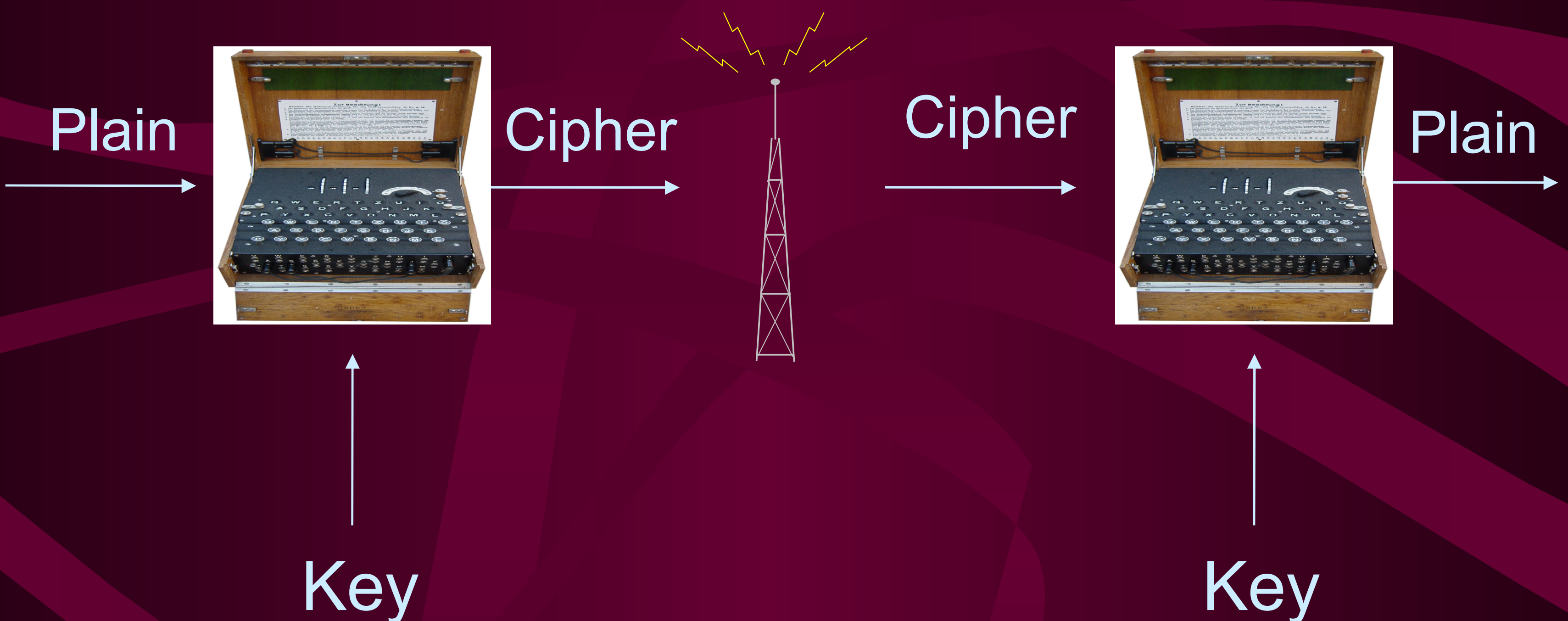
The Growth and Development of Public Key Cryptography

By

Clifford Cocks

Old Style Cryptography



Key Management in the 60s

- Explosion in the need for secure comms
- Key management very labour intensive
- Real concerns of security as net sizes get bigger

- Solution: Do some research into more efficient methods

James Ellis

Encryption

- It is generally regarded as self-evident that….
- ….it is necessary to have some initial information….
- ....kept SECRET from the interceptor

