

Power Analysis on NTRU Prime

Wei-Lun Huang, Jiun-Peng Chen, Bo-Yin Yang

Academia Sinica, Taiwan

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Topics

- ❖ NTRU Prime
- ❖ A Brief Preview
- ❖ Correlation Power Analysis: vertical vs. horizontal in-depth
- ❖ Online Template Attacks
- ❖ Chosen-Input Simple Power Analysis
- ❖ Finale

Topics

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Post-Quantum Cryptography (PQC)

❖ Shor's Algorithm

- solving integer factorization and discrete logarithms efficiently
- Quantum Computers: estimated as arriving in 10~20 years

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❖ The NIST PQC Standardization Project

- key encapsulation mechanisms (KEM) + digital signatures
- lattices / error correction codes / multivariate quadratic equations / ...

NTRU Prime: lattice-based KEM

❖ Streamlined NTRU Prime / NTRU LPRime: 653 / 761 / 857

p and q prime; $R := \mathbb{Z}[x]/(x^p - x - 1)$

small: a ternary polynomial from R

short: **small** with exactly w nonzero coefficients

$R/3 := (\mathbb{Z}/3)[x]/(x^p - x - 1)$ and $R/q := (\mathbb{Z}/q)[x]/(x^p - x - 1)$

Alice

Bob

KeyGen

small g s.t. g^{-1} in $R/3$ exists

short $f \Rightarrow$ compute $f_{\text{inv}} := (3f)^{-1}$ in R/q

$h \leftarrow g \times f_{\text{inv}}$ in R/q

public key h

Encap

short r as the session key

$c \leftarrow \text{Round}(h \times r)$ in R/q

ciphertext c

Decap

$e \leftarrow c \times 3f$ in R/q

$r \leftarrow e \times g^{-1}$ in $R/3$

Interesting Multiplication in NTRU Prime

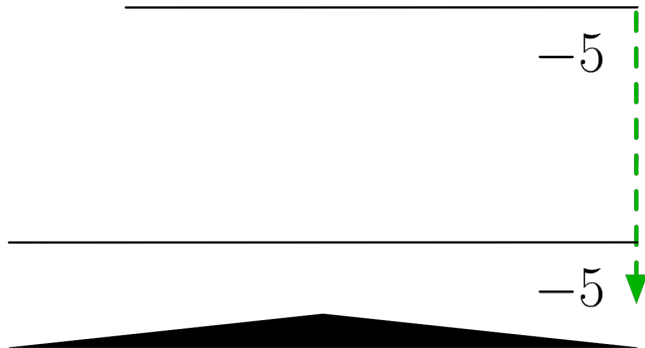
❖ The Product Scanning Method

- Inputs: known c in R/q and secret **short** f
- Output: $e = (c \times f) \bmod q$
- Decap / Encap / KeyGen (**only** `ntrulpr*`)

$$\begin{array}{r} -2x^2 + 3x - 5 \\ \times) -1x^2 + 0x + 1 \\ \hline \end{array}$$

-5

-5



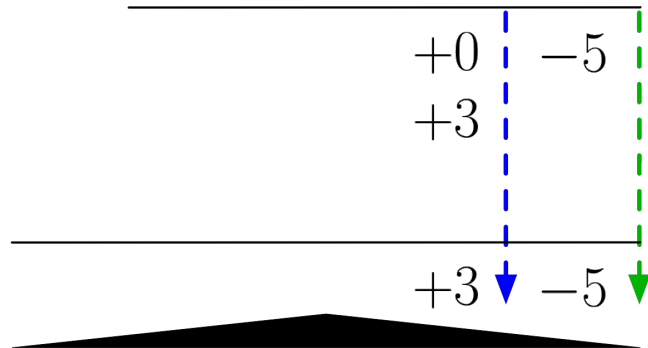
Side-Channel Informative:

Least

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Side-Channel Informative:


More

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$$\begin{array}{r}
 -2x^2 + 3x - 5 \\
 \times) \quad -1x^2 + 0x + 1 \\
 \hline
 +5 \quad +0 \quad -5 \\
 +0 \quad +3 \quad \\
 -2 \quad \quad \quad \\
 \hline
 +3 \downarrow \quad +3 \downarrow \quad -5 \downarrow
 \end{array}$$


Most

Side-Channel Informative:

