Disjunctions for Hash Proof Systems: New Constructions and Applications

Michel Abdalla, *Fabrice Benhamouda*, and David Pointcheval

École Normale Supérieure, CNRS, INRIA, PSL, Paris, France

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Introduction

Hash Proof System / Smooth Projective Hash Function (SPHF)

- Introduced by Cramer and Shoup [CS02]
  - → Implicit designated-verifier proofs,
  - → IND-CCA encryption scheme [CS98],
- Applications:
  - Password-Authenticated Key Exchange (PAKE) [KOY01, GL03, KV11],
  - Oblivious Transfer [Kal05, ABB+13]
  - Relatively-Sound / Dual-System NIZK [JR12, JR14a]
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  - Witness Encryption [GGSW13]

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DDH, Paillier/DCR, QR, ...

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More Complex Languages

Introduction

Languages

Over a cyclic group $\mathbb{G}$ of prime order p (g, h: generators):

[CS02] DDH:

$$\{(u, e) \mid \exists r \in \mathbb{Z}_p,\ u = g^r \text{ and } e = h^r\};$$

[ACP09] Conjunction of two languages L_1 and L_2:

$$L = \{(C_1, C_2) \mid C_1 \in L_1 \text{ and } C_2 \in L_2\} = L_1 \times L_2;$$

[BBC+13] ElGamal/Cramer-Shoup-like ciphertexts of $M_1, \ldots, M_n \in \mathbb{G}$ satisfying:

- a system of multi-exponentiation equations;
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(under some condition and with bilinear groups)

- + other tools for SPHF: 2-smoothness, PrPHF, ...
- Applications:
  - Quasi-Adaptive NIZK [JR13]
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    - even for one-time simulation-soundness
    - application: threshold Cramer-Shoup-like encryption scheme
  - First one-round PAKE for $k \geq 3$ players;
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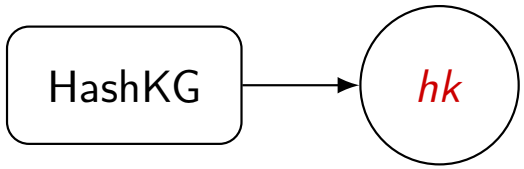
SPHF
Definition

L: NP language, i.e.: $C \in L \subseteq \mathcal{X} \iff \exists w, \mathcal{R}(C, w) = 1.$



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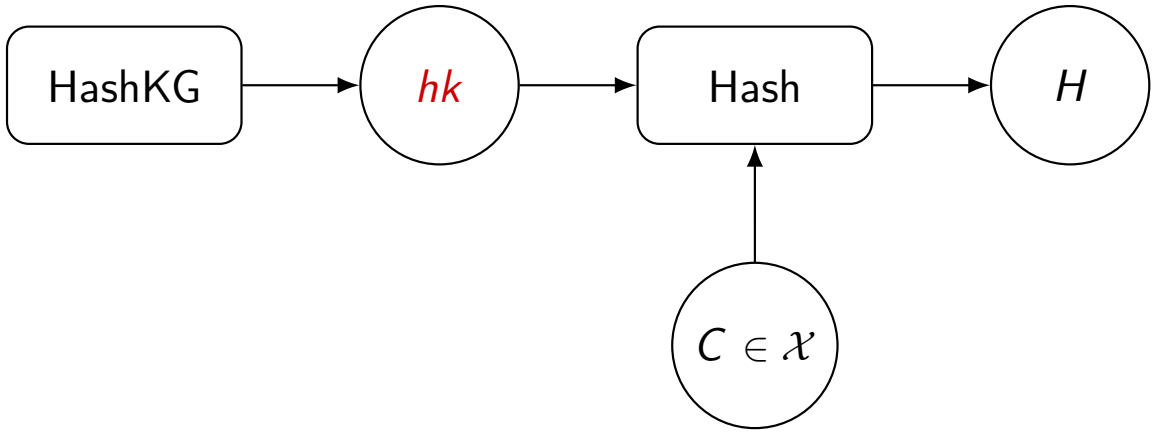

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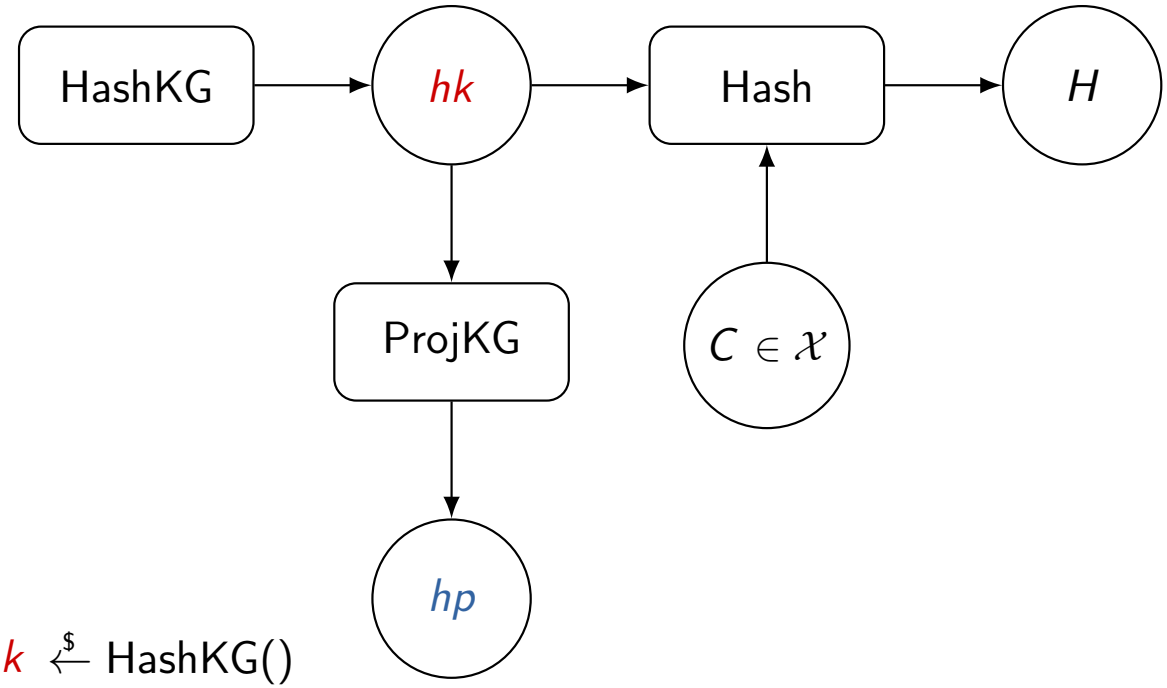

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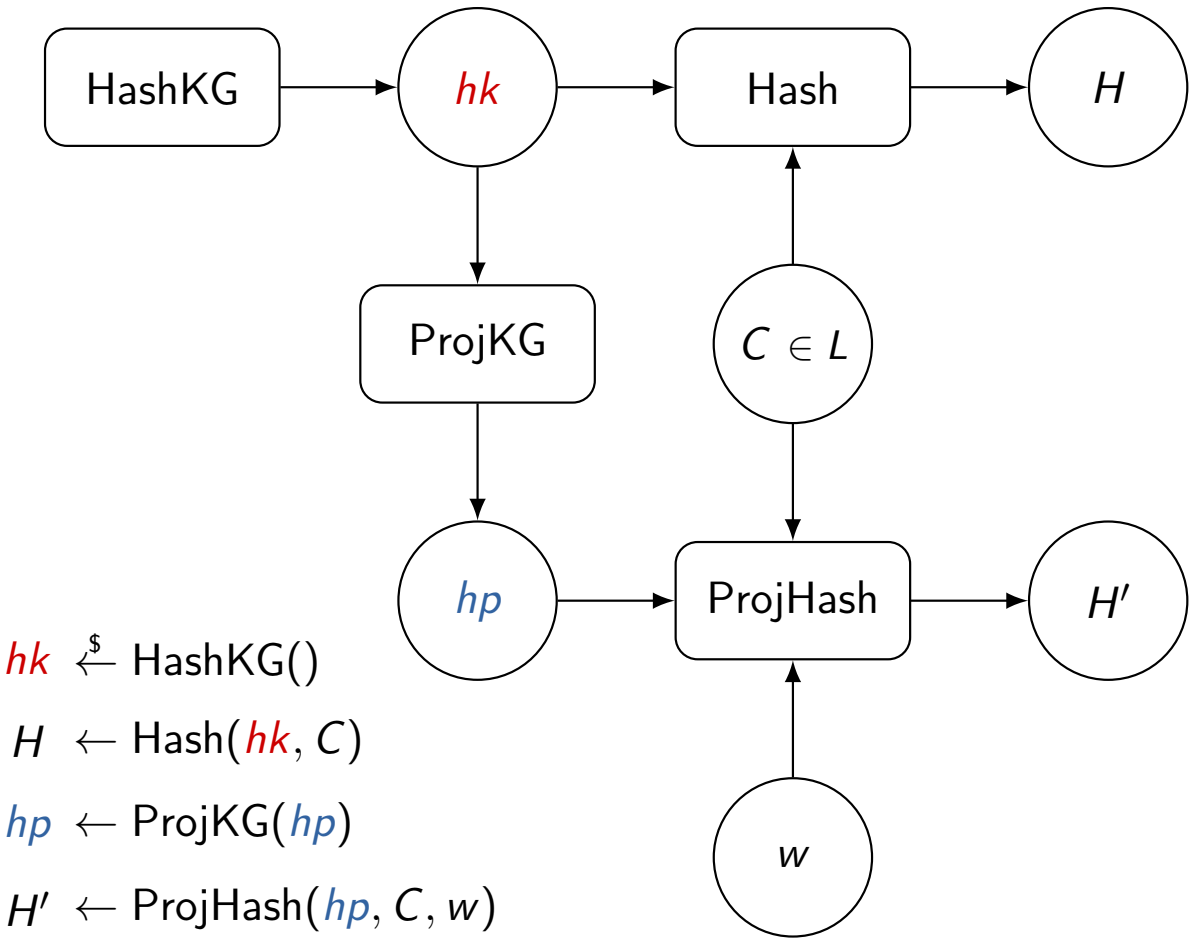

