How to Obfuscate Programs Directly

Joe Zimmerman

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- Weaker definition: indistinguishability obfuscation (*iO*) [BGI+01]

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  – Fundamental building block: multilinear maps
    [BS03, GGH13a, CLT14, GGH14, CLT15]

  – VBB security in generic multilinear map model
    [GGH+13b, BGK+13]

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	Degree of multilinearity	Obfuscation size (# ring elements)	Evaluation time (# ring operations)
Via Barrington's Thm. [GGH+13b, BR14, BGK+14]	$O(4^d n + n^2)$	$O(4^d n + n^2)$	$O(4^d n + n^2)$
[AGIS14]	$O(2^d n + n^2)$	$O(8^d n + n^2)$	$O(8^d n + n^2)$
[AGIS14] + [Gie01]	$O(2^{(1+\varepsilon)d} n + n^2)$	$O(2^{(1+\varepsilon)d} 4^{2/\varepsilon} n + n^2)$	$O(2^{(1+\varepsilon)d} 4^{2/\varepsilon} n + n^2)$
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- Prove VBB obfuscation in generic model

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- Concurrent work:
  - [AB15]: also obfuscates circuits without converting to branching programs; achieves iO in generic model

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(assuming AB^2C is the top-level index set U)

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– Intuitively, encodings $[x]_S$ hide original scalars x in Z_N

– Formally: generic model only exposes map operations [GGH+13a, BR13, BGK+13]

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    - M → M' adds only two components to modulus N

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For encodings $[x]_S$, with x in $Z_N = Z_{N_1 \ldots N_k}$:

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– Crucial property: adversary does not know $N_1, \ldots, N_k$,
cannot act independently on components

Our construction

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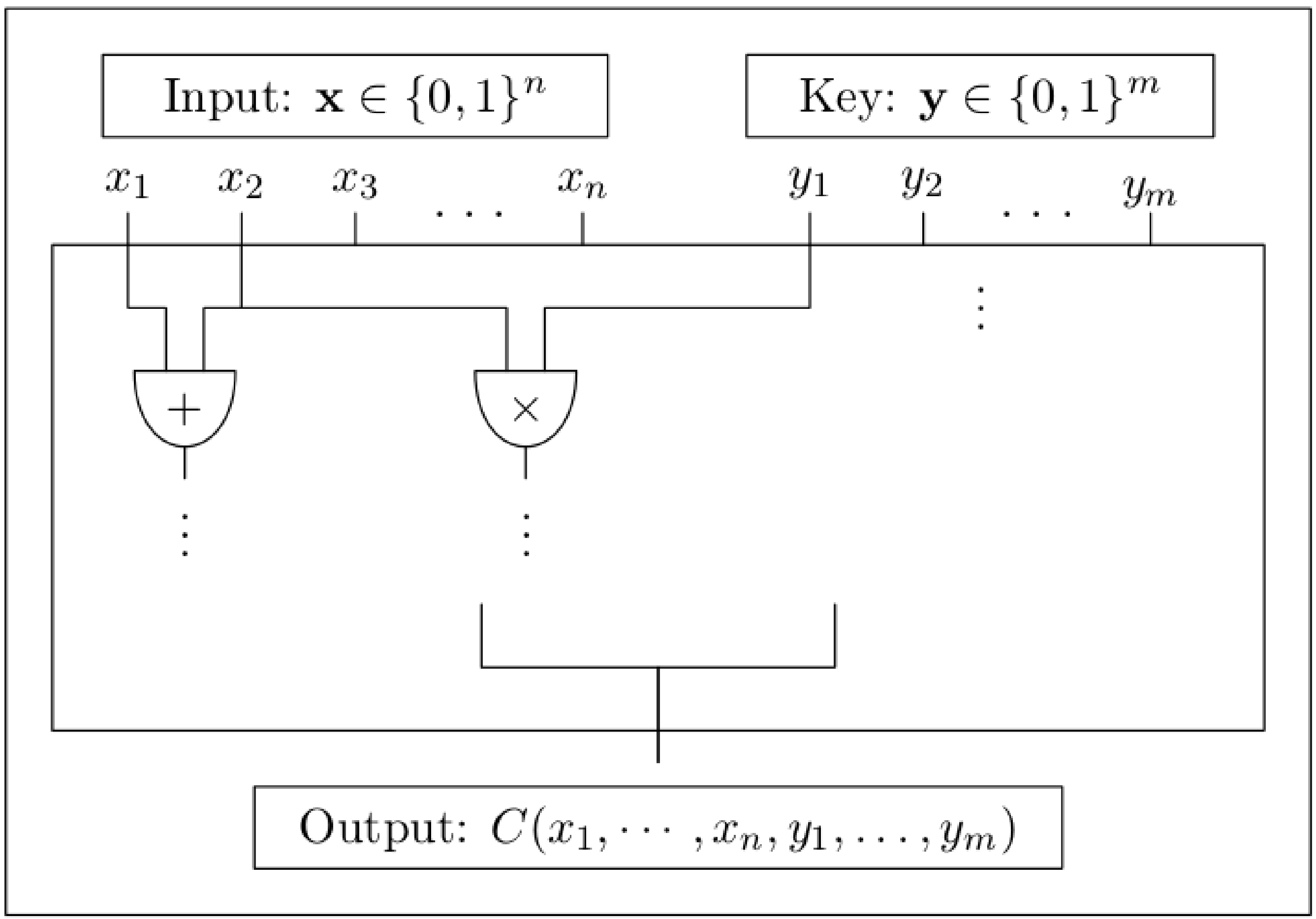