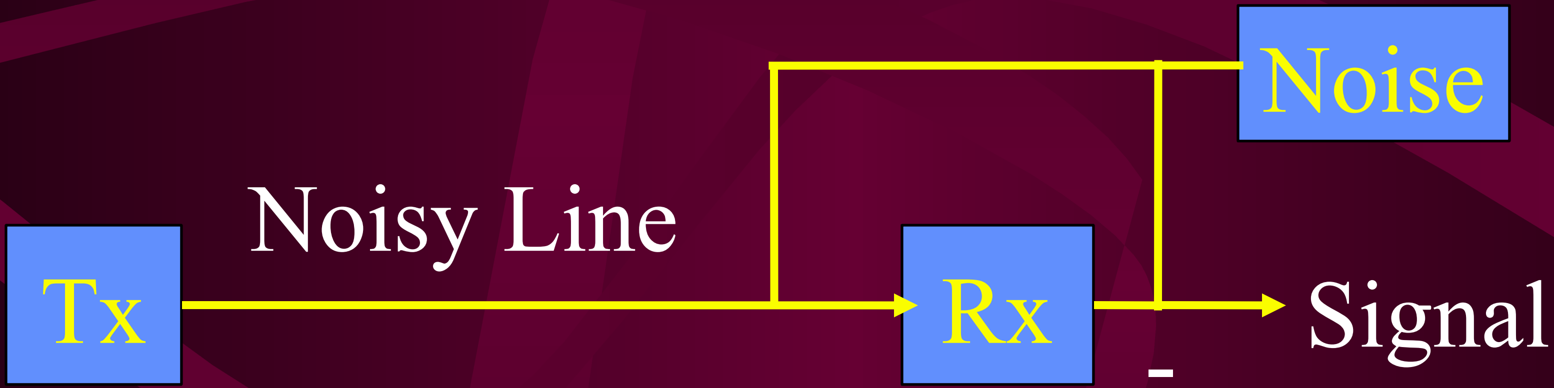
Non-Secret Encryption

- Secure messages sent even though
  - the method of encipherment and
  - all transmissions
- are known to the interceptor

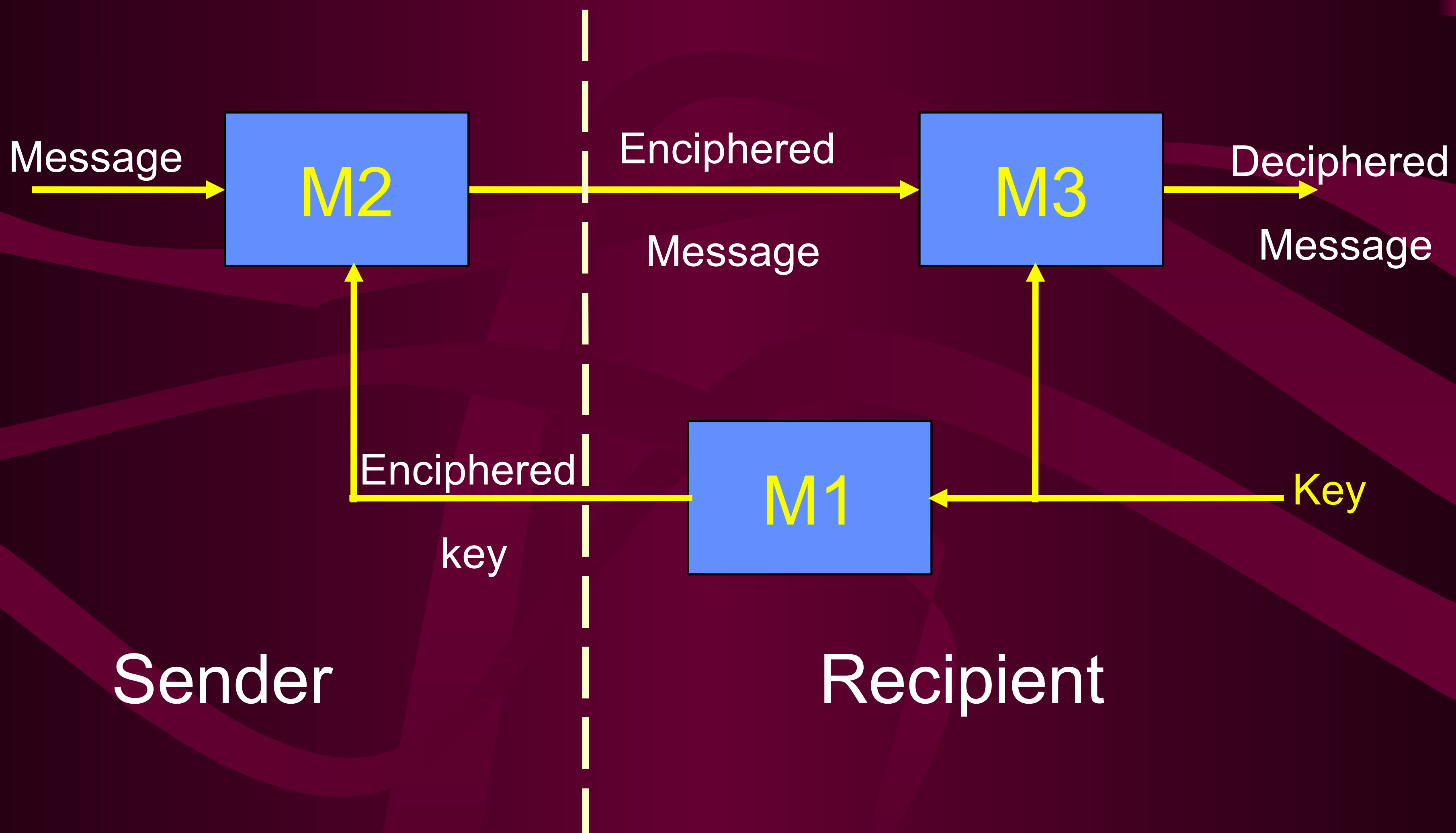
Clue from the Past

1944 Bell Labs Technical report

For a short wire connection:
Recipient adds Random noise to the line which (since he knows it) he can subtract again