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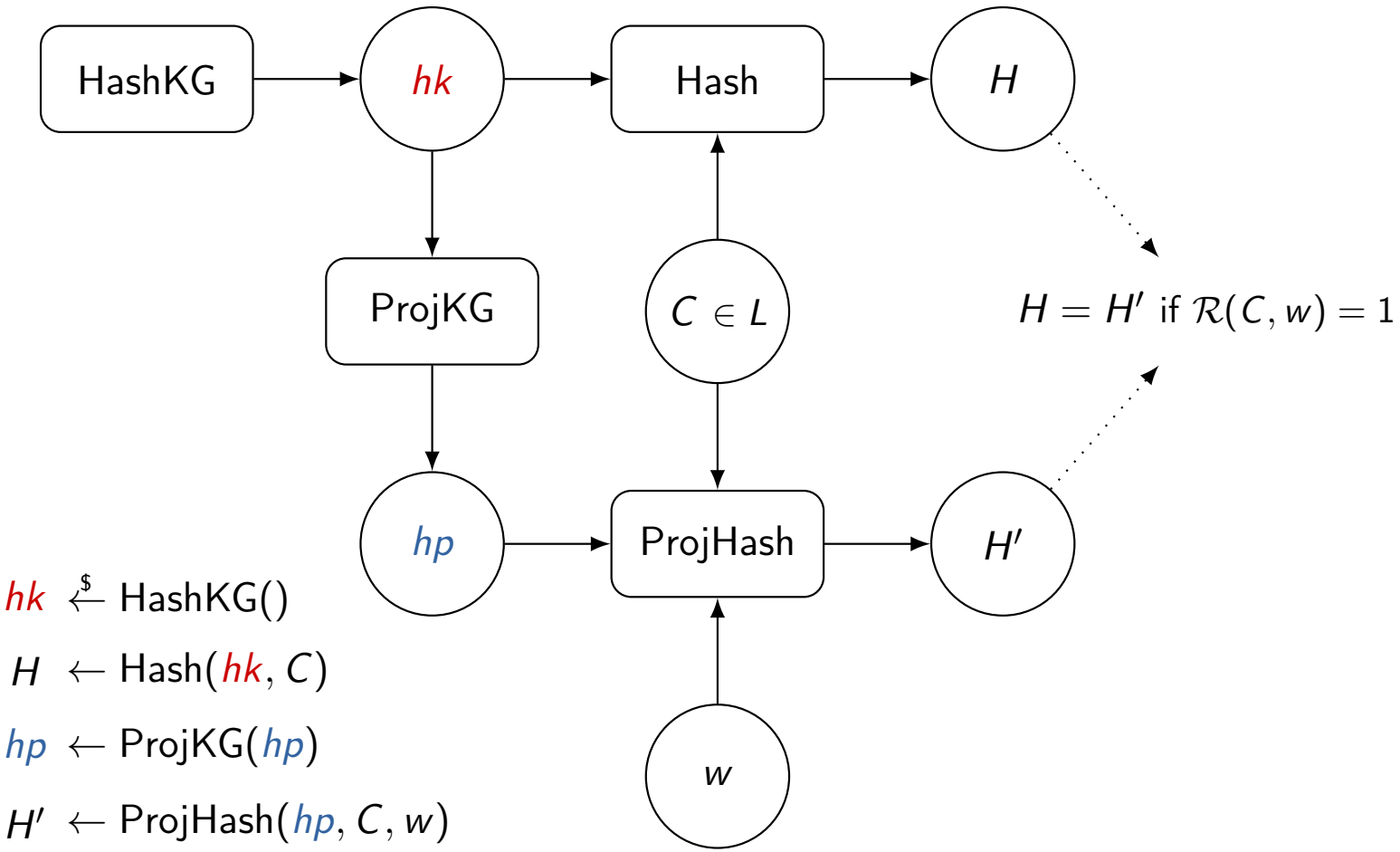

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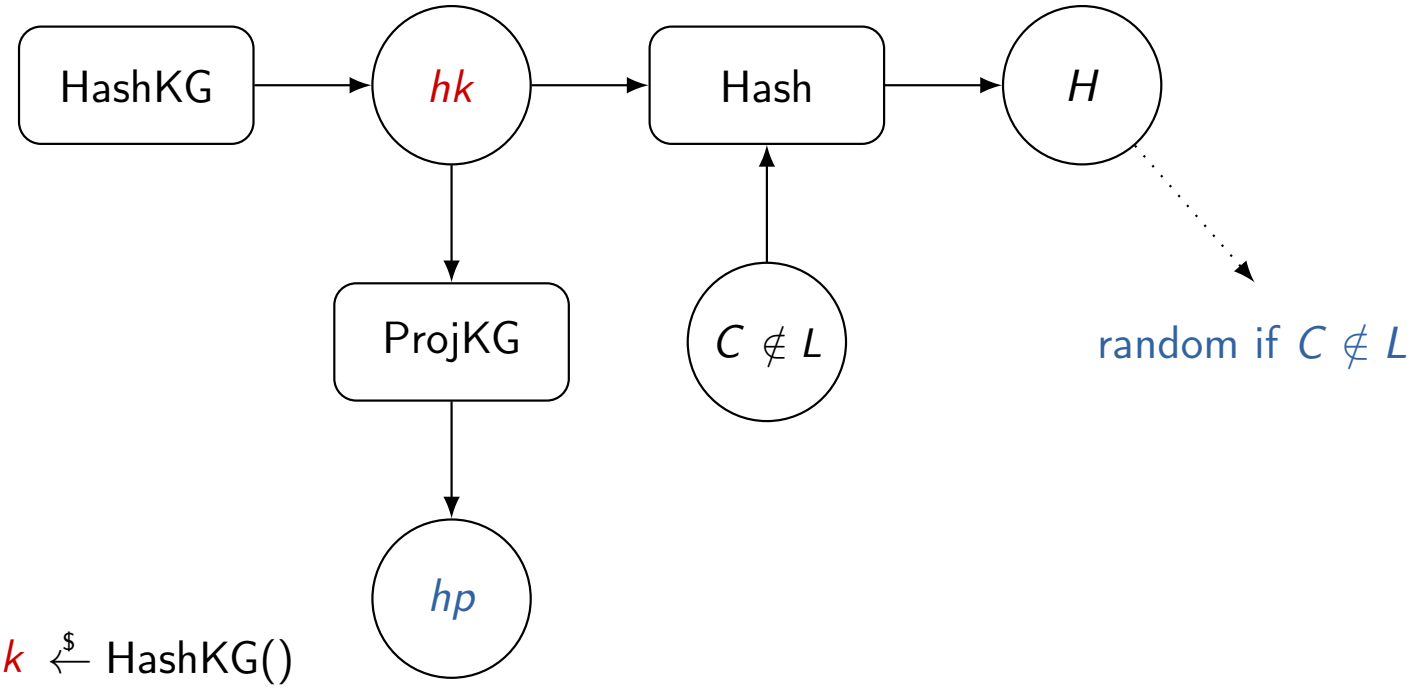

