Fully-Secure MPC with Minimal Trust

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Abstract. The task of achieving *full security* (with guaranteed output delivery) in secure multiparty computation (MPC) is a long-studied problem. Known impossibility results (Cleve, STOC 86) rule out general solutions in the dishonest majority setting. In this work, we consider solutions that use an *external trusted party* (TP) to bypass the impossibility results, and study the *minimal* requirements needed from this trusted party. In particular, we restrict ourselves to the extreme setting where the size of the TP is *independent* of the size of the functionality to be computed (called "small" TP) and this TP is invoked *only once* during the protocol execution. We present several positive and negative results for fully-secure MPC in this setting.

- For a natural class of protocols, specifically, those with a *universal output decoder*, we show that the size of the TP must necessarily be exponential in the number of parties. This result holds irrespective of the computational assumptions used in the protocol. The class of protocols to which our lower bound applies is broad enough to capture prior results in the area, implying that the prior techniques necessitate the use of an exponential-sized TP. We additionally rule out the possibility of achieving information-theoretic full security (without the restriction of using a universal output decoder) using a "small" TP in the plain model (i.e., without any setup).
- In order to get around the above negative result, we consider protocols without a universal output decoder. The main positive result in our work is a construction of such a fully-secure MPC protocol assuming the existence of a succinct Functional Encryption scheme. We also give evidence that such an assumption is likely to be necessary for fully-secure MPC in certain restricted settings.
- Finally, we explore the possibility of achieving full-security with a semi-honest TP that could collude with other malicious parties (which form a dishonest majority). In this setting, we show that even fairness is impossible to achieve regardless of the "small TP" requirement.

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1 Introduction

Secure Multiparty Computation (MPC) allows a set of mutually distrusting parties to compute a joint function of their private inputs such that only the output of the function is revealed. Security of MPC protocols is required to hold even if the participating parties are controlled by a centralized *malicious* adversary, who may instruct them to deviate from the protocol specification.

Two desired properties for MPC protocols are *fairness* and *full security* (a.k.a guaranteed output delivery). Fairness requires that if the adversary learns the output of the functionality, then all the honest parties also learn this output. Full security strengthens fairness by requiring that the adversary cannot prevent the honest parties from learning the output of the functionality. Unfortunately, a classical impossibility result of Cleve [Cle86] shows that many functions cannot be fairly computed in the presence of an adversary corrupting a majority of the parties. Two ways to bypass this impossibility result are to restrict the adversary to corrupt only a minority of the parties, or to make use of some external help. In this work, we focus on the second approach, referring to the external help as a *trusted party* (TP).[6] A trusted party can be realized via different standard mechanisms, such as trusted execution environments, hardware tokens, blockchain based approaches, or cloud service providers.

Size of the TP. TPs are useful in circumventing the above impossibility result as they can be used as an ideal functionality that takes inputs from the parties and provides them outputs. A simple way to obtain protocols that satisfy full security in the TP model is for the TP to perform the entire computation on the private inputs of the parties and provide them outputs. However, this approach is less desirable as the size of the TP grows with the size of the function to be computed. Fitzi et al. [FGMO01] showed how to make the TP in the above solution *universal*, in the sense that it is independent of the function being computed. They also showed that to achieve full security, it is necessary to use TPs that take inputs from all the parties. However, this negative result does not rule out a TP which is independent of circuit size of the functionality. Thus, an interesting line of inquiry is to construct protocols where the size of the TP is independent of the circuit size of the functionality to be computed.

Apart from being a theoretically interesting question, it is also motivated by the practical goal of minimizing the use of trustworthy resources. For instance, if a trusted party service is implemented by a cloud service provider who charges fees for the use of its computational resources, it is obviously desirable (for the clients) to minimize the fees. The same holds if the TP is emulated via the use of a large-scale honest-majority MPC protocol. We refer to a setting of a trusted party whose size is independent of the circuit size of the function as the *small-TP* model. This problem is not new to our work and has already been considered

[6] This notion differs from the line of work on token-based cryptography initiated by Katz [Kat07], where the tamper-proof tokens are generated locally, and the main challenge is to guarantee security even when tokens can be maliciously generated.

in the works of Gordon et al. [GIM+10] and Ishai et al. [IOS12] for the case of fairness and full security respectively. The state of the art result from [IOS12] gave a protocol that achieves guaranteed output delivery with statistical security (in the OT-hybrid model) with a small TP, where the parties make n sequential calls to this TP. In the same work, the authors gave a protocol where the parties make a single call to the TP but where the size of the TP grows exponentially in the number of parties (and is otherwise independent of the size of the function to be computed).

Number of Calls to TP. In this work, in addition to considering a small-TP model, we are interested in designing *fully-secure* protocols that make a single call to the TP. Theoretically, one call is the minimal requirement to circumvent the impossibility of [Cle86] for fair and fully-secure MPC. It further opens up the possibility of protocols in a minimal model, reminiscent of private simultaneous message (PSM) [FKN94] model, where given a common randomness, the parties communicate one-shot message to the TP and compute the output on receiving the reply from the TP. One call as opposed to many calls is also likely to generate more practical solution in the real world settings where, for instance, the TP is replaced with a cloud service provider, or a blockchain based approach.

The question which is the main focus to our work is:

Can we construct efficient protocols that make a single call to a "small" TP and achieve full security?

1.1 Our Results

We obtain both positive and negative results on the existence of fully-secure MPC protocols using a small TP. We first discuss the negative results below.

Impossibility with a Universal Output Decoder. We give evidence that the prior approaches to this problem necessarily require a TP whose size is exponential in the number of parties. To show this, we abstract out the key features of prior protocols and show that any protocol having these features requires an exponential-sized TP (irrespective of the computational assumptions used in the protocol). More concretely, we consider the class of protocols where the parties could interact with each other (in an arbitrary number of rounds), then they make a single call to the trusted party, get a reply from TP, and then apply a *universal decoder* on this reply and their state to compute the output. By universal decoder, we mean that the size of the decoder is independent of the size of the functionality to be computed (considering single bit output functionalities). This model is interesting because it is quite natural and, more importantly, it captures prior approaches of realizing TP-aided MPC protocols [IOS12]. We show that for such protocols, the size of the TP necessarily grows exponentially with the number of parties. Our result holds irrespective of the computational assumptions used by the protocol. Additionally, our result holds even if the size of the TP is allowed to grow with the size of the function output.

Theorem 1 (Informal). *For any fully-secure MPC protocol with a universal output decoder, the size of the TP must necessarily be exponential in the number of parties.*

Necessity of Setup or Computational Assumptions. The above result naturally leads to the question of whether we can have small TP-aided fully secure MPC protocols once the restriction of using a universal decoder is relaxed. In this regard, we prove that any statistically secure protocol (without any trusted setup or correlated randomness) that makes a single call to a small TP cannot be even *semi-honest* secure. This impossibility holds even against protocols that may not have a universal output decoder. This shows that to achieve full security it is necessary to resort to computational assumptions, or assume some sort of setup (such a correlated randomness).

Theorem 2 (Informal). *There exists no MPC protocol that achieves information-theoretic security against semi-honest adversaries in the plain model with a TP whose size is a fixed polynomial in the input size of the functionality.*

Positive Results. We now focus on the problem of achieving fully-secure MPC protocols using a small TP based on computational assumptions. Our main positive result is captured by the following theorem:

Theorem 3 (Informal). *Assuming a single-key succinct Functional Encryption (FE) scheme, there exists a fully secure efficient MPC protocol that makes a single call to the small TP.*

A single-key succinct Functional Encryption is an FE scheme [SW05, O'N10, BSW11] where the size of the encryption algorithm does not grow with size of the function for which a secret key is released. Using known instantiations of these primitives from various assumptions, we get the following corollary (building on [GKP+13, GGSW13, Wat15]).

Corollary 1 (Informal). *There exists a fully secure efficient MPC protocol that makes a single call to a TP, assuming:*

1. *Learning with Errors (with sub-exponential modulus-to-noise ratio) [GKP+13] if the size of the TP is allowed to only grow with the depth and the output length of the functionality.*
2. *Witness Encryption scheme [GGSW13] and FHE if the size of the TP is allowed to only grow with the output length of the functionality.*
3. *Indistinguishability Obfuscation (iO) [BGI+01, GGH+13, JLS21] and one-way functions, where the size of the TP is independent of the depth and the output length.*

We also give evidence that this assumption might be necessary in certain restricted settings. Specifically, consider a restricted model of computation where the parties do not interact with each other, but make a single-call to the TP and could compute the output of the functionality based on the reply from

the TP. This model is reminiscent of the Private Simulataneous Messages setting [FKN94]. It is not too hard to see that this restricted model is equivalent to an MPC protocol with a succinct online phase. Specifically, the computation done by the parties before the TP call can be thought of as the pre-processing phase and this could grow with the circuit-size of the functionality. The messages sent to the TP and the computation performed by the TP correspond to the online phase of the protocol. Since we restrict the size of the TP to be small, it follows that the computation and the communication cost of the online phase is independent of the size of the functionality (i.e., the protocol has succinct online phase). The post-processing phase could grow with the size of the functionality to be computed (this is in fact necessary considering our impossibility with a universal output decoder).

Currently, the only known constructions of an MPC protocol with a succinct online phase are based on Laconic Functional Evaluation [QWW18] (LFE), which is known to imply succinct FE. This suggests that such assumptions are *likely* to be necessary in the restricted setting outlined above. In fact, an MPC protocol with a succinct online phase implies a weaker flavor of LFE with the following property: unlike standard LFE where the size of the encryption algorithm only grows with the input size, the encryption algorithm in this weaker notion of LFE has two components: (i) a pre-processing algorithm which takes the input and the size of the functionality and produces a hint that only grows with the input size, and (ii) a second algorithm that takes the input and the hint and outputs a ciphertext (the size of the second algorithm only grows with the input size). Finally, in this restricted model, we give a positive result by constructing a fully-secure MPC protocol with a single call to a small TP based on LFE.

(Im)Possibility of Reducing the Trust in TP. Finally, we explore the possibility of weakening the security requirements from the TP. Interestingly, our above solutions maintain privacy against the TP, which is an additional desirable feature. More specifically, our constructions are secure if the adversary corrupts the TP in a semi-honest manner (but does not corrupt any of the parties). This led us to explore what happens if we allow the semi-honest TP to collude with the other malicious parties. We showed that irrespective of the size of the TP, such a model would not be enough to circumvent Cleve's impossibility of fairness. This impossibility holds even if we restrict the malicious parties to be fail-stop.[7]

Our results are summarized in Table 1.

1.2 Open Directions

Our work opens up several interesting research directions. We highlight some of them below.

[7] The notion of fail-stop corruption lies between semi-honest and malicious corruption, where eavesdropping like semi-honest corruption is allowed and the only possible malicious corruption is stopping the execution of the protocol.