Interesting Multiplication in NTRU Prime

❖ The Product Scanning Method

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$$\begin{array}{r}
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 \times) \quad -1x^2 + 0x + 1 \\
 \hline
 \begin{array}{cccc}
 & +5 & +0 & -5 \\
 -3 & +0 & +3 & \\
 +0 & -2 & & \\
 \hline
 -3 & +3 & +3 & -5
 \end{array}
 \end{array}$$

Side-Channel Informative:

Less

Interesting Multiplication in NTRU Prime

❖ The Product Scanning Method

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↓ ↓ ↓ ↓ ↓

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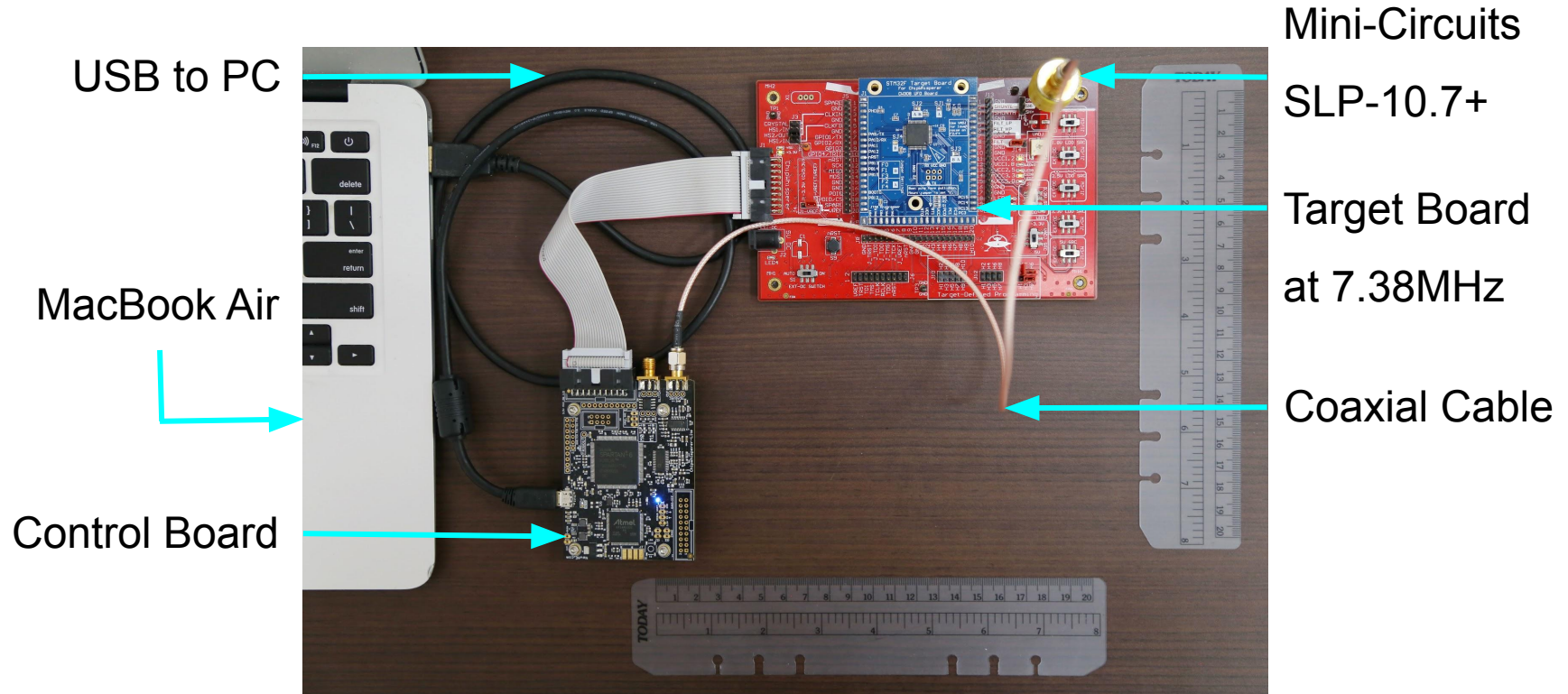
Experiment Settings

- ❖ `snttrup761 Decap` on ARM Cortex-M4
 - $p = 761$, $q = 4591$, and $w = 286$
 - on STM32F303RCT7 and STM32F415RGT6

Experiment Settings

- ❖ `sntrup761 Decap` on ARM Cortex-M4
 - $p = 761$, $q = 4591$, and $w = 286$
 - on STM32F303RCT7 and STM32F415RGT6
- ❖ ChipWhisperer-Lite Two-Part Version
 - random input generation + measurement + data collection
- ❖ Statistical Analysis: in Python 3.6.1 or C++ on a MacBook Air

