Structural Evaluation by Generalized Integral Property

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For Eurocrypt 2015 (@Sofia)

Overview

Integral Distinguisher (Daemen 97, Knudsen and Wagner 02)

- Exploit the set of chosen plaintexts that the XOR of the corresponding ciphertexts always becomes 0.
- Already have two methods to create distinguisher.

1. Integral property	mainly exploit the diffusion part.
2. Degree estimation	mainly exploit the confusion part.

Propose a new method to create integral distinguisher

- Propose a new property called “Division Property.”
- This property can exploit both confusion and diffusion.

Summarization of Structural Evaluation

Innovative R&D by NTT

Structural Evaluation (Generic Attack)

Exploit only the feature of the network.
It is applicable to large classes of block ciphers.

Add some natural assumptions.

(ℓ, d)-Feistel ℓ-bit F-function with degree d.
(ℓ, d, m)-SPN m concatenating ℓ-bit S-boxes with degree d.

Structure	F-function	vulnerable rounds		Example
(16,2)-Feistel	Non-bijection	9R	2^{31} CPs	Simon 32
(64,2)-Feistel	Non-bijection	14R	2^{127} CPs	Simon 128
(4,3,32)-SPN	-	7R	2^{124} CPs	Serpent
(5,2,320)-SPN	-	15R	2^{1595} CPs	Keccak-f[1600]

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Outline

1. **Background**
   - **What's Integral Attack?**
   - **Higher-Order Differential?**
2. Division Property
3. Vectorial and Collective Division Properties
4. Application to Feistel Cipher
5. Conclusion

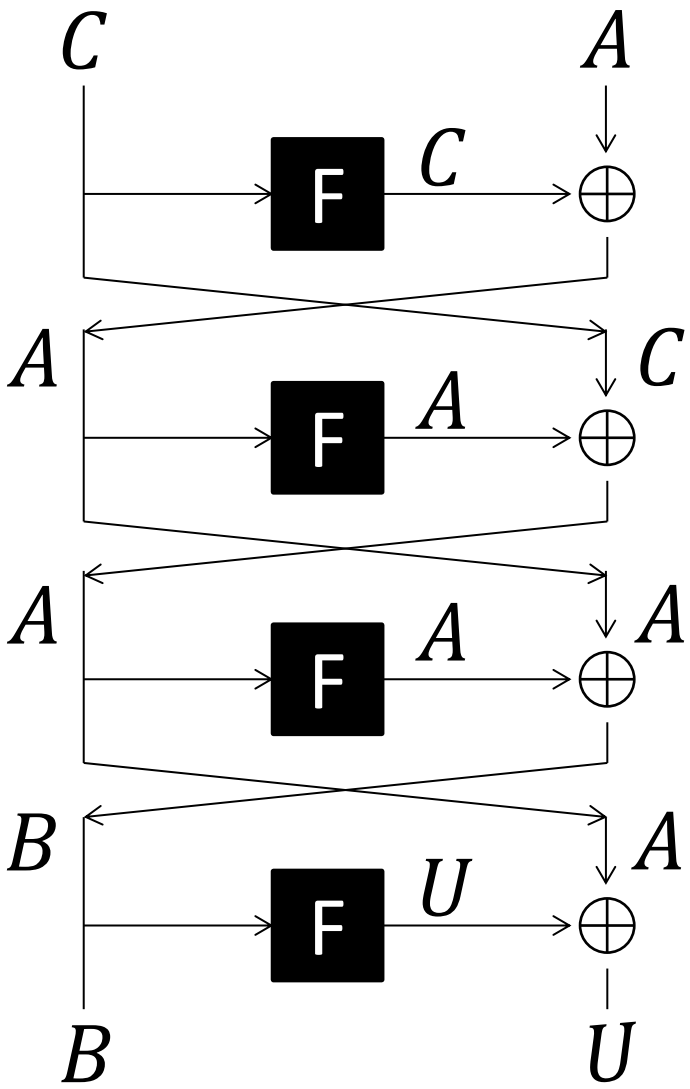
What's Integral Attack?

- Classical differential attack observes the propagation of differences between two values.
- Integral attack treats the propagation of the sum of many values.
- History.
  - It has several names,
    - Higher-Order Differential attack [Lai, 94]
    - Square attack [Daemen et al., FSE97],
    - Saturation attack [Lucks, FSE01],
    - Integral attack [Knudsen, FSE02].
  - We use "integral attack" in my talk.

Integral Property

- When we search for the integral distinguisher, we often use the propagation of the integral property.
- Integral property.
  - A : Every value appears the same number.
  - B : The XOR of all texts is 0.
  - C : The value is fixed to a constant.
  - U : The set does not have useful property.

Propagation of Integral Property



Higher-Order Differential?

- Outline is the same as that of integral attack.
- Exploit the algebraic degree of a function.
  - Choose a set of chosen plaintexts whose (d+1) bits of the input are active.
  - If the algebraic degree is at most d, the sum of the output is always 0.
- How estimate the accurate degree?
  - It is very difficult to estimate the accurate algebraic degree.

Outline

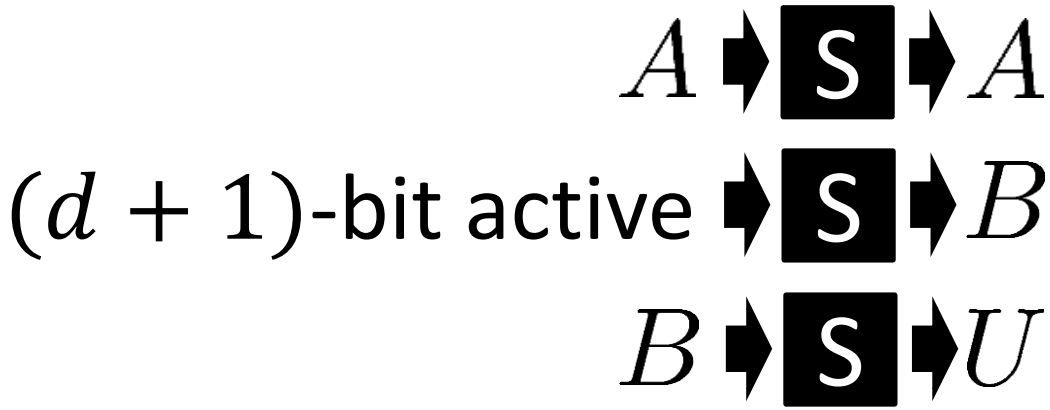
1. Background
2. **Division Property**
   - **Concept**
   - **How to evaluate multi-set by using π_u**
   - **Redefinition of integral property**
   - **Propagation characteristic**
3. Vectorial and Collective Division Properties
4. Application to Feistel Cipher
5. Conclusion

Concept

- Intuition.
  - Integral property.
    - It treats S-box as a black box.
    - It does not clearly exploit the degree of S-box.
    - It mainly exploits the **diffusion** of block cipher.
  - Degree estimation.
    - It treats the algebraic degree of S-box.
    - It is very difficult to estimate accurate degree.
    - It mainly exploits the **confusion** of block cipher.
- Motivation.
  - How can we exploit both confusion and diffusion?