Security	No. of calls	Setup	Pre-TP call interaction	Universal Output Decoder	Possible?	Reference
Statistical	1	Plain	Yes	No	No	Theorem 7
Computational	1	C.R.	Yes	Yes	No	Theorem 6
Computational	1	CRS	No	No	Yes (based on LFE)	Theorem 4
Computational	1	Plain	Yes	No	Yes (based on succint FE)	Theorem 5
Statistical	n	C.R	Yes	Yes	Yes	[IOS12]
Computational	n	Plain	Yes	Yes	Yes (based on OT)	[IOS12]
Statistical	1	C.R	Yes	No	**Open**	

Table 1: Results on fully-secure MPC in dishonest majority using *small* TP under different kinds of setup (plain model i.e. no setup / C.R. i.e. correlated randomness setup / CRS i.e. common random string), security guarantees (statistical / computational) and different TP computation models (with / without the restrictions on pre-TP call interaction and universal output decoder).

- **Showing Necessity of Succinct FE.** In this work, we argued that any protocol in the restricted model (where the parties do not communicate with each other before and after the TP invocation) is equivalent to an MPC protocol with a succinct online phase. However, we are unable to extend this to the setting where the parties could potentially communicate with each other before making the TP call. Can we show that such a weaker model also implies some weakening of an MPC protocol with a succinct online phase? This would justify the necessity of a succinct FE assumption.
- **Making more than a Single Call to TP.** As our goal was to minimize the requirements from the TP as much as possible, we considered the extreme setting where a single call is made to the TP. A fascinating direction is to explore the possibility of constructing fully-secure MPC protocols from weaker assumptions which could make more than one call but less than n calls. The key challenge here is to design protocols using a stateless TP. If we allow the TP to be stateful, we can realize a construction based on FHE that makes two calls to a stateful TP.
- **Characterization of Fair Computation in the Colluding TP model.** As mentioned previously, in this work we show that it is impossible to achieve fairness in the colluding TP model (where the adversary can corrupt the TP in a semi-honest manner, in addition to corrupting majority of the parties maliciously) for general functions. However, it is still possible to achieve fairness for restricted classes of (non-trivial) functions such as coin-tossing (by using the TP to directly compute the desired function). It is an interesting open question to give a complete characterization of which function classes can be fairly computed in the colluding TP model.

1.3 Technical Highlights and Discussion

In this section, we present a high-level technical overview of our results.

1.3.1 Positive Results. We present two protocols based on LFE [QWW18] and single-key succinct FE [SW05, BSW11] respectively utilizing a *single* call to a stateless "small" TP. We start off with their trade-offs below.

LFE-based Construction. LFE's 2-round minimal communication pattern leads to an MPC in a minimal communication setting that is reminiscent of PSM-style [FKN94] communication. Here, the parties start off with a common randomness. Based on the respective inputs and this randomness, the parties communicate a single message to the TP, which performs certain computation and returns a message to each party. In the end, each party recovers the output receiving the message from the TP. Further, the encryption algorithm of LFE enjoys computation that is only dependent on the depth and the output length (and not size) of the function to be computed. This allows our TP to be "small". Here with the best known realizations of LFE, we can achieve a TP of size $\texttt{poly}(n, \kappa, d, m)$, where d denotes depth of the circuit and m denotes input and output size of the circuit, n denotes the number of parties and κ denotes the security parameter. Removing m from the complexity of the TP seems hard, intuitively because the parties never communicate with each other and they communicate only once via the TP. Achieving depth and input-size independence in this minimal communication setting is left as an interesting open question which can possibly contribute back to the LFE regime. In particular, a solution in our setting where TP is of size $\texttt{poly}(n, \kappa, m)$ will lead to a LFE where the encryption scheme and size of the ciphertext are completely independent of the depth of the function under consideration.

FE-based Construction. Unlike the LFE-based construction, our FE-based construction requires communication amongst the parties before making the TP call. While it loses on this front, there are two positive features that it brings to the table: (a) possibly weaker assumption (b) the TP's computation can be independent of d, m. Elaborating further, LFE is seemingly a stronger assumption than FE, since it is known to imply FE, while the other way is not known [QWW18]. Based on the realization of FE under various assumptions, we achieve multiple variants of the protocol where the TP's computation ranges from being completely independent of input, output and function to linearly dependent on output size (yet independent of the function) to linearly dependent on the output size and the depth of the function. To be specific, under iO and OWFs, our FE based construction leads to a TP of size $\texttt{poly}(n, \kappa)$, completely independent of the function to be computed.

Construction Overview. Our constructions follow a three-phase structure as follows: (a) phase 1: here the parties, on holding a common randomness and respective inputs, prepare a (message, state) pair, where the message is sent to the TP and the state is saved; (b) phase 2: the TP, on receiving messages from the parties, performs some computation and returns a message to every party; and (c) phase 3: the parties, on receiving the message from the TP, uses its state to recover the output. Phase 1 involves communication amongst the parties in

the FE-based construction. We provide an informal overview behind the idea for each construction below.

Overview of LFE-based Solution. We present here a simplified version of our LFE-based construction of fully-secure MPC for ease of exposition. The actual construction, detailed in Section 3.3, is significantly more nuanced and uses several techniques to achieve full security against malicious corruptions of parties. In the simplified treatment presented here, we focus on the case of semi-honest corruption, with the aim of highlighting how we manage to keep the TP size small (i.e., independent of the function size). Note that throughout this paper, we assume that each party communicates with the TP via a separate secure channel, and hence an adversary (corrupting a subset of the parties) cannot eavesdrop on the communication between the TP and any honest party.

Given this model, a simplified version of our LFE-based protocol works as follows. Each party first uses a common randomness to (locally) derive a CRS for the LFE scheme and a digest corresponding to the function f. Each party then sends the LFE CRS and the function digest to the TP, along with its own input. The TP uses the CRS and the digest to compute an LFE ciphertext encapsulating the inputs of all of the parties, and sends this ciphertext back to the parties. Finally, each party uses the LFE CRS and its local randomness of digest generation to recover the function output. Observe that the size of the messages to the TP and the computation done by the TP are independent of the size of the function f; this follows immediately from the succinctness properties of the underlying LFE scheme. Finally, we can invoke the privacy guarantees of LFE to argue that the parties learn no more information than the output of the MPC protocol, as desired.

As mentioned earlier, our actual LFE-based protocol uses additional techniques to guarantee full security in the presence of malicious corruptions. This includes techniques that enable the TP to "partition" the parties into various sets depending on their messages to the TP, and to substitute default input values for (malicious) parties not in the partition when preparing partition-specific LFE ciphertexts. Further, we augment the construction to achieve privacy against the TP. We refer to Section 3.3 for the detailed description and analysis of our construction.

Overview of FE-based Solution. We now present a simplified version of our FE-based construction of fully-secure MPC. Once again, our actual protocol, detailed in Section 3.3 uses additional techniques to achieve full security against malicious corruptions of parties; we avoid detailing all of these in the simplified treatment for ease of exposition and focus on the setting of semi-honest corruptions. As in the LFE-base solution, we again assume that each party communicates with the TP via a separate secure channel, and hence an adversary (corrupting a subset of the parties) cannot eavesdrop on the communication between the TP and any honest party.

Given this model, the simplified version of our FE-based protocol works as follows. The parties initially engage in an MPC protocol (with identifiable abort

security) to decide on a common set of public parameters and a common master public key for the FE scheme. The MPC protocol additionally outputs to each party a functional secret key for the function f to be evaluated. Each party then simply sends the master public key and its own input to the TP. The TP uses the master public key to compute an FE ciphertext encapsulating the inputs of all of the parties, and sends this ciphertext back to the parties. Finally, each party uses the functional secret key to recover the function output. Observe that the size of the messages to the TP and the computation done by the TP are independent of the size of the function f as long as the FE scheme is succinct. Finally, we can invoke the privacy guarantees of FE to argue that the parties learn no more information than the output of the MPC protocol, as desired.

Note that in the above simplified exposition, the TP incurs an overhead that grows with the size of the inputs and output of the function f to be evaluated. In our actual protocol, we use additional techniques to get rid of this dependence. In particular, we use a carefully designed indirection mechanism that allows the TP to simply partition the set of parties (depending on their messages to the TP) and encapsulate this partition information into the FE ciphertext, while delegating all computation dependent on the input/function size entirely to the parties. These techniques serve two purposes: (a) making the TP size independent of the function input/output size (and thereby asymptotically smaller than the TP size for our LFE-based solution) and (b) achieving full security against malicious corruptions of parties. Interestingly, this solution also achieves privacy against the TP. We refer to Section 3.4 for the detailed description and analysis of our construction.

1.3.2 Negative Results. We present two impossibility results for fully-secure MPC that utilizes a small TP. Our two results are as follows: **(1)** First, we show that it is impossible to achieve a fully secure TP-aided MPC utilizing a single call to a small TP, for a class of protocols that have an universal output decoder. This result holds irrespective of computational assumptions used in the protocol. The universal output decoder is independent of the function to be computed and only performs $\mathtt{poly}(n, \kappa)$ computation. **(2)** Second, we show an impossibility in the plain model, for any statistically-secure MPC even in the semi-honest setting. This result does not assume that the protocol uses an universal output decoder. We present the high-level intuition of both the impossibility arguments.

Impossibility of Fully-Secure MPC protocols with universal output decoder in the Correlated Randomness Model. We now present a simplified argument of our impossibility result and refer to Section 4.1 for the details. Consider an execution of an MPC protocol with full security, where the adversary behaves honestly until the TP call. During the TP call, he can choose to make any subset of corrupt parties, say S, abort; where the number of such subsets is exponential in the number of parties. Since the protocol achieves full security, it must be the case that the TP is able to enable output computation by the parties, no matter which subset S the adversary chooses. Further, the output

must be such that it is computed on the default input of the corrupt parties in S and the honest inputs of others (i.e. the input used until and including the TP call). Intuitively, this means that the information given to the TP is such that it can be used to recover 2^n output values (one for each possible subset). Since the TP is small, this information must be 'short' and can therefore be perceived as a 'compression' of the 2^n output values. Building on the above intuition, we show that a fully secure protocol with universal output decoder would imply an (encoding, decoding) scheme which can produce an encoding that is smaller than the size of the message domain of the encoding scheme. This breaches the known incompressibility argument. Precisely, we use a result of De et al. [DTT10], which formalizes the notion that it is impossible to compress every element in a set X to a string less than $\log |X|$ bits long.

Impossibility of Statistical MPC in the Plain Model. At a high-level, we show this impossibility by demonstrating that such a protocol would imply a semi-honest information-theoretic oblivious transfer (OT) extension, which is known to be impossible [Bea96]. Here, OT extension refers to a protocol that allows a sender and a receiver to extend a relatively small number of base OTs (say k) to a larger number of OTs (say $k+1$) using only symmetric-key primitives.