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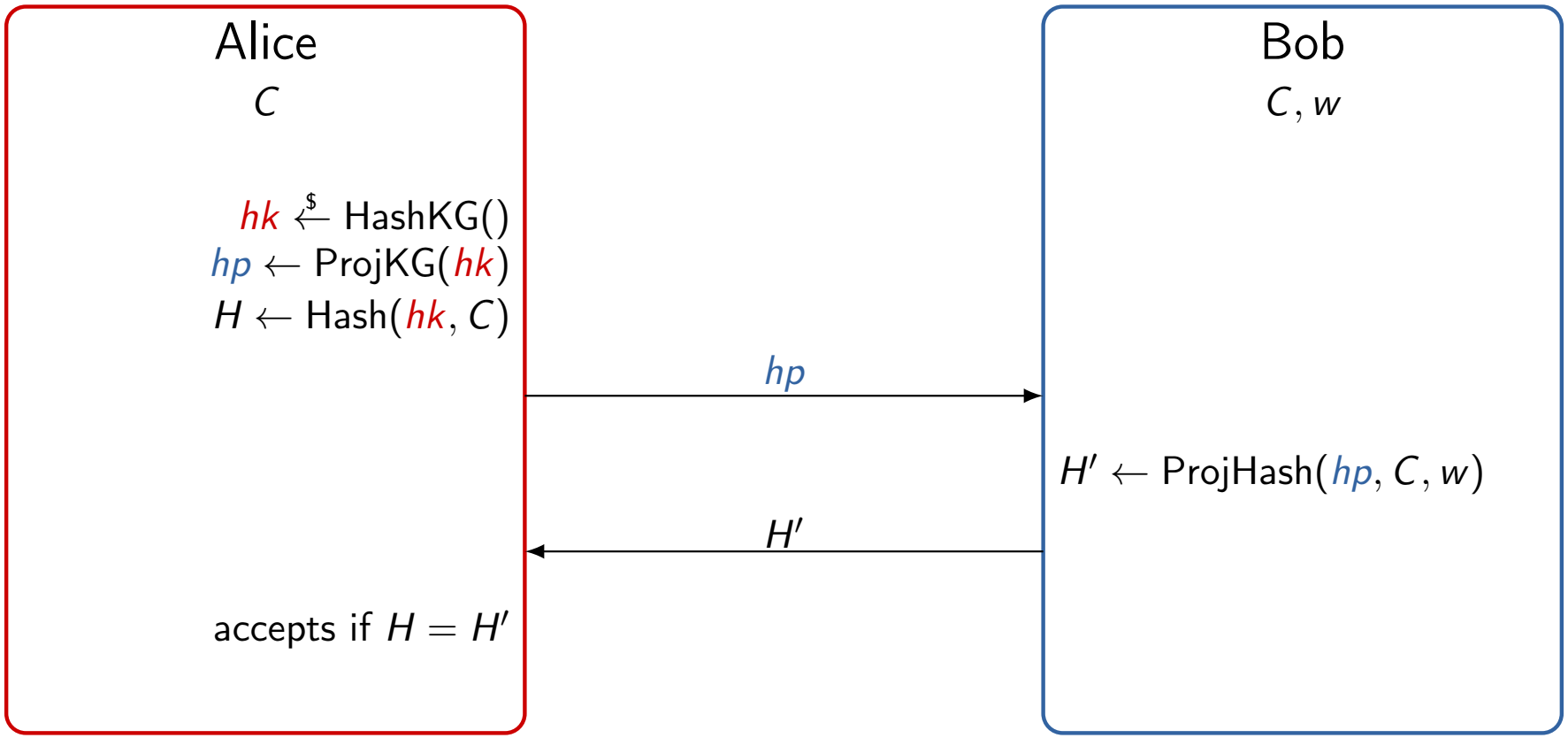
Direct Applications of SPHF

Honest-Verifier Zero-Knowledge Proof

Bob wants to prove to Alice that $C \in L$.



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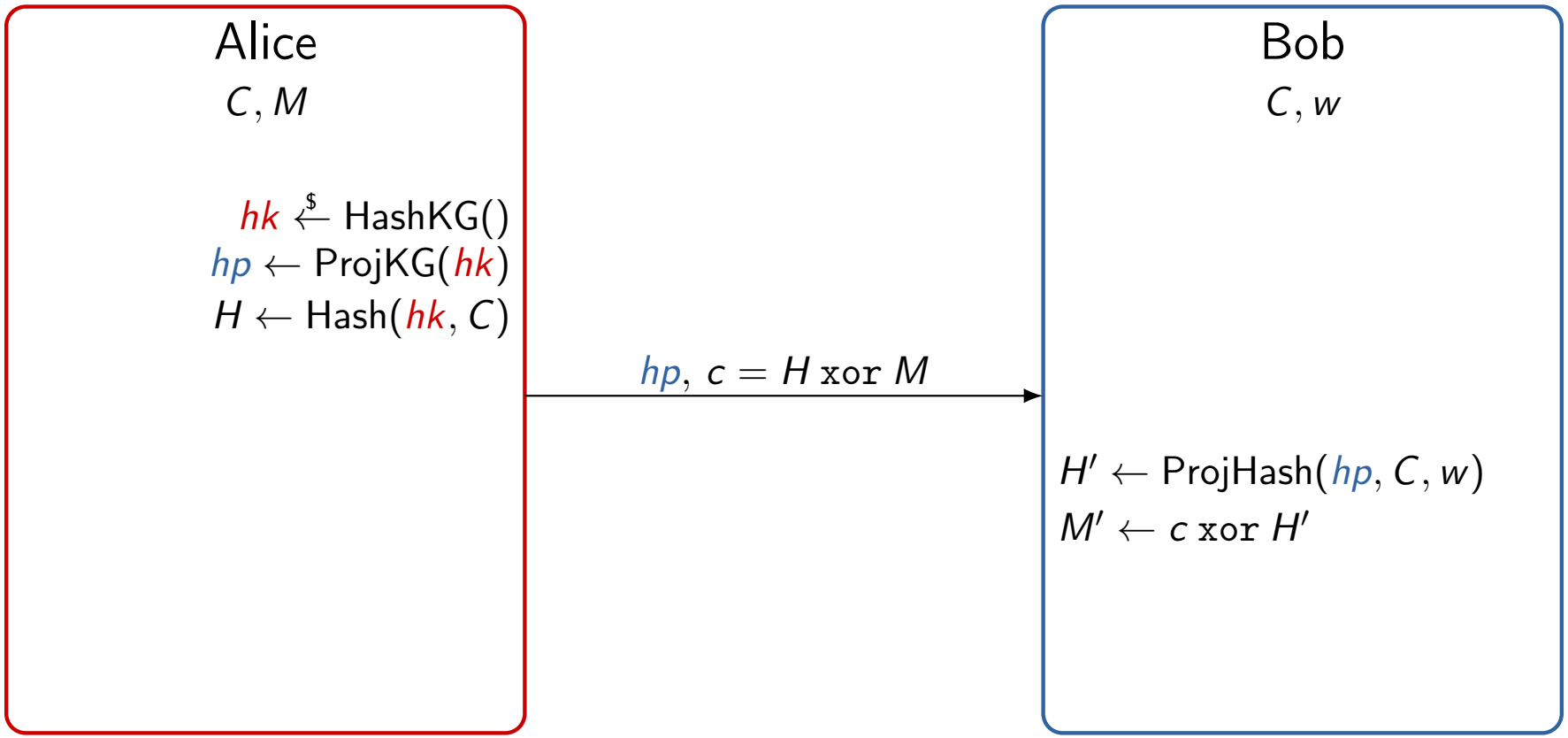
Alice wants to send M to Bob if $C \in L$.