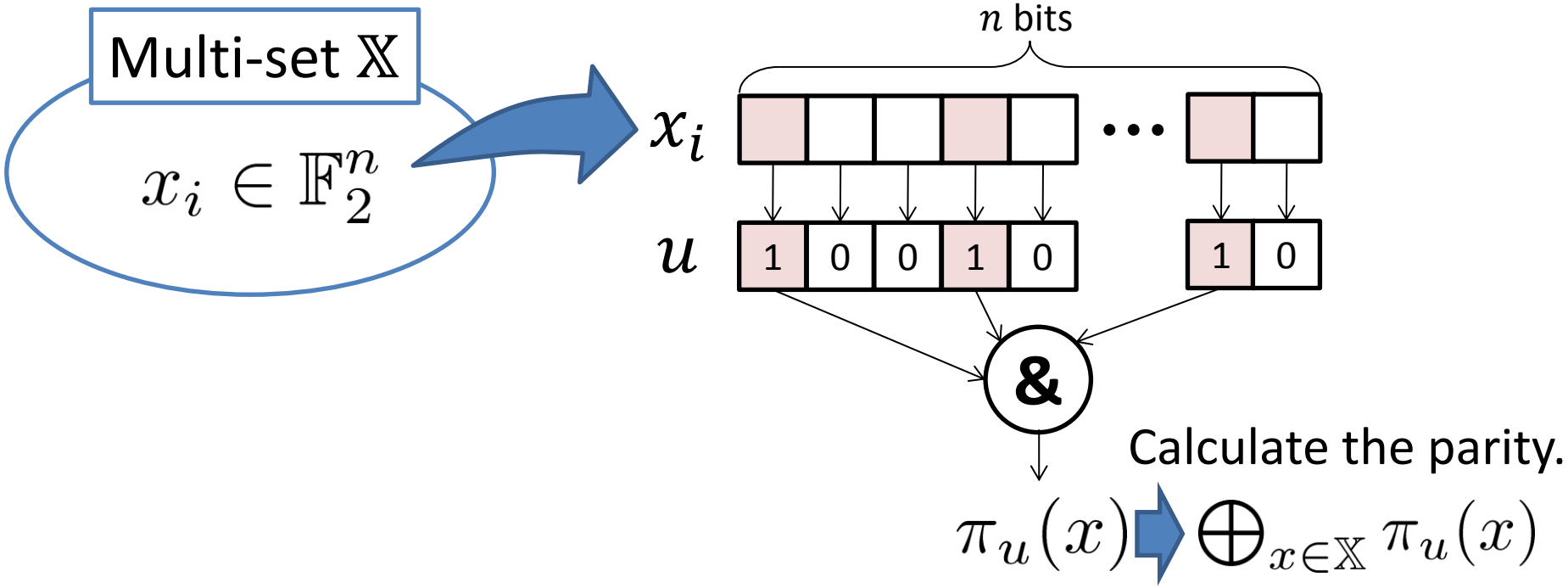
How do we exploit both properties?

- Assume that the degree of S-box is at most d.



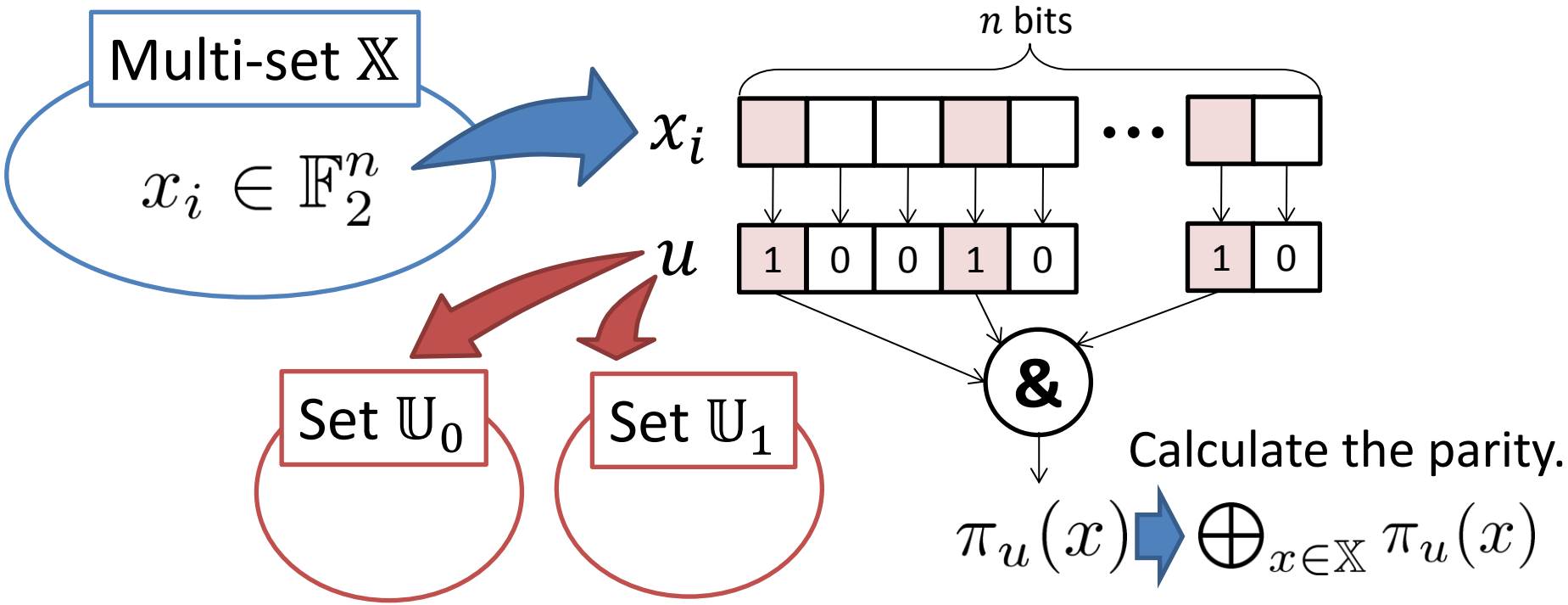
- Believe that some useful properties are hidden between "A" and "B".
- To exploit the useful property, we redefine each property by the same statement, and reveal the useful property.

Bit Product Function π_u



- Choose bits that corresponding bits of u are 1, and output the AND.
- Evaluate the parity of $\pi_u(x)$ for all elements.

Bit Product Function π_u



- Evaluate whether the parity becomes 0 or 1.
- If the parity is 0, the value of u belongs to $\mathbb{U}_0$.
- If the parity is 1, the value of u belongs to $\mathbb{U}_1$.

Example

- Assume that elements of $\mathbb{X}$ take 3-bit value.

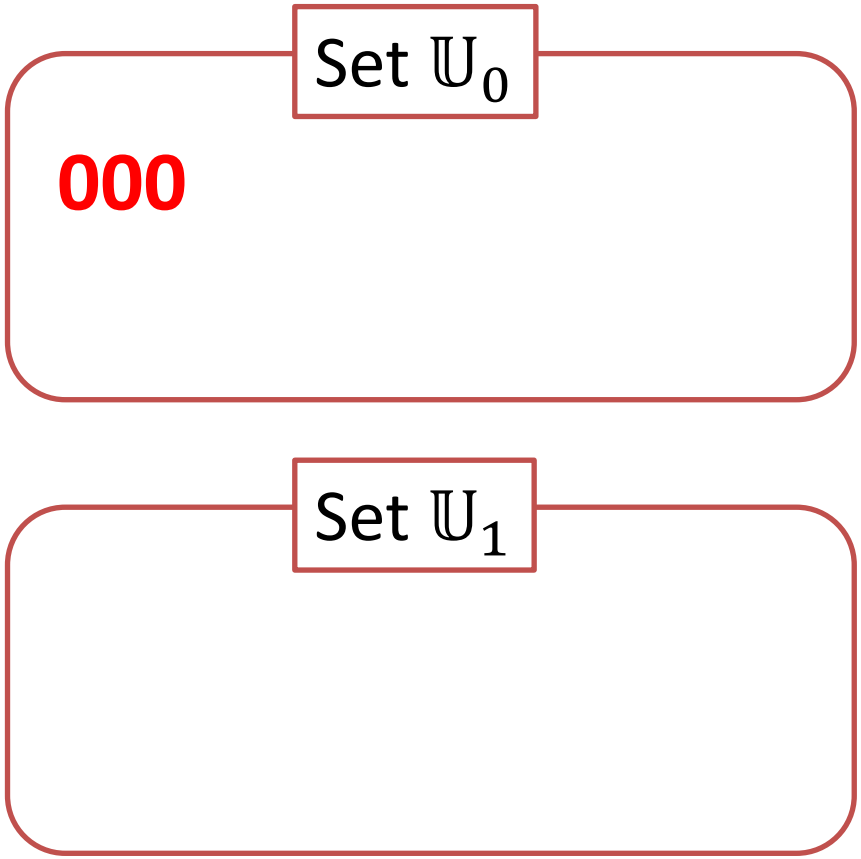
Set $\mathbb{X}$	$\pi_u(x)$
000	
001	
010	
011	
100	
101	
110	
111	
Parity	

- All values appear only once in the set $\mathbb{X}$.
- The set $\mathbb{X}$ has the integral property A.
- Calculate the parity of $\pi_u(x)$, and evaluate whether u belongs to $\mathbb{U}_0$ or $\mathbb{U}_1$.

Example

When u of π_u is equal to **000**.

Set $\mathbb{X}$	$\pi_{000}(x)$
000	0
001	0
010	0
011	0
100	0
101	0
110	0
111	0
Parity	**0**



Example

When u of π_u is equal to **001**.