The main idea of the proof is to construct an OT extension protocol using the semi-honest statistically-secure protocol, say Π, as follows. We choose the functionality computed by Π as computing $(k+1)$ oblivious transfer instances. Since the TP is small, its size must be strictly less than the circuit computing $(k+1)$ oblivious transfer instances. Roughly speaking, Π can thus be viewed as a protocol that enables the parties to generate $(k+1)$ OTs, by having access to the TP whose functionality can be realized by strictly less than $(k+1)$ OTs (say k OTs). We build on this idea to construct an information-theoretic semi-honest OT extension protocol where the parties begin with k base OTs and use Π to generate $(k+1)$ OTs.

1.3.3 Impossibility of Fair MPC with Colluding TP.

Our results show that small TP is sufficient for positive results in the computational security regime. But what happens when the TP is no longer a stand-alone entity, but behaves as another party that can not only eavesdrop but also collude with the corrupt parties (while remaining semi-honest by itself)? This is a model where the adversary controls a majority of the parties maliciously (or even fail-stop fashion) and *simultaneously* corrupts the TP semi-honestly. For this model, we ask the questions: *Can such a TP circumvent Cleve's [Cle86] impossibility result?*

We show a negative result for the above question even for fail-stop adversaries (i.e., the malicious parties still follow the protocol specification but may choose to stop arbitrarily). At a high level, we take the following route. Note that the colluding adversarial model can be viewed more generally, in terms of the general mixed adversarial model that has been studied in works such as [HMZ08, FHM99, BFH+08]. We then use the characterization proposed in [HMZ08] for

fair and fully-secure MPC tolerating mixed adversaries to rule out a fair protocol in the colluding model even when malicious corruption is replaced with fail-stop corruption. In particular, we define an adversarial structure complying with the colluding security model and show that this structure is ruled out by the characterization provided in [HMZ08].

In light of this generic negative result, we also explore whether a TP can be used in the colluding model to realize fair MPC protocols for certain *specific* classes of non-trivial functions such as randomized functions without inputs (e.g. coin-tossing). A naïve solution uses the TP to directly compute the desired function; however, such a TP can no longer be small. We give evidence that a better solution using a small TP is unlikely to exist.

1.4 Related Work

There are several fascinating works in the MPC literature that attempt to bypass fundamental feasibility results using external aid. Impossibility of fair MPC in dishonest majority [Cle86] is one such classic impossibility result that has received noteworthy attention. We focus on three broad categories of related works. First is the most closely related line of work to ours which studies the 'minimal help' required to compute all functions fairly, where the helper is characterized as a 'complete' primitive. Second, we outline the line of works that circumvent the impossibility of [Cle86] by considering non-standard notions of fairness. Lastly, we outline the works that circumvent yet another classical impossibility, namely, impossibility of secure computation of general functionalities within the universal composability (UC) framework in presence of dishonest majority in the plain model [CF01] by using hardware tokens and physically unclonable functions (PUFs).

The work of [FGMO01] initiated the study of minimal complete primitives for secure computation, focusing on the minimal cardinality of complete primitives for various thresholds. In particular, they showed that cardinality n is necessary for any complete primitive in dishonest majority and proposed Universal Black Box (UBB) as one such primitive. Subsequently, the work of [GIM+10] proposed a simpler complete primitive for fairness in dishonest majority, namely 'fair reconstruction'. While [GIM+10] focused on the computational setting, [IOS12] presented the first unconditional construction of a complete primitive for full security, whose complexity does not grow with the complexity of the function being evaluated (in contrast to the UBB solution of [FGMO01]). However, this unconditional construction of [IOS12] utilizes number of calls that scales with the circuit size. To improve the number of calls, [IOS12] also proposes another construction where the number of calls depends only on the number of parties (n) and the output size of the circuit but settles for computational security in the plain model. Finally, they also have a variant where the number of calls is reduced to 1 at the price of increasing the complexity of the computation done by the complete primitive exponentially in n.

As mentioned earlier, an interesting feature that our constructions satisfy is to maintain privacy against the TP. We note that the unconditional variant

of [IOS12] (that utilizes number of calls scaling with circuit size) leaks the inputs of the parties to the TP. With respect to the computational variants in [IOS12] that only leak the output of the computation to the TP, we note that it can be tweaked to maintain privacy of the output by adopting the technique of [GIM+10].

Other works related to breaking barriers imposed by the impossibility of [Cle86] include the works of [GK09, GHKL11, ABMO15] that achieve fairness in dishonest majority for restricted functionalities. Some other works explore non-standard notions of fairness such as [GK12, BOO15, BLOO20] that considers partial fairness, [BK14, KB14, ADMM14] that enforce fairness by imposing penalties, [CGJ+17] that use bulletin boards and [EGL85, GMPY11, PST17] that explore resource-fairness.

The sequence of works of [Kat07, CKS+14, DMRV13, CGS08, CCOV19, HPV16] study UC-security with tamper-proof hardware token, both in the stateful and stateless variants. Another interesting utility of hardware tokens is reflected in designing Non-Interactive Secure Computation (NISC) protocols using minimal assumptions. The work of [BJOV18] proposes a UC-secure NISC protocol based on the minimal assumption of one-way functions using hardware token. Lastly, the works of [BFSK11, OSVW13, BKOV17] explore UC-secure computation assuming access to PUFs.

Paper Outline. We formally define TP-aided MPC protocols in Section 2. Our positive results appear in Section 3. Our negative results for TP-aided MPC appear in Section 4. Our negative result for fair MPC in the colluding TP model is briefly summarized in Section 5. Due to lack of space, we defer certain proof details and extensions of the above results to the full version of our paper.

2 Security Model

In this section, we present our definitions in the UC-framework [Can01]. We denote by $[p]$ the set $\{1, \ldots, p\}$, for a positive integer p.

The Real World. An n-party protocol Π with n parties $\mathcal{P} = (P_1, \ldots, P_n)$ is an n-tuple of probabilistic polynomial-time (PPT) interactive Turing machines (ITMs), where each party P_i is initialized with input $x_i \in \{0,1\}^*$ and random coins $r_i \in \{0,1\}^*$. These parties interact in synchronous rounds. In every round parties can communicate either over a broadcast channel or a fully connected point-to-point (P2P) network, where we additionally assume all communication to be private and ideally authenticated. Further, we assume that there exists a special party P^* called a "trusted party" (abbreviated henceforth as TP) such that each party P_i can interact with P^* via private and authenticated point-to-point channels. The TP P^* does not typically hold any inputs, and also does not obtain any output at the end of the protocol. Further, the TP is *stateless* in the sense that it does not keep any state between calls.

We let $\mathcal{A}$ denote a special ITM that represents the adversary. $\mathcal{A}$ is coordinated by another special non-uniform ITM environment $\mathcal{Z} = \mathcal{Z}_\kappa$. At setup, $\mathcal{Z}$ gives

input $(1^\kappa, x_i)$ to each party P_i. At the same time, $\mathcal{Z}$ provides to $\mathcal{A}$ the tuple $(\mathcal{C}, \{x_i\}_{i\in\mathcal{C}}, \mathsf{aux})$, where $\mathcal{C} \subset [n] \cup \{P^*\}$ denotes the set of all corrupt parties, and aux denotes some auxiliary input.

During the execution of the protocol, the maliciously corrupt parties (sometimes referred to as 'active') receive arbitrary instructions from the adversary $\mathcal{A}$, while the honest parties and the semi-honestly corrupt (sometimes referred to as 'passive') parties faithfully follow the instructions of the protocol. We consider the adversary $\mathcal{A}$ to be rushing, i.e., during every round the adversary can see the messages the honest parties sent before producing messages from corrupt parties.

At the conclusion of the protocol, $\mathcal{A}$ gives to the environment $\mathcal{Z}$ an output which is an arbitrary function of $\mathcal{A}$'s view throughout the protocol. $\mathcal{Z}$ is additionally given the outputs of the honest parties. Finally, $\mathcal{Z}$ outputs a bit. We let $\mathsf{real}_{\pi,\mathcal{A},\mathcal{Z}}(\kappa)$ be a random variable denoting the value of this bit.

Definition 1 (Real-world execution). *Let Π be an n-party protocol amongst $(P_1, \ldots, P_n)$ computing an n-party function $f : (\{0,1\}^*)^n \to (\{0,1\}^*)^n$ and let $\mathcal{C} \subseteq [n] \cup \{P^*\}$ denote the set of indices of the corrupted parties. The execution of Π under $(\mathcal{Z}, \mathcal{S}, \mathcal{C})$ in the real world, on input vector $\vec{x} = (x_1, \ldots, x_n)$, auxiliary input aux and security parameter κ, denoted $\mathsf{real}_{\Pi,\mathcal{C},\mathcal{A}(\mathsf{aux})}(\vec{x}, \kappa)$, is defined as the output of $\mathcal{Z}$ resulting from the protocol interaction.*

The Ideal World. We describe ideal world executions with unanimous abort (un-abort), identifiable abort (id-abort), fairness (fairness) and full security aka. guaranteed output delivery (full).