Model NSE System



Existence Proof

M1, M2 and M3 are huge look-up tables

Say -

M1 is a 2**100 long 1 dimensional table

M2 is a 2**100 x 2**100 2 dimensional table

M3 is the appropriate 2 dimensional table to make the whole thing work

Table Construction

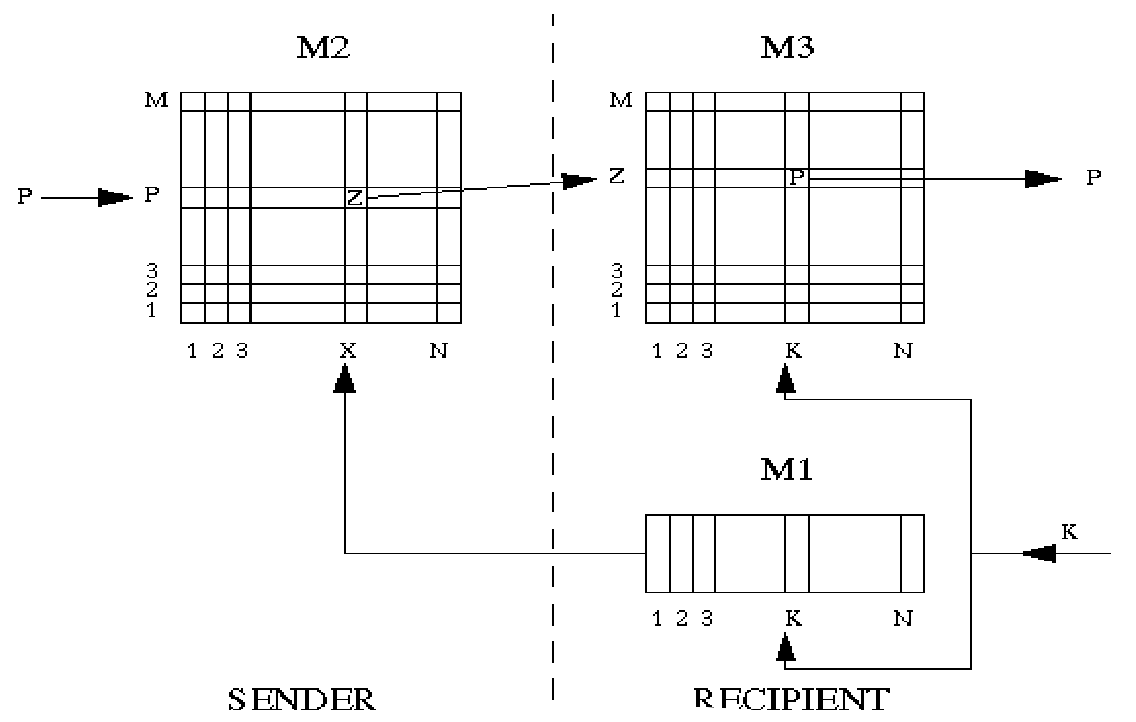
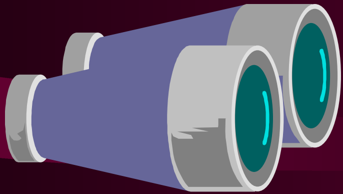


Fig. 2

- M3[M2[P,M1[K]],K] = P



The Search is on!

- It is easy to see that such machines can be represented as look-up tables

- The question is, can we find realisable machines with the required functionality (ie computable functions with the right properties)

Early Reactions

- 1969 Chief Mathematician comments
  - No reason in principle against the scheme
  - but can’t think of implementation
  - impressed by James’ ingenuity
  - but uncertain how to take advantage of it
- 1970 -1973 Several studies by mathematicians and engineers
- But no useful results!

