtt{k}_\mathsf{e}, \mathsf{m})$*: On input a secret key $\mathtt{k}_\mathsf{e}$ for epoch e and a message $\mathsf{m} \in \mathcal{M}$, it outputs a ciphertext ct_e.*
- $\mathsf{ct}_{\mathsf{e}+1} \xleftarrow{\$} \mathsf{UE.upd}(\Delta_{\mathsf{e}+1}, \mathsf{ct}_\mathsf{e})$*: On input a ciphertext ct_e from epoch e and the update token $\Delta_{\mathsf{e}+1}$, it returns the updated ciphertext $\mathsf{ct}_{\mathsf{e}+1}$.*
- $\mathsf{m}'/\perp \leftarrow \mathsf{UE.dec}(\mathtt{k}_\mathsf{e}, \mathsf{ct}_\mathsf{e})$*: On input a ciphertext ct_e and a secret key $\mathtt{k}_\mathsf{e}$ of some epoch e, it returns a message m' or $\perp$.*

$\underline{\mathsf{Setup}(1^\lambda)}$:

$\mathtt{k}_0 \xleftarrow{\$} \mathsf{UE.setup}(1^\lambda)$
$\mathsf{e} := 0; \mathsf{phase} := 0$
$\mathcal{L}, \widetilde{\mathcal{L}}, \mathcal{K}, \mathcal{T}, \mathcal{C} \xleftarrow{\$} \emptyset$

$\underline{\mathcal{O}.\mathsf{enc}(\mathsf{m})}$:

$\mathsf{ct} \xleftarrow{\$} \mathsf{UE.enc}(\mathtt{k}_\mathsf{e}, \mathsf{m})$
$\mathcal{L} := \mathcal{L} \cup \{(\mathsf{e}, \mathsf{ct})\}$
return ct

$\underline{\mathcal{O}.\mathsf{next}}$:

$\mathsf{e} := \mathsf{e} + 1$
$(\mathtt{k}_\mathsf{e}, \Delta_\mathsf{e}) \xleftarrow{\$} \mathsf{UE.next}(\mathtt{k}_{\mathsf{e}-1})$
if $\mathsf{phase} = 1$ **then**
$\widetilde{\mathsf{ct}}_\mathsf{e} \xleftarrow{\$} \mathsf{UE.upd}(\Delta_\mathsf{e}, \widetilde{\mathsf{ct}}_{\mathsf{e}-1})$
$\widetilde{\mathcal{L}} := \widetilde{\mathcal{L}} \cup \{(\mathsf{e}, \widetilde{\mathsf{ct}}_\mathsf{e})\}$

$\underline{\mathcal{O}.\mathsf{upd}(\mathsf{ct}_{\mathsf{e}-1})}$:

if $(\mathsf{e}-1, \mathsf{ct}_{\mathsf{e}-1}) \notin \mathcal{L}$ **then**
return $\perp$
$\mathsf{ct}_\mathsf{e} \xleftarrow{\$} \mathsf{UE.upd}(\Delta_\mathsf{e}, \mathsf{ct}_{\mathsf{e}-1})$
$\mathcal{L} := \mathcal{L} \cup \{(\mathsf{e}, \mathsf{ct}_\mathsf{e})\}$
return ct_e

$\underline{\mathcal{O}.\mathsf{corr}(\mathsf{inp}, \widehat{\mathsf{e}})}$:

if $\widehat{\mathsf{e}} > \mathsf{e}$ **then**
return $\perp$
if $\mathsf{inp} = \mathsf{key}$ **then**
$\mathcal{K} := \mathcal{K} \cup \{\widehat{\mathsf{e}}\}$
return $\mathtt{k}_{\widehat{\mathsf{e}}}$
if $\mathsf{inp} = \mathsf{token}$ **then**
$\mathcal{T} := \mathcal{T} \cup \{\widehat{\mathsf{e}}\}$
return $\Delta_{\widehat{\mathsf{e}}}$

$\underline{\mathcal{O}.\mathsf{chall\text{-}IND\text{-}ENC}(\overline{\mathsf{m}}_0, \overline{\mathsf{m}}_1)}$:

if $|\overline{\mathsf{m}}_0| \neq |\overline{\mathsf{m}}_1|$ **then**
return $\perp$
$\mathsf{phase} := 1; \widetilde{\mathsf{e}} := \mathsf{e}$
$\widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}} \xleftarrow{\$} \mathsf{UE.enc}(\mathtt{k}_{\widetilde{\mathsf{e}}}, \overline{\mathsf{m}}_\mathrm{b})$
$\mathcal{C} := \mathcal{C} \cup \{\widetilde{\mathsf{e}}\}$
$\widetilde{\mathcal{L}} := \widetilde{\mathcal{L}} \cup \{(\widetilde{\mathsf{e}}, \widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}})\}$
return $\widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}}$

$\underline{\mathcal{O}.\mathsf{chall\text{-}IND\text{-}UPD}(\overline{\mathsf{ct}}_0, \overline{\mathsf{ct}}_1)}$:

if $(\mathsf{e}-1, \overline{\mathsf{ct}}_0) \notin \mathcal{L}$ or $(\mathsf{e}-1, \overline{\mathsf{ct}}_1) \notin \mathcal{L}$ or $|\overline{\mathsf{ct}}_0| \neq |\overline{\mathsf{ct}}_1|$ **then**
return $\perp$
$\mathsf{phase} := 1; \widetilde{\mathsf{e}} := \mathsf{e}$
$\widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}} \xleftarrow{\$} \mathsf{UE.upd}(\Delta_{\widetilde{\mathsf{e}}}, \overline{\mathsf{ct}}_\mathrm{b})$
$\mathcal{C} := \mathcal{C} \cup \{\widetilde{\mathsf{e}}\}$
$\widetilde{\mathcal{L}} := \widetilde{\mathcal{L}} \cup \{(\widetilde{\mathsf{e}}, \widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}})\}$
return $\widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}}$

$\underline{\mathcal{O}.\mathsf{chall\text{-}IND\text{-}UE}(\overline{\mathsf{m}}, \overline{\mathsf{ct}})$:

if $(\mathsf{e}-1, \overline{\mathsf{ct}}) \notin \mathcal{L}$ **then**
return $\perp$
$\mathsf{phase} := 1; \widetilde{\mathsf{e}} := \mathsf{e}$
if $\mathrm{b} = 0$ **then**
$\widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}} \xleftarrow{\$} \mathsf{UE.enc}(\mathtt{k}_{\widetilde{\mathsf{e}}}, \overline{\mathsf{m}})$
else
$\widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}} \xleftarrow{\$} \mathsf{UE.upd}(\Delta_{\widetilde{\mathsf{e}}}, \overline{\mathsf{ct}})$
$\mathcal{C} := \mathcal{C} \cup \{\widetilde{\mathsf{e}}\}$
$\widetilde{\mathcal{L}} := \widetilde{\mathcal{L}} \cup \{(\widetilde{\mathsf{e}}, \widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}})\}$
return $\widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}}$

$\underline{\mathcal{O}.\mathsf{upd}\widetilde{\mathsf{C}}}$:

if $\mathsf{phase} = 0$ **then**
return $\perp$
$\mathcal{C} := \mathcal{C} \cup \{\mathsf{e}\}$
return $\widetilde{\mathsf{ct}}_\mathsf{e}$

Fig. 4: Oracles in security games for updatable encryption.