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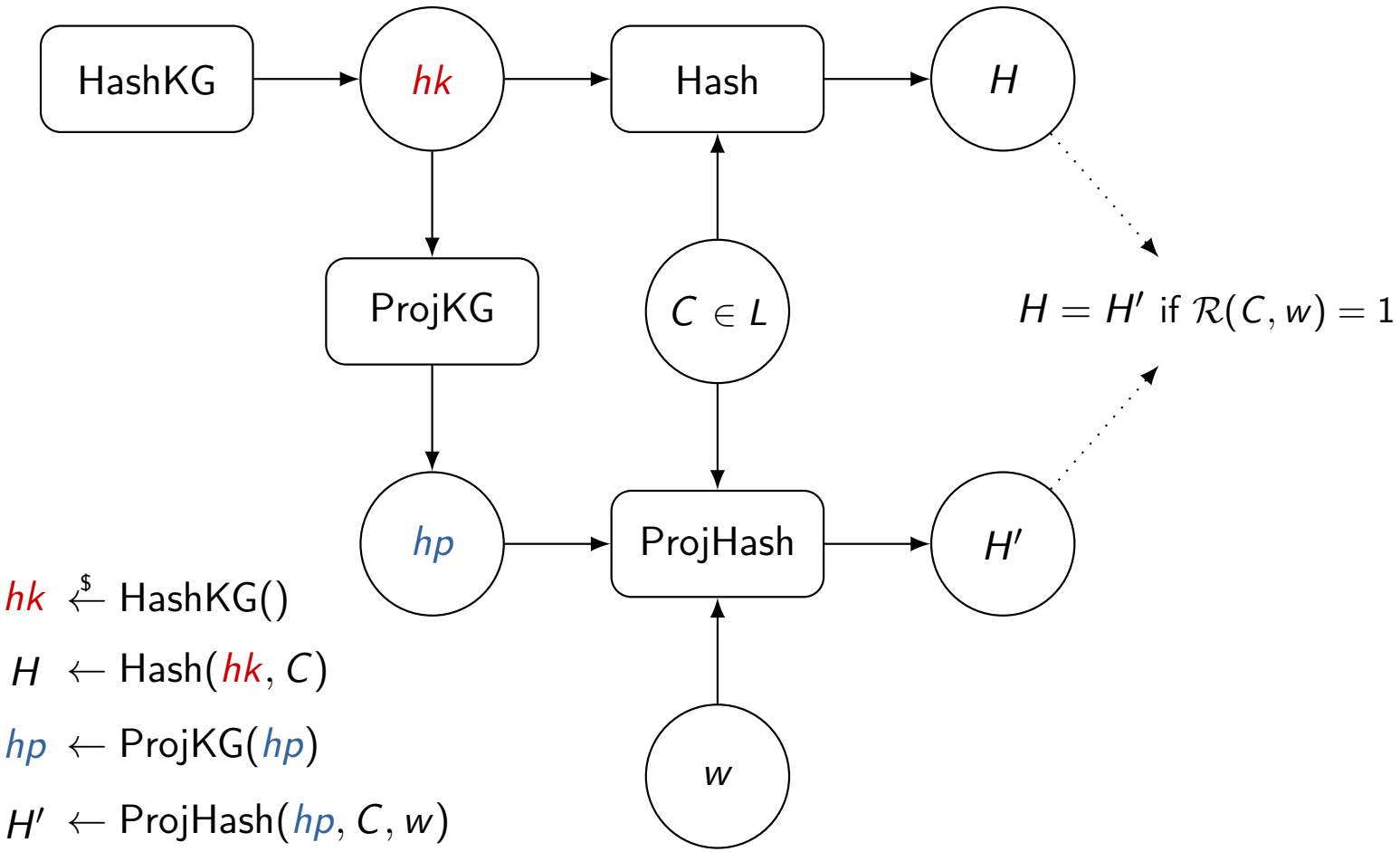
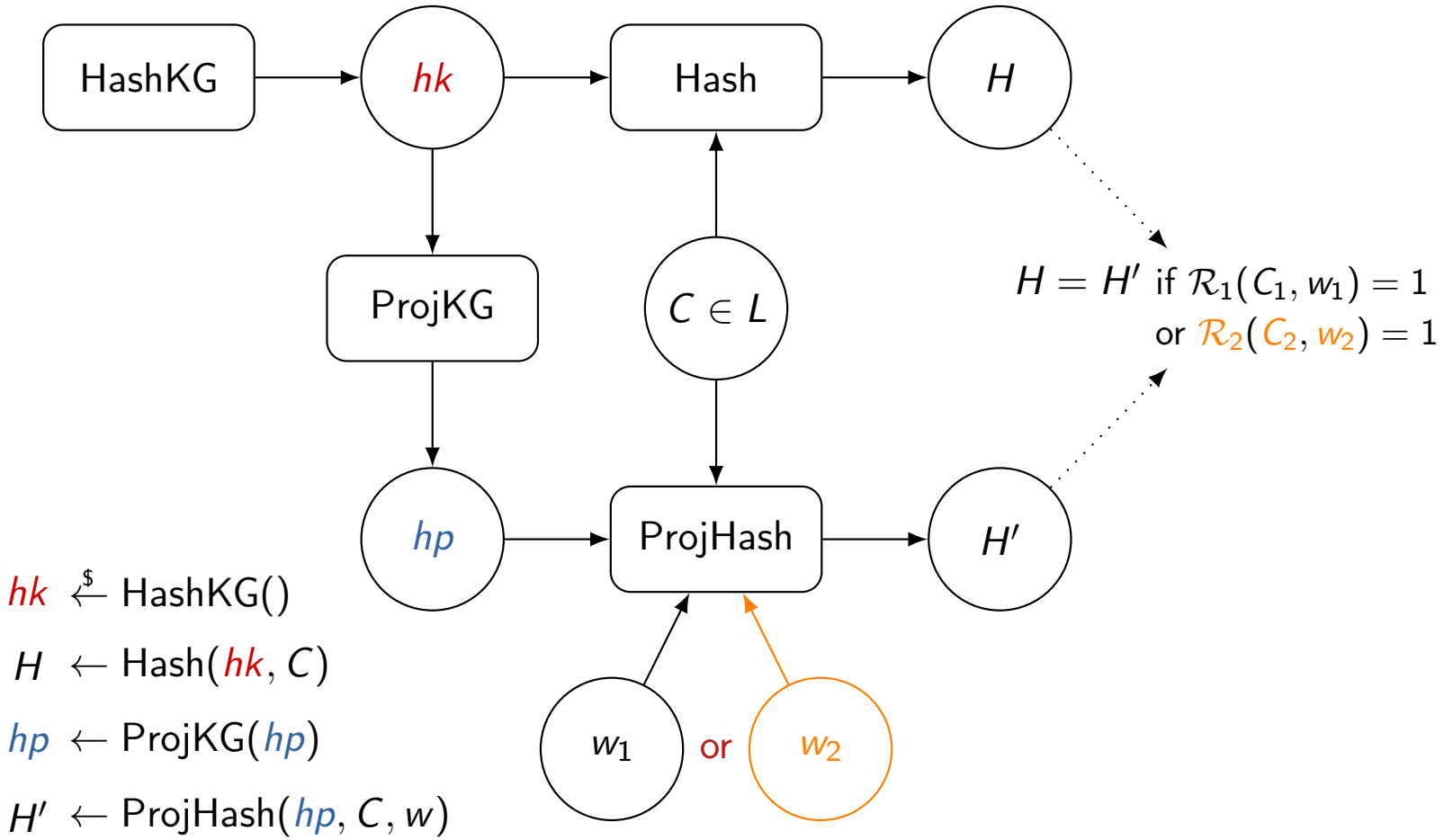


Disjunction

$$C = (C_1, C_2) \in L \iff C_1 \in L_1 \text{ or } C_2 \in L_2$$
$$\iff \exists w_1, \mathcal{R}_1(C_1, w_1) = 1 \text{ or } \exists w_2, \mathcal{R}_2(C_2, w_2) = 1.$$



How to use that? Why is it useful?

In all applications except PAKE:

- L_1:
  - original language;
  - word C_1 for the proof;
- L_2:
  - used to add functionnality;
  - word C_2 in the CRS;
  - DDH for example

$$\{C_2 = (u, e) \mid \exists r \in \mathbb{Z}_p,\ u = g^r \text{ and } e = h^r\};$$

  or k-Lin or any MDDH assumption;
- 3 ways to compute H:
  - knowing hk;
  - knowing hp and witness w_1 for C_1;
  - knowing hp and witness w_2 for C_2.

