Ciphers for MPC and FHE

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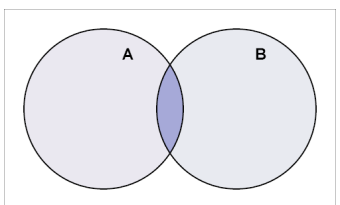
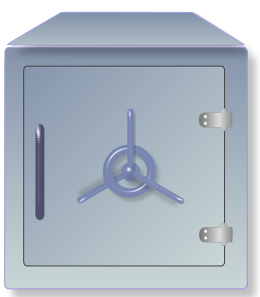
MPC applications using block ciphers

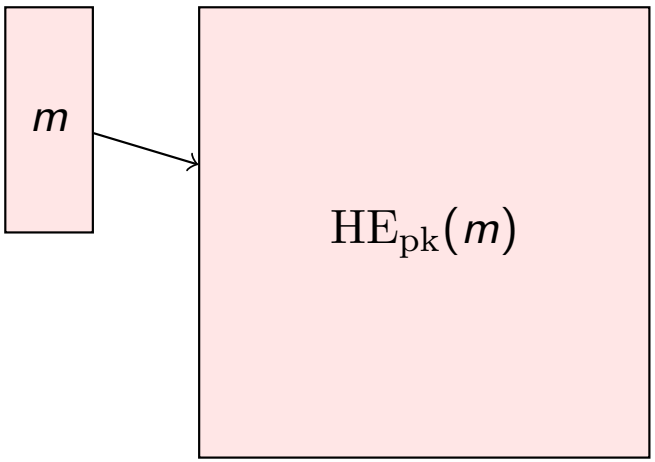
Block ciphers have various applications in MPC



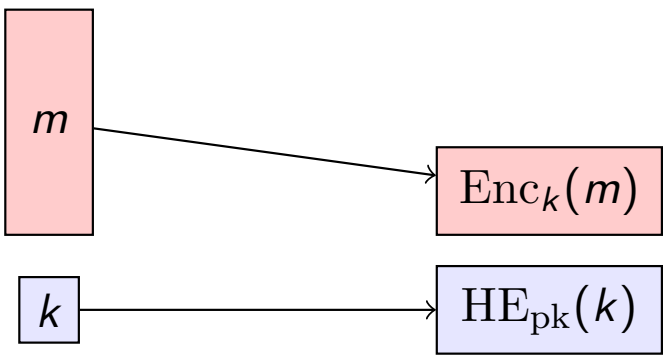


- **Server-side one-time passwords**, commercialized by Dyadic Security (server-side derivation of one-time passwords via MPC)
- Oblivious Pseudorandom Functions (OPRFs) for **privacy-preserving keyword search**, **private set intersection**, **secure database join**, etc.
- **Secure storage**: store symmetrically encrypted intermediate MPC values in untrusted storage

FHE Motivation: Avoid ciphertext expansion

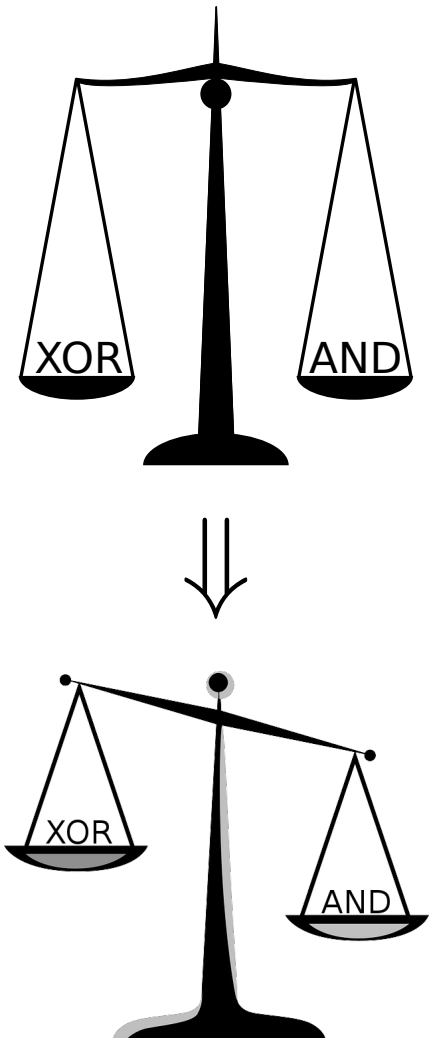


FHE schemes typically come with a ciphertext expansion in the order of **1000s** to **1000000s**.