Breakthrough

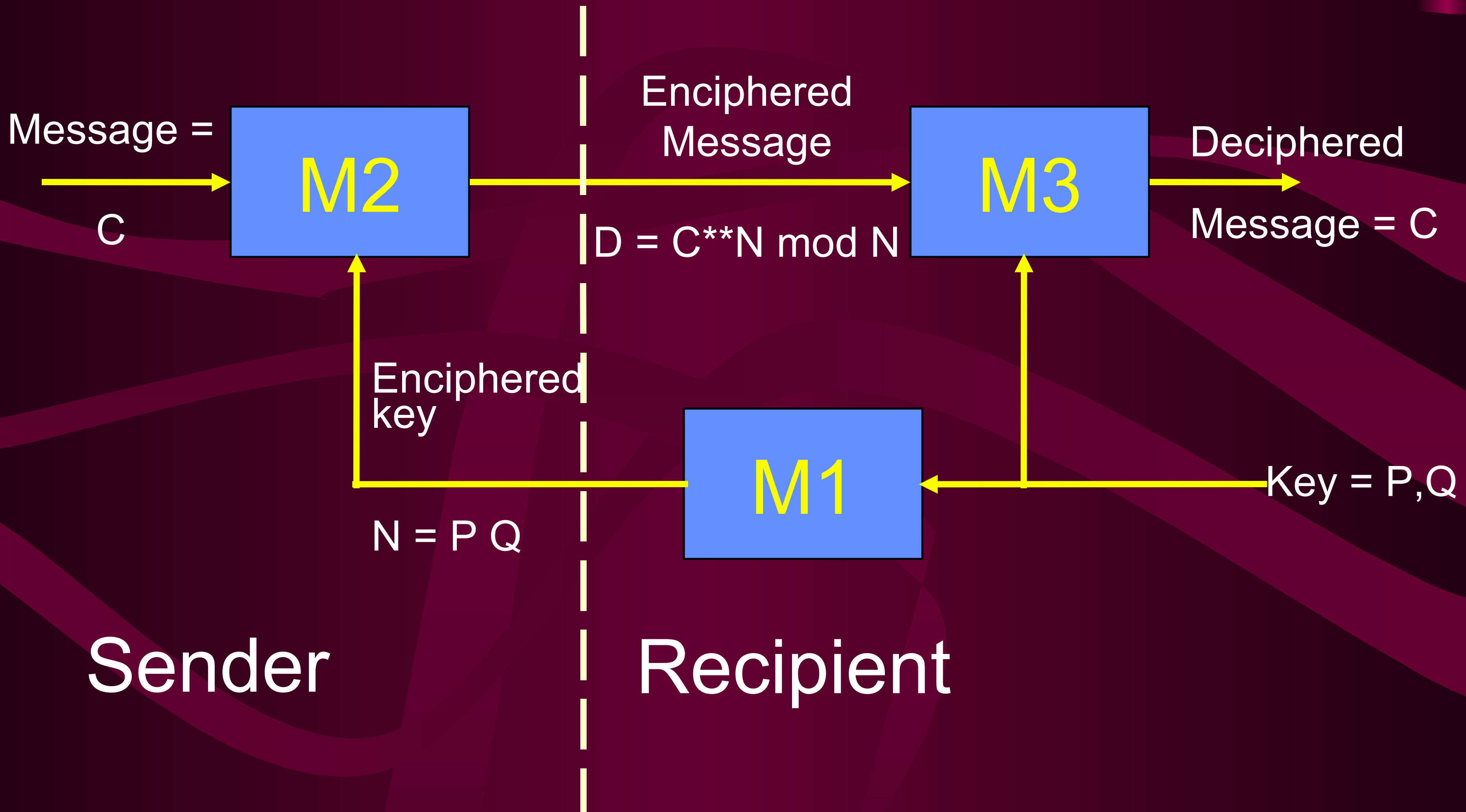
Tunnel vision

Ellis Model

Solution

Nov 1973 1st practical solution

Cocks Implementation



Malcolm Williamson

Shift Registers

- Used as components of many cryptologics of the time
- Distance Problem: Find number of steps between fills
- Natural representation as Finite Fields

Williamson’s 1st Method

January 1974

Message a: Fill of shift register of cycle length p

Sender

Random k

b=a**k

Receiver

c = b**l = a**kl

Random l

Sender

Receiver

d = c**K = a**l

a = d**L

Williamson's 2nd Method

Autumn 1974, written up August 1976

Sender

X**a

Recipient

X**b

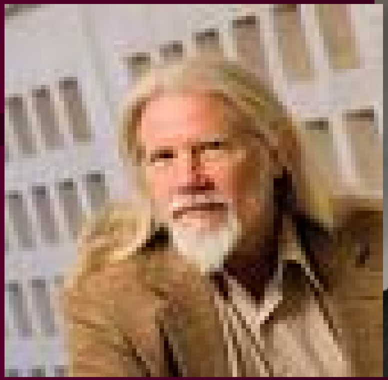
Both can calculate X**ab

same as Diffie Hellman

Reactions to Real NSE

- CESG investigates implementation
  - Williamson preferred for engineering reasons
  - Concern about authentication
- 1970's technology not up to the job
- Debate on 'To patent or Not to patent'

Rediscovery I (Diffie-Hellman)



- Jan 1976 ‘Multiuser Cryptographic techniques’
  - introduces PKC but no example or existence proof
  - Ellis: ‘They are where I started in 1969’
  - shows Ellis solution to Authentication problem
- Nov 1976 ‘New Directions in Cryptography’
  - Williamson 2nd Method

Rediscovery II (RSA)

- Apr 1977 ‘A method for obtaining Digital Signatures and Public-key Cryptosystems’

What Then?

- Developing Theory
  - New Attacks
  - Mathematical Rigour
  - New Primitives
- Practical Uses
  - Cryptographic Products
  - Technology needed to catch up
  - Standards Emerge

Developing Theory

A hard mathematical problem does not guarantee a secure cryptosytem

KNAPSACKS I

Merkle & Hellman (1978): the subset sum problem

Given S, $\{M_i\}$

finding $b_i \in \{0,1\}$ such that $\Sigma b_i M_i = S$

is hard

but

if $M_k > \Sigma M_i$ (superincreasing)

it is easy

KNAPSACKS II

Hide the superincreasing sequence

N, L, $\{M_i\}$ is the secret key

$\{K_i = L\, M_i \bmod N\}$ is the public key

To encrypt $\{b_i\}$

compute $S = \Sigma b_i K_i$ (so finding b_i is hard)