We stress that UE.next *generates a new key along with an update token, which follows from the definition in the work of Lehmann and Tackmann [LT18]. In our constructions, the update token* $\Delta_{\mathsf{e}+1}$ *can also be generated from* $\mathtt{k}_\mathsf{e}$ *and* $\mathtt{k}_{\mathsf{e}+1}$.

Definition 6 (Correctness). *Let* $\mathsf{UE} = (\mathsf{UE.setup}, \mathsf{UE.next}, \mathsf{UE.enc}, \mathsf{UE.upd}, \mathsf{UE.dec})$ *be an updatable encryption scheme. We say* UE *is* correct *if for any* $\mathsf{m} \in \mathcal{M}$*, any* $\mathtt{k}_0 \xleftarrow{\$} \mathsf{UE.setup}(1^\lambda)$*, any sequence of* $(\mathtt{k}_1, \Delta_1), \ldots, (\mathtt{k}_\mathsf{e}, \Delta_\mathsf{e})$ *generated as* $(\mathtt{k}_i, \Delta_i) \xleftarrow{\$}$

$\mathsf{UE.next}(\mathtt{k}_{i-1})$ *for all* $i \in [\mathsf{e}]$*, and for any* $0 \leq \widehat{\mathsf{e}} \leq \mathsf{e}$*, let* $\mathsf{ct}_{\widehat{\mathsf{e}}} \xleftarrow{\$} \mathsf{UE.enc}(\mathtt{k}_{\widehat{\mathsf{e}}}, \mathsf{m})$ *and* $\mathsf{ct}_j \xleftarrow{\$} \mathsf{UE.upd}(\Delta_j, \mathsf{ct}_{j-1})$ *for all* $j = \widehat{\mathsf{e}}+1, \ldots, \mathsf{e}$*, then* $\mathsf{UE.dec}(\mathtt{k}_{\mathsf{e}}, \mathsf{ct}_{\mathsf{e}}) = \mathsf{m}$.

Confidentiality. The adversary $\mathcal{A}$ has access to the oracles defined in Figure 4. We follow the bookkeeping techniques of [LT18, KLR19, BDGJ20, Jia20], using the following sets to keep track of the generated and updated ciphertexts, and the epochs in which the adversary corrupted a key or a token, or learned a version of the challenge-ciphertext.

- $\mathcal{L}$: Set of non-challenge ciphertexts $(\mathsf{e}, \mathsf{ct}_{\mathsf{e}})$ produced by calls to the $\mathcal{O}.\mathsf{enc}$ or $\mathcal{O}.\mathsf{upd}$ oracle. $\mathcal{O}.\mathsf{upd}$ only updates ciphertexts obtained in $\mathcal{L}$.
- $\widetilde{\mathcal{L}}$: Set of updated versions of the challenge ciphertexts $(\mathsf{e}, \widetilde{\mathsf{ct}}_{\mathsf{e}})$. $\widetilde{\mathcal{L}}$ is initiated with the challenge ciphertext $(\widetilde{\mathsf{e}}, \widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}})$. Any call to the $\mathcal{O}.\mathsf{next}$ oracle automatically updates the challenge ciphertext to the new epoch, which the adversary can fetch via a call to $\mathcal{O}.\mathsf{upd}\widetilde{\mathsf{C}}$.
- $\mathcal{K}$: Set of epochs e in which the adversary corrupted the secret key $\mathtt{k}_{\mathsf{e}}$ (from $\mathcal{O}.\mathsf{corr}$).
- $\mathcal{T}$: Set of epochs e in which the adversary corrupted the update token Δ_{e} (from $\mathcal{O}.\mathsf{corr}$).
- $\mathcal{C}$: Set of epochs e in which the adversary learned a version of the challenge ciphertext (from $\mathcal{O}.\mathsf{chall}$ or $\mathcal{O}.\mathsf{upd}\widetilde{\mathsf{C}}$).

We further define the epoch identification sets $\mathcal{C}^*, \mathcal{K}^*, \mathcal{T}^*$ as the extended sets of $\mathcal{C}, \mathcal{K}, \mathcal{T}$ in which the adversary learned or inferred information. We focus on *no-directional* key updates and *uni-directional* ciphertext updates.

$$
\begin{aligned}
&\mathcal{K}^* := \mathcal{K} \\
&\mathcal{T}^* := \{\mathsf{e} \in \{0, \ldots, \mathsf{e}_{\mathsf{end}}\} | (\mathsf{e} \in \mathcal{T}) \vee (\mathsf{e}-1 \in \mathcal{K}^* \wedge \mathsf{e} \in \mathcal{K}^*)\} \\
&\mathcal{C}^* := \{\mathsf{e} \in \{0, \ldots, \mathsf{e}_{\mathsf{end}}\} | \mathsf{ChallEq}(\mathsf{e}) = \mathsf{true}\} \\
&\quad \text{where } \mathsf{true} \leftarrow \mathsf{ChallEq}(\mathsf{e}) \iff (\mathsf{e} \in \mathcal{C}) \vee (\mathsf{ChallEq}(\mathsf{e}-1) \wedge \mathsf{e} \in \mathcal{T}^*)
\end{aligned}
$$

Remark 1. The constructions we present later will in fact permit *backward-leak key updates*. At first glance the *backward-leak key updates* notion proposed by Nishimaki [Nis21] is seemingly weaker than *no-directionality key updates*. However, as mentioned in the introduction, this notion is essentially equivalent to no-directional key updates because backward-leak derivation of $\mathtt{k}_{\mathsf{e}-1}$ does not increase the adversary's advantage in breaking the scheme. In particular, if the adversary obtains a challenge ciphertext $\widetilde{\mathsf{ct}}_{\mathsf{e}-1}$ and corrupts Δ_{e} and $\mathtt{k}_{\mathsf{e}}$, then it does *not* matter if the adversary can derive $\mathtt{k}_{\mathsf{e}-1}$ or not, as it can always update the ciphertext to $\widetilde{\mathsf{ct}}_{\mathsf{e}}$ and decrypt it using $\mathtt{k}_{\mathsf{e}}$.

Definition 7 (IND-ENC, IND-UPD, IND-UE security). *Let* $\mathsf{UE} = (\mathsf{UE.setup}, \mathsf{UE.next}, \mathsf{UE.enc}, \mathsf{UE.upd}, \mathsf{UE.dec})$ *be an updatable encryption scheme. We say* UE *is* notion*-secure for* notion $\in \{\mathsf{IND\text{-}ENC}, \mathsf{IND\text{-}UPD}, \mathsf{IND\text{-}UE}\}$ *if for all PPT adversary* $\mathcal{A}$ *it holds that*

$$
\left| \Pr\left[\mathsf{Exp}^{\mathsf{notion}}_{\mathcal{A},\mathsf{UE}}(1^\lambda) = 1\right] - \frac{1}{2} \right| \leq \mathrm{negl}(\lambda)
$$

for some negligible function $\mathrm{negl}(\cdot)$.