STM32F415RGT6 + ChipWhisperer-Lite



- Low Pass Filter - Mini Circuits. <https://www.minicircuits.com/pdfs/SLP-10.7+.pdf>

Power Analysis Methods

CPA: Correlation Power Analysis

SPA: Simple Power Analysis

- ❖ Vertical CPA: robust and fast
- ❖ Horizontal In-Depth CPA: using one single short trace
- ❖ Online Template Attacks: fast profiling with few template traces
- ❖ Chosen-Input SPA: with the naked eye

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Vertical CPA

e_{p-1} initialized to be 0

$$\begin{array}{l}
 + c_0 \times 0 \\
 \vdots \\
 + c_{p-1-b_w} \times f_{b_w} \\
 + c_{p-1-b_w+1} \times 0 \\
 \vdots \\
 + c_{p-1-b_{w-1}-1} \times 0 \\
 + c_{p-1-b_{w-1}} \times f_{b_{w-1}} \\
 + c_{p-1-b_{w-1}+1} \times 0 \\
 \vdots \\
 + c_{p-1-b_1-1} \times 0 \\
 + c_{p-1-b_1} \times f_{b_1} \\
 \vdots \\
 + c_{p-1} \times 0
 \end{array}$$

$$e_{p-1,1} = (c_{p-1-b_w} \times f_{b_w}) \bmod q$$

$$e_{p-1,2} = (c_{p-1-b_w} \times f_{b_w} + c_{p-1-b_{w-1}} \times f_{b_{w-1}}) \bmod q$$

\approx

$$\begin{aligned}
 e_{p-1,w} = & (c_{p-1-b_w} \times f_{b_w} \\
 & + c_{p-1-b_{w-1}} \times f_{b_{w-1}} \\
 & \vdots \\
 & + c_{p-1-b_1} \times f_{b_1}) \bmod q
 \end{aligned}$$

Vertical CPA

Reveal f_{b_w} and $f_{b_{w-1}}$
at a time.

e_{p-1} initialized to be 0

$$\begin{array}{l}
 + c_0 \times 0 \\
 \vdots \\
 + c_{p-1-b_w} \times f_{b_w} \\
 + c_{p-1-b_w+1} \times 0 \\
 \vdots \\
 + c_{p-1-b_{w-1}-1} \times 0 \\
 + c_{p-1-b_{w-1}} \times f_{b_{w-1}} \\
 + c_{p-1-b_{w-1}+1} \times 0 \\
 \vdots \\
 + c_{p-1-b_1-1} \times 0 \\
 + c_{p-1-b_1} \times f_{b_1} \\
 \vdots \\
 + c_{p-1} \times 0
 \end{array}$$

likely to confuse with c_{p-1-b_w}

$$e_{p-1,1} = (c_{p-1-b_w} \times f_{b_w}) \bmod q$$

$$e_{p-1,2} = (c_{p-1-b_w} \times f_{b_w} + c_{p-1-b_{w-1}} \times f_{b_{w-1}}) \bmod q$$

\approx

$$\begin{aligned}
 e_{p-1,w} = & (c_{p-1-b_w} \times f_{b_w} \\
 & + c_{p-1-b_{w-1}} \times f_{b_{w-1}} \\
 & \vdots \\
 & + c_{p-1-b_1} \times f_{b_1}) \bmod q
 \end{aligned}$$

Vertical CPA

Reveal f_{b_w} and $f_{b_{w-1}}$
at a time.

Reveal $f_{b_{w-2}}, \dots, f_{b_1}$
one by one.

e_{p-1} initialized to be 0

$$\begin{array}{l}
 + c_0 \times 0 \\
 \vdots \\
 + c_{p-1-b_w} \times f_{b_w} \\
 + c_{p-1-b_w+1} \times 0 \\
 \vdots \\
 + c_{p-1-b_{w-1}-1} \times 0 \\
 + c_{p-1-b_{w-1}} \times f_{b_{w-1}} \\
 + c_{p-1-b_{w-1}+1} \times 0 \\
 \vdots \\
 + c_{p-1-b_1-1} \times 0 \\
 + c_{p-1-b_1} \times f_{b_1} \\
 \vdots \\
 + c_{p-1} \times 0
 \end{array}$$

likely to confuse with c_{p-1-b_w}

$$e_{p-1,1} = (c_{p-1-b_w} \times f_{b_w}) \bmod q$$

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\approx

$$\begin{array}{l}
 e_{p-1,w} = (c_{p-1-b_w} \times f_{b_w} \\
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 \vdots \\
 + c_{p-1-b_1} \times f_{b_1}) \bmod q
 \end{array}$$