To decrypt

$L^{-1} S \bmod N = \Sigma b_i M_i \bmod N$ (so finding b_i is easy)

KNAPSACKS III

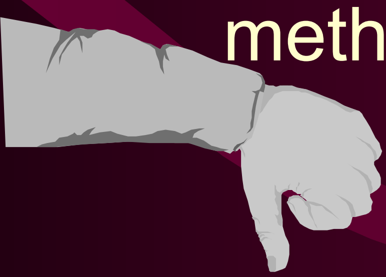
Shamir Crypto'82:

M_1, M_2 …. are very small,

$K_i = L\ M_i \bmod N$ and so for many i,

- $(L^{-1} \bmod N)\ K_i$ is close to a multiple of N
- $K_i\ ((L^{-1} \bmod N) / N)$ is close to an integer

Can use Lenstra's smallest vector in lattice methods to get approximation to $(L^{-1} \bmod N)/N$



Adi Shamir, Ron Rivest, Len Adleman, Ralph Merkle, Martin Hellman, and Whit Diffie receive an award from IEEE at Crypto 2000

Provable Security

Goldwasser & Micali 1984

Security provably equivalent to quadratic residuosity.

Alice: N=PQ c s.t. (c/P)=(c/Q)=-1

Bob: To send m $\in$ {0,1}

Choose random y

Send y^2 c^m (mod N)

Security Proofs

- Theory now well developed
  - Fundamental part of subject
- Allows for clarity
  - What security properties are claimed
  - What mathematical/algorithmic primitives underly security?
- Needed for cryptosystem to be accepted