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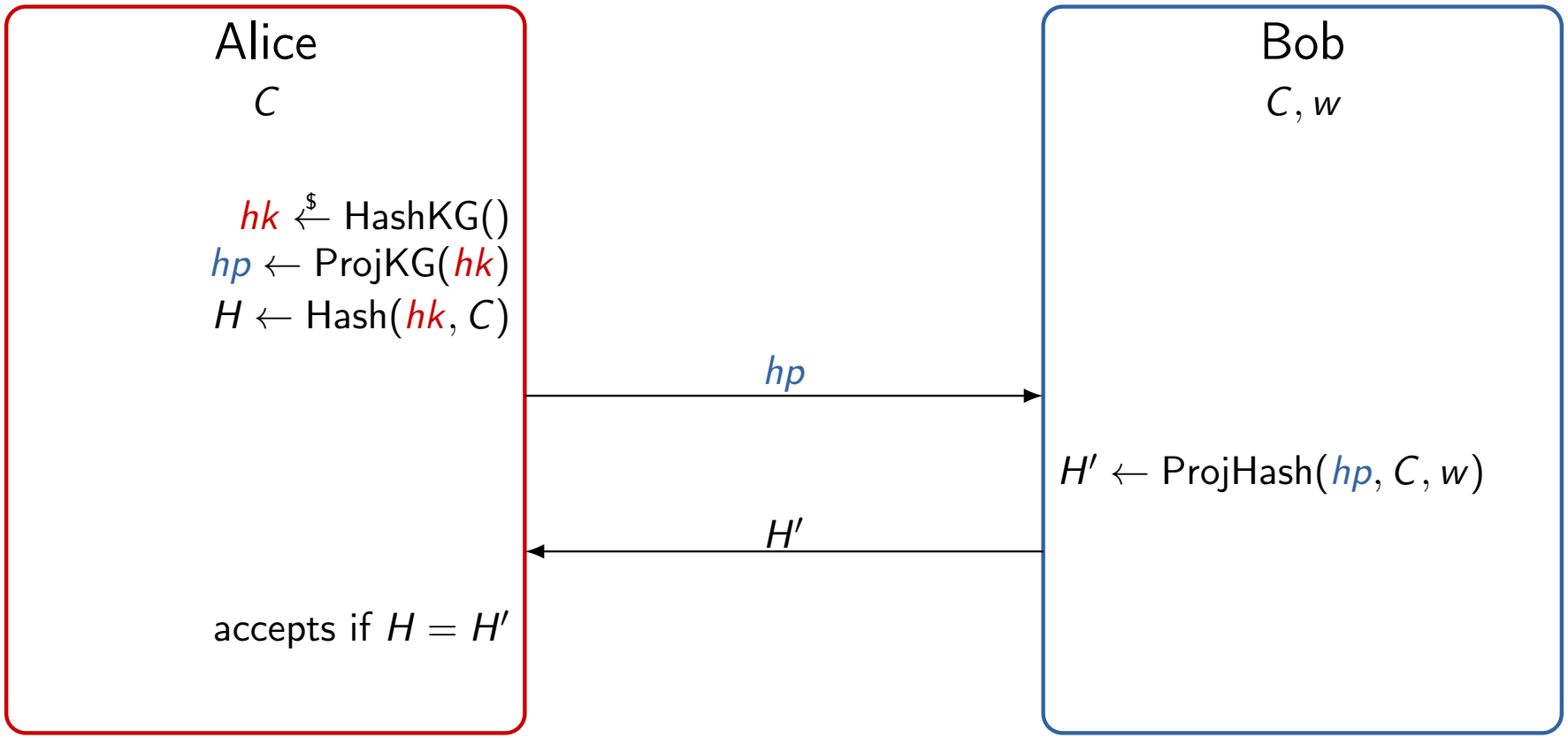

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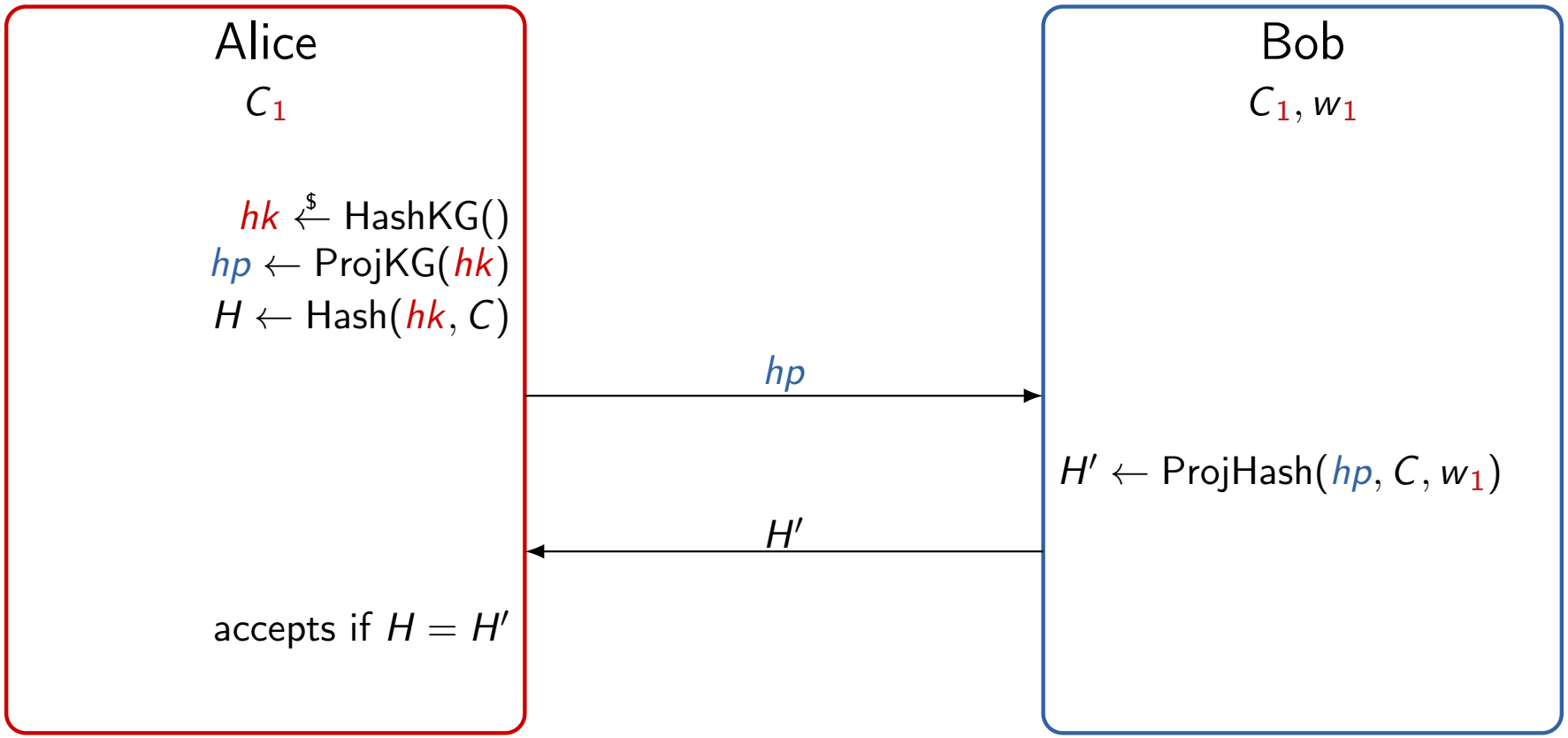
Zero-Knowledge Argument

Bob wants to prove to Alice that $C_1 \in L_1$ in zero-knowledge.

