# Privacy Amplification in the Isolated Qubits Model

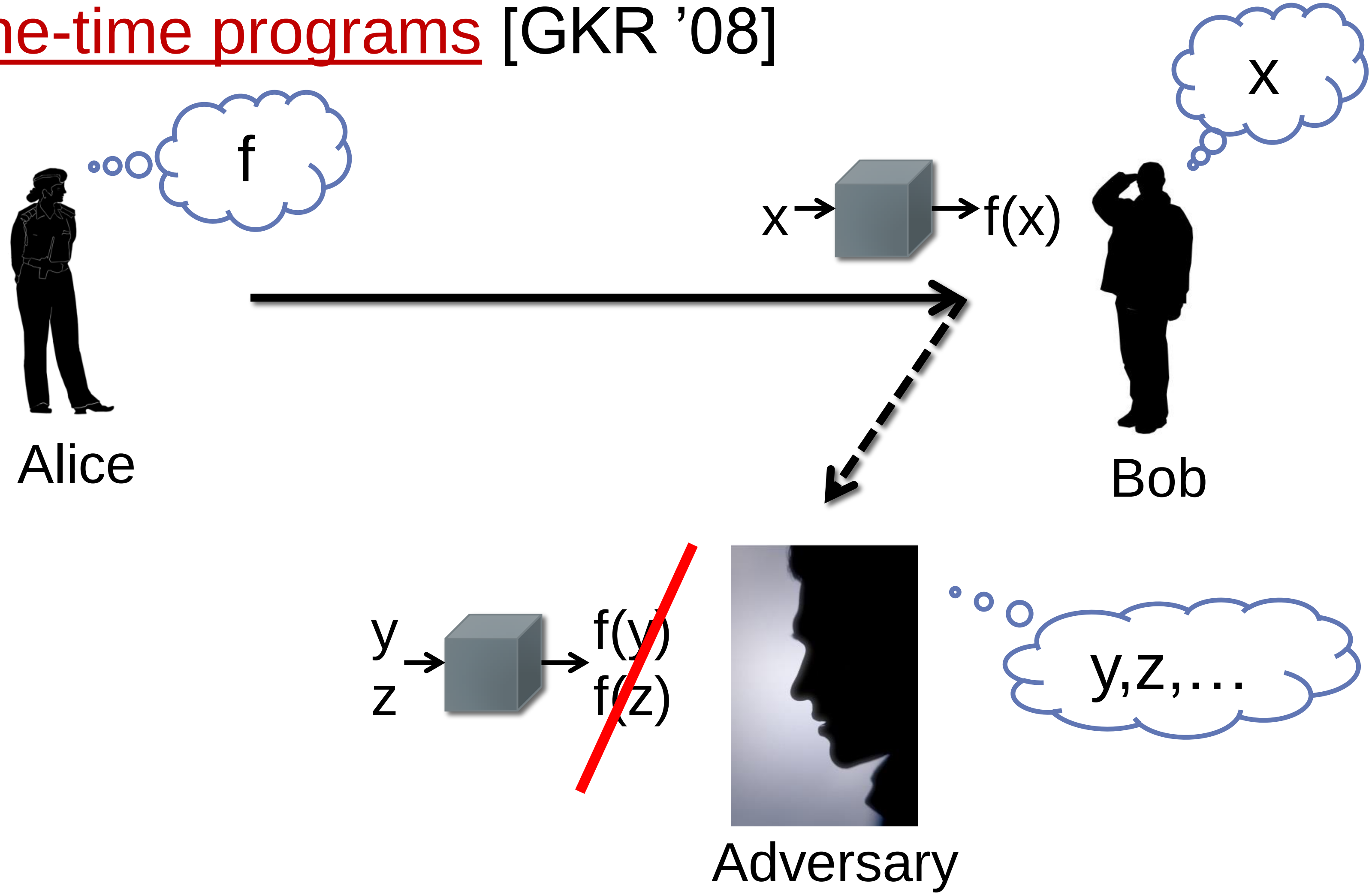
Yi-Kai Liu

National Institute of Standards and Technology (NIST)
Gaithersburg, MD, USA

Joint Center for Quantum Information and Computer Science (QuICS)
NIST / University of Maryland

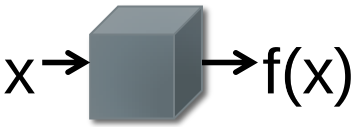
# Tamper-Resistant Hardware

- One-time programs [GKR ’08]