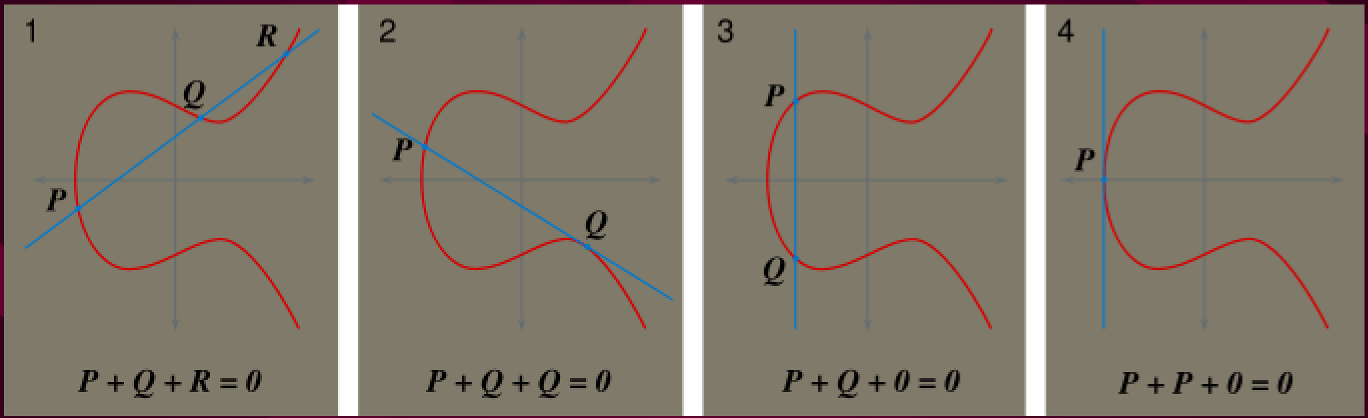
Developing Theory

Elliptic Curves

Neal Koblitz, Victor Miller 1985

$$Y^2 = X^3 + AX + B$$

Defines group structure on points



Elliptic Curves

Neal Koblitz, Victor Miller 1985

$$Y^2 = X^3 + AX + B \text{ on Finite Field}$$

Defines finite group structure on points

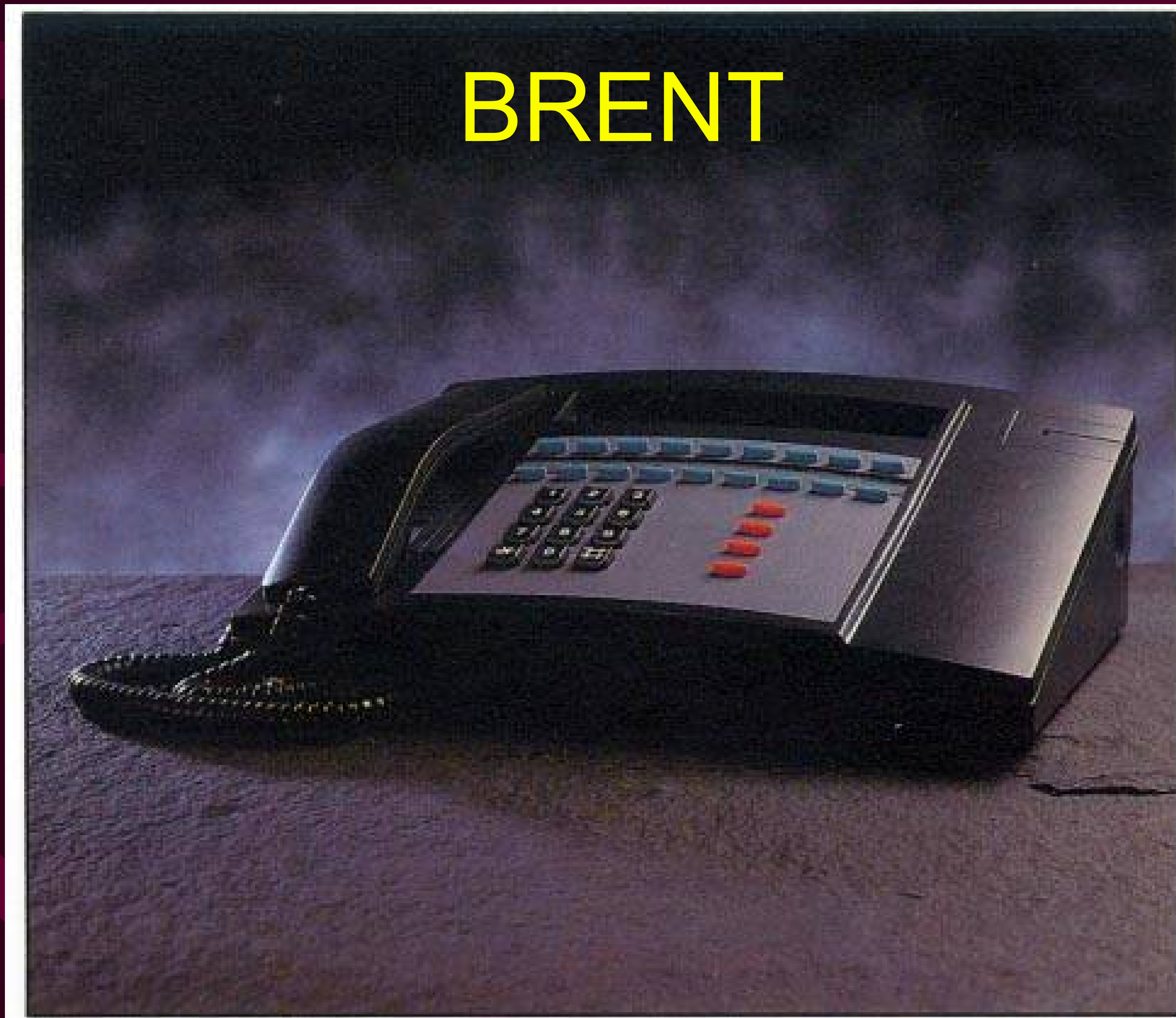
Alternative group for Diffie-Hellman

- N bits of security require only 2N bits of key
- much shorter transmissions than other methods