Experiment $\mathsf{Exp}^{\mathsf{notion}}_{\mathcal{A},\mathsf{UE}}(1^\lambda)$:

Run $\mathsf{Setup}(1^\lambda)$
$(\mathsf{state}, \mathsf{Chall}_0, \mathsf{Chall}_1) \xleftarrow{\$} \mathcal{A}^{\mathcal{O}.\mathsf{enc},\mathcal{O}.\mathsf{next},\mathcal{O}.\mathsf{upd},\mathcal{O}.\mathsf{corr}}(1^\lambda)$
$\mathrm{b} \xleftarrow{\$} \{0,1\}$
$\widetilde{\mathsf{ct}} \xleftarrow{\$} \mathcal{O}.\mathsf{chall\text{-}notion}(\mathsf{Chall}_0, \mathsf{Chall}_1)$
Proceed only if $\widetilde{\mathsf{ct}} \neq \perp$
$\mathrm{b}' \xleftarrow{\$} \mathcal{A}^{\mathcal{O}.\mathsf{enc},\mathcal{O}.\mathsf{next},\mathcal{O}.\mathsf{upd},\mathcal{O}.\mathsf{corr},\mathcal{O}.\mathsf{upd}\widetilde{\mathsf{C}}}(\mathsf{state}, \widetilde{\mathsf{ct}})$
return 1 if $\mathrm{b} = \mathrm{b}'$ and $\mathcal{C}^* \cap \mathcal{K}^* = \emptyset$

3.2 IND-ENC Secure Unidirectional UE

We begin by showing that any KPHE scheme with $2n$-bit secret keys that satisfies distributional semantic security with respect to the distribution $\mathcal{U}_n$, as well as public key and ciphertext blinding as described in Section 2 implies an IND-ENC secure unidirectional UE scheme.

Construction. Given a KHPE scheme of the form

$$\mathsf{KPHE} = (\mathsf{KPHE.Setup}, \mathsf{KPHE.SKGen}, \mathsf{KPHE.PKGen}, \mathsf{KPHE.Enc}, \mathsf{KPHE.Dec}, \mathsf{KPHE.Eval}),$$

with $2n$-bit secret keys, we construct a unidirectional UE scheme

$$\mathsf{UE} = (\mathsf{UE.setup}, \mathsf{UE.next}, \mathsf{UE.enc}, \mathsf{UE.upd}, \mathsf{UE.dec}),$$

with message space $\mathcal{M} = \{0,1\}^{2n}$ as follows:

- $\mathsf{UE.setup}(1^\lambda)$: Generate $\mathsf{pp} \xleftarrow{\$} \mathsf{KPHE.Setup}(1^\lambda)$, $\mathsf{sk}_0 \xleftarrow{\$} \mathsf{KPHE.SKGen}(\mathsf{pp}, \mathcal{U}_n)$, and output
$$\mathtt{k}_0 = (\mathsf{pp}, \mathsf{sk}_0).$$
- $\mathsf{UE.next}(\mathtt{k}_\mathsf{e})$: Parse $\mathtt{k}_\mathsf{e} = (\mathsf{pp}, \mathsf{sk}_\mathsf{e})$. Generate $\mathsf{sk}_{\mathsf{e}+1} \xleftarrow{\$} \mathsf{KPHE.SKGen}(\mathsf{pp}, \mathcal{U}_n)$ and $\mathsf{pk}_{\mathsf{e}+1} \xleftarrow{\$} \mathsf{KPHE.PKGen}(\mathsf{pp}, \mathsf{sk}_{\mathsf{e}+1})$. Output
$$\mathtt{k}_{\mathsf{e}+1} = (\mathsf{pp}, \mathsf{sk}_{\mathsf{e}+1}), \quad \Delta_{\mathsf{e}+1} = (\mathsf{pk}_{\mathsf{e}+1}, \mathsf{KPHE.Enc}(\mathsf{pk}_{\mathsf{e}+1}, \mathsf{sk}_\mathsf{e})).$$
- $\mathsf{UE.enc}(\mathtt{k}_\mathsf{e}, \mathsf{m})$: Parse $\mathtt{k}_\mathsf{e} = (\mathsf{pp}, \mathsf{sk}_\mathsf{e})$. Generate $\mathsf{pk}_\mathsf{e} \xleftarrow{\$} \mathsf{KPHE.PKGen}(\mathsf{pp}, \mathsf{sk}_\mathsf{e})$ and compute $\mathsf{ctx}_\mathsf{e} \xleftarrow{\$} \mathsf{KPHE.Enc}(\mathsf{pk}_\mathsf{e}, \mathsf{m})$. Output
$$\mathsf{ct}_\mathsf{e} = (0, (\mathsf{pk}_\mathsf{e}, \mathsf{ctx}_\mathsf{e})).$$
- $\mathsf{UE.upd}(\Delta_{\mathsf{e}+1}, \mathsf{ct}_\mathsf{e})$: Parse the update token and the ciphertext as
$$\Delta_{\mathsf{e}+1} = (\mathsf{pk}_\Delta, \mathsf{ctx}_\Delta), \quad \mathsf{ct}_\mathsf{e} = (t, (\overline{\mathsf{pk}}_{\mathsf{e}-t}, \overline{\mathsf{ctx}}_{\mathsf{e}-t}), \ldots, (\overline{\mathsf{pk}}_{\mathsf{e}-1}, \overline{\mathsf{ctx}}_{\mathsf{e}-1}), (\mathsf{pk}_\mathsf{e}, \mathsf{ctx}_\mathsf{e}))$$

Sample a uniform random permutation $\pi : [2n] \to [2n]$. Also, let $\pi_{\mathsf{id}} : [2n] \to [2n]$ denote the *identity* permutation. Compute

$$(\overline{\mathsf{pk}}_{\mathsf{e}}, \overline{\mathsf{ctx}}_{\mathsf{e}}) \xleftarrow{\$} \mathsf{KPHE.Eval}(\mathsf{pk}_{\mathsf{e}}, \mathsf{ctx}_{\mathsf{e}}, \pi, \pi_{\mathsf{id}}), \quad (\mathsf{pk}_{\mathsf{e}+1}, \mathsf{ctx}_{\mathsf{e}+1}) \xleftarrow{\$} \mathsf{KPHE.Eval}(\mathsf{pk}_{\Delta}, \mathsf{ctx}_{\Delta}, \pi_{\mathsf{id}}, \pi).$$

and output the updated ciphertext as:

$$\mathsf{ct}_{\mathsf{e}+1} = (t+1, (\overline{\mathsf{pk}}_{\mathsf{e}-t}, \overline{\mathsf{ctx}}_{\mathsf{e}-t}), \ldots, (\overline{\mathsf{pk}}_{\mathsf{e}}, \overline{\mathsf{ctx}}_{\mathsf{e}}), (\mathsf{pk}_{\mathsf{e}+1}, \mathsf{ctx}_{\mathsf{e}+1})).$$

– $\mathsf{UE.dec}(\mathtt{k}_{\mathsf{e}}, \mathsf{ct}_{\mathsf{e}})$: Parse $\mathtt{k}_{\mathsf{e}} = (\mathsf{pp}, \mathsf{sk}_{\mathsf{e}})$ and the ciphertext as

$$\mathsf{ct}_{\mathsf{e}} = (t, (\overline{\mathsf{pk}}_{\mathsf{e}-t}, \overline{\mathsf{ctx}}_{\mathsf{e}-t}), \ldots, (\overline{\mathsf{pk}}_{\mathsf{e}-1}, \overline{\mathsf{ctx}}_{\mathsf{e}-1}), (\mathsf{pk}_{\mathsf{e}}, \mathsf{ctx}_{\mathsf{e}})).$$

If $t = 0$, then output $\mathsf{m} \leftarrow \mathsf{KPHE.Dec}(\mathsf{sk}_{\mathsf{e}}, \mathsf{ctx}_{\mathsf{e}})$.
Otherwise, compute $\overline{\mathsf{sk}}_{\mathsf{e}-1} \leftarrow \mathsf{KPHE.Dec}(\mathsf{sk}_{\mathsf{e}}, \mathsf{ctx}_{\mathsf{e}})$. Then for each j from $(\mathsf{e}-1)$ downto $(\mathsf{e}-t+1)$, compute

$$\overline{\mathsf{sk}}_{j-1} \leftarrow \mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_{j}, \overline{\mathsf{ctx}}_{j}).$$

Finally, output the message $\mathsf{m} \leftarrow \mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_{\mathsf{e}-t}, \overline{\mathsf{ctx}}_{\mathsf{e}-t})$.