Solution:
Encrypt message symmetrically,
transfer key homomorphically.
Cloud decrypts homomorphically then.

New computational models require new designs



- Cost of XOR gate is (almost) negligible compared to AND gate in MPC or FHE setting
- But since 1970s: balance between linear and non-linear operations
- Idea: Explore **extreme** trade-offs

Question

What would an efficient cipher look like if linear operations were for free?

Possible metrics for optimisation

There are three possible metrics to minimise:

1. ANDs per bit of encrypted text (**ANDs/bit**)
2. multiplicative depth of the encryption circuit (**ANDdepth**)
3. total number of ANDs per encryption (**ANDs**)

Question

Can we design a cipher that can be optimized with regard to any combination of these metrics?

Related work

Minimization of multiplicative complexity also relevant in side-channel countermeasures. Designs much less extreme though:

- Noekeon
- Fantomas
- Robin

Joan Daemen, Michaël Peeters, Gilles Van Assche, and Vincent Rijmen. Nessie proposal: Noekeon. In *First Open NESSIE Workshop*, 2000.

Vicente Grosso, Gaëtan Leurent, François-Xavier Standaert, and Kerem Varici. LS-designs: Bitslice encryption for efficient masked software implementations. In *Fast Software Encryption (FSE 2014)*, LNCS. Springer.

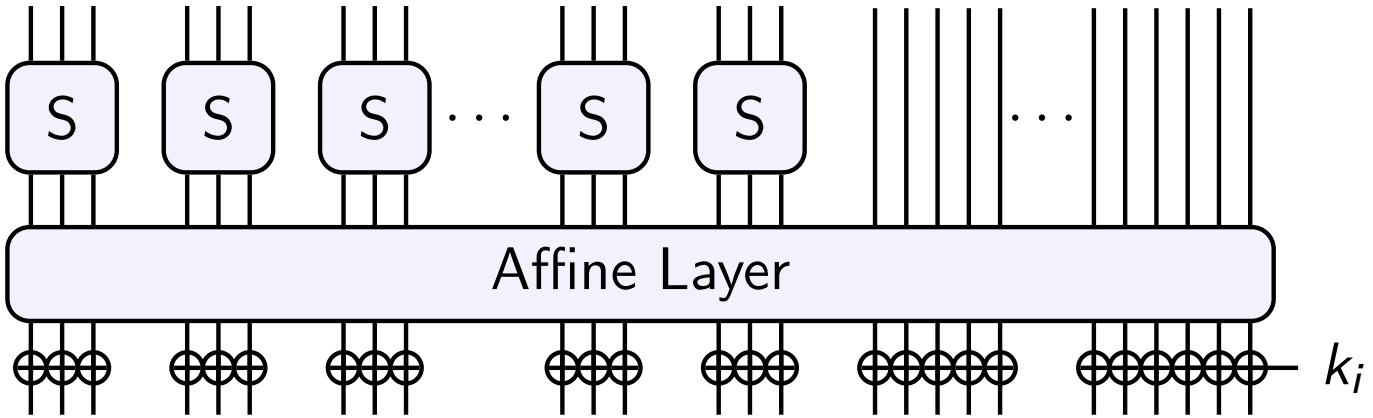
Design Strategy

Design Ideas

Minimise ANDs needed for confusion, maximise diffusion.

- Use an SPN
- Use small Sboxes with low multiplicative complexity
- Maximize diffusion in affine layer
- Utilize a partial substitution layer

The LowMC round function and parameters



Size parameters

- **block size** n bits
- **number** m **of Sboxes** in substitution layer

Security parameters

- **key size** k
- allowed **data complexity** d

Number of **rounds** r is then calculated as a function of the above.

Choice of the Sbox

Properties of Sbox

- Maximum differential probability 2^{-2}
- Maximum squared correlation 2^{-2}
- Circuit needs only 3 AND gates and has ANDdepth 1
- Any combination of output bits has algebraic degree 2

Algebraic Normal Form of Sbox:

$$S_0(A, B, C) = A \oplus BC$$
$$S_1(A, B, C) = A \oplus B \oplus AC$$
$$S_2(A, B, C) = A \oplus B \oplus C \oplus AB$$

Maximise diffusion in affine layer

How do we maximise diffusion in affine layer?