In-Depth CPA

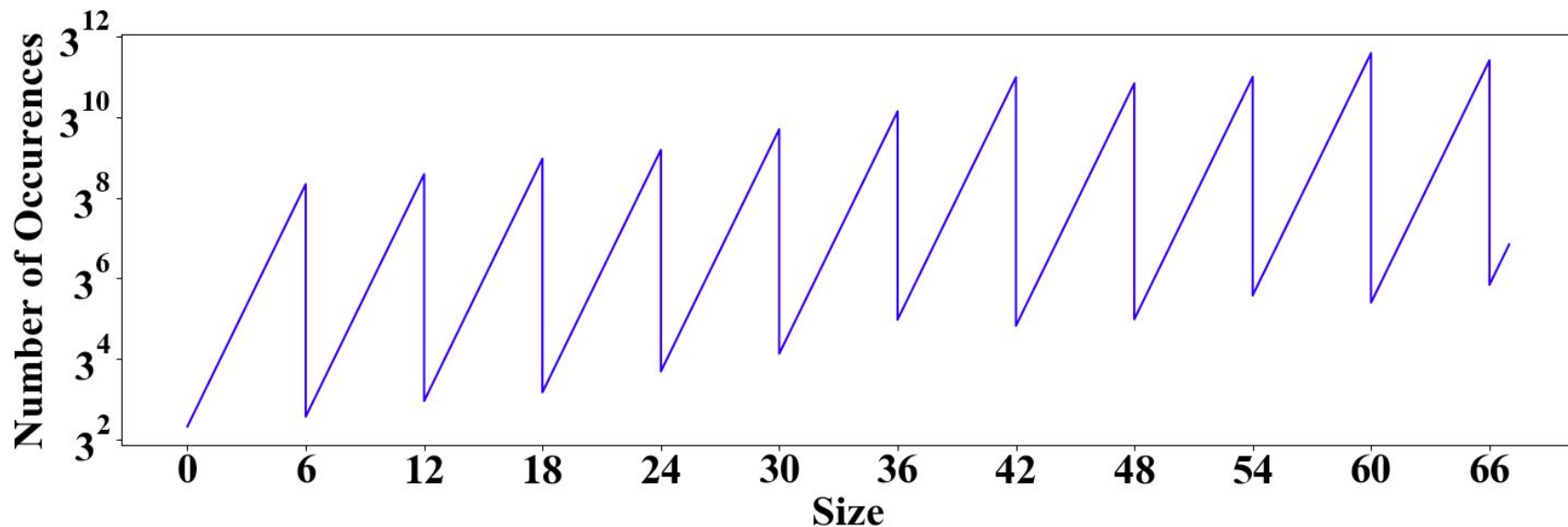
- ❖ Vertical CPA: one coefficient at a time with multiple short traces
 - How to squeeze more information from each short trace?

In-Depth CPA

- ❖ Vertical CPA: one coefficient at a time with multiple short traces
 - How to squeeze more information from each short trace?
- ❖ In-Depth CPA: multiple coefficients at a time with one short trace
 - The intermediate state of e_{p-1} depends on the current (c_j, f_{p-1-j})
 - and **all the previous** (c_j, f_{p-1-j}) . → Extend-and-Prune