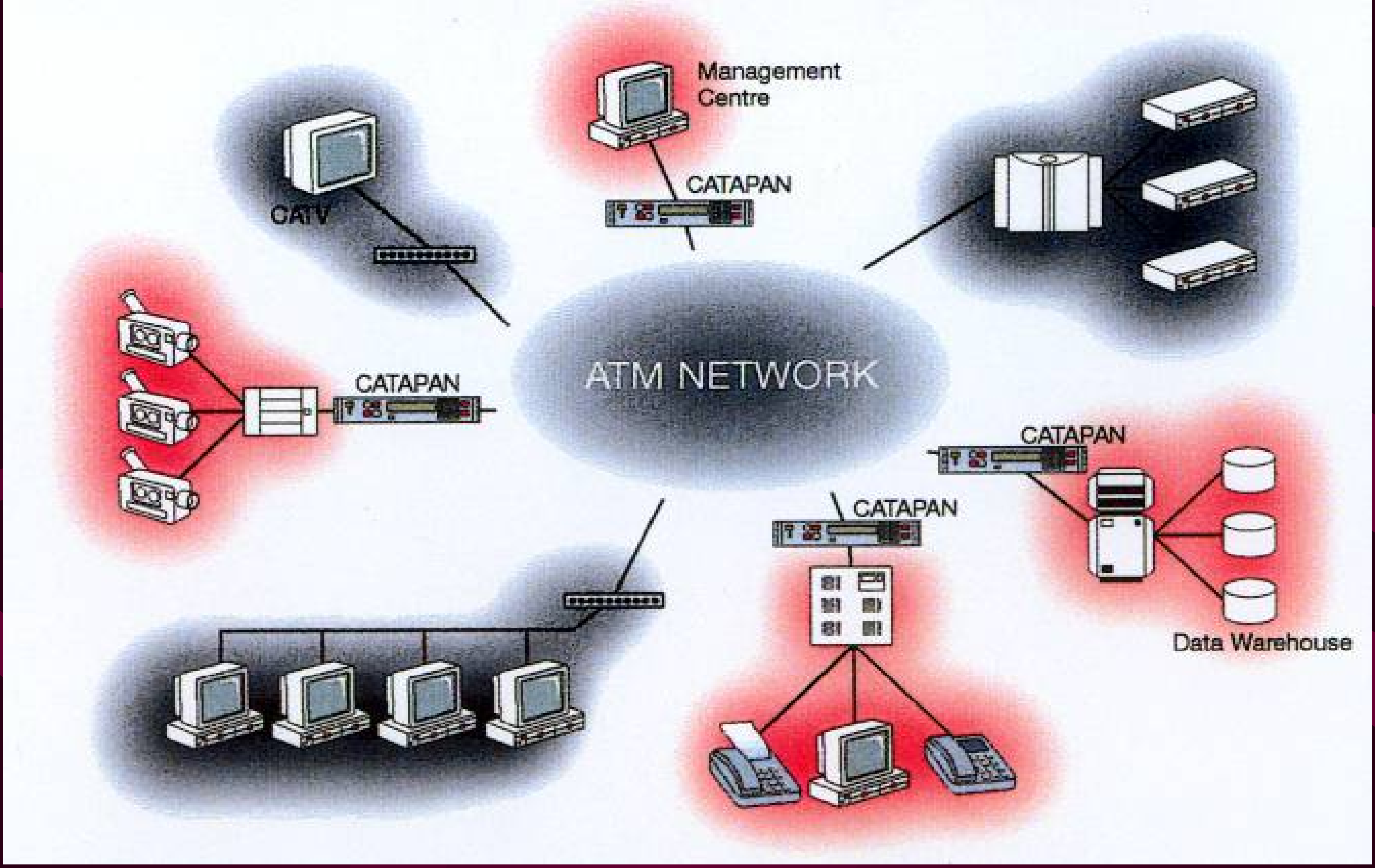
Products

- Experimental Hardware ~1980
  - Sandia, MIT
- Commercial Products from ~1985
  - Cylink CY1024
  - Racal Datacryptor
  - STU III

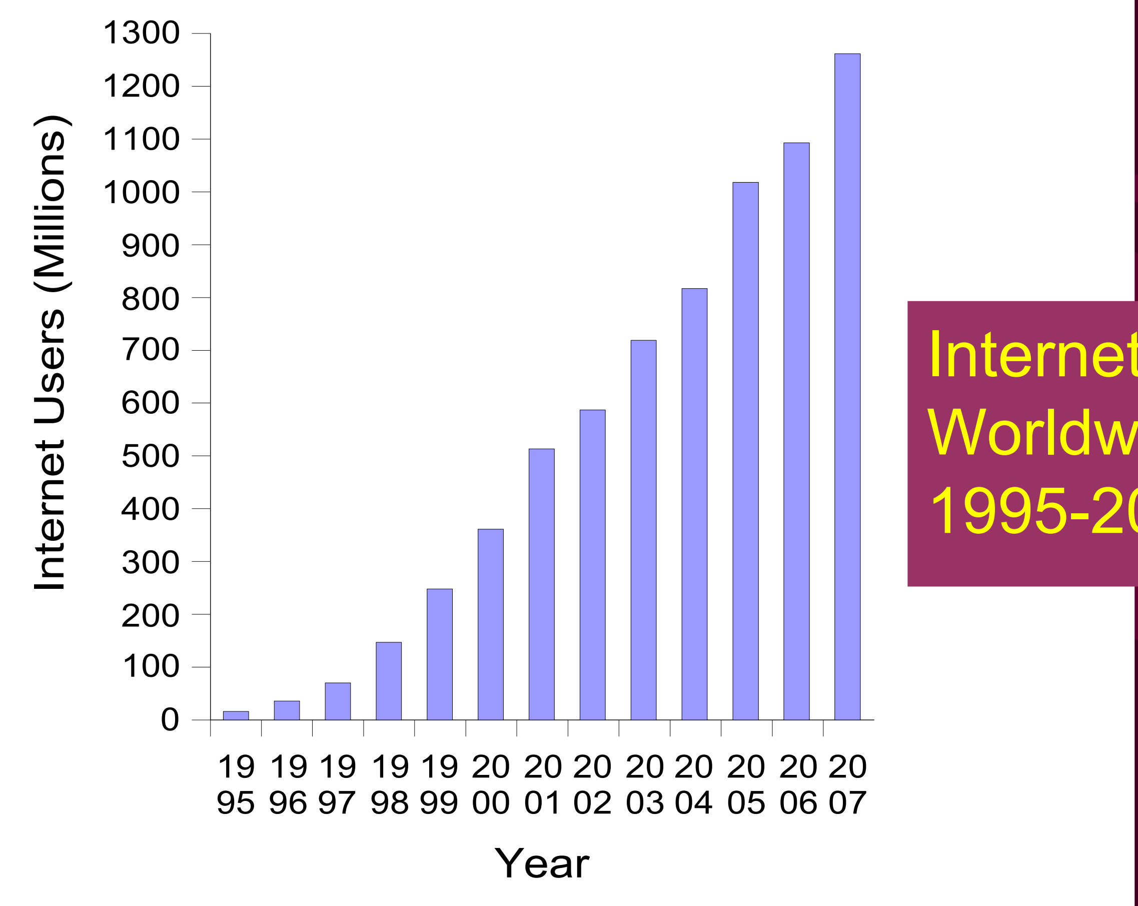
BRENT



CATAPAN & THAMER



Software Era – Internet Growth



Internet Users Worldwide 1995-2007

Software Era - PGP

- PGP
  - Published 1991 by Phil Zimmerman
  - Email encryption and signatures
  - Introduces “web of trust” to manage public keys