$C = (C_1, C_2)$ — CRS: $C_2 \rightarrow \begin{cases} \text{stat. zero-knowledge} & \text{if } C_2 \in L_2 \\ \text{stat. soundness} & \text{if } C_2 \notin L_2 \end{cases}$



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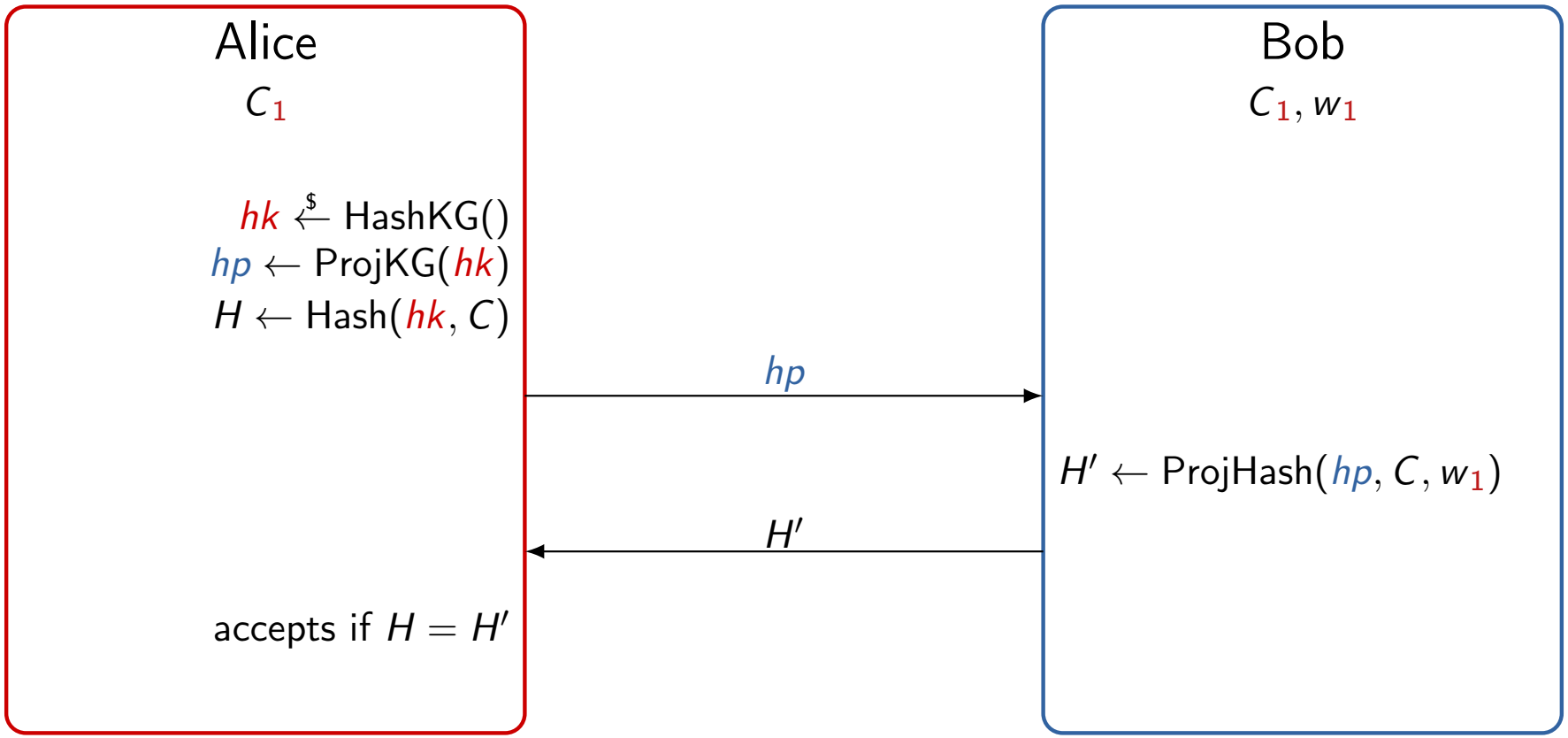

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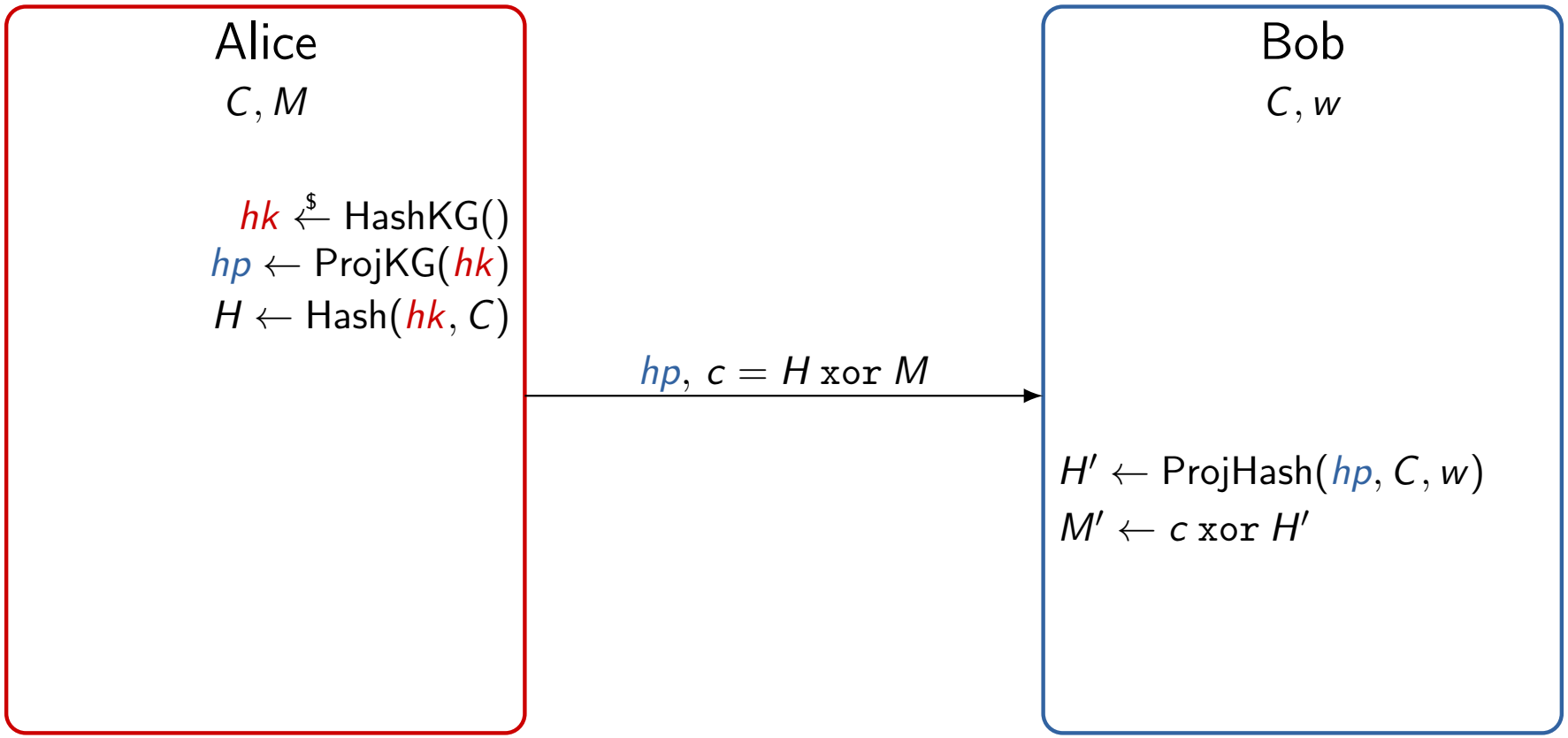