- **Choose most general affine layer**: multiplication with quadratic $n \times n$ matrix over $\mathbb{F}_2$ and addition of constant $\mathbb{F}_2$ vector of length n.

How do we choose good matrices and vectors?

- Unfortunately, determining branch number of a binary matrix is hard in practice and theory.

We thus choose to

- **Choose random matrix** uniformly from all invertible $n \times n$ matrices over $\mathbb{F}_2$.
- **Choose random constant vector** uniformly from $\mathbb{F}_2^n$.

Bonus: This allows novel security arguments.

Instantiation of affine layers and round key matrices

Problem: How do you accountably instantiate the random matrices and vectors?

- instance of cipher cannot use "random" matrices but must use fixed ones
- how choose them in an accountable way ("nothing up the sleeve")?

Our solution:

- **Use Grain LFSR as self-shrinking generator** to produce random bit string
- Then use this string to generate the matrices.

To determine round number cryptanalysis necessary

Two factors determine the number of rounds

1. Maximal length of a distinguisher
2. Number of rounds that can be peeled off

Look at the following distinguishers:

- Statistical distinguishers: linear and differential characteristics
- Low-degree attacks
- Combined attacks, special case: Boomerang attacks

Resistance Against Differential attacks

Standard method to determine probability of best differential characteristic:

1. Determine minimal number of active Sboxes.
2. Combine with maximal differential probability of Sbox to determine lower bound on best possible characteristic.

To determine the minimal number of active Sboxes the branch number would be helpful.

Problem

We do not know the branch number of the randomly chosen matrix.

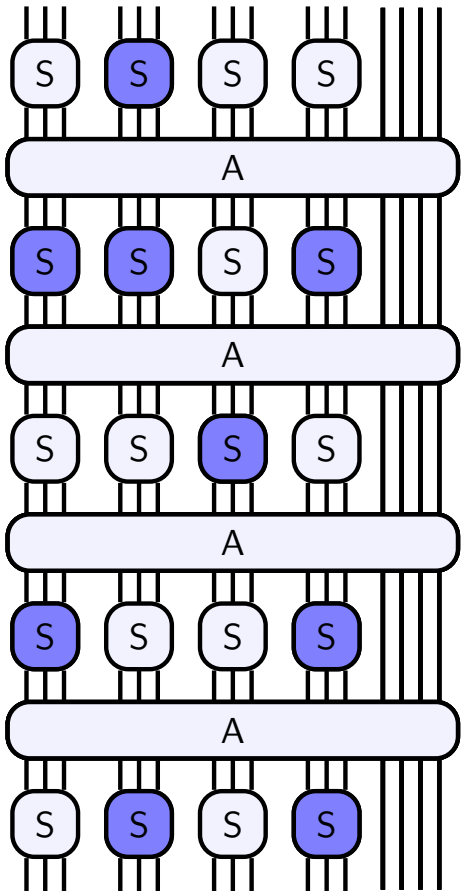
Determining probability of best differential characteristics

Idea

Calculate for each possible good differential characteristic probability that it is realized in instantiation of LowMC. Sum all these probabilities to get upper bound for probability that at least one is realized.

C set of possible good characteristics.

$$\sum_{c \in C} \Pr(c \text{ exists in cipher})$$
$$\leq \Pr(\text{good characteristic exists})$$



Higher Order Attacks

Question: What is the minimal number of rounds needed to reach a given algebraic degree?