Correctness. We first prove the correctness of the UE scheme. For any $\mathsf{m} \in \mathcal{M}$, any $\mathtt{k}_0 \leftarrow \mathsf{UE.setup}(1^\lambda)$, any sequence of $(\mathtt{k}_1, \Delta_1), \ldots, (\mathtt{k}_{\mathsf{e}}, \Delta_{\mathsf{e}})$ generated as $(\mathtt{k}_i, \Delta_i) \leftarrow \mathsf{UE.next}(\mathtt{k}_{i-1})$ for all $i \in [\mathsf{e}]$, let $\mathsf{ct}_0 \leftarrow \mathsf{UE.enc}(\mathtt{k}_0, \mathsf{m})$ and $\mathsf{ct}_i \leftarrow \mathsf{UE.upd}(\Delta_i, \mathsf{ct}_{i-1})$ for all $j \in [\mathsf{e}]$, then the final ciphertext is of the form $\mathsf{ct}_{\mathsf{e}} = (\mathsf{e}, (\overline{\mathsf{pk}}_0, \overline{\mathsf{ctx}}_0), \ldots, (\overline{\mathsf{pk}}_{\mathsf{e}-1}, \overline{\mathsf{ctx}}_{\mathsf{e}-1}), (\mathsf{pk}_{\mathsf{e}}, \mathsf{ctx}_{\mathsf{e}}))$. All the secret keys are of the form $\mathtt{k}_0 = (\mathsf{pp}, \mathsf{sk}_0), \ldots, \mathtt{k}_{\mathsf{e}} = (\mathsf{pp}, \mathsf{sk}_{\mathsf{e}})$. Let π_j be the random permutation sampled in $\mathsf{UE.upd}(\Delta_{j+1}, \mathsf{ct}_j)$ and let $\overline{\mathsf{sk}}_j = \pi_j(\mathsf{sk}_j)$ for all $j = 0, 1, \ldots, \mathsf{e}-1$. We can prove by induction that $\mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_0, \overline{\mathsf{ctx}}_0) = \mathsf{m}, \mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_1, \overline{\mathsf{ctx}}_1) = \overline{\mathsf{sk}}_0, \ldots, \mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_{\mathsf{e}-1}, \overline{\mathsf{ctx}}_{\mathsf{e}-1}) = \overline{\mathsf{sk}}_{\mathsf{e}-2}, \mathsf{KPHE.Dec}(\mathsf{sk}_{\mathsf{e}}, \mathsf{ctx}_{\mathsf{e}}) = \overline{\mathsf{sk}}_{\mathsf{e}-1}$. Therefore, $\mathsf{UE.dec}(\mathtt{k}_{\mathsf{e}}, \mathsf{ct}_{\mathsf{e}})$ outputs m. This argument is for any ciphertext starting from epoch 0. The same argument holds for any ciphertext starting from any epoch $\widehat{\mathsf{e}}$ where $0 \le \widehat{\mathsf{e}} \le \mathsf{e}$.

Confidentiality. Next we prove the IND-ENC security of our UE scheme. More formally, we state and prove the following theorem:

Theorem 3 (IND-ENC Security). *Assuming that* KPHE *satisfies distributional security with respect to the distribution $\mathcal{U}_n$, as well as public key and ciphertext blinding as described in Section 2, the above* UE *construction is an IND-ENC secure unidirectional UE scheme.*

Proof. The proof proceeds via a hybrid argument.

Hyb_0 The challenger plays the real game with the adversary.

Hyb_1 Same as Hyb_0 but for $\mathsf{UE.upd}(\Delta_{\mathsf{e}}, \mathsf{ct}_{\mathsf{e}-1})$ in $\mathcal{O}.\mathsf{upd}$ and $\mathsf{UE.upd}(\Delta_{\mathsf{e}}, \widetilde{\mathsf{ct}}_{\mathsf{e}-1})$ in $\mathcal{O}.\mathsf{next}$, do the following:
  – Let $\mathtt{k}_{\mathsf{e}-1} = (\mathsf{pp}, \mathsf{sk}_{\mathsf{e}-1})$ and $\mathtt{k}_{\mathsf{e}} = (\mathsf{pp}, \mathsf{sk}_{\mathsf{e}})$.

– Parse the ciphertext $\mathsf{ct}_{\mathsf{e}-1}$ or $\widetilde{\mathsf{ct}}_{\mathsf{e}-1}$ as

$$(t, (\overline{\mathsf{pk}}_{\mathsf{e}-1-t}, \overline{\mathsf{ctx}}_{\mathsf{e}-1-t}), \ldots, (\overline{\mathsf{pk}}_{\mathsf{e}-2}, \overline{\mathsf{ctx}}_{\mathsf{e}-2}), (\mathsf{pk}_{\mathsf{e}-1}, \mathsf{ctx}_{\mathsf{e}-1})),$$

where $\mathsf{ctx}_{\mathsf{e}-1} = \mathsf{KPHE.Enc}(\mathsf{pk}_{\mathsf{e}-1}, x)$. Note that if $t = 0$, then $x = \mathsf{m}$ for some message, otherwise $x = \overline{\mathsf{sk}}_{\mathsf{e}-2}$ that is the KPHE secret key corresponding to $\overline{\mathsf{pk}}_{\mathsf{e}-2}$.

– Sample a uniform random permutation $\pi : [2n] \to [2n]$, let $\overline{\mathsf{sk}}_{\mathsf{e}-1} = \pi(\mathsf{sk}_{\mathsf{e}-1})$, and sample $\overline{\mathsf{pk}}_{\mathsf{e}-1} \xleftarrow{\$} \mathsf{KPHE.PKGen}(\mathsf{pp}, \overline{\mathsf{sk}}_{\mathsf{e}-1})$. Compute $\overline{\mathsf{ctx}}_{\mathsf{e}-1} \xleftarrow{\$} \mathsf{KPHE.Enc}(\overline{\mathsf{pk}}_{\mathsf{e}-1}, x)$.

– Sample $\mathsf{pk}_{\mathsf{e}} \xleftarrow{\$} \mathsf{KPHE.PKGen}(\mathsf{pp}, \mathsf{sk}_{\mathsf{e}})$ and compute $\mathsf{ctx}_{\mathsf{e}} \xleftarrow{\$} \mathsf{KPHE.Enc}(\mathsf{pk}_{\mathsf{e}}, \overline{\mathsf{sk}}_{\mathsf{e}-1})$.