Definition 2 (Ideal Computation). *Consider $\mathsf{type} \in \{\mathsf{un\text{-}abort}, \mathsf{id\text{-}abort}, \mathsf{fairness}, \mathsf{full}\}$. Let $f : (\{0,1\}^*)^n \to (\{0,1\}^*)^n$ be an n-party function. Once again, we have a non-uniform environment $\mathcal{Z} = \mathcal{Z}_\kappa$ that gives (at setup) input $(1^\kappa, x_i)$ to each party P_i, while also providing to the simulator $\mathcal{S}$ the tuple $(\mathcal{C}, \{x_i\}_{i\in\mathcal{C}}, \mathsf{aux})$, where $\mathcal{C} \subset [n] \cup \{P^*\}$ denotes the set of all corrupt parties, and aux denote some auxiliary input. Then, the ideal execution of f under $(\mathcal{Z}, \mathcal{S}, \mathcal{C})$ on input vector $\vec{x} = (x_1, \ldots, x_n)$, auxiliary input aux to $\mathcal{S}$ and security parameter κ, denoted $\mathsf{ideal}^{\mathsf{type}}_{f,\mathcal{C},\mathcal{S},(\mathsf{aux})}(\vec{x}, \kappa)$, is defined as the output bit of $\mathcal{Z}$ resulting from the following ideal process.*

1. Parties send inputs to trusted party*: An honest party P_i sends its input x_i to the trusted party. The simulator $\mathcal{S}$ may send to the trusted party arbitrary inputs for the corrupt parties. Let x'_i be the value actually sent as the input of party x_i.*
2. Trusted party speaks to simulator*: The trusted party computes $(y_1, \ldots, y_n) = f(x'_1, \ldots, x'_n)$. If there are no corrupt parties or $\mathsf{type} = \mathsf{full}$, proceed to step 4.*
   (a) If $\mathsf{type} \in \{\mathsf{un\text{-}abort}, \mathsf{id\text{-}abort}\}$: *The trusted party sends $\{y_i\}_{i\in\mathcal{C}}$ to $\mathcal{S}$.*
   (b) If $\mathsf{type} = \mathsf{fairness}$: *The trusted party sends* `ready` *to $\mathcal{S}$.*
3. Simulator $\mathcal{S}$ responds to trusted party*:*
   (a) If $\mathsf{type} \in \{\mathsf{un\text{-}abort}, \mathsf{fairness}\}$: *The simulator can send* `abort` *to the trusted party.*

(b) If $\mathsf{type} = \mathsf{id\text{-}abort}$: *If it chooses to abort, the simulator $\mathcal{S}$ can select a corrupt party $i^* \in \mathcal{C}$ who will be blamed, and send $(\mathtt{abort}, i^*)$ to the trusted party.*

4. Trusted party answers parties*:*
   (a) *If the trusted party got* abort *from the simulator $\mathcal{S}$,*
      i. *It sets the abort message* $\mathsf{abortmsg}$*, as follows:*
         – *if* $\mathsf{type} \in \{\mathsf{un\text{-}abort}, \mathsf{fairness}\}$*, we let* $\mathsf{abortmsg} = \perp$.
         – *if* $\mathsf{type} = \mathsf{id\text{-}abort}$*, we let* $\mathsf{abortmsg} = (\perp, i^*)$.
      ii. *The trusted party then sends* $\mathsf{abortmsg}$ *to every party P_j, $j \in [n] \setminus \mathcal{C}$. Note that, if* $\mathsf{type} = \mathsf{full}$*, we will never be in this setting, since $\mathcal{S}$ was not allowed to ask for an abort.*
   (b) *Otherwise, it sends y_j to every P_j, $j \in [n]$.*
5. Outputs*: Honest parties always output the message received from the trusted party while the corrupt parties output nothing. At the conclusion of the above execution, $\mathcal{S}$ provides $\mathcal{Z}$ with an output which is an arbitrary function of $\mathcal{S}$'s view throughout the protocol. $\mathcal{Z}$ is additionally given the outputs of the honest parties. Finally, $\mathcal{Z}$ outputs a bit. We let* $\mathsf{ideal}^{\mathsf{type}}_{f,\mathcal{S},\mathcal{Z}}(\kappa)$ *be a random variable denoting the value of this bit.*

Security Definitions. We now define the security notions used in this paper.

Definition 3 (Colluding and Non-colluding Security). *Consider* $\mathsf{type} \in \{\mathsf{un\text{-}abort}, \mathsf{id\text{-}abort}, \mathsf{fairness}, \mathsf{full}\}$*. Let $f : (\{0,1\}^*)^n \to (\{0,1\}^*)^n$ be an n-party function. A protocol Π securely computes the function f in the* colluding model *with* type *security if for any adversary $\mathcal{A}$, there exists a simulator $\mathcal{S}$ such that for any security parameter κ and any circuit family $\mathcal{Z} = \{\mathcal{Z}_\kappa\}$ corrupting any $\mathcal{C} \subset [n]$ maliciously and the TP P^* semi-honestly simultaneously, we have*

$$\left\{\mathsf{real}_{\Pi,\mathcal{C},\mathcal{A}(\mathsf{aux})}(\vec{x},\kappa)\right\}_{\vec{x}\in(\{0,1\}^*)^n,\kappa\in\mathbb{N}} \equiv \left\{\mathsf{ideal}^{\mathsf{type}}_{f,\mathcal{C},\mathcal{S}(\mathsf{aux})}(\vec{x},\kappa)\right\}_{\vec{x}\in(\{0,1\}^*)^n,\kappa\in\mathbb{N}}.$$

When the corruption is non-simultaneous i.e. either *any subset of $[n]$ are maliciously corrupt* or *the TP P^* is semi-honestly corrupt, we denote the security by* non-colluding*. Therefore we need the above indistinguishability to hold in two corruption cases: (a) $\mathcal{C} \subset [n]$ malicious corruption (b) $\mathcal{C} = P^*$ semi-honest corruption.*

A protocol achieves computational security, if the above distributions are computationally close in the presence of the parties, $\mathcal{A}$, $\mathcal{S}$, $\mathcal{Z}$ that are PPT. A protocol achieves statistical (resp. perfect) security if the distributions are statistically close (resp. identical).

3 Fully-secure MPC with Single Call to Small TP

Here, we present TP-aided MPC protocols that make a single call to a small TP and achieve full security in the non-colluding setting against malicious corruption of majority of parties and semi-honest corruption of the TP. We present two flavors of protocols– one based on laconic function evaluation (LFE) [QWW18] and the other based on succinct single-key functional encryption (FE) [GKP+13]. We begin by recalling the definitions for these primitives.

3.1 Laconic Function Evaluation (LFE)

We recall the definition of LFE – a primitive introduced in [QWW18].

Definition 4 (Laconic Function Evaluation). *An* LFE *scheme for a class of circuits* $\mathcal{H} = \{\mathcal{H}_m\}_{m\in\mathbb{N}}$ *(represented as Boolean circuits with* m*-bit inputs) is a tuple* $(\mathsf{LFE.Setup}, \mathsf{LFE.Compress}, \mathsf{LFE.Enc}, \mathsf{LFE.Dec})$ *defined below.*

- $\mathsf{LFE.Setup}(1^\kappa) \to \mathsf{LFE.crs}$*: On input the security parameter* 1^κ*, the generation algorithm returns a common random string* $\mathsf{LFE.crs}$*.*
- $\mathsf{LFE.Compress}(\mathsf{LFE.crs}, h) \to (\mathsf{digest}, r)$*: On input* $\mathsf{LFE.crs}$ *and a circuit* h*, the compression algorithm returns a digest* digest *and a decoding information* r*.*
- $\mathsf{LFE.Enc}(\mathsf{LFE.crs}, \mathsf{digest}, x) \to \mathsf{ct}$*: On input* $\mathsf{LFE.crs}$*, a digest* digest*, and a message* x*, the encryption algorithm returns a ciphertext* ct*.*
- $\mathsf{LFE.Dec}(\mathsf{LFE.crs}, \mathsf{ct}, r) \to y$*: On input* $\mathsf{LFE.crs}$*, a ciphertext* ct*, and a decoding string* r*, the decoding algorithms returns a message* y*.*

In this work, we use LFE schemes that satisfy correctness, simulation-security and function-hiding security, as defined formally below.

Definition 5 (Correctness). *Let* $\mathsf{LFE} = (\mathsf{LFE.Setup}, \mathsf{LFE.Compress}, \mathsf{LFE.Enc}, \mathsf{LFE.Dec})$ *be an LFE scheme for a class of functions* $\mathcal{H} = \{\mathcal{H}_m\}_{m\in\mathbb{N}}$*. We say that* LFE *is a correct LFE scheme if for any* $m = \mathtt{poly}(\kappa)$*, for all* $h \in \mathcal{H}_m$*, and for all* $x \in \{0,1\}^m$*, letting* $\mathsf{LFE.crs} \leftarrow \mathsf{LFE.Setup}(1^\kappa)$*, and letting*

$$(\mathsf{digest}, r) \leftarrow \mathsf{LFE.Compress}(\mathsf{LFE.crs}, h), \quad \mathsf{ct} \leftarrow \mathsf{LFE.Enc}(\mathsf{LFE.crs}, \mathsf{digest}, x),$$

the following holds:

$$\Pr[\mathsf{LFE.Dec}(\mathsf{LFE.crs}, \mathsf{ct}, r) = h(x)] = 1 - \mathtt{negl}(\kappa),$$

where the probability is taken over the random coins of $\mathsf{LFE.Setup}$*,* $\mathsf{LFE.Compress}$*, and* $\mathsf{LFE.Enc}$*.*

Definition 6 (Simulation-Security). *Let* $\mathsf{LFE} = (\mathsf{LFE.Setup}, \mathsf{LFE.Compress}, \mathsf{LFE.Enc}, \mathsf{LFE.Dec})$ *be an LFE scheme for a class of functions* $\mathcal{H} = \{\mathcal{H}_m\}_{m\in\mathbb{N}}$*. For every non-uniform PPT adversary* $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ *and every PPT simulator* $\mathcal{S}$*, consider the following two experiments (*κ *being the security parameter):*

Experiment $\mathsf{Expt}^{\mathsf{real}}_{\mathsf{LFE},\mathcal{A}}(1^\kappa)$*:*

$\mathsf{LFE.crs} \leftarrow \mathsf{LFE.Setup}(1^\kappa)$
$(x, h, s, \mathtt{st}_{\mathcal{A}}) \leftarrow \mathcal{A}_1(1^\kappa, \mathsf{LFE.crs})$
$(\mathsf{digest}, r) \leftarrow \mathsf{LFE.Compress}(\mathsf{LFE.crs}, h; s)$
$\mathsf{ct} \leftarrow \mathsf{LFE.Enc}(\mathsf{LFE.crs}, \mathsf{digest}, x)$
Output $b \leftarrow \mathcal{A}_2(\mathtt{st}_{\mathcal{A}}, \mathsf{ct})$

Experiment $\mathsf{Expt}^{\mathsf{ideal}}_{\mathsf{LFE},\mathcal{A},\mathcal{S}}(1^\kappa)$*:*

$\mathsf{LFE.crs} \leftarrow \mathsf{LFE.Setup}(1^\kappa)$
$(x, h, s, \mathtt{st}_{\mathcal{A}}) \leftarrow \mathcal{A}_1(1^\kappa, \mathsf{LFE.crs})$
$(\mathsf{digest}, r) \leftarrow \mathsf{LFE.Compress}(\mathsf{LFE.crs}, h; s)$
$\widetilde{\mathsf{ct}} \leftarrow \mathcal{S}(\mathsf{LFE.crs}, \mathsf{digest}, h, h(x))$
Output $b \leftarrow \mathcal{A}_2(\mathtt{st}_{\mathcal{A}}, \widetilde{\mathsf{ct}})$

The LFE scheme LFE is said to satisfy (semi-malicious)-simulation-security if for any security parameter $\kappa \in \mathbb{N}$, there exists a PPT simulator $\mathcal{S}$ such that for every non-uniform PPT adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$, the outcomes of the real and ideal experiments are computationally indistinguishable, i.e., we have

$$\left|\Pr[\mathsf{Expt}^{\mathsf{real}}_{\mathsf{LFE},\mathcal{A}}(1^\kappa) = 1] - \Pr[\mathsf{Expt}^{\mathsf{ideal}}_{\mathsf{LFE},\mathcal{A},\mathcal{S}}(1^\kappa) = 1]\right| \leq \mathtt{negl}(\kappa),$$

where $\mathcal{A}$ is admissible if $h \in \mathcal{H}_m$ for some $m = \mathtt{poly}(\kappa)$, and the probability is taken over the random coins of $\mathsf{LFE.Setup}$, $\mathsf{LFE.Compress}$, $\mathsf{LFE.Enc}$, $\mathcal{A}_1$, and $\mathcal{S}$.