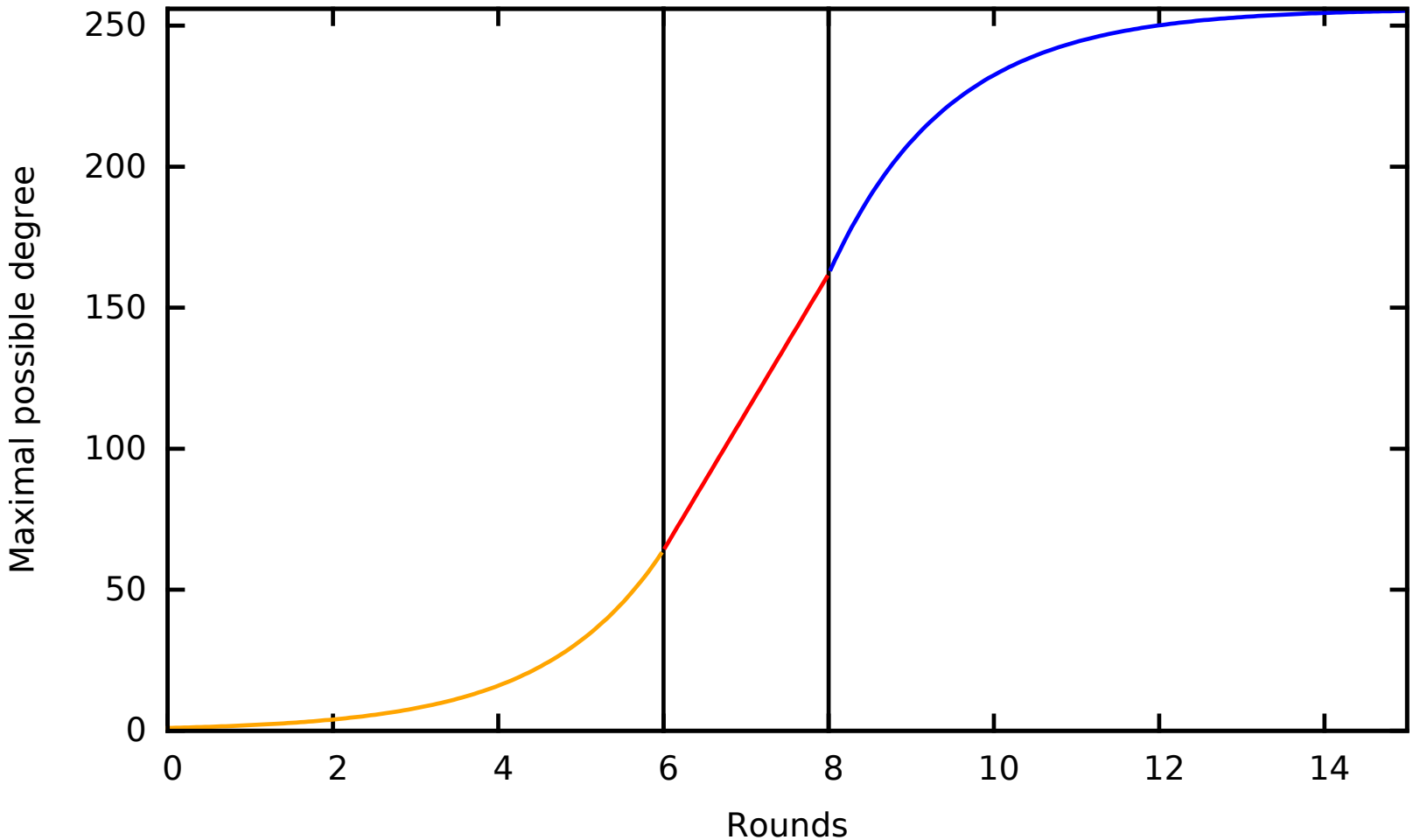
Lemma

If algebraic degree is d_r after r rounds, max. degree in round $r+1$ is

$$\min\left(2d_r, m + d_r, \frac{n}{2} + \frac{d_r}{2}\right).$$

- The first bound is trivial.
- Third bound was proven by Boura, Canteaut, and De Cannière [BCC11]
- Second bound is new.

Growth of the degree



Formula to calculate number of rounds

Round formula

$$r \geq \max(r_{\text{stat}}, r_{\text{deg}}, r_{\text{cmbnd}}) + r_{\text{outer}}$$

r_{stat}: bound for differential and linear distinguishers
r_{deg}: bound for sufficient degree
r_{cmbnd}: bound for combined distinguishers
r_{outer}: bound for rounds that can be peeled off