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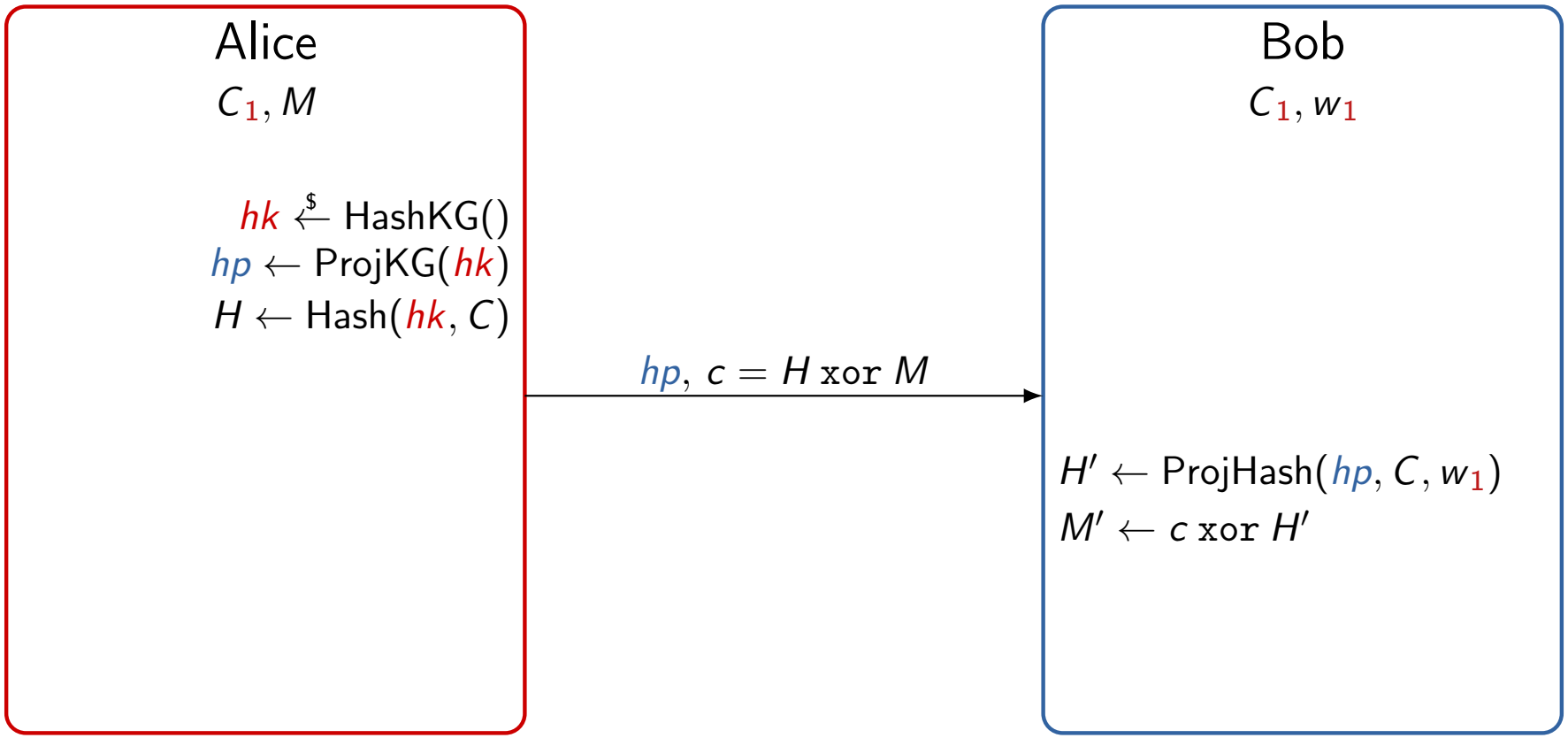

TSPHF

Previous ideas captured by

$$\text{Trapdoor SPHF} = \text{TSPHF}$$

- introduced in [BBC$^+$13];
- here: clean explanation.

Disjunction of two SPHFs?

- Ideally fully generic

  $$\text{SPHF for } L_1 + \text{SPHF for } L_2 \longrightarrow \text{SPHF for } L$$

  L: disjunction of L_1 and L_2

  $\longrightarrow$ seems hard

- Here:
  - Specific languages L_1 and L_2 over cyclic group $\mathbb{G}_1$ and $\mathbb{G}_2$
  - Use bilinear map $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_t$

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Framework [BBC+13]: SPHF when

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Diverse Vector Space

encompasses most SPHF over cyclic groups.

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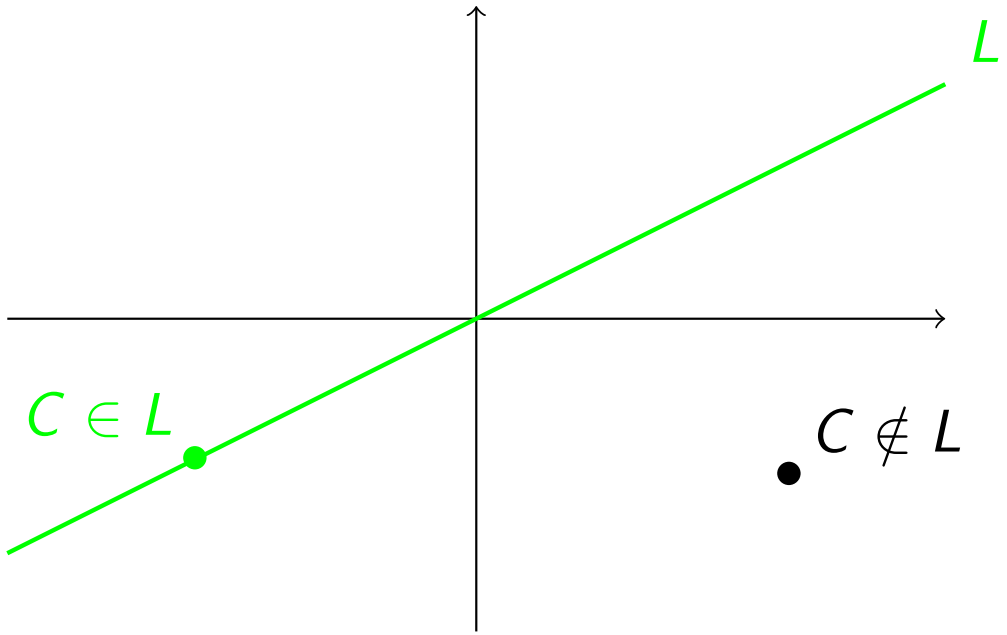

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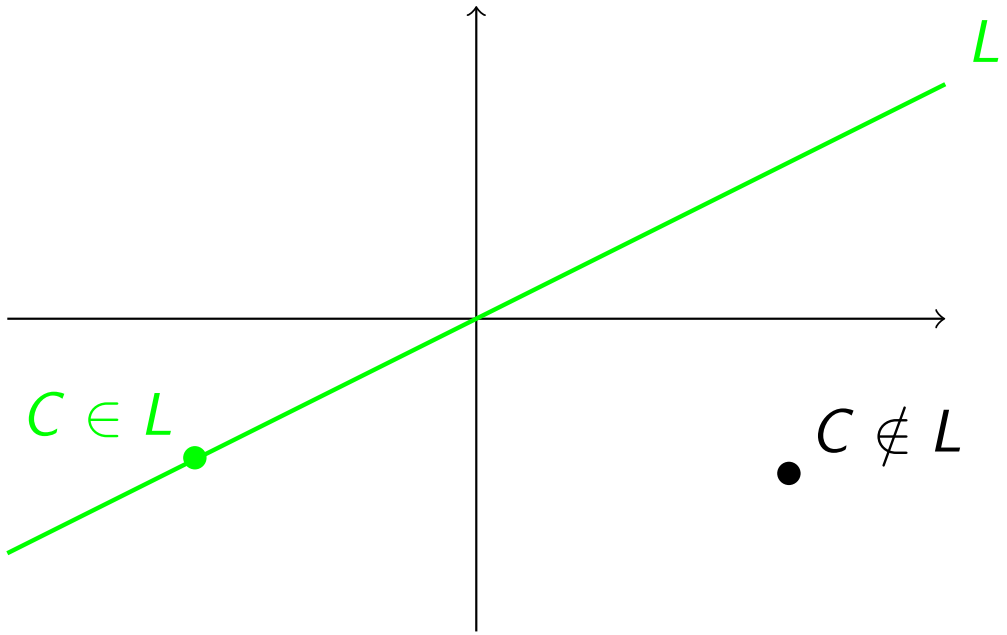

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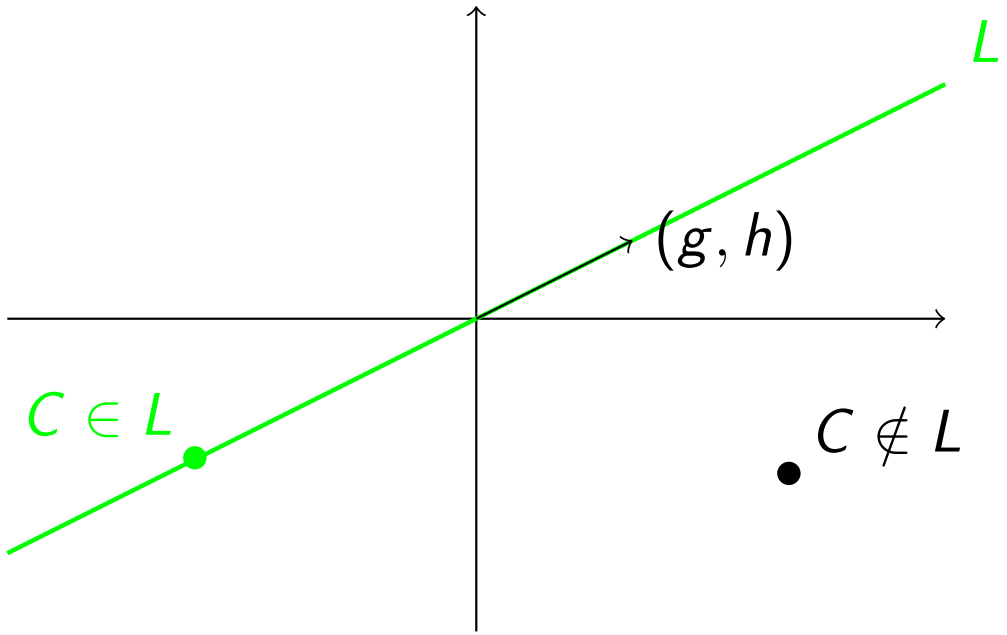

