

From 5-pass \mathcal{MQ} -based identification to \mathcal{MQ} -based signatures

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Post-quantum signatures

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Solutions:

- ▶ Hash-based: SPHINCS [BHH+15], XMSS [BDH11, HRS16]
 - ▶ Slow or stateful
- ▶ Lattice-based: (Ring-)TESLA [ABB+16, ABB+15], BLISS [DDL+13], GLP [GLP12]
 - ▶ Large keys, or additional structure
- ▶ MQ : ?
 - ▶ Unclear security: many broken (except HFEv-, UOV)

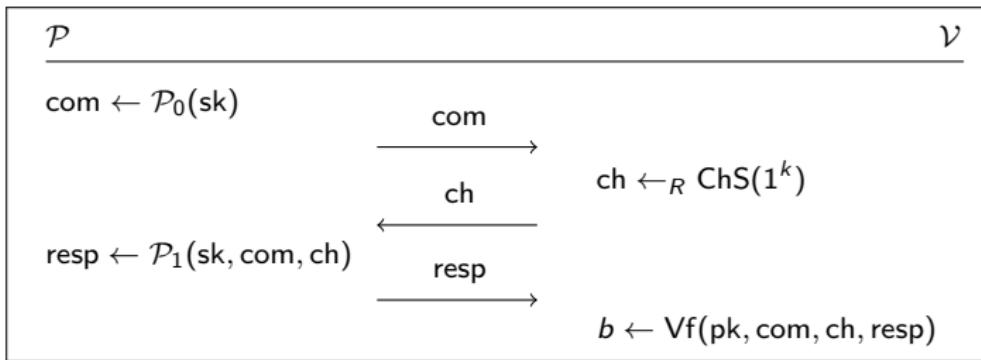
This work

- ▶ Transform class of 5-pass IDS to signature schemes
 - ▶ Extend Fiat Shamir transform
- ▶ Prove an earlier attempt [EDV+12] vacuous
 - ▶ Amended in [DGV+16]
- ▶ Propose MQDSS
 - ▶ Obtained by performing transform
 - ▶ Hardness of \mathcal{MQ}
- ▶ Instantiate and implement as MQDSS-31-64

But also:

- ▶ Reduction in the ROM (not in QROM)
- ▶ No tight proof

Canonical Identification Schemes



Informally:

1. Prover commits to some (random) value derived from sk
2. Verifier picks a challenge 'ch'
3. Prover computes response 'resp'
4. Verifier checks if response matches challenge

Security of the IDS

- ▶ Passively secure IDS

Soundness: the probability that an adversary can convince is 'small'

Honest-Verifier Zero-Knowledge: simulator can 'fake' transcripts

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Soundness: the probability that an adversary can convince is 'small'

- ▶ Shows knowledge of secret
- ▶ Adversary \mathcal{A} can 'guess right': soundness error κ

$$\Pr \left[\begin{array}{l} (\text{pk}, \text{sk}) \leftarrow \text{KGen}(1^k) \\ \langle \mathcal{A}(1^k, \text{pk}), \mathcal{V}(\text{pk}) \rangle = 1 \end{array} \right] \leq \kappa + \text{negl}(k).$$

Honest-Verifier Zero-Knowledge: simulator can 'fake' transcripts

- ▶ Shows that transcripts do not leak the secret

Fiat-Shamir transform

- ▶ First transform IDS with soundness error κ to $\text{negl}(k)$
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 - ▶ Transcript is signature

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- ▶ Generalize to 5-pass
 - ▶ Benefit from lower soundness error

5-pass Fiat-Shamir transform

- ▶ Attempt in [EDV+12] incorrect
 - ▶ ‘ n -soundness’
 - ▶ Two transcripts agree up to last challenge \Rightarrow extract sk
- ▶ Vacuous assumption: satisfying schemes reduce to 3-pass
 - ▶ HVZK: combine first 3 messages into 1
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- ▶ Existing schemes do not satisfy n -soundness
- ▶ n -soundness fixed in [DGV+16]
 - ▶ Still does not apply to existing schemes

5-pass Fiat-Shamir transform

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5-pass Fiat-Shamir transform

- ▶ Restrict to challenge spaces of size q resp. 2
 - ▶ ‘ $q2$ -IDS’
- ▶ Prove EU-CMA using dedicated forking lemma
 - ▶ Assuming a successful forgery ..
 - ▶ .. generate 4 signatures fulfilling pattern on challenges
 - ▶ .. obtain 4 traces with same commitments, pattern on challenges
 - ▶ Use $q2$ -IDS that allow extracting sk

\mathcal{MQ} problem

The function family $\mathcal{MQ}(n, m, \mathbb{F}_q)$:

$$\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_m(\mathbf{x})), \text{ where } f_s(\mathbf{x}) = \sum_{i,j} a_{i,j}^{(s)} x_i x_j + \sum_i b_i^{(s)} x_i$$

for $a_{i,j}^{(s)}, b_i^{(s)} \in \mathbb{F}_q, s \in \{1, \dots, m\}$

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Problem: For given $\mathbf{y} \in \mathbb{F}_q^m$, find $\mathbf{x} \in \mathbb{F}_q^n$ such that $\mathbf{F}(\mathbf{x}) = \mathbf{y}$.