Definition 7 (Function-Hiding Security). *Let* $\mathsf{LFE} = (\mathsf{LFE.Setup}, \mathsf{LFE.Compress}, \mathsf{LFE.Enc}, \mathsf{LFE.Dec})$ *be an LFE scheme for a class of functions* $\mathcal{H} = \{\mathcal{H}_m\}_{m\in\mathbb{N}}$. *For every non-uniform PPT adversary* $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ *and every PPT simulator* $\mathcal{S}$*, consider the following two experiments (*κ *being the security parameter):*

Experiment $\mathsf{Expt}^{\mathsf{real,FH}}_{\mathsf{LFE},\mathcal{A}}(1^\kappa)$*:*

$\mathsf{LFE.crs} \leftarrow \mathsf{LFE.Setup}(1^\kappa)$
$(h, \mathtt{st}_{\mathcal{A}}) \leftarrow \mathcal{A}_1(1^\kappa, \mathsf{mpk})$
$(\mathsf{digest}, r) \leftarrow \mathsf{LFE.Compress}(\mathsf{LFE.crs}, h)$
Output $b \leftarrow \mathcal{A}_2(\mathtt{st}_{\mathcal{A}}, \mathsf{digest})$

Experiment $\mathsf{Expt}^{\mathsf{ideal,FH}}_{\mathsf{LFE},\mathcal{A},\mathcal{S}}(1^\kappa)$*:*

$\mathsf{LFE.crs} \leftarrow \mathsf{LFE.Setup}(1^\kappa)$
$\widetilde{(h\mathtt{st}_{\mathcal{A}})} \leftarrow \mathcal{A}_1(1^\kappa, \mathsf{LFE.crs})$
$\mathsf{digest} \leftarrow \mathcal{S}(\mathsf{LFE.crs}, \mathcal{F})$
Output $b \leftarrow \mathcal{A}_2(\mathtt{st}_{\mathcal{A}}, \widetilde{\mathsf{digest}})$

The LFE scheme LFE is said to satisfy function-hiding simulation-security if for any security parameter $\kappa \in \mathbb{N}$, there exists a PPT simulator $\mathcal{S}$ such that for every non-uniform PPT adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$, the outcomes of the real and ideal experiments are computationally indistinguishable, i.e., we have

$$\left|\Pr[\mathsf{Expt}^{\mathsf{real,FH}}_{\mathsf{LFE},\mathcal{A}}(1^\kappa) = 1] - \Pr[\mathsf{Expt}^{\mathsf{ideal,FH}}_{\mathsf{LFE},\mathcal{A},\mathcal{S}}(1^\kappa) = 1]\right| \leq \mathtt{negl}(\kappa),$$

where $\mathcal{A}$ is admissible if $h \in \mathcal{H}_m$ for some $m = \mathtt{poly}(\kappa)$, and the probability is taken over the random coins of $\mathsf{LFE.Setup}$, $\mathsf{LFE.Compress}$, $\mathcal{A}_1$, and $\mathcal{S}$.

3.2 Succinct Single-key Functional Encryption

We now recall the definition of succinct single-key functional encryption (FE).

Definition 8 (Functional Encryption). *A functional encryption scheme* FE *for a class of functions* $\mathcal{H} = \{\mathcal{H}_m\}_{m\in\mathbb{N}}$ *(represented as Boolean circuits with* m*-bit inputs), is a tuple of four PPT algorithms* $(\mathsf{FE.Setup}, \mathsf{FE.KeyGen}, \mathsf{FE.Enc}, \mathsf{FE.Dec})$ *such that:*

- $\mathsf{FE.Setup}(1^\kappa) \to (\mathsf{mpk}, \mathsf{msk})$*: On input the security parameter* κ*, the setup algorithm outputs a master public key* mpk *and a master secret key* msk*.*
- $\mathsf{FE.KeyGen}(\mathsf{msk}, h) \to \mathsf{sk}_h$*: On input the master secret key* msk *and a function* $h \in \mathcal{H}$*, the key generation algorithm outputs a key* sk_h*.*

– $\mathsf{FE.Enc}(\mathsf{mpk}, x) \to \mathsf{ct}$*: On input the master public key* mpk *and an input* $x \in \{0,1\}^m$ *for some* $m = \mathtt{poly}(\kappa)$*, the encryption algorithm outputs a ciphertext* ct*.*
– $\mathsf{FE.Dec}(\mathsf{sk}_h, \mathsf{ct}) \to y$*: On input a key* sk_h *and a ciphertext* ct*, the decryption algorithm outputs a value* y*.*

In this work, we use single-key FE schemes that satisfy correctness, single-key full-simulation-security and succinctness, as defined formally below.

Definition 9 (Correctness). *Let* $\mathsf{FE} = (\mathsf{FE.Setup}, \mathsf{FE.KeyGen}, \mathsf{FE.Enc}, \mathsf{FE.Dec})$ *be a single-key FE scheme for a class of functions* $\mathcal{H} = \{\mathcal{H}_m\}_{m\in\mathbb{N}}$*. We say that* FE *is a correct single-key FE scheme if for any* $m = \mathtt{poly}(\kappa)$*, for all* $h \in \mathcal{H}_m$*, and for all* $x \in \{0,1\}^m$*, letting*

$$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{FE.Setup}(1^\kappa), \quad \mathsf{sk}_h \leftarrow \mathsf{FE.KeyGen}(\mathsf{msk}, h), \quad \mathsf{ct} \leftarrow \mathsf{FE.Enc}(\mathsf{mpk}, x),$$

the following holds:

$$\Pr[\mathsf{FE.Dec}(\mathsf{sk}_h, \mathsf{ct}) = h(x)] = 1 - \mathtt{negl}(\kappa),$$

where the probability is taken over the random coins of $\mathsf{FE.Setup}$*,* $\mathsf{FE.KeyGen}$*, and* $\mathsf{FE.Enc}$*.*

Definition 10 (Full-Simulation Security). *Let* $\mathsf{FE} = (\mathsf{FE.Setup}, \mathsf{FE.KeyGen}, \mathsf{FE.Enc}, \mathsf{FE.Dec})$ *be a single-key FE scheme for a class of functions* $\mathcal{H} = \{\mathcal{H}_m\}_{m\in\mathbb{N}}$*. For every non-uniform PPT adversary* $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ *and every PPT simulator* $\mathcal{S}$*, consider the following two experiments (*κ *being the security parameter):*

Experiment $\mathsf{Expt}^{\mathsf{real}}_{\mathsf{FE},\mathcal{A}}(1^\kappa)$*:*	**Experiment** $\mathsf{Expt}^{\mathsf{ideal}}_{\mathsf{FE},\mathcal{A},\mathcal{S}}(1^\kappa)$*:*
$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{FE.Setup}(1^\kappa)$	$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{FE.Setup}(1^\kappa)$
$(h, \mathtt{st}_{\mathcal{A}}) \leftarrow \mathcal{A}_1(1^\kappa, \mathsf{mpk})$	$(h, \mathtt{st}_{\mathcal{A}}) \leftarrow \mathcal{A}_1(1^\kappa, \mathsf{mpk})$
$\mathsf{sk}_h \leftarrow \mathsf{FE.KeyGen}(\mathsf{msk}, h)$	$\mathsf{sk}_h \leftarrow \mathsf{FE.KeyGen}(\mathsf{msk}, h)$.
$(x, \mathtt{st}'_{\mathcal{A}}) \leftarrow \mathcal{A}_2(\mathtt{st}_{\mathcal{A}}, \mathsf{sk}_h)$	$(x, \mathtt{st}'_{\mathcal{A}}) \leftarrow \mathcal{A}_2(\mathtt{st}_{\mathcal{A}}, \mathsf{sk}_h)$
$\mathsf{ct} \leftarrow \mathsf{FE.Enc}(\mathsf{mpk}, x)$	$\widetilde{\mathsf{ct}} \leftarrow \mathcal{S}(\mathsf{mpk}, \mathsf{sk}_h, h(x), 1^{\lvert x\rvert})$
Output $(\mathtt{st}'_{\mathcal{A}}, \mathsf{ct})$	*Output* $(\mathtt{st}'_{\mathcal{A}}, \widetilde{\mathsf{ct}})$

The FE scheme FE is said to satisfy (single-key) full-simulation-security if for any security parameter $\kappa \in \mathbb{N}$, there exists a PPT simulator $\mathcal{S}$ such that for every non-uniform PPT adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$, the outcomes of the real and ideal experiments are computationally indistinguishable, i.e., we have

$$\mathsf{Expt}^{\mathsf{real}}_{\mathsf{FE},\mathcal{A}}(1^\kappa) \approx_c \mathsf{Expt}^{\mathsf{ideal}}_{\mathsf{FE},\mathcal{A},\mathcal{S}}(1^\kappa).$$

Definition 11 (Succinctness). *Let* $\mathsf{FE} = (\mathsf{FE.Setup}, \mathsf{FE.KeyGen}, \mathsf{FE.Enc}, \mathsf{FE.Dec})$ *be a single-key FE scheme for a class of functions* $\mathcal{H} = \{\mathcal{H}_m\}_{m\in\mathbb{N}}$*. We say that* FE *is succinct if for any* $m = \mathtt{poly}(\kappa)$*, for all* $h \in \mathcal{H}_m$*, and for all* $x \in \{0,1\}^m$*, letting*

$$(\mathsf{mpk}, \mathsf{msk}) \leftarrow \mathsf{FE.Setup}(1^\kappa), \quad \mathsf{ct} \leftarrow \mathsf{FE.Enc}(\mathsf{mpk}, x),$$

the size of the ciphertext ct *(i.e.,* $|\mathsf{ct}|$*) does not grow with the size of the circuit for* h*, but only with its depth.*

3.3 Fully-secure MPC from Laconic Cryptography

In this subsection, we present our construction of TP-aided MPC from LFE.

Construction Overview. The high-level description of the construct, following the three-phase structure (as discussed in Section 1.3), is presented in two steps. In the first step, we assume an honest TP and allow the parties to hand out the inputs to the TP in the clear. In the second step, input privacy against the TP is put in place via *function-hiding* LFE. Throughout, we assume an LFE with a common *random* string (CRS), as is the case for the construction of LFE in [QWW18].