Set $\mathbb{X}$	$\pi_{001}(x)$
000	0
001	1
010	0
011	1
100	0
101	1
110	0
111	1
Parity	**0**

Set $\mathbb{U}_0$

000, **001**

Set $\mathbb{U}_1$

Example

When u of π_u is equal to **111**.

Set $\mathbb{X}$	$\pi_{111}(x)$
000	0
001	0
010	0
011	0
100	0
101	0
110	0
111	1
Parity	**1**

Set $\mathbb{U}_0$

000, 001, 010, 011,
100, 101, 110

Set $\mathbb{U}_1$

111

Another Example

Similarly, divide the set of u into $\mathbb{U}_0$ and $\mathbb{U}_1$.

Set $\mathbb{X}$	$\pi_{110}(x)$
000	0
001	0
001	0
011	0
100	0
110	1
110	1
111	1
Parity	**1**

Set $\mathbb{U}_0$

000, 001, 010, 011, 100,

Set $\mathbb{U}_1$

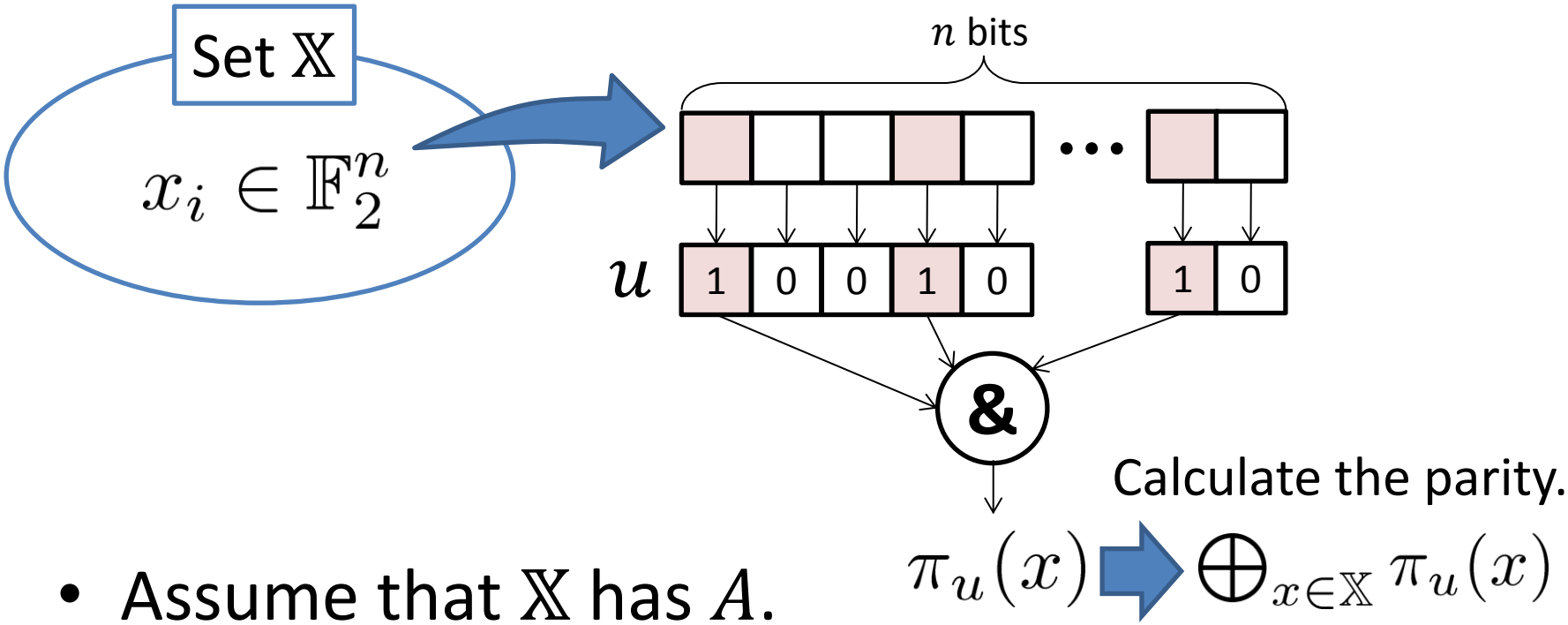
101, 110, 111

How to Evaluate Multi-Set with Unknown Elements

- In previous simple example, we know all elements of the multi-set.
- However, attackers can't know elements, but they can only know the property of the multi-set.
  - Every value appears the same number.
  - The XOR of all values becomes 0.
- We use a set $\mathbb{U}_?$ instead of $\mathbb{U}_1$.
  - If attackers can know that the parity is always 0, u belongs to $\mathbb{U}_0$.
  - Otherwise, u belongs to $\mathbb{U}_?$.

Redefinition of ALL

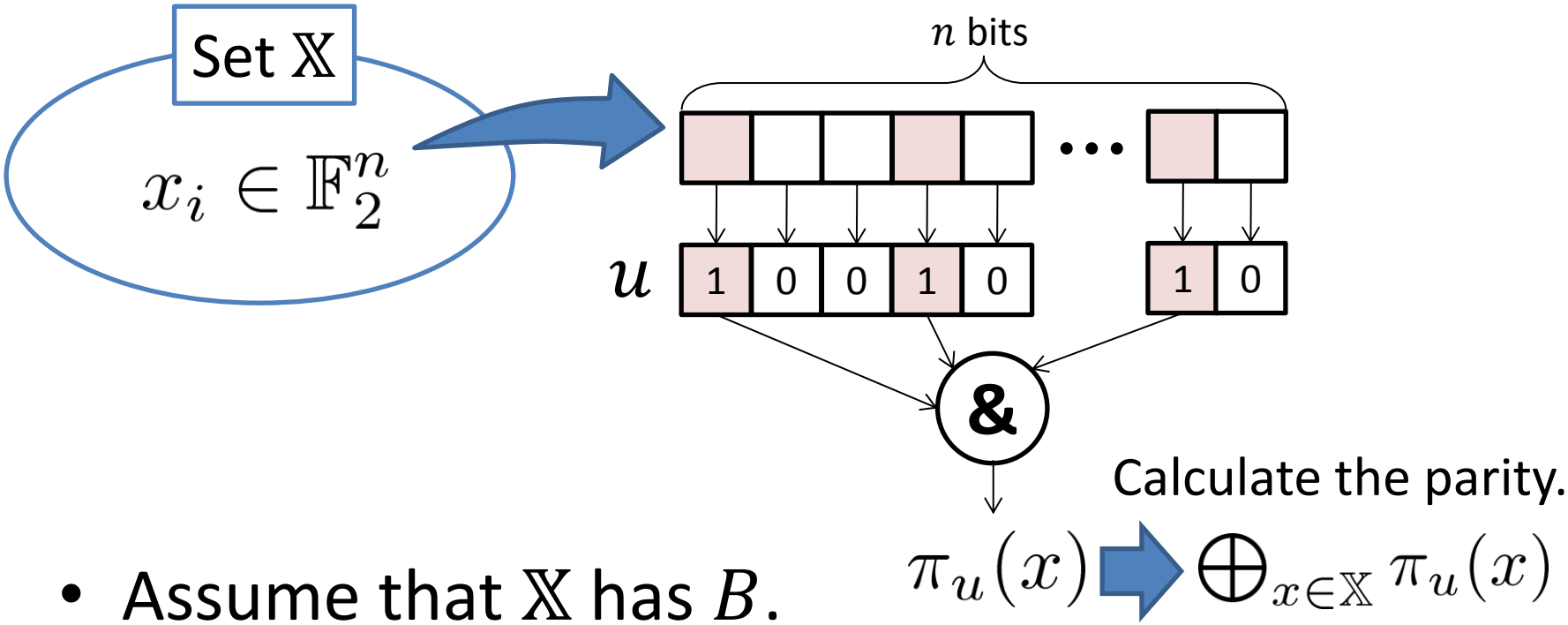
Evaluate the set by a bit product function π_u.



- Assume that $\mathbb{X}$ has A.
  - If $\boldsymbol{wt(u) < n}$, the parity is always **0**.
  - If $\boldsymbol{wt(u) = n}$, the parity becomes **unknown**.

Redefinition of BALANCE

Evaluate the set by a bit product function π_u.