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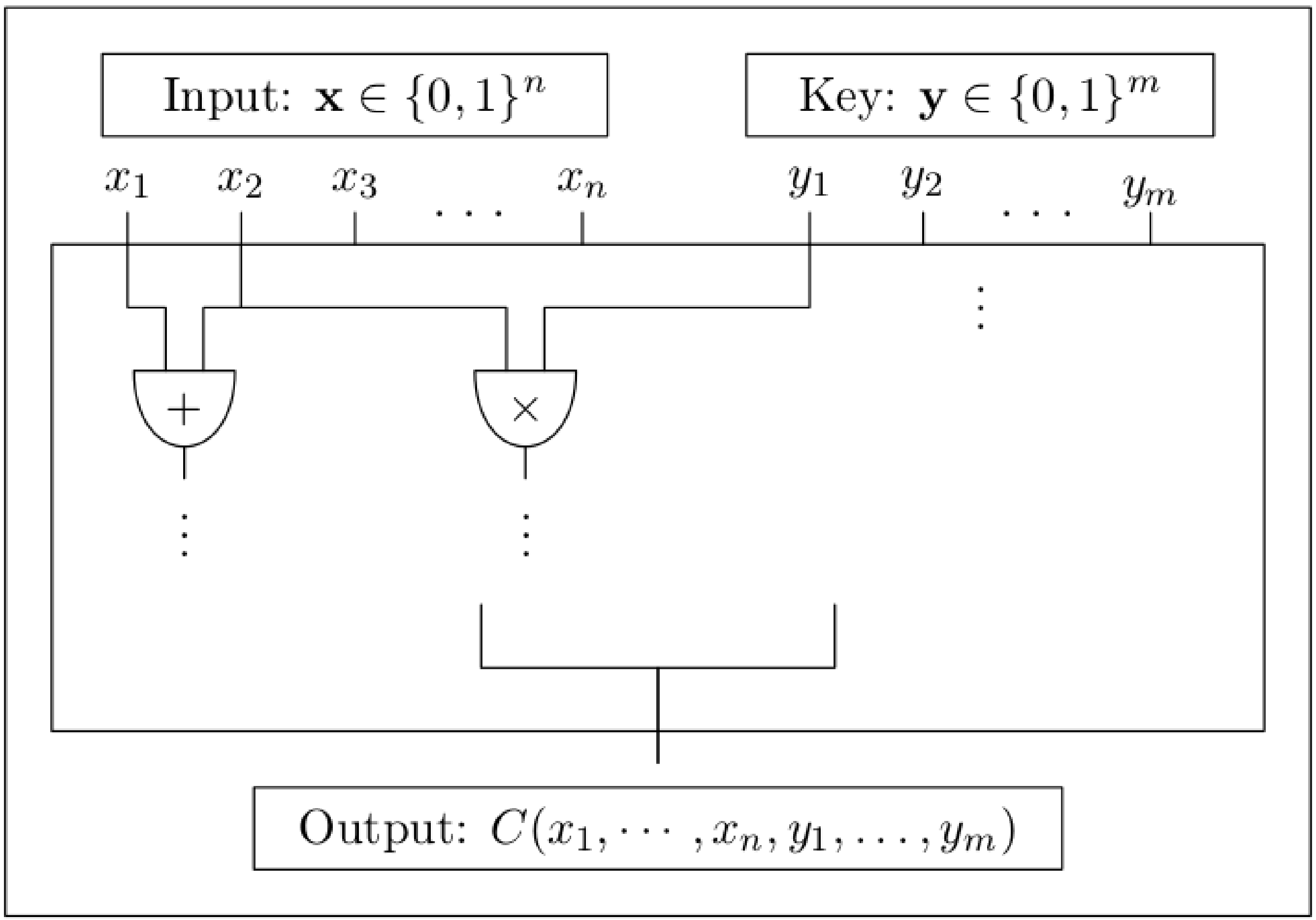
  – e.g., AES



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  – Rich design space of data-oblivious algorithms

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    - Test whether C(x, y) = 0 using the map's ZeroTest

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  - Standard approach [GGH+13b]:
    - Convert to branching program [Bar86]
    - “Garble” using Kilian's protocol [Kil88]

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- Adversary's C' will not pass ZeroTest unless C' = C (as a polynomial)

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    - Theorem: “clean” multilinear maps imply succinct obfuscation (assuming hardness of factoring)

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