Review of the book

"*Applications of Group Theory to Combinatorics*"

by Jack Koolen, Jin Ho Kwak and Ming-Yao Xu (Eds.)
CRC Press, Taylor & Francis Group 2008

ISBN: 978-0-415-47184-8

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March 31, 2011

1 Summary of the review

This book is definitely a reference style book that a graduate student might find useful, especially if he/she is working on a combinatorial problem that could benefit from the use of an algebraic tool.

2 Summary of the book

This is a collection of 11 surveys on the applications of group theory to combinatorics. The surveys are the exposition of the invited talks given at the Com^{2}MaC International Workshop on the topic, held at Pohang University of Science and technology in July, 2007.

The eleven papers could be classified into basically four sections.

2.1 Problems in Graph Theory

The first four papers as well as papers 9 and 11 deal with problems in graph theory.

Combinatorial and Computational group-theoretic methods in the study of graphs, maps and polytopes with maximal symmetry.
In this paper, M. Conder starts with some symmetries of discrete structures outlining symmetric graphs, regular maps and abstract polytopes. For example a symmetric graph is one which is arc-transitive, with an arc being an ordered edge. Combinatorial group theoretic tools such as the Schreier coset diagrams and Shur's theorem are then used to prove for example that there are infinitely many 5-arc-transitive connected finite cubic graphs. Conder also gives a brief of the use of computational techniques, using mainly MAGMA, for investigating groups with a small number of generators and defining relations.

Automorphism groups of Cayley digraphs
In their paper on the automorphism groups of Cayley digraphs, Y. Feng, Z. Lu and M. Xu give a brief survey of results on the normality of these digraphs – determining the automorphism groups of Cayley digraphs allows one to investigate the symmetry properties of digraphs.

Symmetrical covers, decompositions and factorisations of graphs
Paper 3, by M. Giudici, C.H. Li and C.E. Praeger, introduces new combinatorial structures – symmetrical covers, symmetrical decompositions and symmetrical factorisations of graphs. These new structures generalise objects such as 2-designs, regular maps and near-polygon graphs and linear spaces. The paper develops the theory giving examples of symmetrical covers for well known graphs and posing a number of research problems.

Complete bipartite maps, factorisable groups and generalised Fermat curves
This paper on complete bipartite maps, by G. Jones, also fits in this section, and looks at the combinatorial problem of classifying orientably regular embeddings of complete bipartite graphs. The first part of the paper discusses the problem and the methods used to solve it, while the second part links the problem to other topics, for example Riemann surfaces, algebraic geometry and Galois theory.

Injectivity radius of traingle group representations, with application to regular embeddings of hypermaps
M. Mačaj, J. Širáň and M. Ipolyiová survey the algebraic background for constructing representations of triangle groups in linear groups over certain algebras. This leads to among other things, applications to the planar width of regular hypermaps.

Belyi functions: examples, properties and applications
A.K. Zvonkin points out that the study of Belyi functions relates to Riemann surfaces, Galois theory as well as to the combinatorics of maps. This paper provides a number of examples of maps and hypermaps, and is easy to read.

2.2 Properties of Groups

Separability properties of groups
Paper 5 by G. Kim and C.Y.Tang talks about the separability of groups. They look at groups that are $\{1\}$- separable, subgroup separable and conjugacy separable. For example $\{1\}$- separable groups are said to be residually finite. Residual finiteness has applications in group theory and topology.

Genus parameters and sizings of groups
This paper by T.W. Tucker looks at the various genus parameters for finite groups. The sizing of a function is one such parameter and the paper looks at some natural questions for sizing, ending with the concept of sizing for a graph.

2.3 Hurwitz Problems

Papers 6 and 7 look at the Hurwitz Enumeration problem: "determine the number of branched coverings of a given Riemann surface with prescribed ramification type".

Coverings, enumeration and Hurwitz problems
Paper 6, by J.H. Kwak, J. Lee and A. Mednykh, devotes itself to solutions of this problem as well as more generally, for graphs, manifolds and orbifolds with finitely generated fundamental group, indicating a general approach to solve the problem in the higher dimensional case.

Combinatorial facets of Hurwitz numbers
Paper 7 on the other hand looks at Hurwitz numbers – the number of ramified coverings of two-dimensional surfaces – and other manifestations of these numbers. The author, S.K. Lando, gives the recent progress in the understanding of these numbers, giving among other things, some formulae

for rational Hurwitz numbers.

2.4 Groups and Designs

H. Li's paper with the above title looks at application of finite permutation groups to combinatorial designs. Starting with definitions of 2-designs, projective planes and affine geometries, Li goes on to give elementary properties of the automorphism groups of designs and then goes on to the classification of certain transitive designs, and finally the actions of certain groups on designs.

3 Style of the Book

The book is written for an advanced audience, in the style of a "handbook" and it is a good reference text but only for an expert who is looking for tools from group theory, particularly combinatorial group theory. Some papers jump straight to the research problems assuming the reader knows the area, while others give definitions and examples.

4 Would you recommend this book?

I would recommend this book only to researchers in the area of combinatorics and design theory, who are interested in the use of group theory tools.

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