For r_{outer}, we use the ad-hoc formular

$$r_{\text{outer}} = r_{\text{stat}}.$$

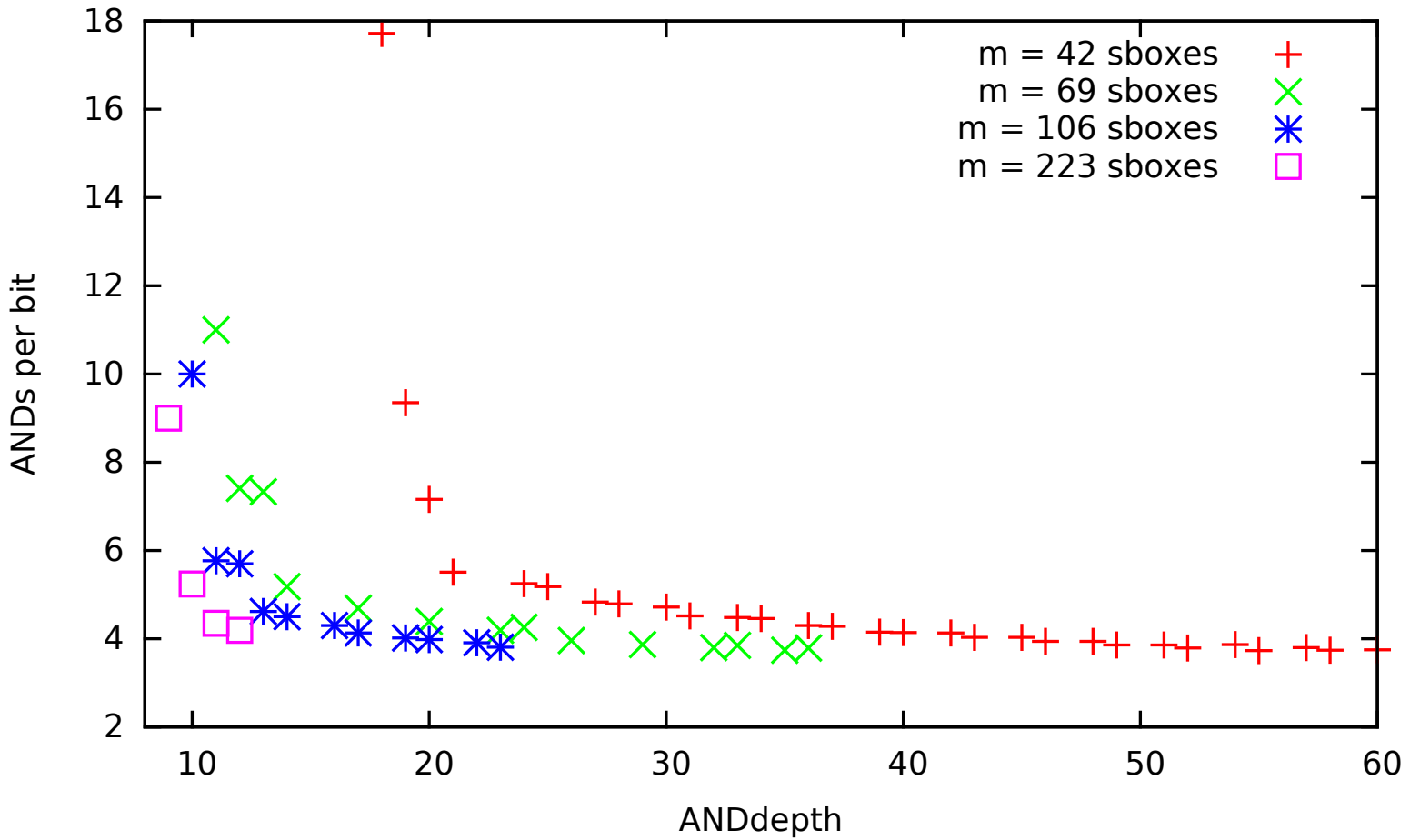
We thank Dmitry Khovratovich for pointing out that combined attacks can be more effective than others.