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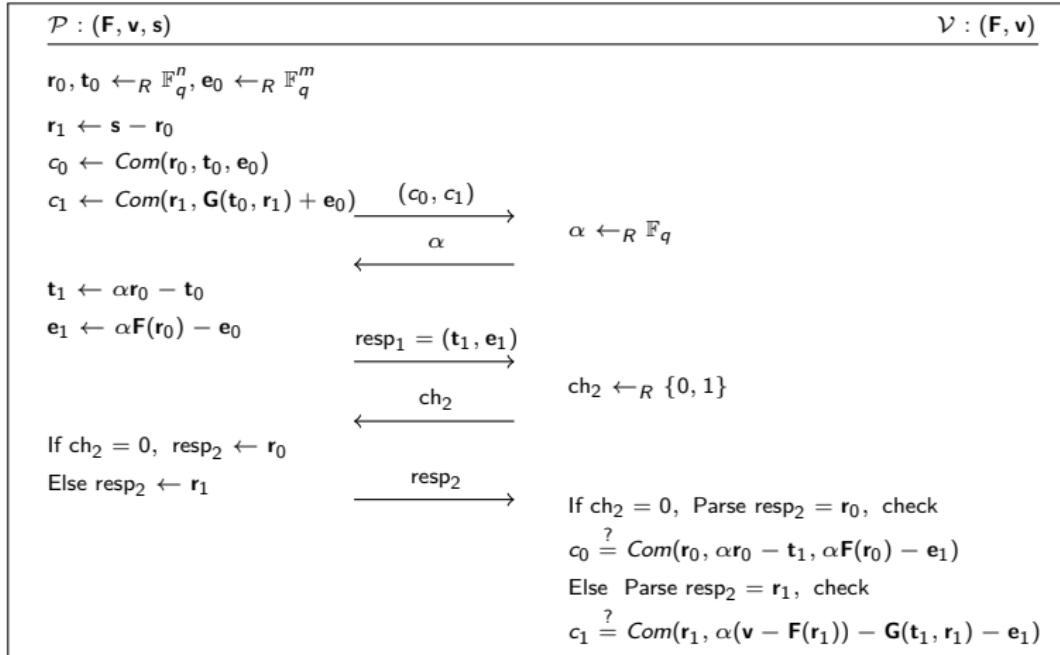
i.e., solve the system of equations:

$$y_0 = a_{0,0}^{(0)} x_0 x_0 + a_{0,1}^{(0)} x_0 x_1 + \dots + a_{n,n}^{(0)} x_n x_n + b_0^{(0)} x_0 + \dots + b_n^{(0)} x_n$$

\vdots

$$y_m = a_{0,0}^{(m)} x_0 x_0 + a_{0,1}^{(m)} x_0 x_1 + \dots + a_{n,n}^{(m)} x_n x_n + b_0^{(m)} x_0 + \dots + b_n^{(m)} x_n$$

Sakumoto et al. 5-pass IDS [SSH11]



Sakumoto et al. 5-pass IDS [SSH11]

- ▶ Relies only on \mathcal{MQ} , not IP
- ▶ Key technique: cut-and-choose for \mathcal{MQ}
 - ▶ Analogously, consider DLP: $s = r_0 + r_1 \Rightarrow g^s = g^{r_0} \cdot g^{r_1}$
- ▶ Bilinear map $\mathbf{G}(\mathbf{x}, \mathbf{y}) = \mathbf{F}(\mathbf{x} + \mathbf{y}) - \mathbf{F}(\mathbf{x}) - \mathbf{F}(\mathbf{y})$
 - ▶ Split \mathbf{s} and $\mathbf{F}(\mathbf{s})$ into $\mathbf{r}_0, \mathbf{r}_1$ and $\mathbf{F}(\mathbf{r}_0), \mathbf{F}(\mathbf{r}_1)$
 - ▶ Split again into $\mathbf{t}_0, \mathbf{t}_1$ resp. $\mathbf{e}_0, \mathbf{e}_1$, using α
 - ▶ See [SSH11] for details
- ▶ Result: reveal either $(\mathbf{r}_0, \mathbf{t}_1, \mathbf{e}_1)$ or $(\mathbf{r}_1, \mathbf{t}_1, \mathbf{e}_1)$