Candidate Pruning

❖ Block Size $m = 67$ + Pruning Period $n = 6$



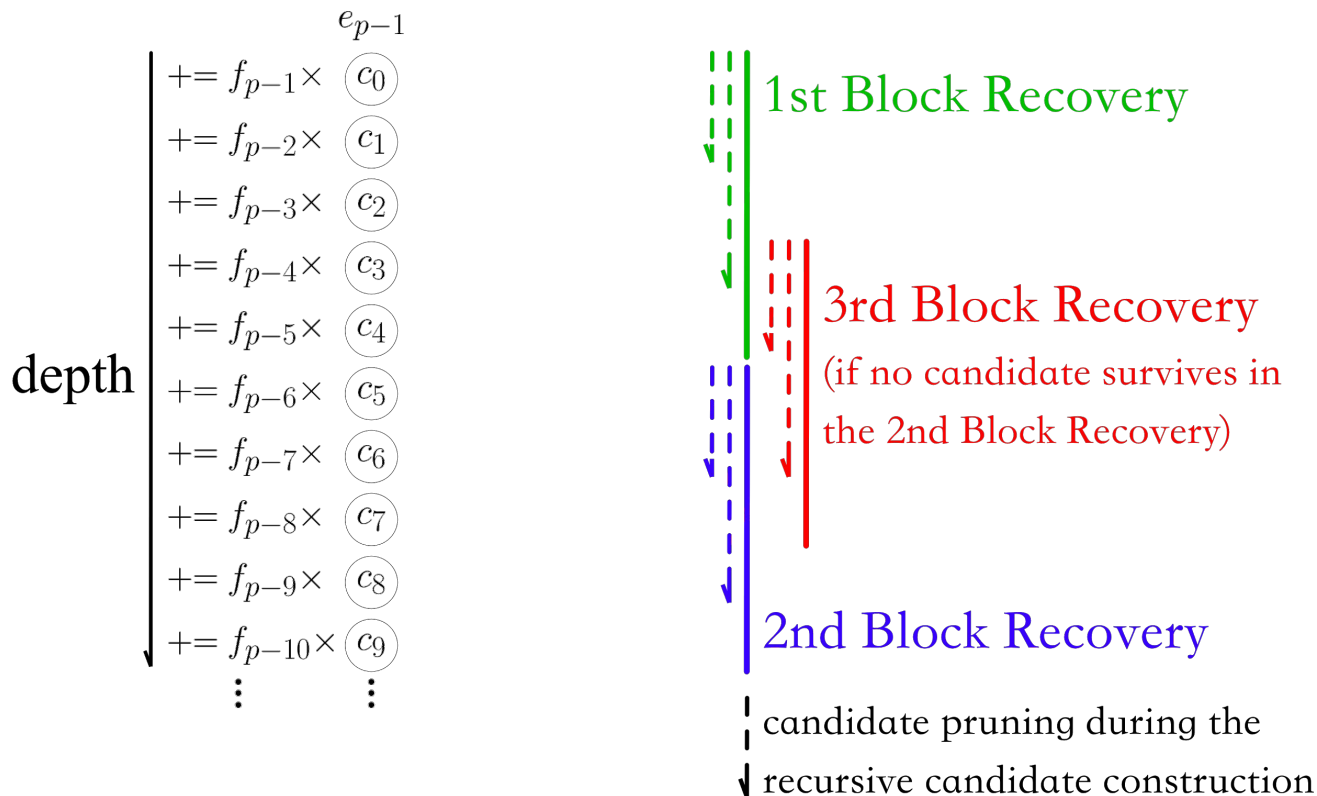
Tail-Error Removal

- ❖ Tail Errors: at the end of the block
 - In the current block recovery, the correlation still looks great.
 - In the next block recovery, no hypotheses survive.