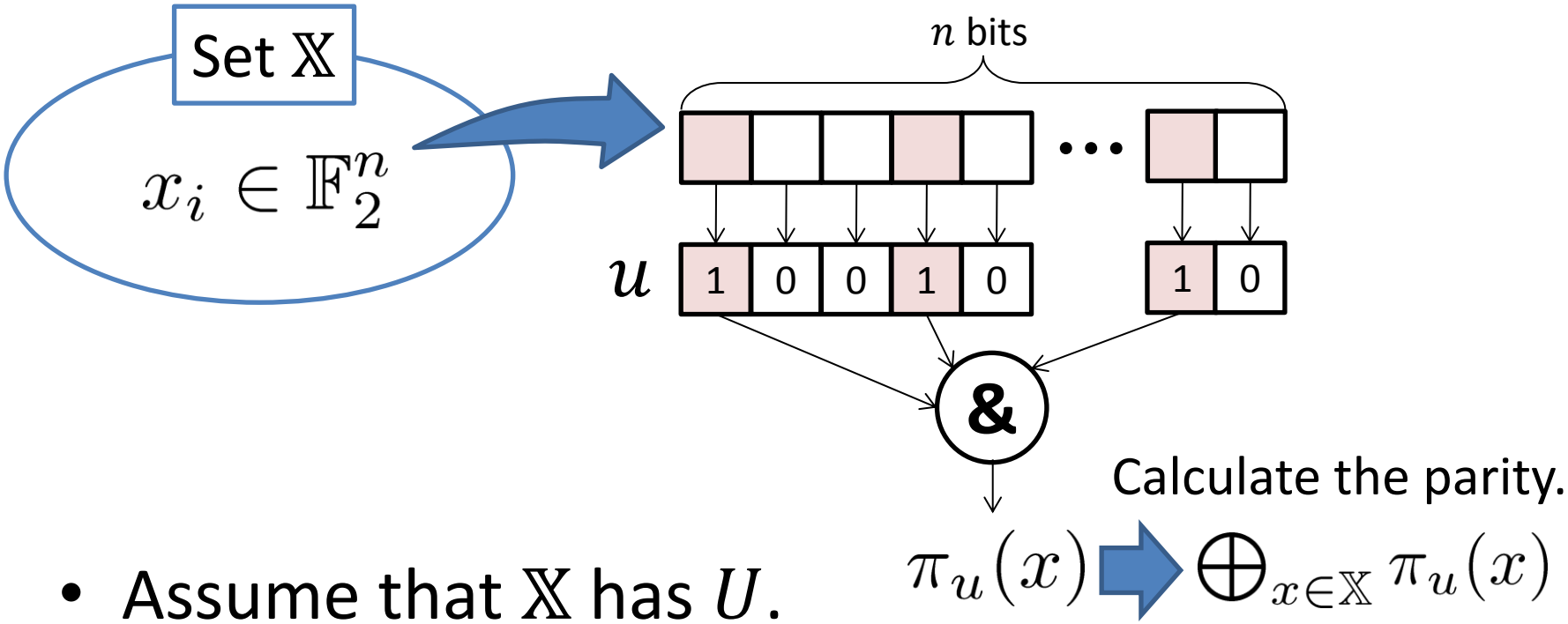


- Assume that $\mathbb{X}$ has B.
  - If $\boldsymbol{wt(u) < 2}$, the parity is always **0**.
  - If $\boldsymbol{wt(u) \geq 2}$, the parity becomes **unknown**.

Redefinition of UNKNOWN

Innovative R&D by NTT

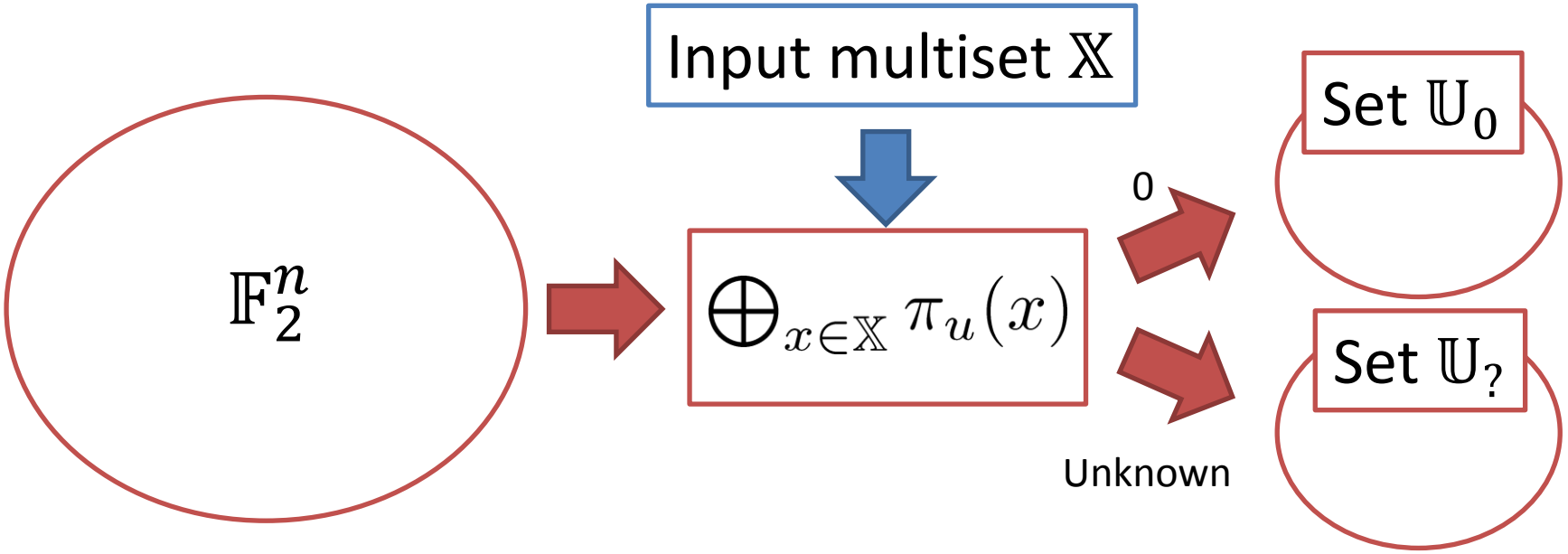
Evaluate the set by a bit product function π_u.



- Assume that $\mathbb{X}$ has U.
  - If $\boldsymbol{wt(u) < 1}$, the parity is always **0**.
  - If $\boldsymbol{wt(u) \geq 1}$, the parity becomes **unknown**.

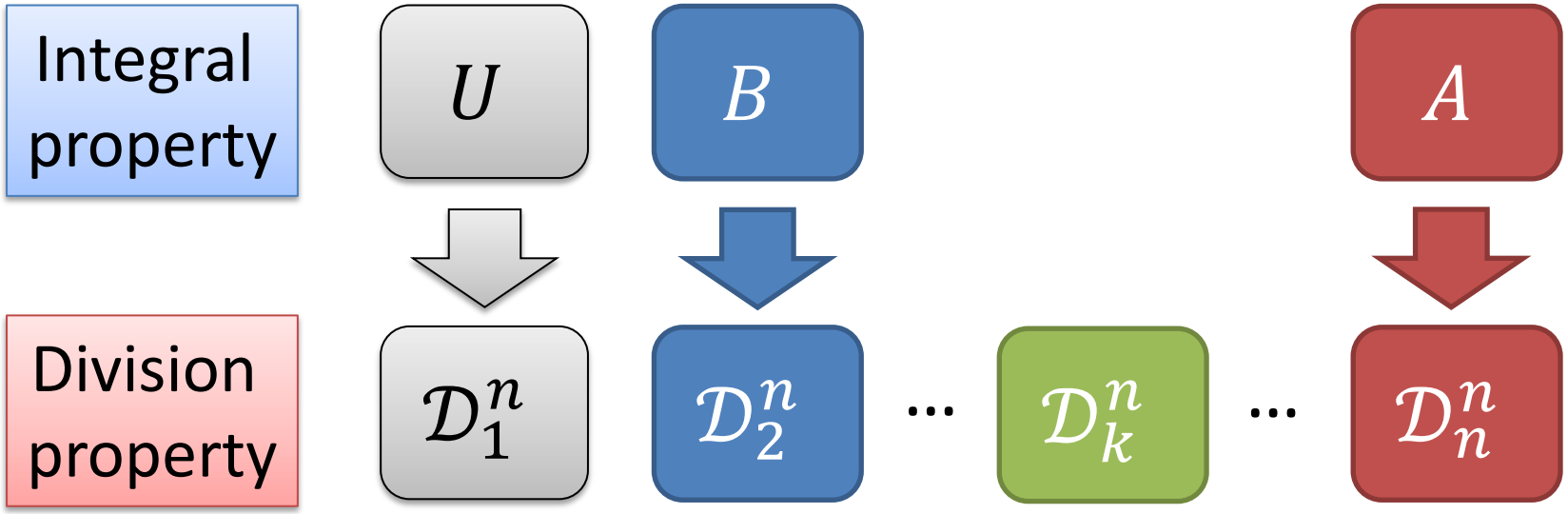
Division Property

- Division property $\mathcal{D}_k^n$
  - All n-bit values are divided into elements of the set $\mathbb{U}_0$ and those of the set $\mathbb{U}_?$.