In the first phase, every party uses the common randomness to derive a CRS for the LFE and subsequently computes a digest of f (the function to be computed) using the CRS. It sends the CRS, the digest and its input to the TP. The TP needs to compute an encryption of the collective inputs under the correct digest and CRS. However, a malicious party may send a incorrect digest, say for a function that leaks an honest party's input. The TP can verify the correctness of the digest, since the compress function of the LFE scheme is deterministic. But this amounts to a computation that is dependent on the circuit size, breaking the promise of small TP. To tackle this issue without recomputing the function digest, the TP partitions the set of parties based on the CRS and digest. For every set that sends the same copy of both, gets an encryption under the digest, of the message that consists of the real inputs received from that set and default inputs for those outside that set. This trick ensures that a corrupt party does not get encryption of the inputs of the honest parties under its ill-formed digest. Lastly, on receiving the encryption from the TP, a party simply uses the CRS to learn the function output.

To additionally ensure input privacy against the TP, the function f for LFE is replaced with a related function g that hard-codes n random masks and takes as input n masked inputs of the parties. It first unmasks the masked inputs and then performs the f-computation. The masks are derived from common randomness and thus are known to all. We can use one-time pad for masking. This implies every party has the knowledge of g and can generate a digest that is supposed to be the same. Now, every party uses its respective mask to mask its input before sending to the TP. The TP performs the same computation as before, but now on the received masked inputs, digest for g and CRS. To hide the random masks that are hard-coded inside g from the TP who will learn the digest, we switch to *function-hiding* LFE. This makes sure the TP learns neither about the inputs, not about the output. The LFE security ensures the parties learn nothing but the output of g. The detailed construction is as described below.

Protocol Π_{LFE}

Inputs: Each party P_i has input x_i. All parties share a common randomness of the form $r\|r'$.
Output: $f(x_1, \ldots x_n)$

Primitive: The following building blocks are used
- An LFE scheme $\mathsf{LFE} = (\mathsf{LFE.Setup}, \mathsf{LFE.Compress}, \mathsf{LFE.Enc}, \mathsf{LFE.Dec})$.

Phase 1 (Pre-TP Call): Each party P_i does the following:
- Set $\mathsf{LFE.crs} := r$, where r is obtained from the common randomness $r\|r'$.
- Derive n random pads $\{r_j\}_{j\in[n]}$, where $|r_j| = |x_j|$, using r' obtained from the common randomness $r\|r'$.
- Compute $(\mathsf{digest}^g, r^g) \leftarrow \mathsf{LFE.Compress}\Big(g, \mathsf{LFE.crs}\Big)$, where function g is as follows and send $(\mathsf{LFE.crs}, \mathsf{digest}^g, z_i = x_i \oplus r_i)$ to the TP.
  - g hard-codes the n pads $\{r_j\}_{j\in[n]}$
  - it takes n inputs $z_1, \ldots, z_n$
  - it computes f on input $\{z_j \oplus r_j\}_{j\in[n]}$.

  We note that $(\mathsf{LFE.crs}, \mathsf{digest}^g, r^g)$ is supposed to be the same for all parties, since they use the common randomness r and f.

Phase 2(TP Call): The TP carries out the following computation:
- Initialize the set $\mathcal{Z} = \varnothing$. Add P_j to $\mathcal{Z}$ if nothing (or syntactically incorrect message) is received from P_j.
- Partition the set $\mathcal{P} \setminus \mathcal{Z}$ into subsets $S_1, S_2 \ldots S_\ell$ according to the values of ($\mathsf{LFE.crs}, \mathsf{digest}^g$) received from the parties i.e. all parties in a subset have sent the same ($\mathsf{LFE.crs}, \mathsf{digest}^g$).
- For each S_α for $\alpha \in \{1, \ldots, \ell\}$
  - Let $\mathsf{LFE.crs}_\alpha, \mathsf{digest}^g_\alpha$ denote the common values submitted by parties in S_α.
  - For each $j \in \{1, \ldots, n\}$, set $\bar{z}_j = z_j$ if $j \in S_\alpha$, and $\bar{z}_j = z'_j$ otherwise, where z_j is received from P_j and $\{z'_j\}_{j\in\{1,\ldots,n\}}$ are the default (masked) inputs sampled randomly by the TP.
  - Send $\mathsf{ct}_\alpha, S_\alpha$ to every party in S_α, where $\mathsf{ct}_\alpha \leftarrow \mathsf{LFE.Enc}\Big(\mathsf{digest}^g_\alpha, (\bar{z}_1, \ldots, \bar{z}_n)\Big)$.

Phase 3 (Post-TP Call): A party P_i, on receiving ct, computes output y as

$$y \leftarrow \mathsf{LFE.Dec}\Big(\mathsf{LFE.crs}, \mathsf{ct}, r^g\Big),$$

using $\mathsf{LFE.crs}, r^g$ from Phase 1.

Fig. 1: Fully-secure MPC with single TP call based on LFE

Our result can be summarized via the following theorem.

Theorem 4 (TP-Aided MPC from LFE). *Assuming the existence of a laconic function evaluation (LFE) scheme that satisfies correctness, simulation-security and function-hiding security, there exists a TP-aided MPC protocol Π_{LFE} for any functionality f that:*

- *utilizes a single call to a stateless TP of size $\mathtt{poly}(n, \kappa, m, \alpha, \beta)$ (where n is the number of parties, κ is the security parameter, m is the size of each party's input to f, and α and β denote the sizes of a single digest and a single ciphertext, respectively, in the LFE scheme), and*
- *achieves full security against malicious corruption of up to $(n-1)$ parties and semi-honest corruption of the TP in the non-colluding model (see Definition 3).*

We defer the formal proof of this theorem to the full version of our paper.

3.4 Fully-secure MPC from Single-Key Succinct FE

In this subsection, we show how to construct TP-aided MPC from single-key succinct FE.

Construction Overview. The high-level description of the construct, following the three-phase structure (as discussed in Section 1.3), is presented in two steps. In the first step, we assume an honest TP and allow the parties to hand out the inputs to the TP. In the second step, input privacy is put in place via a SKE.

For our construction, in the first phase, the parties execute an MPC protocol with identifiable abort[8] amongst the n parties that establishes the setup of the FE and gives the parties sk_f (corresponding to the function f desired to be computed) to aid in output computation. Since this execution may result in abort (where only corrupt parties may get the output), we cannot allow the MPC to output the FE ciphertext corresponding to the parties' inputs directly. Instead, the ciphertext is computed by the TP to whom the parties submit their inputs when Phase 1 is successful (which may need repeated run of the MPC with identifiable abort). To enable the TP to do so, the parties additionally submit mpk (obtained in Phase 1) to the TP. In order to ensure that privacy of honest parties' inputs is maintained against a corrupt party who sends mpk distinct from the one obtained in Phase 1, the TP does the following: partition the set of parties based on the value of mpk they submitted. For each partition, the TP returns ciphertext based on actual inputs of parties within the partition and default otherwise. This ensures that a corrupt party who submits an incorrect mpk (say mpk' which is distinct from the one obtained from Phase 1) never get access to a ciphertext computed using mpk' that involves an honest party's input. Lastly, the parties use the ciphertext obtained from the TP and sk_f to obtain the output.

Note that the above protocol is not secure in the non-colluding model as it does not achieve input privacy against a semi-honest TP. Further, the computation done by the TP grows with the size of the parties' inputs. In order to achieve security against a semi-honest TP and make the computation of the TP independent of the size of the parties' inputs, we make the following modifications. First, the input of each party is hidden in a ciphertext of a SKE. The MPC with identifiable abort now takes as input the inputs of the parties, computes distinct ciphertexts for the inputs, each under a distinct secret key, and delivers only the ith secret key to P_i. Instead of the inputs, these keys are sent to the TP, who performs similar computation as before, but with respect to these keys. To make the both ends meet, the function to be computed by FE is changed to a related function g (instead of the function to be computed f) that hard-codes the ciphertexts of the inputs and takes the n keys as inputs. The function g

[8] Some of the protocols in the literature realizing this functionality for general functions are [GS18].

first decrypts the ciphertexts and then compute f on the decrypted messages. The MPC with identifiable abort now prepares and gives out the secret key of FE corresponding to g. To prevent the parties from tampering the secret keys for SKE while sending to the TP, we use a signature scheme. The MPC samples a (public, secret) key pair for a digital signature scheme and delivers signed messages meant for TP (SKE key and mpk in this case) and the public key for verification to a party. The parties forward this to the TP, who now discards the parties whose verification fails, partitions the parties based on the verification key and proceeds as before. The detailed construction is as described below.

Protocol Π_{FE}

Inputs: Each P_i participates with input x_i.
Output: $f(x_1, \dots x_n)$
Primitive: The following building blocks are used
- An MPC protocol Π_{idua} that achieves security with identifiable abort.
- A succinct single-key simulation-secure FE scheme $\mathsf{FE} = (\mathsf{FE.Setup}, \mathsf{FE.KeyGen}, \mathsf{FE.Enc}, \mathsf{FE.Dec})$.
- An IND-CPA secure symmetric-key encryption scheme $\mathsf{SKE} = (\mathsf{SKE.Gen}, \mathsf{SKE.Enc}, \mathsf{SKE.Dec})$.
- A digital signature scheme $(\mathsf{Sign}, \mathsf{Vrfy})$.

Phase 1 (Pre-TP Call): Each P_i invokes an instance of Π_{idua} with input x_i to compute a function that does the following:
- Generate a default input x_i' for every P_i.
- Generate a secret key $\mathsf{k}_i \leftarrow \mathsf{SKE.Gen}(1^\kappa)$ for every party P_i.
- Generate $(\mathsf{msk}, \mathsf{mpk}) \leftarrow \mathsf{FE.Setup}(1^\kappa)$.
- Generate $e_i \leftarrow \mathsf{SKE.Enc}(\mathsf{k}_i, x_i)$ for every P_i.
- Generate $\mathsf{sk}_g = \mathsf{FE.KeyGen}(\mathsf{msk}, g)$, where g is a function defined as follows:
  - g embeds the ciphertexts $\{e_j\}_{j\in[n]}$ and default inputs $\{x_j'\}_{j\in[n]}$.
  - g takes as input a set of keys $\{\mathsf{k}_j\}_{j\in[n]}$ and an n-length bit vector $\{b_j\}_{j\in[n]}$.
  - g outputs $f(\bar{x}_1, \dots, \bar{x}_n)$ where for each $j \in [n]$, $\bar{x}_j = \mathsf{SKE.Dec}(\mathsf{k}_i, e_j)$ if $b_i = 1$ and $\bar{x}_j = x_j'$ otherwise.
- Generate $(\mathsf{sk}, \mathsf{vk})$ for the digital signature scheme.
- For each $i \in [n]$, output $(\mathsf{vk}, \mathsf{mpk}, \mathsf{k}_i, \sigma_i, \mathsf{sk}_g)$ to P_i where $\sigma_i = \mathsf{Sign}(\mathsf{sk}, (i, \mathsf{mpk}, \mathsf{k}_i))$.