# One-Time Programs

- Non-interactive
- Need trusted hardware




- Previous work: assume <u>one-time memories</u>
  - Abstract functionality, like oblivious transfer
  - [GKR ’08] [Goyal et al] [Bellare et al]
- This work: assume <u>isolated qubits</u>
  - A special class of quantum-mechanical devices,
    with a natural restriction on the power of the adversary
  - [Liu ’14]

# Privacy Amplification

- Needed because real devices are never perfect
  - Some information always leaks (e.g., via side channels)
- Usual recipe: use a hash function h
  - Randomly chosen from a 2-universal family

- This doesn't work for us!
  - One-time programs are non-interactive
  - Need to announce h at the beginning of the protocol
  - Adversary knows h before he attacks the scheme

# This Talk

- Deterministic privacy amplification
  - Secure in isolated qubits model
  - Non-interactive: uses a single fixed hash function, is secure against all adversaries simultaneously
- => One-time memories using isolated qubits
  - Only leak an exponentially small amount of information

# Quantum Mechanics

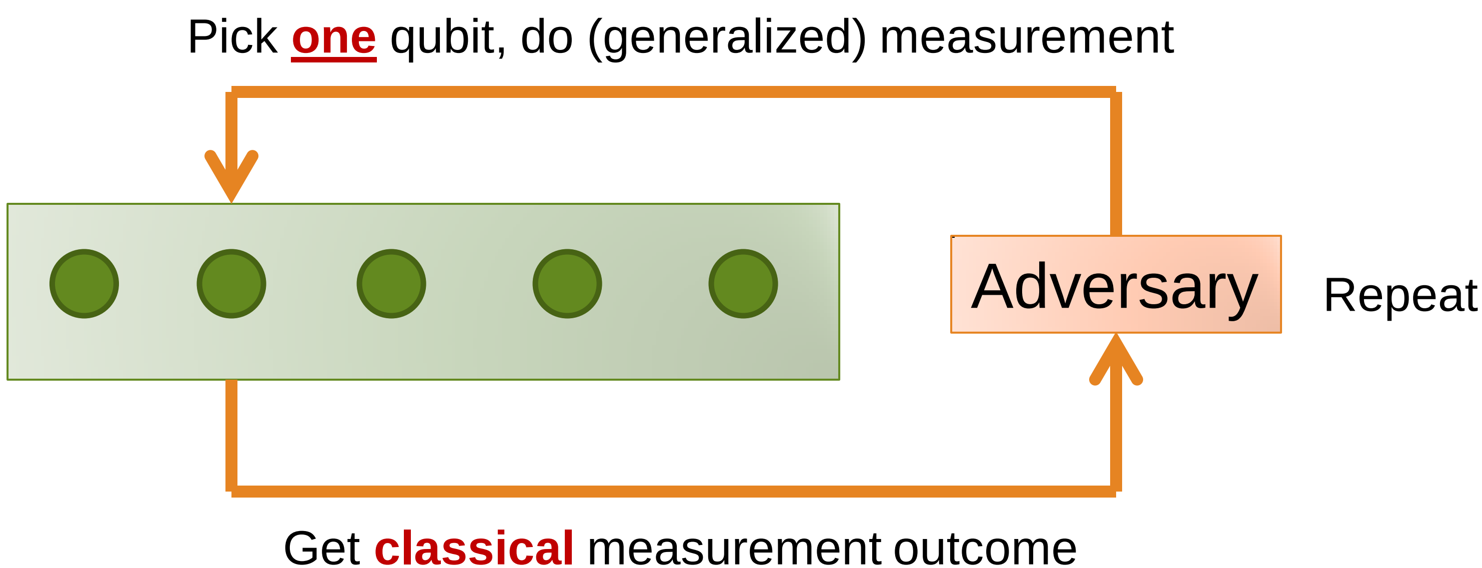
- Limits the power of an adversary
  - No-cloning theorem
  - Measurement disturbs the quantum state

- However…
  - Adversary can do entangled measurements on many qubits at once
  - Quantum bit-commitment, oblivious transfer are impossible

# Isolated Qubits Model

Real-world examples:
solid-state nuclear spins,
Si defects, NV centers

- Assume adversary cannot do entangling operations
  - Can only do adaptive single-qubit operations
  - "LOCC" = local operations and classical communication



# Related Work

- “Nonlocality without entanglement” [Bennett et al, 1999]
  - There exist quantum operations that are “one-way,” in a world where everyone is restricted to LOCC operations

- Quantum bit-commitment secure against k-local adversaries [Salvail, 1998]

- Quantum bounded storage model [Damgaard et al, 2005]

- Quantum tokens [Pastawski et al, 2012]

- Password-based identification [Bouman et al, 2012]