The set of u whose $wt(u) < k$ belongs the set $\mathbb{U}_0$.

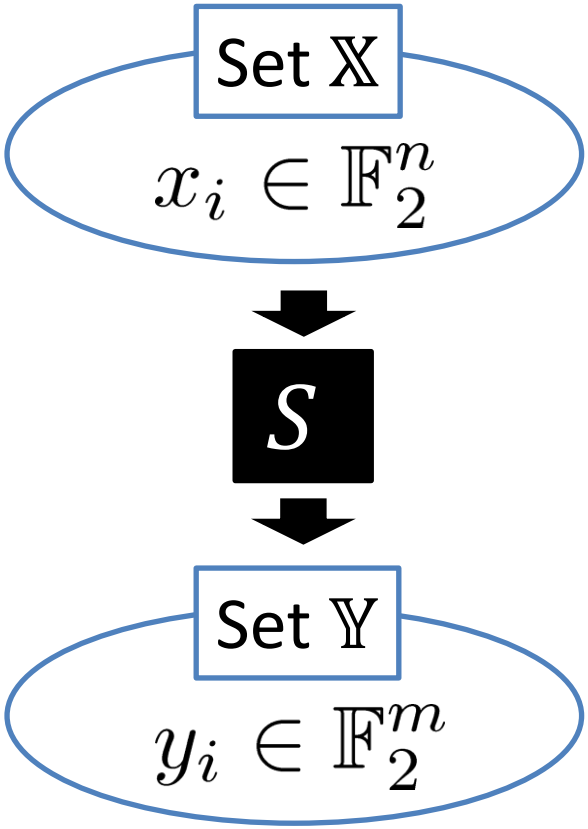
Relation between Integral and Division



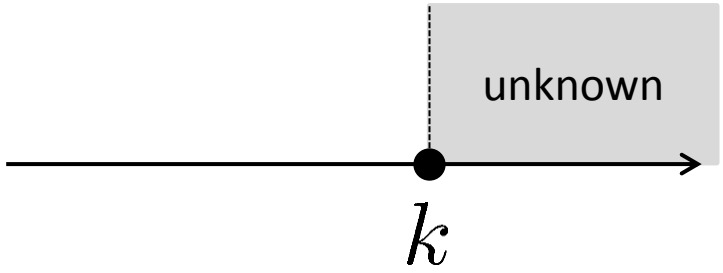
- The division property can treat the hidden property between A and B by using from $\mathcal{D}_3^n$ to $\mathcal{D}_{n-1}^n$.

Propagation

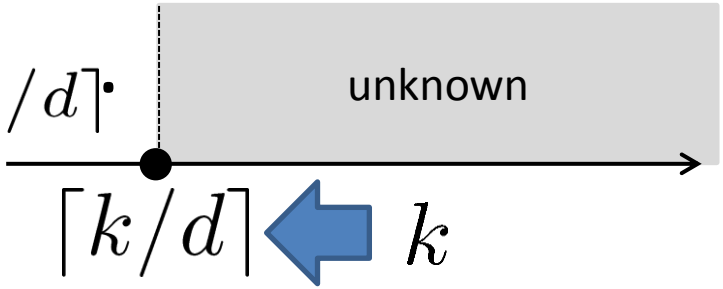
Let S be a functions from n bits to m bits, and the algebraic degree is d.



$\mathbb{X}$ has $\mathcal{D}_k^n$.

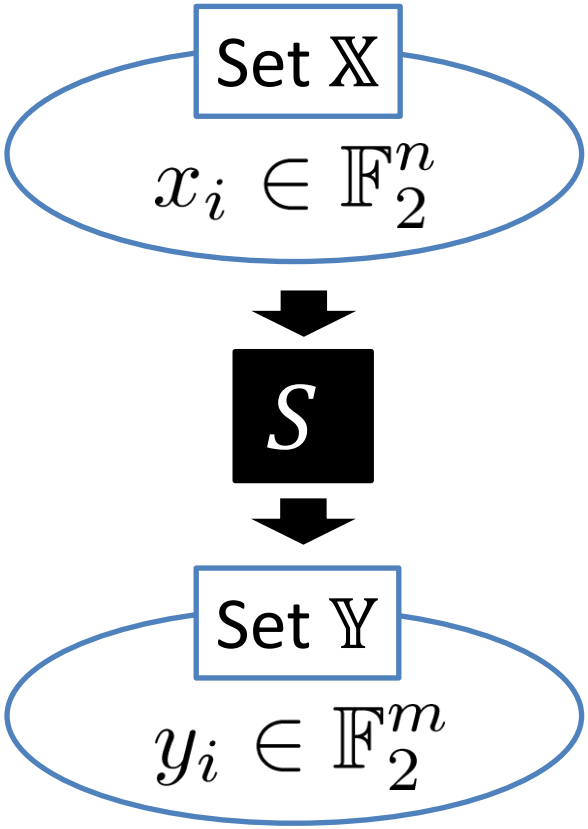


$\mathbb{Y}$ has $\mathcal{D}_{\lceil k/d \rceil}^m$.

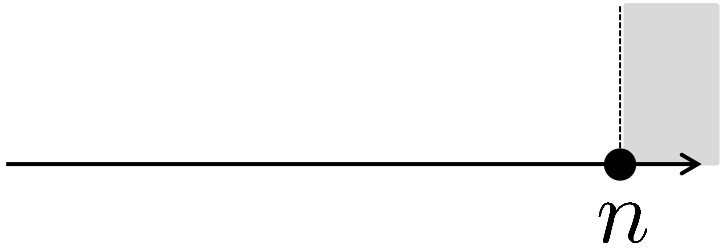


Propagation – Special Case

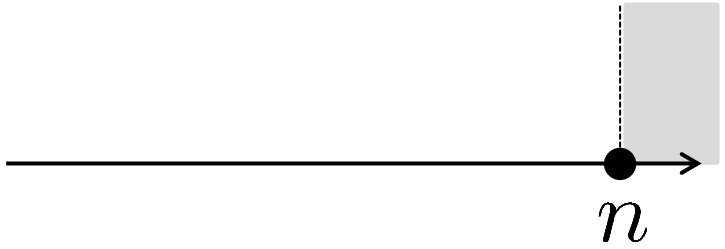
Moreover, if the function S is bijective,



$\mathbb{X}$ has $\mathcal{D}_n^n$.



$\mathbb{Y}$ has $\mathcal{D}_n^n$.