The parameter set

Sboxes	blocksize	data	r_{stat}	r_{bmrg}	r_{deg}	total rounds
49	256	2^{64}	5	6	6	11
63	256	2^{128}	5	6	7	12

- But LowMC is **not limited** to this parameter set
- Dependent on optimization metric, size parameters and security parameters other parameter sets can be calculated
- As **few as 9 rounds** possible for data security of 128 bits

Parameter space for AES-like security



Comparison with most competitive other ciphers

AES-like security

Cipher	Key size	Block size	Data sec.	ANDdepth	ANDs/bit
AES-128	128	128	128	40 (60)	43 (40)
Simon	128	128	128	68	34
Noekeon	128	128	128	32	16
Robin	128	128	128	96	24
Fantomas	128	128	128	48	16.5
LowMC	128	256	128	12	8.85

Comparison with most competitive other ciphers

Lightweight security

Cipher	Key size	Block size	Data sec.	ANDdepth	ANDs/bit
PrintCipher-96	160	96	96	96	96
PrintCipher-48	80	48	48	48	48
Present	80 or 128	64	64	62 (93)	62 (31)
Simon	96	64	64	42	21
Simon	64	32	32	32	16
Prince	128	64	64	24	30
KATAN64	80	64	64	74	36
KATAN32	80	32	32	64	24
DES	56	64	56	261	284
LowMC	80	256	64	11	6.31

Benchmark results for multiple blocks of total size 12.8 Mbit in GMW

Lightweight Security

Cipher	Present		Simon		LowMC	
Comm. [GB]	7.4		5.0		**2.5**	
	LAN	WAN	LAN	WAN	LAN	WAN
Total [s]	216.88	488.24	272.22	605.41	**45.36**	**155.75**

Long-Term Security

Cipher	AES		Simon		LowMC	
Comm. [GB]	16		13		**3.5**	
	LAN	WAN	LAN	WAN	LAN	WAN
Total [s]	555.91	947.79	447.27	761.90	**64.37**	**215.01**

Benchmark results FHE using HELib by Halevi & Shoup

d	n	ANDdepth	t_{block}	t_{bit}	Cipher	Ref.	Key Sched.
128	128	40	1.5s	0.0119s	AES-128	[GHS12]	excluded
128	128	40	55s	0.2580s	AES-128	[DHS14]	excluded
128	128	40	22m	10.313s	AES-128	[MS13]	excluded
128	128	40	14m	6.562s	AES-128	[MS13]	excluded
128	256	12	0.8s	0.0033s	LowMC	this work	included
64	size	24	3.3s	0.0520s	PRINCE	[DSES14]	excluded
64	256	11	0.64s	0.0025s	LowMC	this work	included

Conclusion

- Proposed **flexible block cipher** design of **extremely low** number of **ANDs/bit** and **extremely low ANDdepth**
- Provided experimental and theoretical cryptanalysis to ensure soundness of design
- Demonstrate that symmetric design and cryptanalysis can significantly contribute to make applications of MPC and FHE more practical
- Measured **speed-up** factors between 2 and 25

Open problems

- Can the cost of LowMC in the traditional setting be reduced by using a sparser affine layer without reducing security claims?
- Improve implementations of LowMC in MPC and FHE settings
- What designs can minimize the multiplicative complexity over larger fields than GF(2)?
- Further refinement of round number formula, explicitly include key size
- Further cryptanalysis needed

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Questions?

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Key schedule

Reuse random matrix approach for key schedule:

- Derive round keys from general key by multiplication with $n \times k$ binary matrix.
- Choose matrices uniformly at random from all binary $n \times k$ matrices of rank $\min(n, k)$.

Benchmark results for single block in GMW

Lightweight Security						
Cipher	Present		Simon		LowMC	
Communication [kB]	39		**26**		51	
Runtime	LAN	WAN	LAN	WAN	LAN	WAN
Setup [s]	0.003	0.21	**0.002**	0.21	**0.002**	**0.14**
Online [s]	**0.05**	13.86	**0.05**	5.34	0.06	**1.46**
Total [s]	**0.05**	14.07	**0.05**	5.45	0.06	**1.61**

Long-Term Security						
Cipher	AES		Simon		LowMC	
Communication [kB]	170		136		**72**	
Runtime	LAN	WAN	LAN	WAN	LAN	WAN
Setup [s]	0.01	0.27	0.009	0.23	**0.002**	**0.15**
Online [s]	**0.04**	4.08	0.05	6.95	0.07	**1.87**
Total [s]	**0.05**	4.35	0.06	7.18	0.07	**2.02**

Boomerang attacks

- Use good differentials that meet halfway from both sides
- Partial non-linear layers allow probability 1 differentials for a few rounds
- The individual differentials must have higher probability though

Solution

- Calculate length at which no differential is usable for boomerang attacks
- Double this length