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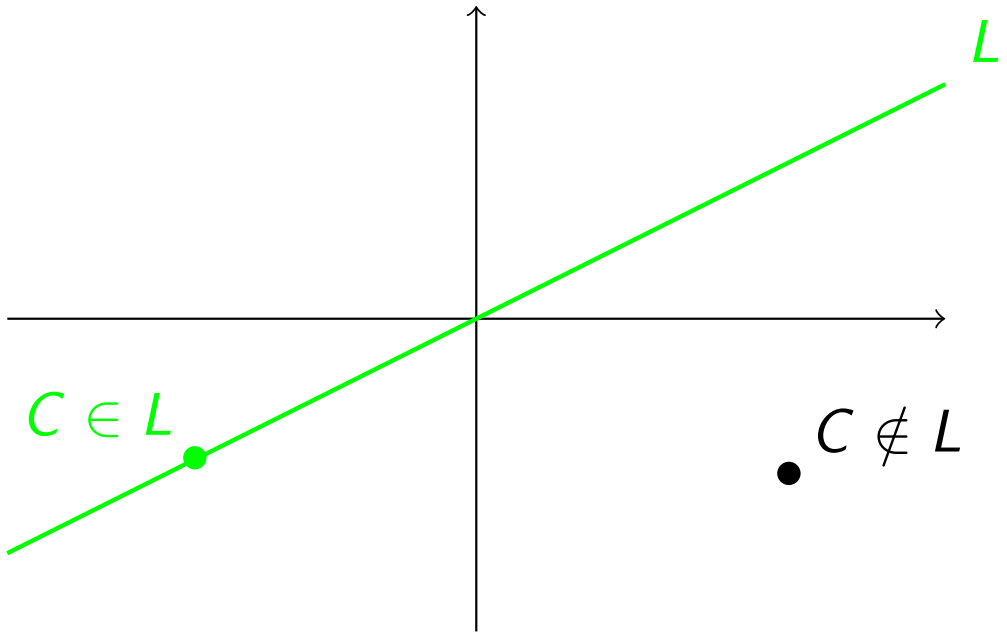

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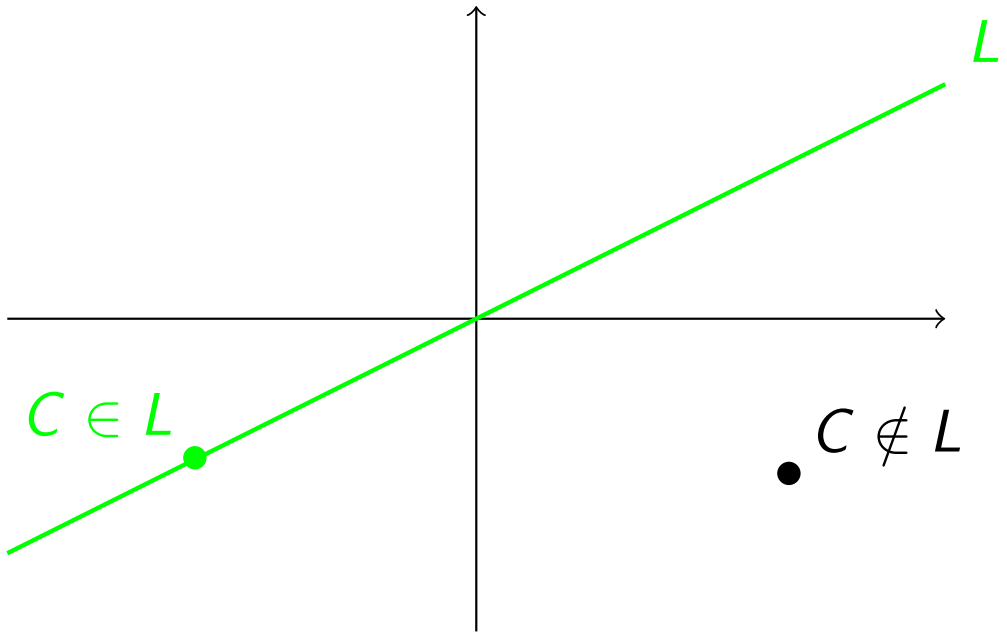

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Construction of an SPHF for a Diverse Vector Space

- hk: random linear map: $\mathcal{X} \to \mathbb{G}$
- hp: hk restricted to L
- Hash value of C:

$$H := hk(C),$$

can be computed from hp, if $C \in L$.

Smoothness

No information on $hk(C)$ for $C \notin L$ (knowing only hp).

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Conjunction and Disjunction

L is a subspace of a vector space $\mathcal{X} \approx \mathbb{Z}_p^n$

- Conjunction of $L_1 \subseteq \mathcal{X}_1$ and $L_2 \subseteq \mathcal{X}_2$: SPHF for

$$L := L_1 \times L_2 \quad \subseteq \quad \mathcal{X}_1 \times \mathcal{X}_2 =: \mathcal{X};$$

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But wait!

L is not a vector space and $\langle L \rangle = \mathcal{X}$!

Idea: Tensor Product

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Quasi-Adaptive Non-Interactive Zero-Knowledge Arguments (QA-NIZK)

- CRS σ (depending on $L = \mathsf{QA}$) + trapdoor $\mathcal{T}$
- Proof of $C_1 \in L_1$ with witness w_1:

$$\pi \xleftarrow{\$} \mathsf{Prove}(\sigma, C_1, w_1);$$

- Verification

$$\mathsf{Ver}(\sigma, C_1, \pi);$$

- Simulation

$$\pi \xleftarrow{\$} \mathsf{Sim}(\sigma, \mathcal{T}, C_1).$$

Construction of QA-NIZK from Disjunction

- $L_2 \subseteq \mathcal{X}_2$: DDH, k-Lin, MDDH, ...
- CRS $\sigma = hp$ + trapdoor $\mathcal{T} = hk$
- Proof π of C_1: hash values of (C_1, e_j)
  e_j basis of $\mathcal{X}_2$
- Verification using hp
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Soundness?

Valid π for $C_1 \longrightarrow$ compute hash value H'' of any (C_1, C_2)
If $C_1 \notin L_1$ and $C_2 \notin L_2$:

$$H'' = \mathsf{Hash}(hk, (C_1, C_2)) \quad \text{unpredictable!}$$

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Comparison: QA-NIZK for Linear Subspaces

- When L_2 = DDH:
  - Same resulting construction as [JR14b];
  - Proof = 1 group element;
  - \+ Even for one-time simulation-soundness!
- When L_2 = MDDH (general case):
  - Same construction as [KW15];
  - \+ Different proof and prior work;
  - − Stronger assumption: MDDH instead of Ker-MDDH.

Contributions

- SPHF for disjunction of two languages L_1 and L_2:

$$L = \{(C_1, C_2) \mid C_1 \in L_1 \text{ or } C_2 \in L_2\};$$

(under some condition and with bilinear groups)

- + other tools for SPHF: 2-smoothness, PrPHF, ...
- Applications:
  - Quasi-Adaptive NIZK [JR13]
    - 1 group element (under DDH), as [JR14b]
    - even for one-time simulation-soundness
    - application: threshold Cramer-Shoup-like encryption scheme
  - First one-round PAKE for $k \geq 3$ players;
  - Link with homomorphic signatures [LPJY13];
  - New construction of Trapdoor SPHF (TSPHF) [BBC+13] (zero-knowledge variant of SPHF).

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- Lightweight alternative to NIZK or Zero-Knowledge proofs;
- Even more applications: PAKE;
- Large family of "algebraic" languages.

To sum up!

Next time you need a proof
think SPHF!

But beyond that...

Hash proof systems are cool!

- Lightweight alternative to NIZK or Zero-Knowledge proofs;
- Even more applications: PAKE;
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To sum up!

Next time you need a proof
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Thank you for your attention!

Questions?

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