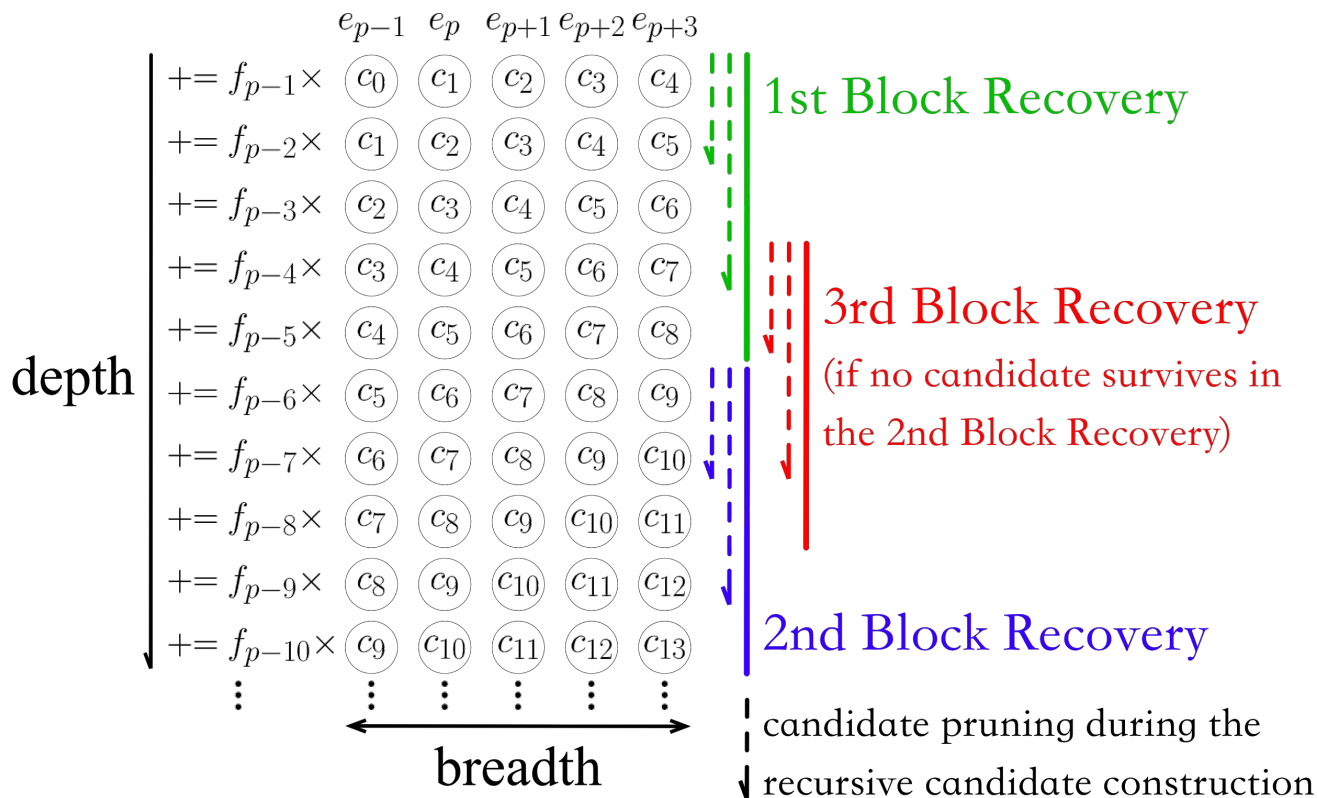
Tail-Error Removal

- ❖ Tail Errors: at the end of the block
 - In the current block recovery, the correlation still looks great.
 - In the next block recovery, no hypotheses survive.
- ❖ Roll Back: by half a block
 - once no hypotheses survive in the current block recovery.
 - tail errors in the final block → exhaustive search

A Toy Example: $m = 5$, $n = 2$, $l = 5$



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Horizontal In-Depth CPA (HIDCPA)

- ❖ In-Depth CPA: inefficient and inaccurate
 - every m coefficients mapped to only m samples
 - the lack of data → ineffective candidate pruning

Horizontal In-Depth CPA (HIDCPA)

- ❖ In-Depth CPA: inefficient and inaccurate
 - every m coefficients mapped to only m samples
 - the lack of data \rightarrow ineffective candidate pruning
- ❖ Learn from horizontal attacks!
 - Observe the calculation of $e_{p-1}, e_p, \dots, e_{p-2+l}$.
 - For $l \ll p$, we have **nearly l times as many data**.

A Real-World Example: $m = 67$, $n = 6$, $l = 5$

The
Top

```
Candidate 25 -> bestCorr = -0.976936
f_559 ~ f_493: [0, 0, 1, -1, 0, 0, -1, 1, 1, 0, 0, 0, 1, -1, 0, 0, -1, 0, 1,
0, -1, 0, -1, -1, -1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0,
1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, -1, 0, -1, 0, -1, 0, 0, 0, 0, 0, 0, 1, -1]
Candidate 26 -> bestCorr = -0.983238
f_559 ~ f_493: [0, 0, 1, -1, 0, 0, -1, 1, 1, 0, 0, 0, 1, -1, 0, 0, -1, 0, 1,
0, -1, 0, -1, -1, -1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0,
1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, -1, 0, -1, 0, -1, 0, 0, 0, 0, 0, 0, 1, 0]
42 candidates reach the final comparison.

- - - - -
Now start with f_492
- - - - -
0 candidates reach the final comparison.

- - - - -
Now start with f_525
- - - - -
Candidate 1 -> bestCorr = -0.946477
f_525 ~ f_459: [0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
-1, 0, -1, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, -1, 0, 0, -1, 0, 0, 1,
0, -1, 1, 0, 0, 0, 0, 0, -1, 0, 0, 0, -1, 0, 0, -1, 1, -1, 1, 0, 1, 1, -1]
```

Tail Error

A Real-World Example: $m = 67$, $n = 6$, $l = 5$

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42 candidates reach the final comparison.
```

The Middle

```
- - - - -
Now start with f_492
- - - - -
0 candidates reach the final comparison.
```

```
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Candidate 1 -> bestCorr = -0.946477
f_525 ~ f_459: [0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
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A Real-World Example: $m = 67$, $n = 6$, $l = 5$