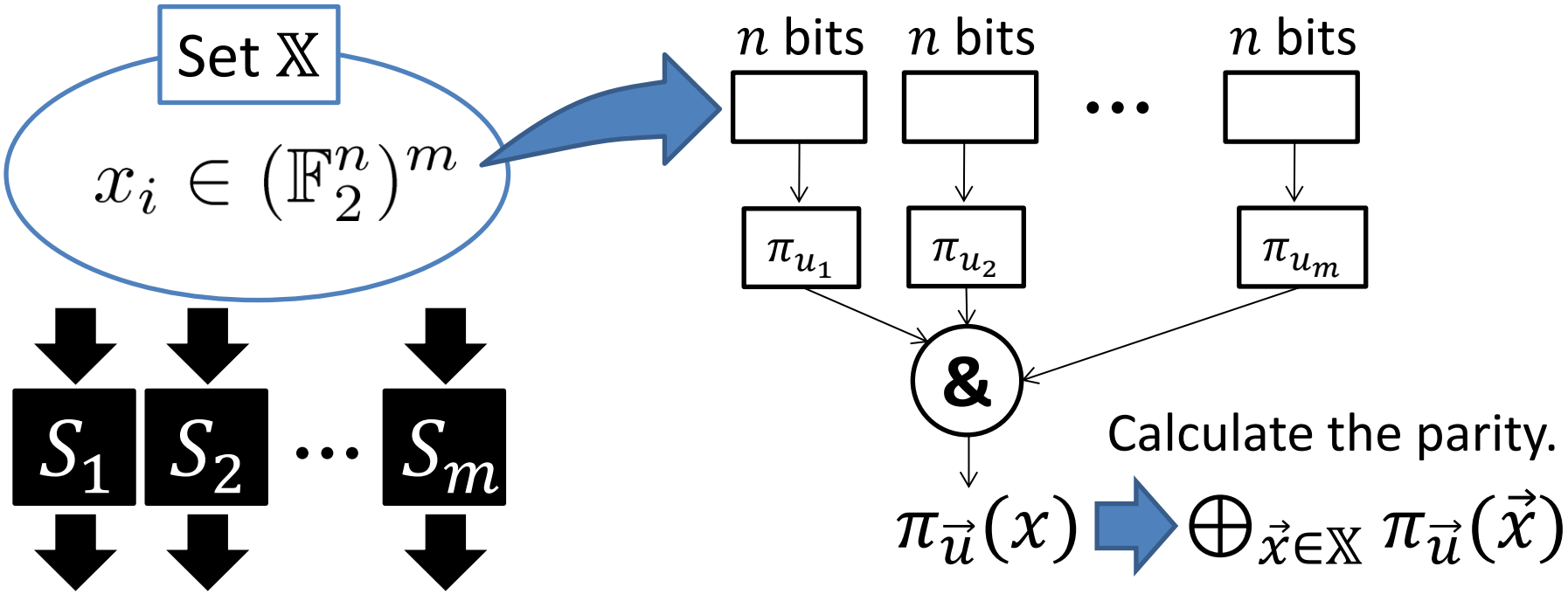


Outline

1. Background
2. Division Property
3. **Vectorial and Collective Division Properties**
   - **Definition of vectorial and collective ones**
   - **Simple example for vectorial division property**
   - **Simple example for collective division property**
4. Application to Feistel Cipher
5. Conclusion

Vectorial and collective Division Property

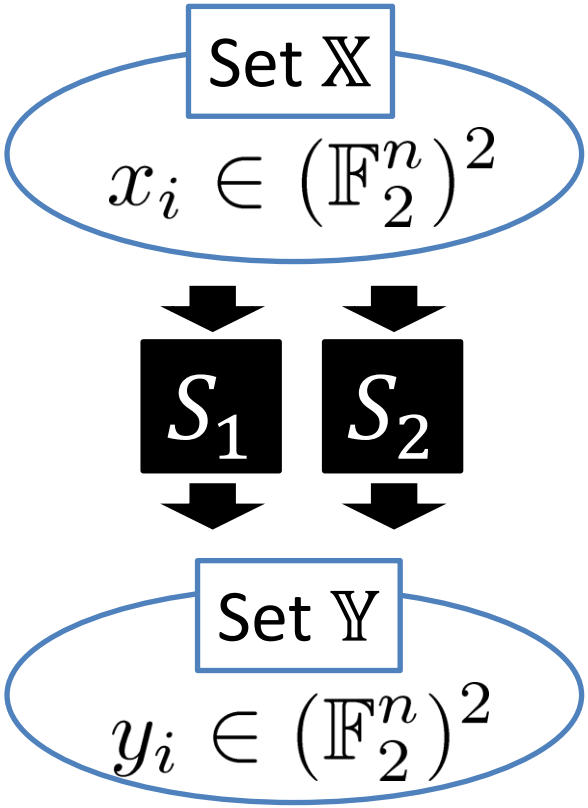
Evaluate the set by the bit product function $\pi_{\vec{u}}$.



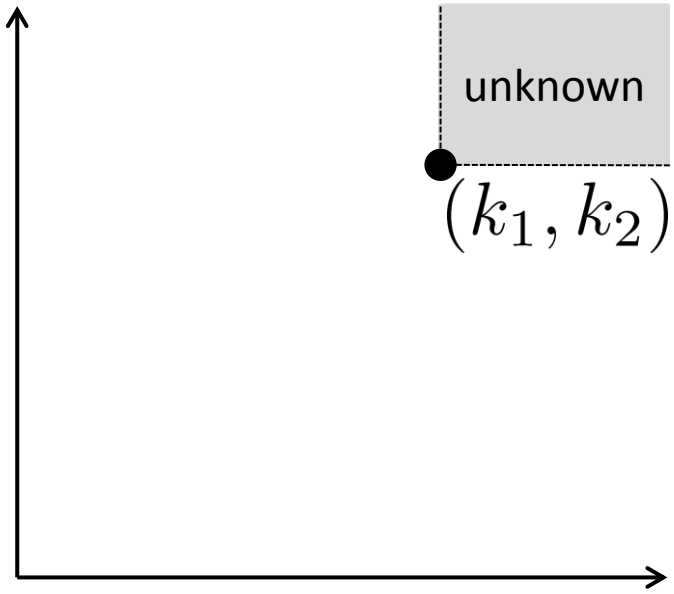
Vectorial and collective Division Properties are a little complicated, so we only explain them by two dimensions.

Vectorial Division Property

The parity becomes unknown when $\pi_{\vec{u}}$ is applied such that $u_1 \geq k_1$ and $u_2 \geq k_2$.

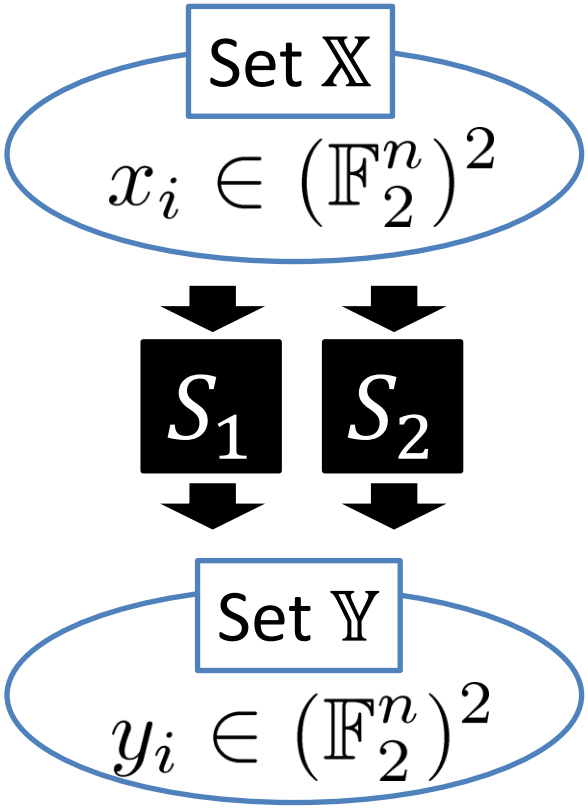


$\mathbb{X}$ has $\mathcal{D}^{n,2}_{[k_1,k_2]}$.

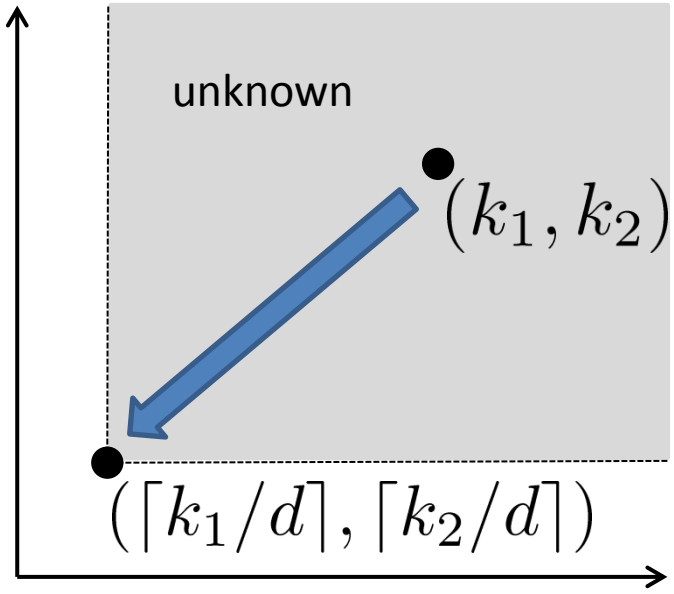


Propagation of Vectorial Division Property

When 2 S-boxes are applied, we evaluate the propagation of each element of vector $[k_1, k_2]$.