If Π_{idua} outputs $(\perp, \mathcal{C})$, re-run **Phase 1** among the set of parties $\mathcal{P} \setminus \mathcal{C}$ (the inputs of parties in $\mathcal{C}$ are substituted using default inputs). Else, continue to the next phase. Each P_i invokes the TP with $\mathsf{in}_i = (\mathsf{vk}, \mathsf{mpk}, \mathsf{k}_i, \sigma_i)$.

Phase 2 (TP Call): The TP carries out the following computation:
- Initialize $\mathcal{Z} = \varnothing$. Add P_j to $\mathcal{Z}$ if nothing is received or $\mathsf{Vrfy}(\mathsf{vk}, (j, \mathsf{mpk}, \mathsf{k}_j, \sigma_j) = 0$, for a tuple $(\mathsf{vk}, \mathsf{mpk}, \mathsf{k}_j, \sigma_j)$ received from P_j.
- Partition the set $\mathcal{P} \setminus \mathcal{Z}$ into subsets $S_1, S_2 \dots S_\ell$ according to the values of vk received from the parties i.e. all parties in a subset have sent the same vk.
- For each S_α for $\alpha \in \{1, \dots, \ell\}$
  - Let mpk_α denote the common mpk submitted by parties in S_α.
  - For each $j \in [n]$, set $\mathsf{k}_{\alpha,j} = \mathsf{k}_j$ and $b_{\alpha,j} = 1$ if $j \in S_\alpha$, and $\mathsf{k}_{\alpha,j} = \perp$ and $b_{\alpha,j} = 0$ otherwise.

- Compute and return ct_α to every party in S_α, where

$$\mathsf{ct}_\alpha \leftarrow \mathsf{FE.Enc}\Big(\mathsf{mpk}_\alpha, \big(\{\mathsf{k}_{\alpha,j}\}_{j\in[n]}, \{b_{\alpha,j}\}_{j\in[n]}\big)\Big).$$

Phase 3 (Post-TP Call): A party computes output $y = \mathsf{FE.Dec}\Big(\mathsf{sk}_g, \mathsf{ct}_\alpha\Big)$ using sk_g obtained from Phase 1 and ct_α obtained from Phase 2.

Fig. 2: Fully-secure MPC with single TP call based on Succinct Single-Key FE

Our result can be summarized via the following theorem:

Theorem 5 (TP-Aided MPC from Single-Key Succinct FE). *Assuming the existence of an FE scheme that satisfies correctness, (single-key) simulation-security and succinctness, there exists a TP-aided MPC protocol Π_{FE} for any functionality f that:*

- *utilizes a single call to a stateless TP of size* $\mathtt{poly}(n, \kappa, \beta)$ *(where n is the number of parties, κ is the security parameter, and β denotes the size of a single ciphertext in the FE scheme), and*
- *achieves full security against malicious corruption of up to $(n-1)$ parties and semi-honest corruption of the TP in the non-colluding model (see Definition 3).*

We defer the formal proof of this theorem to the full version of our paper.

4 Impossibilities in the Non-colluding Model

In this section, we present our negative results for small-TP aided MPC.

4.1 Impossibility in the Correlated Randomness Model for protocols with universal output decoder

Here, we make following assumptions– (a) small TP: the TP performs $\mathtt{poly}(n, \kappa)$ computation, (b) small output decoder: the parties, on receiving the message from the TP, perform $\mathtt{poly}(n, \kappa)$ computation to compute the output. We show that in this model, it is impossible to design a fully secure MPC, even if parties have access to correlated randomness and irrespective of computational assumptions used in the protocol. This holds even if the parties are corrupted in fail-stop fashion in the non-colluding model. Before we begin, we formalize the class of protocols for which the impossibility holds.

Notation. A fully-secure n-party protocol Π in the correlated randomness model that utilizes a single call to a small stateless TP comprises of the following phases.

- **Correlated Randomness Setup.** The setup computes correlated randomness $(\mathsf{cr}_1, \mathsf{cr}_2, \ldots, \mathsf{cr}_n)$ and outputs cr_i to P_i $(i \in [n])$.

– **Pre-TP Computation.** In this phase, the parties may interact with each other (before the TP call), where each P_i participates with input x_i and randomness r_i. Let st_i denotes the state of P_i at the end of this phase, where st_i comprises of its input x_i, randomness r_i, correlated randomness cr_i (received as part of the setup) and in addition, the messages sent / received during this phase, if this phase was interactive. Lastly, each P_i computes algorithm $(\mathsf{in}_i, \mathsf{st}'_i) \leftarrow \mathsf{preTP}_i(\mathsf{st}_i)$ and invokes TP with in_i.

– **TP Computation.** For each $i \in [n]$, the TP computes its response as $\mathsf{out}_i \leftarrow \mathsf{TP}_i(\mathsf{in}_1, \ldots, \mathsf{in}_n; r_{\mathsf{TP}})$, where r_{TP} denotes the internal randomness of the TP and TP_i denotes the algorithm used by the TP to compute its response to P_i.

– **Post-TP Computation.** Each P_i ($i \in [n]$) computes its output as $y \leftarrow \mathsf{postTP}_i(\mathsf{st}'_i, \mathsf{out}_i)$, where postTP_i denotes the algorithm used by P_i to compute its output. We refer to this algorithm as output decoder occasionally.[9]

In our model, (a) each TP_i for $i \in [n]$ is $\mathtt{poly}(n, \kappa)$-time (b) each postTP_i for $i \in [n]$ is $\mathtt{poly}(n, \kappa)$-time.

To show the impossibility, we show that a fully secure protocol would imply a statistically-correct (encoding, decoding) scheme which can produce an encoding that is smaller than the size of the message domain of the encoding scheme. This breaches the known incompressibility argument. Precisely, we use the following proposition of De et al. [DTT10], which formalizes the notion that it is impossible to compress every element in a set X to a string less than $\log |X|$ bits long.

Proposition 1. *[Incompressibility Argument [DTT10]] Let $E : X \times \{0,1\}^\rho \to \{0,1\}^m$ and $D : \{0,1\}^m \times \{0,1\}^\rho \to X$ be randomized encoding and decoding procedures such that, for every $x \in X$, $Pr_{r \in \{0,1\}^\rho}[D(E(x,r),r) = x] \geq \delta$. Then $m \geq \log(|X|) - \log(1/\delta)$.*

Theorem 6. *A general fully secure MPC protocol is impossible in the non-colluding model (see Definition 3), where the parties have access to arbitrary correlated randomness, a single call to a TP of size $\mathtt{poly}(n, \kappa)$, and are allowed to use an output decoder of size $\mathtt{poly}(n, \kappa)$, even when malicious corruption of parties in $\mathcal{P}$ is restricted to fail-stop corruption.*

Proof. Towards a contradiction, assume such a protocol Π computing an arbitrary function f exists (f is defined later) that achieves full security in the correlated randomness model, satisfying correctness with overwhelming probability. Without loss of generality, Π comprises of the phases (Correlated randomness setup, pre-TP computation, TP computation, post-TP computation) described previously.

[9] We believe that a non-interactive post-TP computation phase is essentially without loss of generality. In other words, any fully secure MPC protocol (having access to one TP call) with interaction amongst the parties can be transformed to one where the parties do not communicate at all amongst themselves after receiving TP's response. We give a proof in the full version of our paper.

Below, we show that Π leads to a statistically-correct randomized (encoding, decoding) scheme (E, D) (as defined in Proposition 1).

Algorithm (E, D)

$E : \{0,1\}^{2^{n-1}} \times \{0,1\}^{\rho} \to \{0,1\}^{m}$**:** This algorithm takes as input 2^{n-1} bits, say $(b_1, b_2, \ldots, b_{2^{n-1}})$, an randomness $r \in \{0,1\}^{\rho}$ and computes its encoding as follows:

1. For each $i \in [n]$, choose a pair of inputs (x_i, x_i^*) using r.
2. Consider a set S containing tuples of the form $(x_1, x_2', \ldots, x_n')$ where $x_i' \in \{x_i, x_i^*\}$ for $i \in \{2, \ldots, n\}$. Note that x_1 is fixed in all the tuples and $|S| = 2^{n-1}$.
3. Consider a lexicographic ordering of the elements in S generated as follows. For each $i \in [n]$, map x_i to bit 0 and x_i^* to bit 1. Now each tuple in S can be viewed as an n bit string and the elements in S can be lexicographically ordered. Let us denote the jth element as S_j. Let M be a mapping between S and $(b_1, b_2, \ldots, b_{2^{n-1}})$, where S_j is mapped to b_j for $j \in [2^{n-1}]$.
4. Construct an n-input function $f(X_1, \ldots, X_n)$ that outputs $M\big(X_1, \ldots, X_n\big)$, when $(X_1, \ldots, X_n) \in S$ and $\perp$ otherwise.
5. Suppose Π computes f on input X_i from P_i. Consider an execution of Π where parties $\{P_1, \ldots, P_n\}$ participate using inputs $\{x_i\}_{i\in[n]}$, randomness $\{r_i\}_{i\in[n]}$ and correlated randomness $\{\mathsf{cr}_i\}_{i\in[n]}$ (the latter two picked using r). Further, Π uses x_i^* as the default input of P_i ($i \in [n]$). Emulate the steps of this execution until the pre-TP computation to obtain $\{\mathsf{st}_i', \mathsf{in}_i\}_{i\in[n]}$. Let $\bar{\mathsf{st}}_1'$ denote the subset of st_1' used in postTP_1; with size restricted to $\mathtt{poly}(n, \kappa)$, as dictated by Π (recall that postTP function is allowed to do only $\mathtt{poly}(n, \kappa)$ computation).
6. The encoding of input $(b_1, b_2, \ldots, b_{2^{n-1}})$ is defined as $\{\bar{\mathsf{st}}_1', \mathsf{in}_1, \ldots, \mathsf{in}_n\}$, TP_1 (the algorithm used by the TP to compute its response to P_1) and postTP_1 (the output computation algorithm of P_1).