– Let ct_{e} or $\widetilde{\mathsf{ct}}_{\mathsf{e}}$ be

$$(t+1, (\overline{\mathsf{pk}}_{\mathsf{e}-1-t}, \overline{\mathsf{ctx}}_{\mathsf{e}-1-t}), \ldots, (\overline{\mathsf{pk}}_{\mathsf{e}-1}, \overline{\mathsf{ctx}}_{\mathsf{e}-1}), (\mathsf{pk}_{\mathsf{e}}, \mathsf{ctx}_{\mathsf{e}})).$$

We prove in Lemma 2 that this hybrid is computationally indistinguishable from Hyb_0 to any PPT adversary by the blinding property of KPHE.

Hyb_2 Same as Hyb_1 but for $\mathsf{UE.upd}(\Delta_{\mathsf{e}}, \mathsf{ct}_{\mathsf{e}-1})$ in $\mathcal{O}.\mathsf{upd}$ and $\mathsf{UE.upd}(\Delta_{\mathsf{e}}, \widetilde{\mathsf{ct}}_{\mathsf{e}-1})$ in $\mathcal{O}.\mathsf{next}$, instead of letting $\overline{\mathsf{sk}}_{\mathsf{e}-1} = \pi(\mathsf{sk}_{\mathsf{e}-1})$, sample $\overline{\mathsf{sk}}_{\mathsf{e}-1}$ from the distribution $\mathcal{U}_n$. This hybrid is statistically identical to Hyb_1.

Hyb_3 Let $\widetilde{\mathsf{e}}$ be the challenge epoch, and let $\overline{\mathsf{e}}$ be the last epoch where the adversary corrupts continuous update tokens from $\widetilde{\mathsf{e}}$, namely the adversary corrupts $\Delta_{\widetilde{\mathsf{e}}+1}, \Delta_{\widetilde{\mathsf{e}}+2}, \ldots, \Delta_{\overline{\mathsf{e}}}$ but not $\Delta_{\overline{\mathsf{e}}+1}$. This hybrid is the same as Hyb_2 except that the challenger guesses $\widetilde{\mathsf{e}}^\star$ and $\overline{\mathsf{e}}^\star$ at the beginning of the game and aborts the game if guessing incorrectly. Let E be the upper bound on the number of epochs during the game. If the challenger does not abort, then this hybrid is identical to Hyb_2, which happens with probability at least $\frac{1}{E^2}$. In the remaining hybrids, we assume for simplicity that the challenger guesses $\widetilde{\mathsf{e}}$ and $\overline{\mathsf{e}}$ correctly.

Hyb_4 Same as Hyb_3 except that for each $\mathtt{k}_{\mathsf{e}} = (\mathsf{pp}, \mathsf{sk}_{\mathsf{e}})$, generate a single public key $\widehat{\mathsf{pk}}_{\mathsf{e}} \xleftarrow{\$} \mathsf{KPHE.PKGen}(\mathsf{pp}, \mathsf{sk}_{\mathsf{e}})$. Then whenever $\mathsf{KPHE.Enc}(\mathsf{pk}_{\mathsf{e}}, x)$ is computed for a freshly generated pk_{e} and some x, compute it as $\mathsf{KPHE.Eval}(\widehat{\mathsf{pk}}_{\mathsf{e}}, \mathsf{KPHE.Enc}(\widehat{\mathsf{pk}}_{\mathsf{e}}, x), \pi_{\mathsf{id}}, \pi_{\mathsf{id}})$. That is, instead of generating a fresh pk_{e} from sk_{e} every time, use the same $\widehat{\mathsf{pk}}_{\mathsf{e}}$ to encrypt x and use then KPHE.Eval to re-randomize it.
This hybrid is computationally indistinguishable from Hyb_3 by the blinding property of KPHE. We omit the detailed reduction here, but it is similar to the reduction in the proof of Lemma 2.

Hyb_5 Same as Hyb_4 except that for all $\widetilde{\mathsf{e}}+1 \le \mathsf{e} \le \overline{\mathsf{e}}$, $\mathsf{UE.next}(\mathtt{k}_{\mathsf{e}-1})$ is computed as follows. Generate $\mathsf{sk}_{\mathsf{e}} \xleftarrow{\$} \mathsf{KPHE.SKGen}(\mathsf{pp}, \mathcal{U}_n)$ and let $\widehat{\mathsf{pk}}_{\mathsf{e}} \xleftarrow{\$} \mathsf{KPHE.PKGen}(\mathsf{pp}, \mathsf{sk}_{\mathsf{e}})$ be the single public key for $\mathtt{k}_{\mathsf{e}}$ (that will be used for every KPHE.Enc). Output

$$\mathtt{k}_{\mathsf{e}} = (\mathsf{pp}, \mathsf{sk}_{\mathsf{e}}), \quad \Delta_{\mathsf{e}} = (\widehat{\mathsf{pk}}_{\mathsf{e}}, \mathsf{KPHE.Enc}(\widehat{\mathsf{pk}}_{\mathsf{e}}, 0^{2n})).$$

We prove in Lemma 3 that this hybrid is computationally indistinguishable from Hyb_4 to any PPT adversary based on the distributional semantic security of KPHE.

Hyb_6 Same as Hyb_5 except that for each $\mathtt{k}_\mathsf{e} = (\mathsf{pp}, \mathsf{sk}_\mathsf{e})$, generate a single public key $\widehat{\mathsf{pk}_\mathsf{e}} \xleftarrow{\$} \mathsf{KPHE.PKGen}(\mathsf{pp}, \mathsf{sk}_\mathsf{e})$ and use $\mathsf{KPHE.Eval}(\widehat{\mathsf{pk}_\mathsf{e}}, \mathsf{KPHE.Enc}(\widehat{\mathsf{pk}_\mathsf{e}}, \cdot), \pi_\mathsf{id}, \pi_\mathsf{id})$ for all the computation of $\mathsf{KPHE.Enc}(\mathtt{k}_\mathsf{e}, \cdot)$ (including the computation of Δ_e). The only exception is the challenge ciphertext $\widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}}$, which is computed using $\widehat{\mathsf{pk}_{\widetilde{\mathsf{e}}}}$ without re-randomization, namely

$$\widetilde{\mathsf{ct}}_{\widetilde{\mathsf{e}}} = (0, (\widehat{\mathsf{pk}_{\widetilde{\mathsf{e}}}}, \mathsf{KPHE.Enc}(\widehat{\mathsf{pk}_{\widetilde{\mathsf{e}}}}, \overline{\mathsf{m}}_\mathrm{b}))).$$

This hybrid is computationally indistinguishable from Hyb_5 by the blinding property of KPHE. We omit the detailed reduction here, but it is similar to the reduction in the proof of Lemma 2.

Finally, we argue that in the final hybrid Hyb_6, any PPT adversary cannot distinguish an encryption of $\overline{\mathsf{m}}_0$ or $\overline{\mathsf{m}}_1$ in the challenge epoch $\widetilde{\mathsf{e}}$, which relies on the distributional semantic security of KPHE, which will conclude our proof.