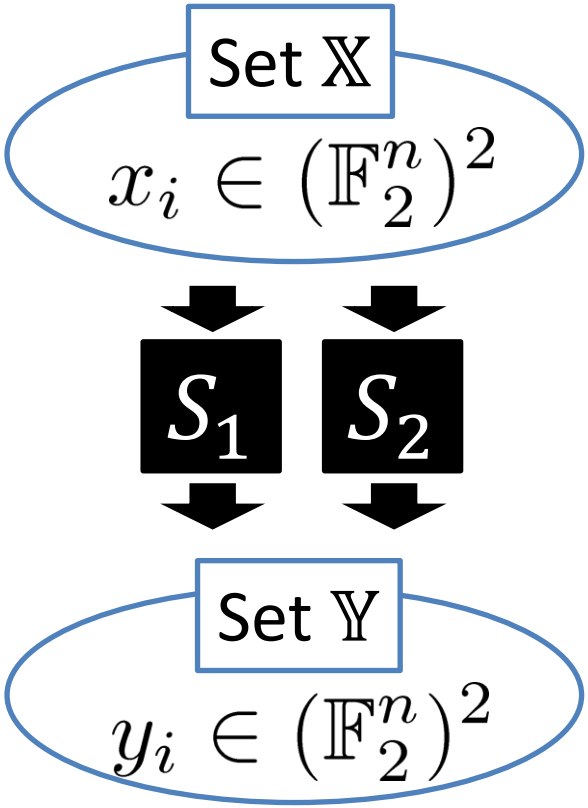


$\mathbb{Y}$ has $\mathcal{D}^{n,2}_{[\lceil \frac{k_1}{d} \rceil, \lceil \frac{k_2}{d} \rceil]}$.

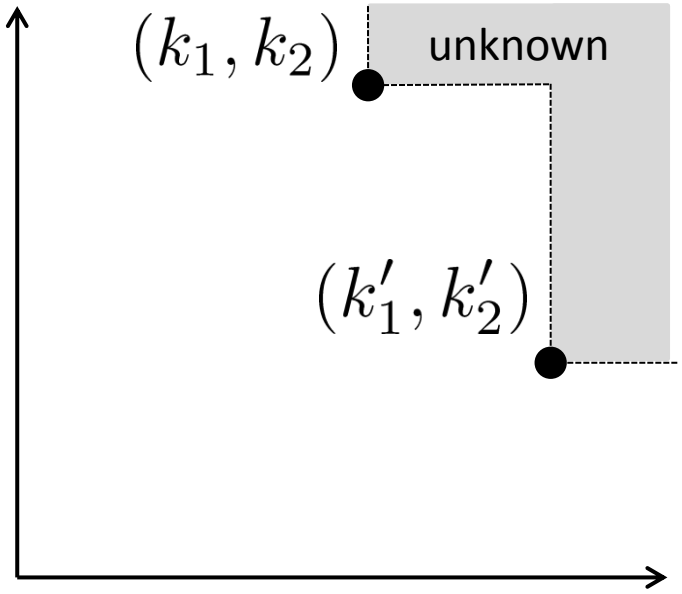


Collective Division Property

When the unknown has to be represented by 2 vectors, we use collective division property.

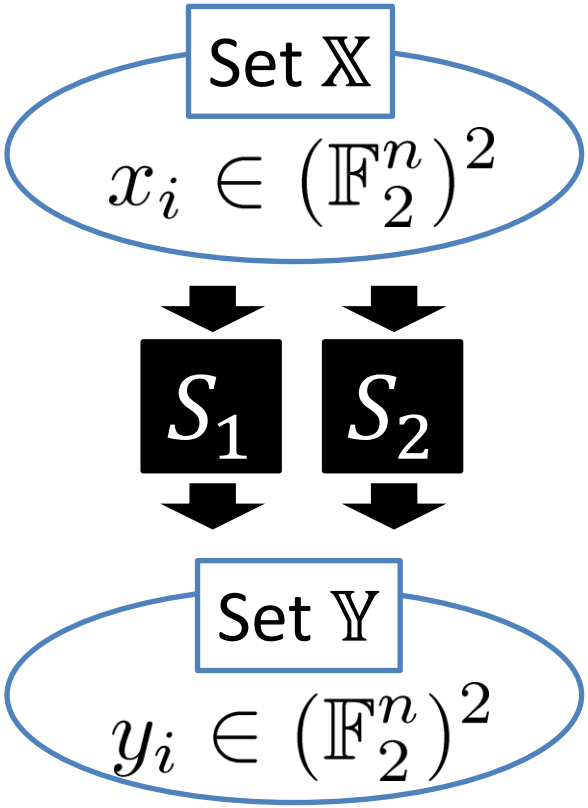


$\mathbb{X}$ has $\mathcal{D}^{n,2}_{[k_1,k_2],[k_1',k_2']}$.

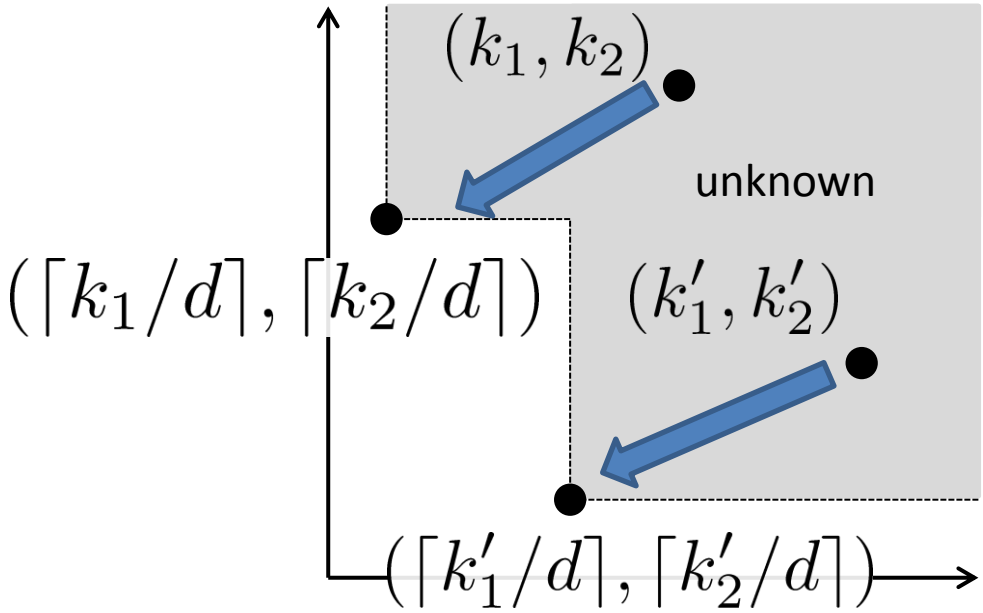


Propagation of Collective Division Property

We evaluate the propagation of the vectorial division property every vector.



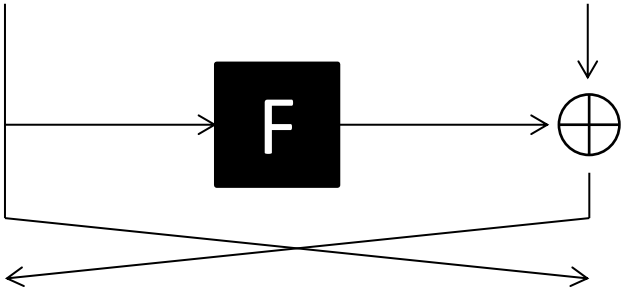
$\mathbb{Y}$ has $\mathcal{D}^{n,2}_{[\lceil \frac{k_1}{d} \rceil, \lceil \frac{k_2}{d} \rceil], [\lceil \frac{k'_1}{d} \rceil, \lceil \frac{k'_2}{d} \rceil]}$.