Standards & Protocols

- X.509 Certificates ISO 1988
- PKCS RSA Data Security 1991 on
- DSA Signature Algorithm 1991
- SSL Netscape 1994
- IPSec 1995
- SMIME RSA Data Security 1995
- Now IETF lead on standards

Public Key Attacks

- Low exponent
  - encrypt Hastad 1985
  - decrypt Wiener 1990
  - non-random padding Coppersmith 1995

Moral: Beware of small exponents

Public Key Attacks

- Low exponent Coppersmith 1995

Polynomial of degree k: $p(x)=0 \bmod N$

has root x_0 where $|x_0| < N^{(1/k)}$

Then short vector in lattice methods find x_0 quickly

Fixed padding and low exponent:

$y = [{*}{*}{*}{*}{*}{*}{*}{*}{*}\ r_1,r_2,\ldots,r_m,\ {*}{*}{*}{*}{*}{*}{*}{*}] = ar+b$

See $y^e = (ar+b)^e \bmod N$

Public Key Attacks

- Timing attacks Kocher 1996

To compute y^x mod N:

set R<-- 1 z<-- y then iterate:

If bit i of x = 1: R<-- (R z) mod N

z <-- z^2 mod N

Time to do modular multiplication may depend on z , R

Lots of samples: recover x bit by bit

Moral: Blind the calculation

Public Key Attacks

- Quantum Computation
  - Shor's Algorithm 1994
  - unitary operation on 2^n states with n qbits
  - Fourier Transform is a unitary operation
  - at end of calculation sample one state by amplitude
  - Can use this to break RSA and Diffie Hellman

Public Key Attacks

- Quantum Computation
  - Shor's Algorithm 1994

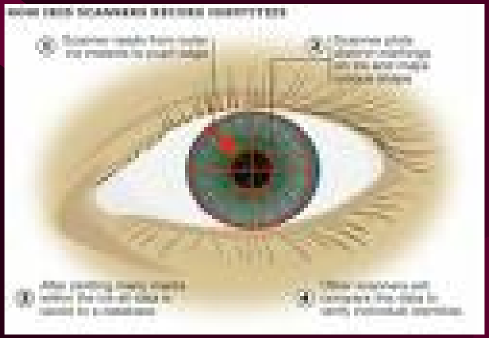
Calculate x^a mod N for a = 1,...,M and M ~ N^2

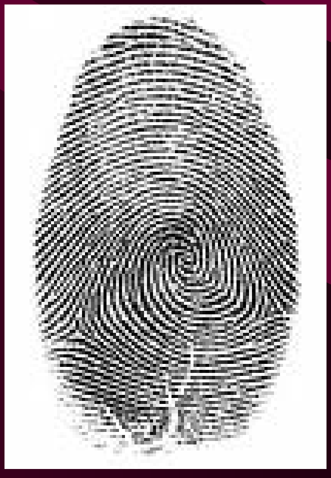
Observe x^a, now have set of values $a=a_0$ mod Phi(N)

Perform Fourier transform on a values to recover Phi(N)

Continuing Developments

- Pairings
- Identifier Based Cryptography





Identifier Based Cryptography

Shamir 1984

Bob's Public Key derived from his identity

Alice encrypts with no need for directory

Bob gets his Private Key from a Trusted Authority

Identifier Based Cryptography

- History
  - Concept Proposed by Shamir 1984
  - False starts: e.g. Tanaka 1987
  - Expensive Scheme: Maurer 1991
  - QR proposals : Cocks 1998, published 2001
  - Pairings method: Boneh & Franklin 2001
  - Improved QR method: Boneh Gentry & Hamburg 2007

Weil Pairing

- E Elliptic curve over F_q
  - e maps E x E --> F_{q^k}
  - e(A+B,C) = e(A,C) e(B,C)
  - e(A,B+C) = e(A,B) e(A,C)
- Originally used to attack proposed curves
- Limited sets of “Pairing Friendly” curves

IDPKC from the Weil Pairing


Boneh & Franklin '01

Trusted Authority

secret: x

global : elliptic curve E / F_p

bilinear map $e: E \times E \dashrightarrow F_{p^2}$

key = e(x.IDA, IDB)

Alice

Bob

x . IDA

x . IDB

Maps identities to points in E

Next Chapter

Implementation Challenge: Make PKI work (better)


Next Chapter
Research Challenge
Find a Convincing & Elegant
Quantum Resistant Public Key Algorithm