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The Bottom

Corrected

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Template Attacks

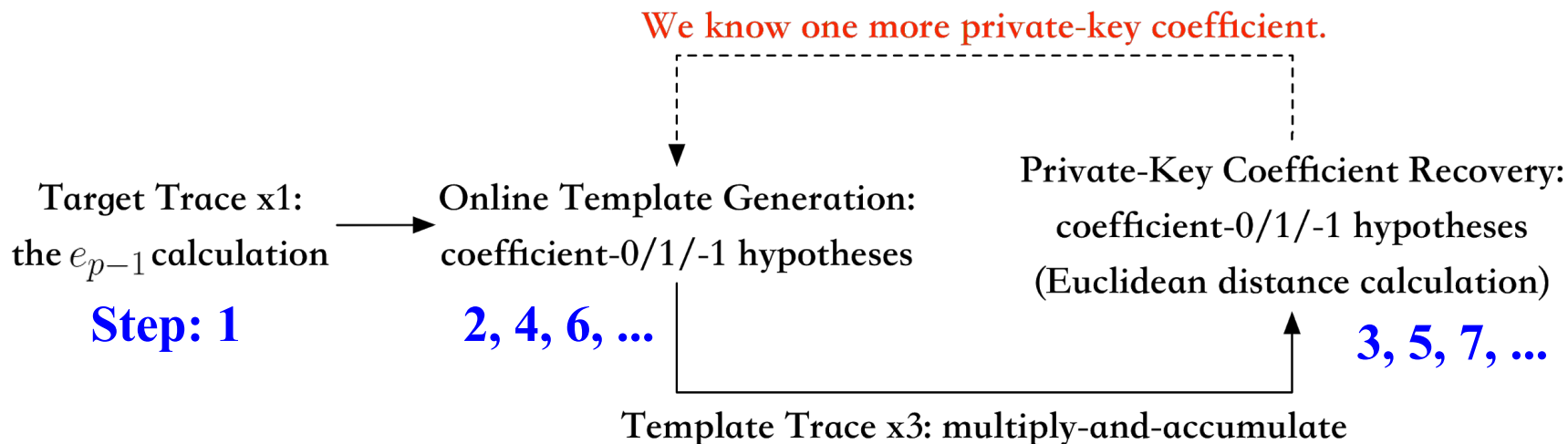
- ❖ What if the assumption of simple power models fails?
 - Classical Correlation Attacks: the Hamming weight/distance models
 - Classical Template Attacks: multivariate normal distribution

Template Attacks

- ❖ What if the assumption of simple power models fails?
 - Classical Correlation Attacks: the Hamming weight/distance models
 - Classical Template Attacks: multivariate normal distribution
- ❖ The Profiling Stage
 - numerous template traces + heavy computational power

Can we mount template attacks with **few template traces**?

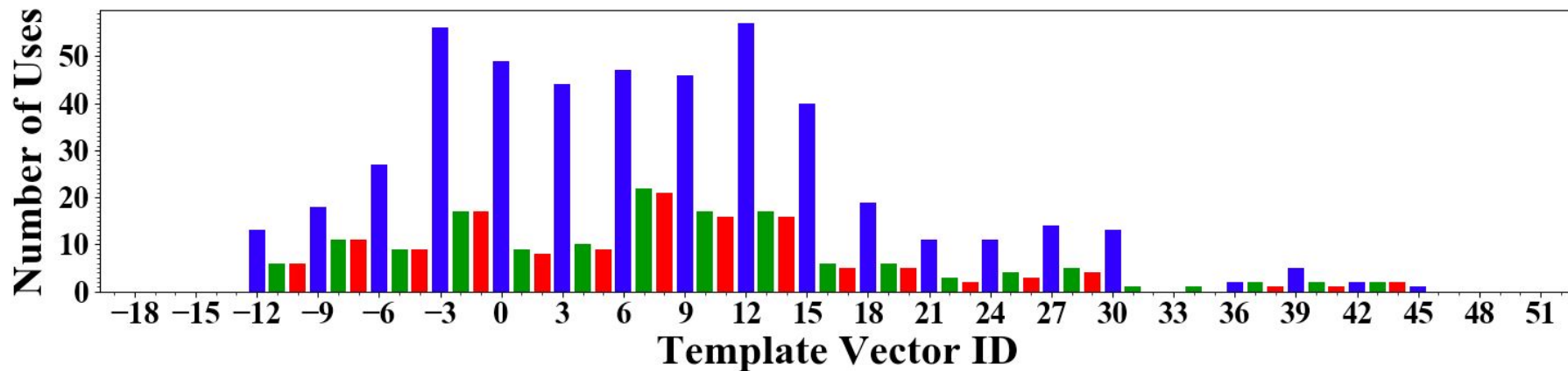
Online Template Attacks (OTA)



Can we mount template attacks with **fewer executions**?