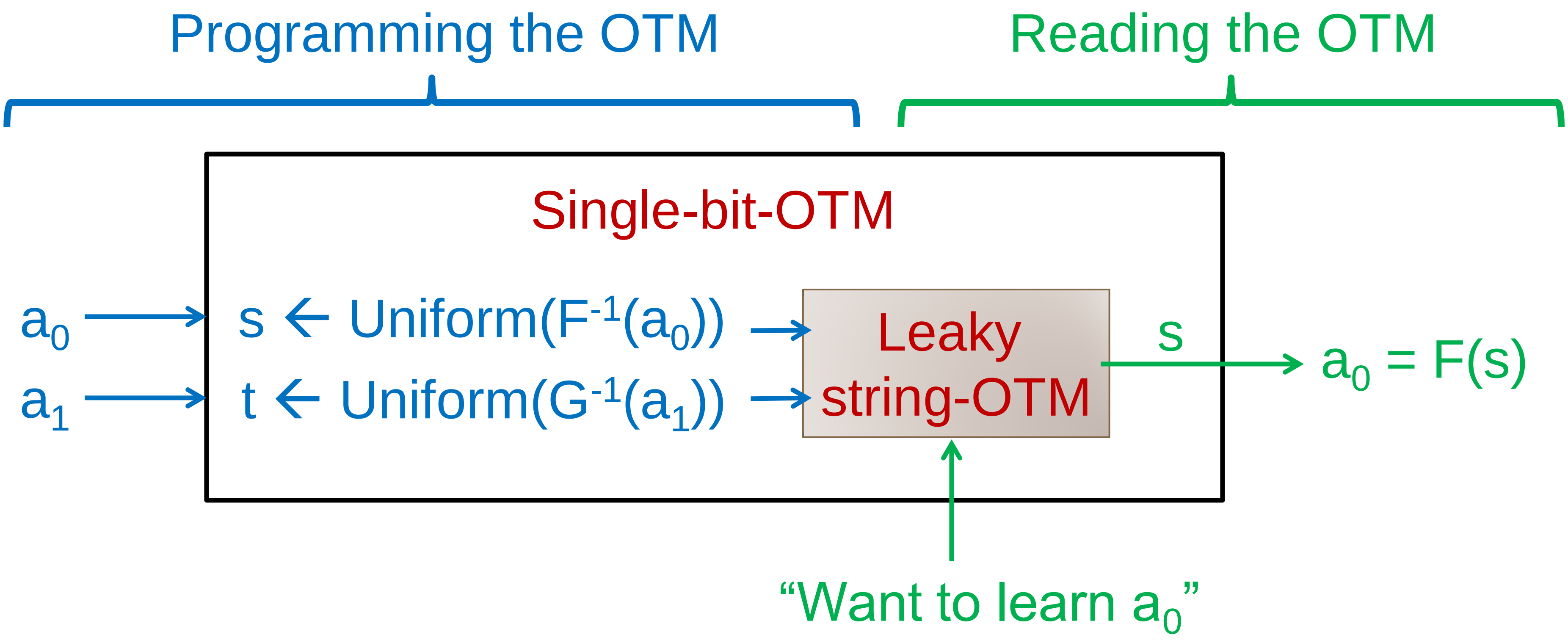
# Main Result

- Deterministic privacy amplification for one-time memories in the isolated qubits model
  - Given a leaky string-OTM
  - Construct a bit-OTM with exponentially-small leakage

Combine with construction of leaky string-OTM using isolated qubits [Liu, CRYPTO 2014]