$D : \{0,1\}^{m} \times \{0,1\}^{\rho} \to \{0,1\}^{2^{n-1}}$**:** It takes as input the encoding $\{\bar{\mathsf{st}}_1', \mathsf{in}_1, \ldots, \mathsf{in}_n\}$ and the $r \in \{0,1\}^{\rho}$ used by E. For each subset $S' \subseteq \{2, \ldots, n\}$ in lexicographic order (starting from $S' = \varnothing$ to $S' = \{2, \ldots, n\}$), do the following (below we abuse the notation and use S' to denote the decimal value corresponding to the binary representation):

1. Compute $\mathsf{out}_1^{(S')} \leftarrow \mathsf{TP}_1(\mathsf{in}_1', \mathsf{in}_2', \ldots, \mathsf{in}_n'; r_{\mathsf{TP}})$, where $\mathsf{in}_i' = \mathsf{in}_i$ for $i \notin S'$ [a] and $\mathsf{in}_i' = \perp$ for $i \in S'$. Here, r_{TP} is derived from r as per the distribution corresponding to the internal randomness of the TP in Π.
2. Compute $b_{(S')} \leftarrow \mathsf{postTP}_1(\bar{\mathsf{st}}_1', \mathsf{out}_1^{(S')})$.

Output $(b_1, b_2, \ldots, b_{2^{n-1}})$.

[a] Note $\mathsf{in}_1' = \mathsf{in}_1$ is always satisfied as S' is defined as subsets of $\{2, \ldots, n\}$.

Fig. 3: A Randomized Encoding and Decoding Scheme

Lemma 1. *(E, D) is a statistically-correct encoding and decoding scheme.*

Proof. We now claim that the above pair (E, D) is statistically correct. That is the following holds good: for every $(b_1, \ldots, b_{2^{n-1}}) \in \{0,1\}^{2^{n-1}}$, $Pr_{r\in\{0,1\}^{\rho}}[D(E((b_1, \ldots, b_{2^{n-1}}), r), r) = (b_1, \ldots, b_{2^{n-1}})] \geq \delta$. This is because Π computes f that, for every input in S, as defined in E, maps to one distinct bit in the sequence $(b_1,$

$\ldots, b_{2^{n-1}}$) (recall that the jth element of S, S_j is mapped to b_j). Further, Π computes f and achieves full security (guaranteed output delivery) and satisfies correctness with overwhelming probability. Specifically, if a subset of parties P_i such that $i \in S'$ do not invoke the TP during Π, then the TP receives $\{\mathsf{in}_i\}$ only from the other parties P_i where $i \notin S'$ and sets $\mathsf{in}_i = \perp$ for parties in S'. The output computed by the TP is on the default input x_i^* for each party P_i with $i \in S'$ and x_i for each party P_i with $i \notin S'$.

Since S' is defined as subsets of $\{2, \ldots, n\}$ and never includes the index 1, the above captures executions of Π where P_1 is honest, participated honestly with input x_1 and invoked the TP with $\mathsf{in}'_1 = \mathsf{in}_1$. This allows us to rely on the correctness of the output computed by postTP_1. We can thus infer that the 2^{n-1} bits computed during decoding indeed correspond to the set of outputs of f for each subset S', namely $(b_1, b_2, \ldots, b_{2^{n-1}})$.

Notice that the above argument holds good even if Π satisfies full security tolerating fail-stop corruption where the parties do not send their message to the TP. Furthermore, Π satisfying fairness is not enough to claim that (E, D) is (statistically) correct, because D may fail to recover $(b_1, \ldots, b_{2^{n-1}})$ always.

By the incompressibility argument of [DTT10] (which is formally stated above), it must hold that $|\bar{\mathsf{st}}'_1| + |\mathsf{in}_1| + \ldots |\mathsf{in}_n| + |\mathsf{out}_1| + |\mathsf{postTP}_1| \geq 2^{n-1}$. We can thus infer that at least one of the terms $\geq \frac{2^{n-1}}{n+3}$. Recall that by our assumption on small output decoder, the terms $|\bar{\mathsf{st}}'_1|$ and $|\mathsf{postTP}_1|$ are bounded by size $\mathtt{poly}(n, \kappa)$. Therefore, it must be the case that one of the terms $\mathsf{in}_1, \ldots, \mathsf{in}_n, \mathsf{out}_1$ must be of size $\geq \frac{2^{n-1}}{n+3}$. However, this contradicts our assumption that the TP has size $\mathtt{poly}(n, \kappa)$ as $\mathsf{in}_1, \ldots, \mathsf{in}_n$ comprises of the input to the TP and out_1 is the algorithm run by the TP to compute its response to P_1. We have thus arrived at a contradiction; completing the proof.

4.2 Impossibility in the Plain Model

In this section, we show that in the plain model (without correlated randomness), it is impossible to design statistically secure MPC with the non-colluding security, even when the parties are only semi-honestly corrupt. That is, we prove that a protocol is impossible when the adversary in the non-colluding TP model can either (a) corrupt majority of the parties $\{P_1, \ldots, P_n\}$ semi-honestly or (b) control the TP semi-honestly). We state the formal theorem below.

Theorem 7. *A general statistically-secure MPC protocol is impossible in the plain and the non-colluding TP model (see Definition 3), where the parties have access to a single call to a small TP of size* $\mathtt{poly}(n, \kappa)$*, even when malicious corruption of parties in* $\mathcal{P}$ *is restricted to semi-honest corruption.*

Proof. Towards a contradiction, assume that there exists a statistically-secure 2-party protocol Π securely computing f against a semi-honest adversary in the non-colluding TP model. Let f be defined as the functionality computing $(k+1)$

oblivious transfer (OT) instances i.e.

$$f\big(x_1 = (m_i^0, m_i^1)_{i\in[k+1]}, x_2 = (b_1, \ldots, b_k, b_{k+1})\big) = (m_1^{b_1}, m_2^{b_2} \ldots, m_{k+1}^{b_{k+1}})$$

Here, the input of P_1 (who acts as the sender) consists of $(k+1)$ pairs of bits and the input of P_2 (who acts as the receiver) consists of $(k+1)$ bits.

Suppose C_{TP} denotes the circuit describing the function $\{\mathsf{TP}\}_{i\in[n]}$ computed by the TP during Π. Based on our assumption that the TP is 'small', it must hold that $|C_{\mathsf{TP}}| \leq \mathtt{poly}(n, \kappa)$ which is independent of the function f being computed. Specifically, this means that the computation done by the TP must be strictly less than computing $(k+1)$ OTs.

We claim that Π computing f can be used to build a semi-honest OT extension protocol Π'. Assume a semi-honest setting where the parties are given k OT correlations generated as the base OTs of the OT extension protocol Π'. Π' proceeds as follows:

1. The parties execute the steps of Π in the pre-TP computation phase.
2. Next, the parties emulate the TP computation phase of Π by executing the perfectly-secure semi-honest GMW protocol [GMW87] to compute the function described by C_{TP}. For this, the parties use the k OT correlations (given as base OTs). Note that these OT correlations must suffice as computing C_{TP} must involve computing fewer than $(k+1)$ OTs (based on our assumption).
3. Finally, the parties use the output of the execution of the GMW protocol (which computes the TP response of Π) to carry out the steps of output computation as per Π. This will result in the parties obtaining the output of f.

We note that Π' does not involve any calls to the stateless TP. Since Π' computes $(k+1)$ OTs starting with k base OTs and involves execution of steps in Π and the GMW protocol, which are both information-theoretically secure; we can conclude that Π' is indeed a semi-honest information-theoretic OT extension protocol. However, this is a contradiction as information-theoretically secure OT extension does not exist in the plain model [Bea96]. This completes the proof.

5 Impossibility of Fair MPC in the Colluding Model

In this section, we briefly summarize our negative results for fair MPC in the colluding security model (see Definition 3). Recall that, in this model, we assume that the adversary controls a majority of the parties among $\{P_1, \ldots, P_n\}$ maliciously and simultaneously corrupt the TP semi-honestly. Our impossibility holds good even when malicious corruption is weakened to fail-stop corruption and the requirement of full security is relaxed to fairness. Our result is summarized by the following theorem.

Theorem 8. *There exists a function f such that it is impossible to design a fair MPC protocol securely computing f in the computational colluding model (see Definition 3) even when malicious corruption of parties in $\mathcal{P}$ is restricted to fail-stop corruption.*

We defer the detailed proof of this theorem to the full version of our paper. At a high level, we follow the following route. We note that the colluding adversarial model can be viewed more generally, in terms of the general mixed adversarial model that has been studied in works such as [HMZ08, FHM99, BFH+08]. Recall that a general mixed adversary is characterized by an adversary structure $\mathbb{Z} = \{(A_1, E_1, F_1), \ldots, (A_m, E_m, F_m)\}$ (for some m), which is a monotone set of triples of party sets. At the beginning of the protocol, the adversary chooses one of these triples $\mathbb{Z}^* = (A^*, E^*, F^*) \in \mathbb{Z}$ and actively corrupts parties in A^*, semi-honestly corrupts the parties in E^* and fail-corrupts the parties in F^*.

Viewing the TP as an additional party P_{n+1} (who can be semi-honestly corrupted) and the party set $\mathcal{P} = \{P_1, \ldots, P_n, P_{n+1}\}$, the adversarial structure for the colluding TP model can be expressed as: $\mathbb{Z} = \{\mathbb{Z}_1, \ldots, \mathbb{Z}_n\}$, where for each $i \in [n]$, we have

$$\mathbb{Z}_i = \Big(A_i = \mathcal{P} \setminus \{P_i, P_{n+1}\}, E_i = \mathcal{P} \setminus \{P_i\}, F_i = \mathcal{P} \setminus \{P_i, P_{n+1}\}\Big).$$

Specifically, the above denotes the maximal class of the adversarial structure of the colluding TP model, since these subsume all other possible corruption scenarios indicated by subsets of the triples in each $\mathbb{Z}_i$, i.e. the adversary can choose to corrupt (A^*, E^*, F^*), such that there exists $(\bar{A}, \bar{E}, \bar{F}) \in \mathbb{Z} : A^* \subseteq \bar{A}, E^* \subseteq \bar{E}, F^* \subseteq \bar{F}$. Now restricting the malicious adversaries to behave in a fail-stop manner, we refine the maximal adversarial structure as $\mathbb{Z}' = \{\mathbb{Z}'_1, \ldots, \mathbb{Z}'_n\}$, where for each $i \in [n]$,

$$\mathbb{Z}'_i = \Big(A_i = \varnothing, E_i = \mathcal{P} \setminus \{P_i\}, F_i = \mathcal{P} \setminus \{P_i, P_{n+1}\}\Big).$$

Given this adversarial structure, we show that our desired impossibility result is implied by the impossibility of fair (non-reactive) MPC shown in [HMZ08]. The proof requires a careful mapping between the maximal adversarial structures between our model of TP-aided MPC and the general mixed adversarial model considered in [HMZ08] (see the full version of our paper for details). Note that in above analysis, we consider the TP to be just another party that can communicate freely with the other parties while maintaining states across the communication. This implies that our impossibility holds even for stateful TPs.

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