Assume for the purpose of contradiction that there exists a PPT adversary $\mathcal{A}$ that can distinguish an encryption of $\overline{\mathsf{m}}_0$ or $\overline{\mathsf{m}}_1$ in the challenge epoch. Then we construct a PPT adversary $\mathcal{B}$ that breaks the distributional semantic security of KPHE. The adversary $\mathcal{B}$ first receives $(\mathsf{pp}, \mathsf{pk})$ from the challenger in the semantic security security game. Then $\mathcal{B}$ plays the UE game with $\mathcal{A}$ as a challenger in Hyb_6. $\mathcal{B}$ uses pp to generate UE keys and update tokens as in Hyb_6 except that for epoch $\widetilde{\mathsf{e}}$, the UE key $\mathtt{k}_{\widetilde{\mathsf{e}}}$ is unknown. When $\mathcal{B}$ receives the challenge messages $(\overline{\mathsf{m}}_0, \overline{\mathsf{m}}_1)$ from $\mathcal{A}$ in the UE game, it forwards the two messages to the KPHE challenger and gets back ctx, and then responds to $\mathcal{A}$ with $\mathsf{ct} = (0, (\mathsf{pp}, \mathsf{ctx}))$. Note that $\mathcal{B}$ doesn't need to know $\mathtt{k}_{\widetilde{\mathsf{e}}}$ because it is never used. In particular, $\mathcal{B}$ can use pk to compute all the $\mathsf{UE.enc}(\mathtt{k}_{\widetilde{\mathsf{e}}}, \cdot)$. Finally, $\mathcal{B}$ outputs whatever $\mathcal{A}$ outputs.

If $\mathcal{A}$ can distinguish between encryptions of $\overline{\mathsf{m}}_0$ and $\overline{\mathsf{m}}_1$ with non-negligible probability, then $\mathcal{B}$ can break the distributional semantic security of KPHE with non-negligible probability, which leads to contradiction.

Lemma 2. $\mathsf{Hyb}_0 \overset{c}{\approx} \mathsf{Hyb}_1$ *in the proof of Theorem 3.*

Lemma 3. $\mathsf{Hyb}_4 \overset{c}{\approx} \mathsf{Hyb}_5$ *in the proof of Theorem 3.*

We defer the formal proofs of Lemmas 2 and 3 to the full version of our paper [MPW22]. These proofs complete the overall proof of Theorem 3. □

Remark 2. In our construction one can derive $\mathtt{k}_{\mathsf{e}-1}$ from Δ_e and $\mathtt{k}_\mathsf{e}$. It is for this reason that our construction permits *backward-leak unidirectional key updates* proposed by Nishimaki [Nis21] where secret keys can be derived in the backward direction but not forward direction. However, as discussed earlier, this notion is essentially equivalent to no-directional key updates (the optimal case) and has no bearing on our security analysis.

3.3 Post-Compromise Secure Unidirectional UE

In this section, we show that any KPHE scheme with $2n$-bit secret keys that satisfies distributional security with respect to the distribution $\mathcal{U}_n$, as well as public key and ciphertext blinding as described in Section 2 implies a post-compromise secure unidirectional UE scheme.

3.3.1 IND-UPD Secure Unidirectional UE

We first show how to construct an UE scheme that satisfies the IND-UPD security definition as proposed in [LT18]. Given a KHPE scheme of the form

$$\mathsf{KPHE} = (\mathsf{KPHE.Setup}, \mathsf{KPHE.KeyGen}, \mathsf{KPHE.Enc}, \mathsf{KPHE.Dec}, \mathsf{KPHE.TransPK}, \mathsf{KPHE.Eval}),$$

with $2n$-bit secret keys, we construct a unidirectional UE scheme

$$\mathsf{UE} = (\mathsf{UE.setup}, \mathsf{UE.next}, \mathsf{UE.enc}, \mathsf{UE.upd}, \mathsf{UE.dec}),$$

that only differs from the IND-ENC construction in $\mathsf{UE.upd}$:

- $\mathsf{UE.upd}(\Delta_{\mathsf{e}+1}, \mathsf{ct}_\mathsf{e})$: Parse the update token and the ciphertext as

  $$\Delta_{\mathsf{e}+1} = (\mathsf{pk}_\Delta, \mathsf{ctx}_\Delta), \quad \mathsf{ct}_\mathsf{e} = (t, (\overline{\mathsf{pk}}_{\mathsf{e}-t}, \overline{\mathsf{ctx}}_{\mathsf{e}-t}), \ldots, (\overline{\mathsf{pk}}_{\mathsf{e}-1}, \overline{\mathsf{ctx}}_{\mathsf{e}-1}), (\mathsf{pk}_\mathsf{e}, \mathsf{ctx}_\mathsf{e}))$$

  Sample $(t+1)$ uniform random permutations $\pi_{\mathsf{e}-t}, \ldots, \pi_\mathsf{e} : [2n] \to [2n]$. Also, let $\pi_{\mathsf{id}} : [2n] \to [2n]$ denote the identity permutation. For each $i \in \{\mathsf{e}-t+1, \ldots, \mathsf{e}-1\}$, compute

  $$(\widetilde{\mathsf{pk}}_i, \widetilde{\mathsf{ctx}}_i) \xleftarrow{\$} \mathsf{KPHE.Eval}(\overline{\mathsf{pk}}_i, \overline{\mathsf{ctx}}_i, \pi_i, \pi_{i-1}).$$

  Additionally, compute

  $$(\widetilde{\mathsf{pk}}_{\mathsf{e}-t}, \widetilde{\mathsf{ctx}}_{\mathsf{e}-t}) \xleftarrow{\$} \mathsf{KPHE.Eval}(\overline{\mathsf{pk}}_{\mathsf{e}-t}, \overline{\mathsf{ctx}}_{\mathsf{e}-t}, \pi_{\mathsf{e}-t}, \pi_{\mathsf{id}}),$$
  $$(\overline{\mathsf{pk}}_\mathsf{e}, \overline{\mathsf{ctx}}_\mathsf{e}) \xleftarrow{\$} \mathsf{KPHE.Eval}(\mathsf{pk}_\mathsf{e}, \mathsf{ctx}_\mathsf{e}, \pi_\mathsf{e}, \pi_{\mathsf{e}-1}),$$
  $$(\mathsf{pk}_{\mathsf{e}+1}, \mathsf{ctx}_{\mathsf{e}+1}) \xleftarrow{\$} \mathsf{KPHE.Eval}(\mathsf{pk}_\Delta, \mathsf{ctx}_\Delta, \pi_{\mathsf{id}}, \pi_\mathsf{e}).$$

  Output the updated ciphertext as:

  $$\mathsf{ct}_{\mathsf{e}+1} = (t+1, (\widetilde{\mathsf{pk}}_{\mathsf{e}-t}, \widetilde{\mathsf{ctx}}_{\mathsf{e}-t}), \ldots, (\widetilde{\mathsf{pk}}_{\mathsf{e}-1}, \widetilde{\mathsf{ctx}}_{\mathsf{e}-1}), (\overline{\mathsf{pk}}_\mathsf{e}, \overline{\mathsf{ctx}}_\mathsf{e}), (\mathsf{pk}_{\mathsf{e}+1}, \mathsf{ctx}_{\mathsf{e}+1})).$$