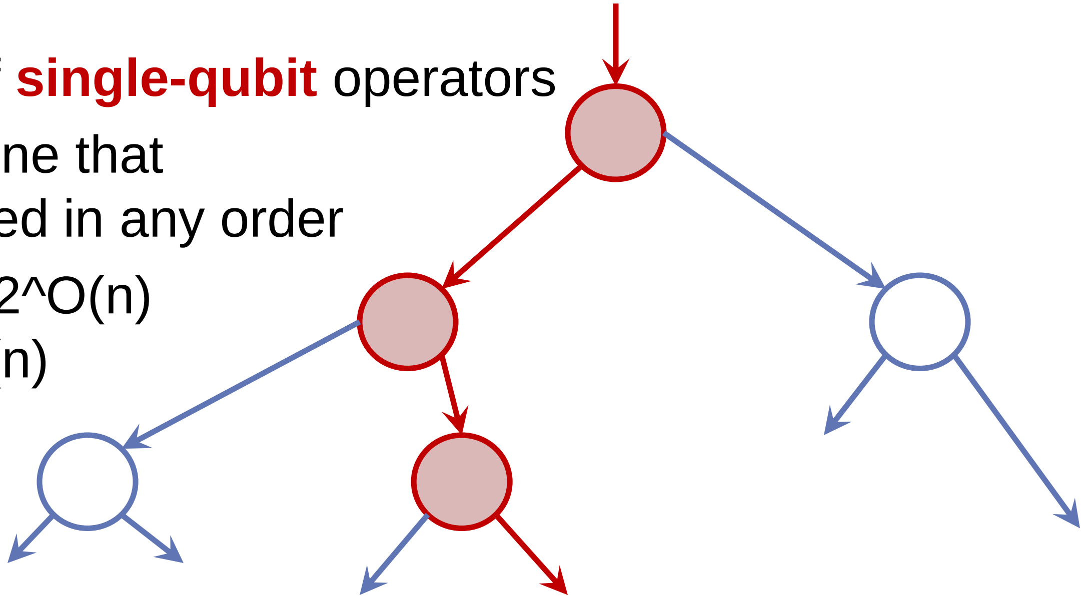
# Deterministic Privacy Amplification

- Use two r-wise independent hash functions
  F, G: $\{0,1\}^\ell \rightarrow \{0,1\}$ $r = poly(k)$



# Proof of Security

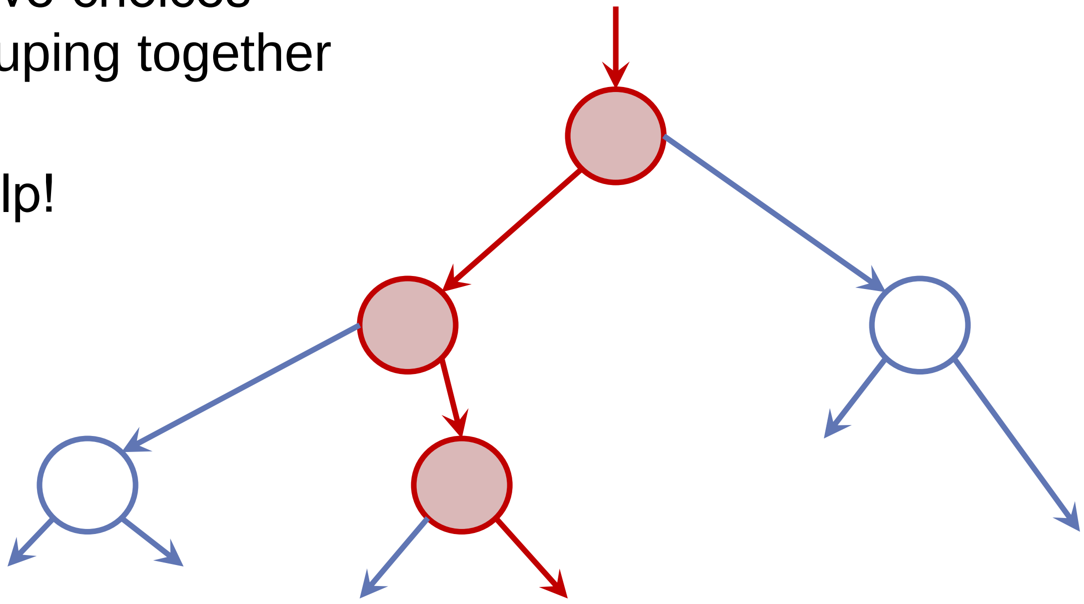
- **Key idea:**
- Don’t analyze LOCC adversaries (decision trees)
- Instead look at **POVM elements** (individual paths through decision trees)
  - Tensor products of **single-qubit** operators
  - Simpler: can imagine that qubits are measured in any order
  - Fewer in number: 2^O(n) rather than 2^2^O(n)



**POVM element**
**$M = M_1 \times M_2 \times \ldots \times M_n$**

# Proof of Security

- **Key idea:**
- Show that **every POVM element** (every path through every decision tree) is bad for the adversary
  - Adversary's adaptive choices = clever ways of grouping together POVM elements
  - But this doesn't help!