MQDSS

- ▶ Generate keys
 - ▶ Sample seed $S_F \in \{0, 1\}^k$, $\mathbf{sk} \in \mathbb{F}_q^n \quad \Rightarrow (S_F, \mathbf{sk})$
 - ▶ Expand S_F to \mathbf{F} , compute $\mathbf{pk} = \mathbf{F}(\mathbf{sk}) \quad \Rightarrow (S_F, \mathbf{pk})$

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 - ▶ $2r$ \mathcal{MQ} evaluations

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 - ▶ Reconstruct challenges from σ_0, σ_1
 - ▶ Verify responses in σ_2

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- ▶ Parameters: k, n, m, \mathbb{F}_q , Com, hash functions, PRGs

MQDSS-31-64

- ▶ Security parameter $k = 256$ (\Rightarrow 128-bit PQ security)
- ▶ Soundness error κ depends on q
 - ▶ $\kappa = \frac{q+1}{2q}$
 - ▶ Determines number of rounds: $r = 269$, $\kappa^{269} < (\frac{1}{2})^{256}$
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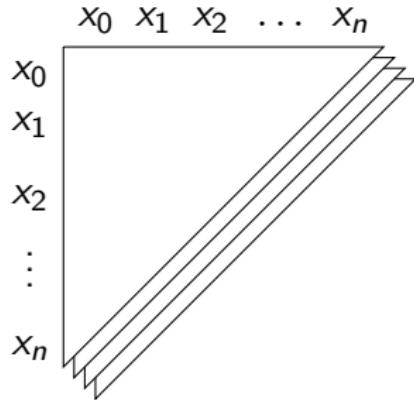
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- ▶ Commitments, hashes, PRGs: SHA3-256, SHAKE-128
- ▶ Signature σ contains:
 - ▶ R , for random digest $\Rightarrow 32B$
 - ▶ Hash $\mathcal{H}(commits)$ $\Rightarrow 32B$
 - ▶ For every round:
 - ▶ Response vectors $\mathbf{t}, \mathbf{e}, \mathbf{r}$ $\Rightarrow 3 \times 40B$
 - ▶ ‘Missing commit’ $\Rightarrow 32B$

Evaluating \mathcal{MQ}

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- ▶ From \mathbf{x} to $\mathbf{F}(\mathbf{x})$ should be easy

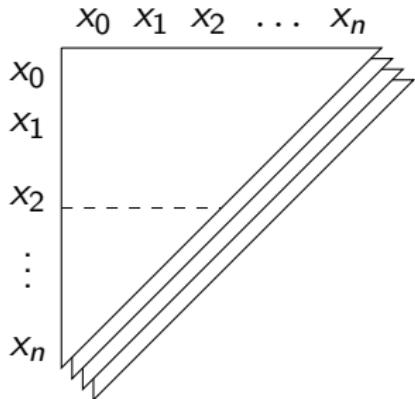
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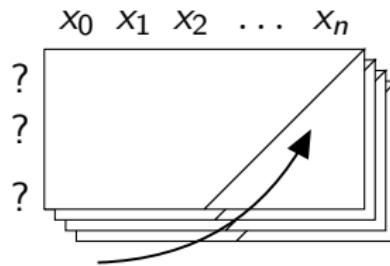
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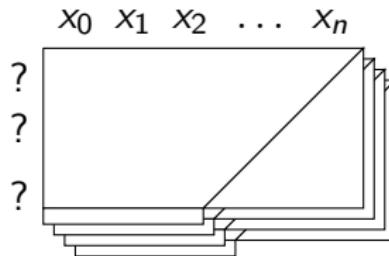
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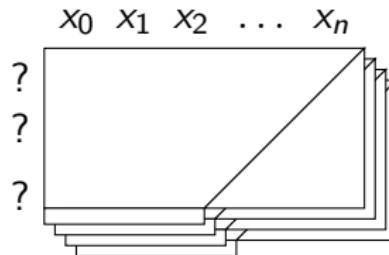
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- ▶ Compute monomials, evaluate polynomials
- ▶ 64 elements in \mathbb{F}_{31} ; 16 (or 32) per 256 bit AVX2 register

Benchmarks & conclusion

- ▶ Signatures: ~40 KB (\approx SPHINCS)
- ▶ Public and private keys: 72 resp. 64 bytes
- ▶ Signing time: ~8.5M cycles (2.43ms @ 3.5GHz)
 - ▶ Verification 5.2M, key generation 1.8M
- ▶ ~6x faster than SPHINCS, >10x slower than lattices

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- ▶ Fiat-Shamir transform for $q2$ -IDS
- ▶ Competitive signatures with (non-tight) reduction to \mathcal{MQ}

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