Correctness. We first prove the correctness of the UE scheme. For any $\mathsf{m} \in \mathcal{M}$, any $\mathtt{k}_0 \leftarrow \mathsf{UE.setup}(1^\lambda)$, any sequence of $(\mathtt{k}_1, \Delta_1), \ldots, (\mathtt{k}_\mathsf{e}, \Delta_\mathsf{e})$ generated as $(\mathtt{k}_i, \Delta_i) \leftarrow \mathsf{UE.next}(\mathtt{k}_{i-1})$ for all $i \in [\mathsf{e}]$, let $\mathsf{ct}_0 \leftarrow \mathsf{UE.enc}(\mathtt{k}_0, \mathsf{m})$ and $\mathsf{ct}_i \leftarrow \mathsf{UE.upd}(\Delta_i, \mathsf{ct}_{i-1})$ for all $j \in [\mathsf{e}]$, then the final ciphertext is of the form $\mathsf{ct}_\mathsf{e} = (\mathsf{e}, (\overline{\mathsf{pk}}_0, \overline{\mathsf{ctx}}_0), \ldots, (\overline{\mathsf{pk}}_{\mathsf{e}-1}, \overline{\mathsf{ctx}}_{\mathsf{e}-1}), (\mathsf{pk}_\mathsf{e}, \mathsf{ctx}_\mathsf{e}))$. All the UE secret keys are of the form $\mathtt{k}_0 = (\mathsf{pp}, \mathsf{sk}_0), \ldots, \mathtt{k}_\mathsf{e} = (\mathsf{pp}, \mathsf{sk}_\mathsf{e})$. We can prove by induction that there exist permutations $\pi_0, \pi_1 \ldots, \pi_{\mathsf{e}-1} : [2n] \to [2n]$ such that $\overline{\mathsf{sk}}_i = \pi_i(\mathsf{sk}_i)$ for all $i = 0, 1, \ldots, \mathsf{e}-1$, and that $\mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_0, \overline{\mathsf{ctx}}_0) = \mathsf{m}, \mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_1, \overline{\mathsf{ctx}}_1) = \overline{\mathsf{sk}}_0, \ldots, \mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_{\mathsf{e}-1}, \overline{\mathsf{ctx}}_{\mathsf{e}-1}) = \overline{\mathsf{sk}}_{\mathsf{e}-2}, \mathsf{KPHE.Dec}(\mathsf{sk}_\mathsf{e}, \mathsf{ctx}_\mathsf{e}) = \overline{\mathsf{sk}}_{\mathsf{e}-1}$. Therefore, $\mathsf{UE.dec}(\mathtt{k}_\mathsf{e}, \mathsf{ct}_\mathsf{e})$ outputs m. This argument is for any ciphertext starting from epoch 0. The same argument holds for any ciphertext starting from any epoch $\widehat{\mathsf{e}}$ where $0 \le \widehat{\mathsf{e}} \le \mathsf{e}$.

Confidentiality. Next we prove the IND-UPD security the UE scheme. More formally, we state and prove the following theorem (the proof is provided in the full version of our paper [MPW22]):

Theorem 4 (IND-UPD Security). *Assuming that* KPHE *satisfies distributional security with respect to the distribution* $\mathcal{U}_n$*, as well as public key and ciphertext blinding as described in Section 2, the above* UE *construction is an IND-UPD secure unidirectional UE scheme.*

3.3.2 IND-UE Secure Unidirectional UE

The basic IND-UPD construction allows ciphertexts from the same epoch e to have different sizes. In particular, a freshly created ciphertext in epoch e can be trivially distinguished from a ciphertext that was created as an update of a ciphertext from epoch $(\mathsf{e}-1)$. So it cannot satisfy the combined security definition of post-compromise security for UE due to Boyd et al. [BDGJ20].

In the full version of our paper [MPW22], we showcase a simple extension of the basic construction wherein we ensure that the size for any ciphertext in epoch e is the same, irrespective of whether it was freshly created, or created as an update of a ciphertext from epoch $(\mathsf{e}-1)$. The overall construction remains exactly the same; the key alteration is in how we generate fresh ciphertexts. At a high level, a freshly created ciphertext in epoch e is made to look exactly like a ciphertext that has undergone e update operations. We do this by having e "dummy wrapper" layers over and above the core ciphertext generated by the basic construction. We defer the detailed construction and security proof to the full version of our paper [MPW22].

4 Unidirectional PRE from Circular-Secure KPHE

In this section, we show how to construct unidirectional PRE from any KPHE scheme that satisfies distributional circular security. We present the simpler construction of IND-HRA unidirectional PRE in Section 4.2. In the full version of our paper [MPW22], we show how to augment it to achieve the stronger notion of strong post-compromise security (PCS) as introduced in a recent work by Davidson et al. [DDLM19].

4.1 Definition of Unidirectional PRE

Definition 8 (Unidirectional Proxy Re-Encryption (PRE)). *A unidirectional PRE scheme is a tuple of PPT algorithms of the form*

$$\mathsf{PRE} = (\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Enc}, \mathsf{ReKeyGen}, \mathsf{ReEnc}, \mathsf{Dec}),$$

described as follows:

- $\mathsf{pp} \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$*: On input the security parameter* λ*, the setup algorithm outputs some public parameters* pp *(these parameters are implicit to all other algorithms).*
- $(\mathsf{sk}, \mathsf{pk}) \xleftarrow{\$} \mathsf{KeyGen}(\mathsf{pp})$*: On input the public parameters* pp*, the key-generation algorithm outputs a secret key-public key pair,* $(\mathsf{sk}, \mathsf{pk})$*.*
- $\mathsf{ct} \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}, \mathsf{m})$*: On input a public key* pk *and a message* m*, the encryption algorithm outputs a ciphertext* ct*.*
- $\mathsf{rk}_{i,j} \xleftarrow{\$} \mathsf{ReKeyGen}((\mathsf{sk}_i, \mathsf{pk}_i), \mathsf{pk}_j)$*: The re-key generation algorithm returns a re-encryption key* $\mathsf{rk}_{i,j}$ *for translation of a ciphertext from a key-pair* $(\mathsf{sk}_i, \mathsf{pk}_i)$ *to a key-pair* $(\mathsf{sk}_j, \mathsf{pk}_j)$*. It takes as input* $(\mathsf{sk}_i, \mathsf{pk}_i)$ *and* pk_j*, and outputs the re-encryption key* $\mathsf{rk}_{i,j}$*.*[5]
- $\mathsf{ct}_j \xleftarrow{\$} \mathsf{ReEnc}(\mathsf{rk}_{i,j}, \mathsf{ct}_i)$*: On input a re-encryption key* $\mathsf{rk}_{i,j}$ *and a ciphertext* ct_i*, the re-encryption algorithm outputs an updated ciphertext* ct_j*.*[6]
- $\mathsf{m}/\perp \leftarrow \mathsf{Dec}(\mathsf{sk}, \mathsf{ct})$*: On input a secret key* sk *and a ciphertext* ct*, the decryption algorithm outputs either a plaintext message or an error symbol.*

Definition 9 (Correctness). *A PRE scheme* $\mathsf{PRE} = (\mathsf{Setup}, \mathsf{KeyGen}, \mathsf{Enc}, \mathsf{ReKeyGen}, \mathsf{ReEnc}, \mathsf{Dec})$ *is said to be correct if for any* $\mathsf{pp} \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$*, for any* $\ell \geq 0$*, for any* $(\ell + 1)$ *key-pairs* $(\mathsf{pk}_0, \mathsf{sk}_0), \ldots, (\mathsf{pk}_\ell, \mathsf{sk}_\ell) \xleftarrow{\$} \mathsf{KeyGen}(\mathsf{pp})$*, and for any plaintext message* m*, letting* $\mathsf{ct}_0 \xleftarrow{\$} \mathsf{Enc}(\mathsf{pk}_0, \mathsf{m})$*, and letting for each* $j \in [\ell]$

$$\mathsf{rk}_j \xleftarrow{\$} \mathsf{ReKeyGen}(\mathsf{sk}_{j-1}, \mathsf{pk}_{j-1}, \mathsf{pk}_j), \quad \mathsf{ct}_j \xleftarrow{\$} \mathsf{ReEnc}(\mathsf{rk}_j, \mathsf{ct}_{j-1}),$$

we have $\mathsf{Dec}(\mathsf{sk}_\ell, \mathsf{ct}_\ell) = \mathsf{m}$ *(with all but negligible probability).*

Confidentiality. We defer the confidentiality definitions to the full version of our paper [MPW22].