**POVM element**
$\mathbf{M = M_1 \times M_2 \times \ldots \times M_n}$

# Definition of Security: Leaky String-OTM

- Store two strings S and T, each ℓ bits long
  - Assume S,T are uniformly distributed
  - Ideal security goal: adversary can learn either S or T, but not both

- "Leaky" security:
  - For any LOCC adversary, have uncertainty about (S,T)
- $H^{\varepsilon}_{\infty}(S,T|Z) \geq (0.5 - \delta)\, \ell$
  - Z = adversary's measurement outcome
  - $\varepsilon \leq \exp(-\Omega(k))$

# Definition of Security: Single-Bit OTM

- Store two bits $A_0$ and $A_1$
- Every LOCC adversary learns at most one of $A_0$, $A_1$
  - There exists a binary random variable C, such that adversary doesn't learn $A_C$ (even if he learns $A_{1-C}$)
- $\Delta( (A_C,A_{1-C},C,Z), (U,A_{1-C},C,Z) ) \leq \varepsilon$

"Classical" security definition

  - $\Delta$ = statistical distance, $\varepsilon \leq \exp(-\Omega(k))$
  - Z = adversary's measurement outcome
  - U = independent uniformly random bit

# Definition of Security: Single-Bit OTM

- NB: our definition of security is mostly classical
  - Justification: isolated qubits can't become entangled with anything else
  - Caveat: security claim only applies after the adversary measures the qubits
  - Question about composability: what if the adversary defers some measurements until later?

# Proof of Security

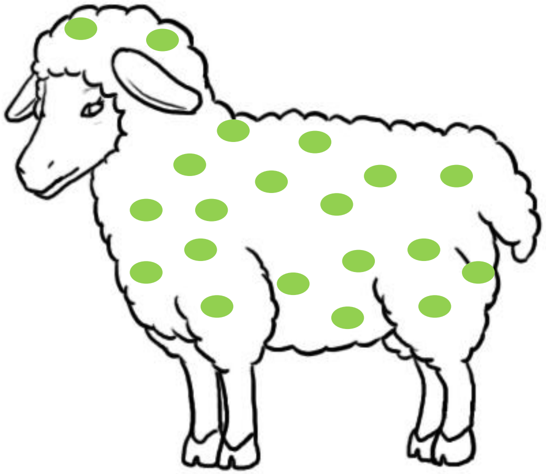
- Recall: hash functions F, G
  - Input bits $(a_0,a_1)$ expanded to strings (s,t) such that $F(s)=a_0$, $G(t)=a_1$
  - Let M be some measurement outcome (POVM element) that the adversary can observe
- First, prove security conditioned on a fixed M
  - For any fixed M, with high probability over F and G, the scheme is secure
  - $E_{ST}[\ (-1)^{A0+A1} \mid \text{Adv. gets outcome } M\ ]$
    $= \Sigma_{st}\ (-1)^{F(s)+G(t)}\ \Pr[\ S=s, T=t \mid \text{Adv. gets outcome } M\ ]$

# Proof of Security

- First, prove security conditioned on a fixed M
  - For any fixed M, with high probability over F and G, the scheme is secure
  - $E_{ST}[\, (-1)^{A0+A1} \mid \text{Adv. gets outcome } M \,]$
    $= \Sigma_{st}\, (-1)^{F(s)+G(t)}\, \Pr[\, S=s, T=t \mid \text{Adv. gets outcome } M \,]$
  - Use large-deviation bounds for sums and quadratic functions of (r-wise) independent rv's (Hoeffding, Hanson-Wright)
  - Security property of leaky OTM =>
    distribution of (S,T) has high entropy =>
    variance is small

# Proof of Security

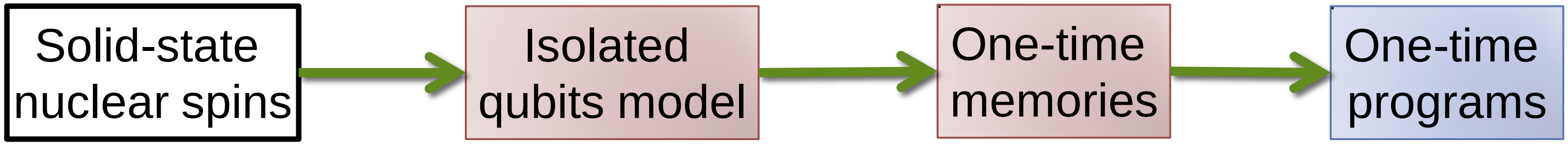
- Covering argument
  - ε-net for the set of all (tensor product) POVM elements
  - This has cardinality $\leq 2^{\text{poly}(k)}$



  - Union bound over all points M in the net
  - "Continuity argument": perturbation of M does not affect security much

  - So with high probability over F and G,
    for all M (simultaneously),
    the scheme is secure

# Outlook




- Experimental implementations?
  - Fault tolerance?
  - Adversaries who can perform noisy entangling gates?
- Composable security?
  - One-time programs? Other protocols?
  - Delayed measurements?
- Leakage resilience using quantum resources?