Outline

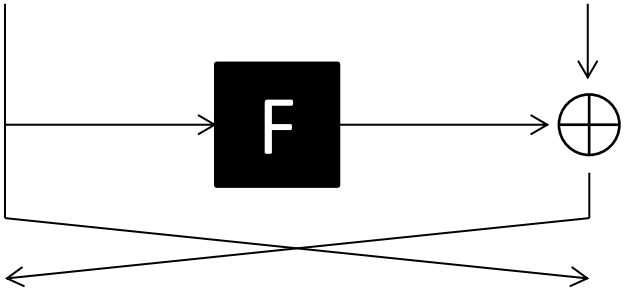
1. Background
2. Division Property
3. Vectorial and Collective Division Properties
4. **Application to Feistel Cipher**
   - **Definition of (ℓ, d)-Feistel**
   - **Propagation characteristic against (ℓ, d)-Feistel**
   - **Integral distinguisher on $(\ell, \ell - 1)$-Feistel**
   - **Integral distinguisher on $(\ell, 2)$-Feistel**
5. Conclusion

Feistel Cipher



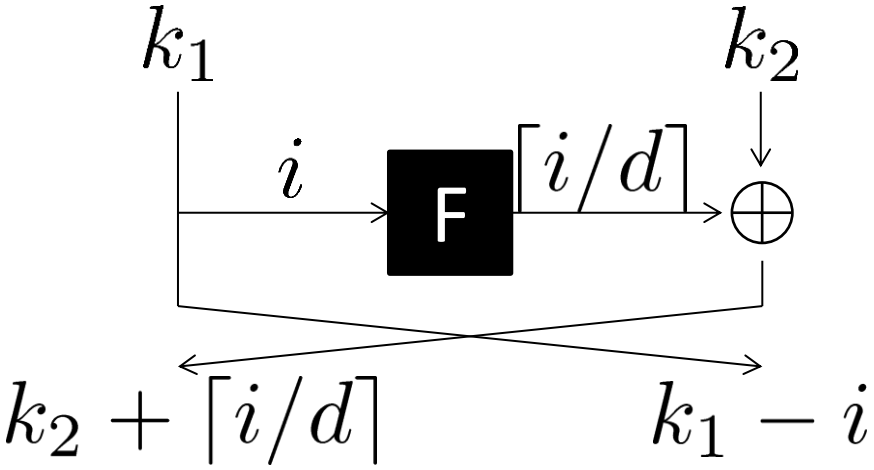
- Famous structure to design block cipher.
- It is widely applied, e.g., DES and Camellia.
- Many results have been known about the structural evaluation.
- Integral attack is strong attack, but previous approach is not effective if F is non-bijective.

(ℓ, d)-Feistel



- The bit length of F-function is ℓ bits.
- The algebraic degree of F-function is at most d.
- The block length is 2ℓ bits.

Propagation on (ℓ, d)-Feistel



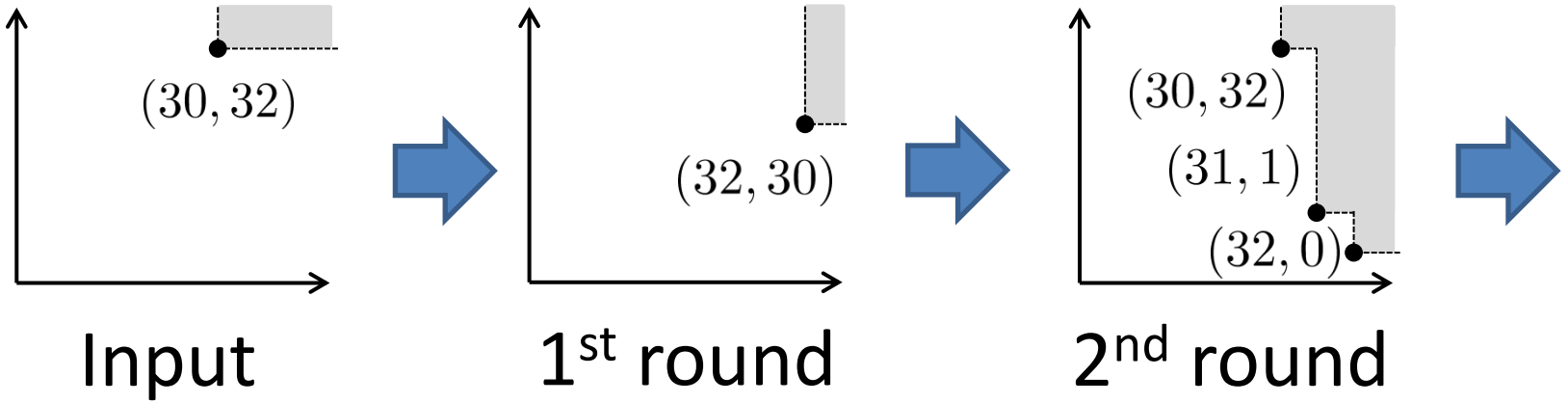
- The division property propagates as follows.

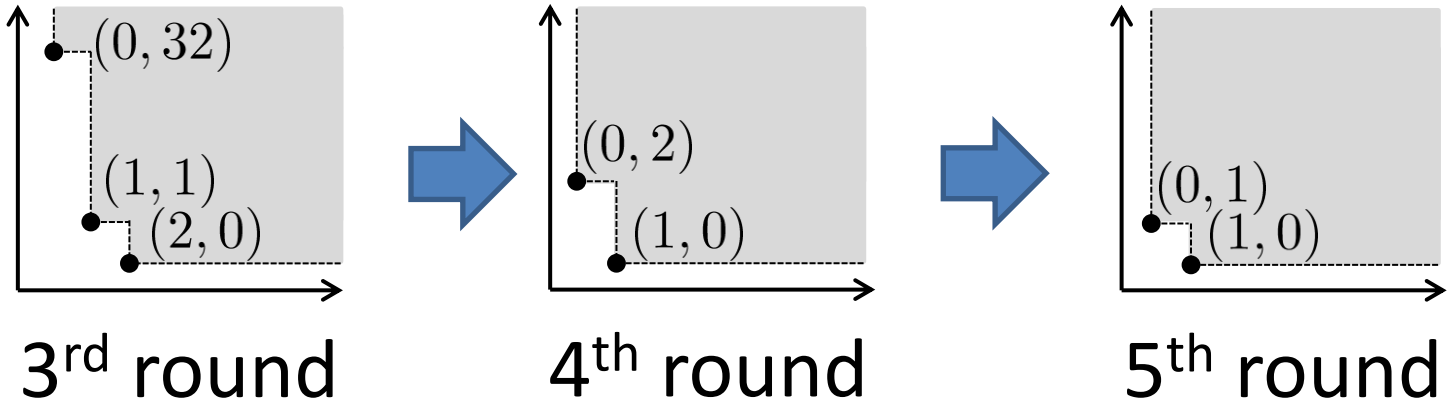
$$\mathcal{D}^{\ell,2}_{[k_1,k_2]} \Rightarrow \mathcal{D}^{\ell,2}_{[k_2,k_1],[k_2+1,k_1-d],\ldots,[k_2+q,k_1-qd]}$$

- Here $k_1 - qd \geq 0$ and $k_2 + q \leq \ell$.

Distinguisher against (32,31)-Feistel

Prepare 2^{62} CPs with left 30 and right 32 bits are active





Distinguisher against $(\ell, 2)$-Feistel

Target [Application]	$\log_2(\#texts)$								Method
	r=6	r=7	r=8	r=9	r=10	r=11	r=12	r=13	
(16,2)-Feistel [Simon 32]	17	25	29	31	-	-	-	-	our
	-	-	-	-	-	-	-	-	degree
(24,2)-Feistel [Simon 48]	17	29	39	44	46	47	-	-	our
	17	-	-	-	-	-	-	-	degree
(32,2)-Feistel [Simon 64]	17	33	49	57	61	63	-	-	our
	17	-	-	-	-	-	-	-	degree
(48,2)-Feistel [Simon 96]	17	33	57	77	87	92	94	95	our
	17	33	-	-	-	-	-	-	degree
(64,2)-Feistel [Simon 128]	17	33	65	97	113	121	125	127	our
	17	33	-	-	-	-	-	-	degree

15-round integral distinguisher against Simon32 were already known, but it was confirmed by experiments and the existence is not proven.

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5. **Conclusion**

Conclusion

- Propose the division property.
  - It is a generalization of the integral property such that it can also exploit the algebraic degree.
- Application
  - I showed structural evaluations for Feistel and SPN by adding some realistic assumptions.
  - Toward to dedicated cryptanalysis, we also show integral distinguishers of AES-Like ciphers.
- Future works
  - I expect the division property is useful to construct integral characteristic of specific block ciphers.
  - I think that all integral distinguishers are looked again by using the division property.