4.2 HRA-Secure Unidirectional PRE

We show that any KPHE scheme with $2n$-bit secret keys and plaintext messages that satisfies: (a) distributional semantic and *circular* security with respect to the distribution $\mathcal{U}_n$, and (b) blinding, implies the existence of a multi-hop IND-HRA secure unidirectional PRE scheme.

Construction. Given a KHPE scheme of the form

$$\mathsf{KPHE} = (\mathsf{KPHE.Setup}, \mathsf{KPHE.SKGen}, \mathsf{KPHE.PKGen}, \mathsf{KPHE.Enc}, \mathsf{KPHE.Dec}, \mathsf{KPHE.Eval}),$$

[5] In a bidirectional PRE scheme, the re-key generation algorithm additionally takes as input the destination secret key sk_j, i.e., it takes as input $(\mathsf{sk}_i, \mathsf{pk}_i)$ and $(\mathsf{sk}_j, \mathsf{pk}_j)$, and outputs the re-encryption key $\mathsf{rk}_{i,j}$.

[6] The re-encryption algorithm could be either deterministic or randomized; in this work, we assume throughout that the re-encryption algorithm is randomized.

with $2n$-bit secret keys, we construct a unidirectional PRE scheme

$$\mathsf{PRE} = (\mathsf{PRE.Setup}, \mathsf{PRE.KeyGen}, \mathsf{PRE.Enc}, \mathsf{PRE.ReKeyGen}, \mathsf{PRE.ReEnc}, \mathsf{PRE.Dec}),$$

with message space $\mathcal{M} = \{0,1\}^{2n}$ as follows:

- $\mathsf{PRE.Setup}(1^\lambda)$: Sample $\mathsf{pp} \xleftarrow{\$} \mathsf{KPHE.Setup}(1^\lambda)$ and output pp.
- $\mathsf{PRE.KeyGen}(\mathsf{pp})$: Sample and output $(\mathsf{pk}, \mathsf{sk})$ where

$$\mathsf{sk} \xleftarrow{\$} \mathsf{KPHE.SKGen}(\mathsf{pp}, \mathcal{U}_n), \quad \mathsf{pk} \xleftarrow{\$} \mathsf{KPHE.PKGen}(\mathsf{pp}, \mathsf{sk}).$$

- $\mathsf{PRE.Enc}(\mathsf{pk}, \mathsf{m})$: Compute $\mathsf{ctx}_0 \xleftarrow{\$} \mathsf{KPHE.Enc}(\mathsf{pk}, \mathsf{m})$ and output

$$\mathsf{ct} = (0, (\mathsf{pk}, \mathsf{ctx}_0)).$$

- $\mathsf{PRE.ReKeyGen}(\mathsf{sk}_i, \mathsf{pk}_i, \mathsf{pk}_j)$: Output $\mathsf{rk}_{i,j} = (\mathsf{pk}_j, \mathsf{ctx}_\Delta)$, where

$$\mathsf{ctx}_\Delta \xleftarrow{\$} \mathsf{KPHE.Enc}(\mathsf{pk}_j, \mathsf{sk}_i).$$

- $\mathsf{PRE.ReEnc}(\mathsf{rk}_{i,j}, \mathsf{ct})$: Parse the reencryption key and the ciphertext as

$$\mathsf{rk}_{i,j} = (\mathsf{pk}_j, \mathsf{ctx}_\Delta), \quad \mathsf{ct} = (t, (\overline{\mathsf{pk}}_0, \overline{\mathsf{ctx}}_0), \ldots, (\overline{\mathsf{pk}}_{t-1}, \overline{\mathsf{ctx}}_{t-1}), (\widehat{\mathsf{pk}}_t, \widehat{\mathsf{ctx}}_t)),$$

  for some $t \geq 0$. Sample a uniformly random permutation $\pi : [2n] \to [2n]$. Also, let $\pi_{\mathsf{id}} : [2n] \to [2n]$ denote the *identity* permutation. Compute

$$(\overline{\mathsf{pk}}_t, \overline{\mathsf{ctx}}_t) \xleftarrow{\$} \mathsf{KPHE.Eval}(\widehat{\mathsf{pk}}_t, \widehat{\mathsf{ctx}}_t, \pi, \pi_{\mathsf{id}}),$$

$$(\widehat{\mathsf{pk}}_{t+1}, \widehat{\mathsf{ctx}}_{t+1}) \xleftarrow{\$} \mathsf{KPHE.Eval}(\mathsf{pk}_j, \mathsf{ctx}_\Delta, \pi_{\mathsf{id}}, \pi),$$

  and output the updated ciphertext as:

$$\mathsf{ct}' = (t+1, (\overline{\mathsf{pk}}_0, \overline{\mathsf{ctx}}_0), \ldots, (\overline{\mathsf{pk}}_t, \overline{\mathsf{ctx}}_t), (\widehat{\mathsf{pk}}_{t+1}, \widehat{\mathsf{ctx}}_{t+1})).$$

- $\mathsf{PRE.Dec}(\mathsf{sk}, \mathsf{ct})$: Parse the ciphertext as

$$\mathsf{ct} = (t, (\overline{\mathsf{pk}}_0, \overline{\mathsf{ctx}}_0), \ldots, (\overline{\mathsf{pk}}_{t-1}, \overline{\mathsf{ctx}}_{t-1}), (\widehat{\mathsf{pk}}_t, \widehat{\mathsf{ctx}}_t)),$$

  for some $t \geq 0$. Compute $\overline{\mathsf{sk}}_{t-1} = \mathsf{KPHE.Dec}(\mathsf{sk}, \widehat{\mathsf{ctx}}_t)$. Next, compute the following for each ℓ from $(t-1)$ to 1 in decreasing order:

$$\overline{\mathsf{sk}}_{\ell-1} = \mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_\ell, \overline{\mathsf{ctx}}_\ell).$$

  Finally, output the message $\mathsf{m} = \mathsf{KPHE.Dec}(\overline{\mathsf{sk}}_0, \overline{\mathsf{ctx}}_0)$.

Correctness. We defer the detailed proof of correctness to the full version of our paper [MPW22]. At a high level, the correctness argument is very similar to that for our unidirectional UE scheme in Section 3.2.

Theorem 5 (IND-HRA Security). *Assuming that* KPHE *satisfies blinding and distributional semantic+circular security with respect to the distribution* $\mathcal{U}_n$*,* PRE *is a multi-hop IND-HRA secure unidirectional PRE scheme.*

We defer the detailed proofs of correctness and IND-HRA security to the full version of our paper [MPW22]. Also, see [MPW22] for our construction of IND-PCS secure unidirectional PRE from any circular-secure KPHE scheme.

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