The Chosen-Input Offline Variant

- ❖ Chosen-Input: $c_0 = c_1 = \dots = c_{p-1}, 3 \mid c_0$
 - enhancing the reusability of template traces



The Chosen-Input **Offline** Variant

- ❖ Illegitimate Private Key: f^* on the template generator
 - generating all the required template traces **within four executions**
 - $c^* = c + (c_1^* - c_0)x$ and e_{p-1} expressed as $c_0 \times t \bmod q$

$c_1^* \backslash f^*$	$x + x^3 + x^5 + \dots + x^{p-2}$	$-x - x^3 - x^5 - \dots - x^{p-2}$
$c_0 \times (-w) \bmod q$	$[+]$ for $t = -w, \dots, -1, 0$ $[\times]$ for $t = -w, \dots, -1, 0$	$[\times]$ for $t = 0, 1, \dots, w$ $[-]$ for $t = 0, 1, \dots, w$
c_0	$[+]$ for $t = 1, 2, \dots, w$ $[\times]$ for $t = 1, 2, \dots, w$	$[\times]$ for $t = -w, \dots, -2, -1$ $[-]$ for $t = -w, \dots, -2, -1$

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Not Countermeasures

- ❖ Apply a random mask to each output coefficient.
 - integer offsets added at the beginning and removed at the end
- ❖ Shuffle multiply-and-accumulates for each output coefficient.
 - input-coefficient pairs accessed in a random order

Not Countermeasures

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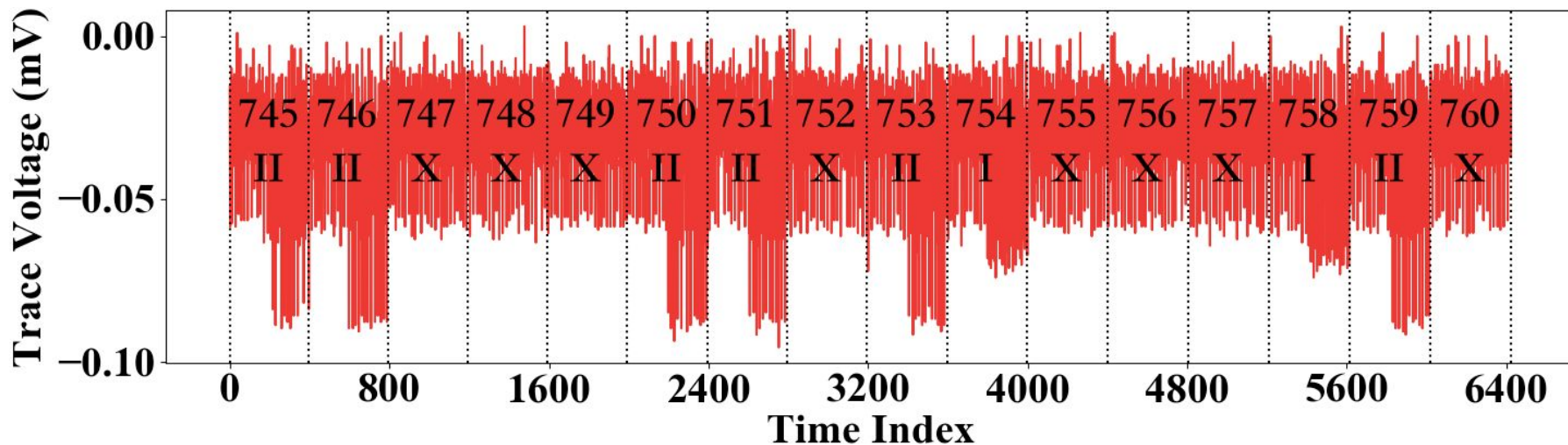
subject to **chosen-input SPA**
(CISPA)
- ❖ Shuffle multiply-and-accumulates for each output coefficient.
 - input-coefficient pairs accessed in a random order

subject to **chosen-input SPA**
(CISPA)

CISPA on Countermeasure 2

$c = c_0$, where $c_0 \neq 0$ and $3 \mid c_0$

one sample per 64 clock cycles

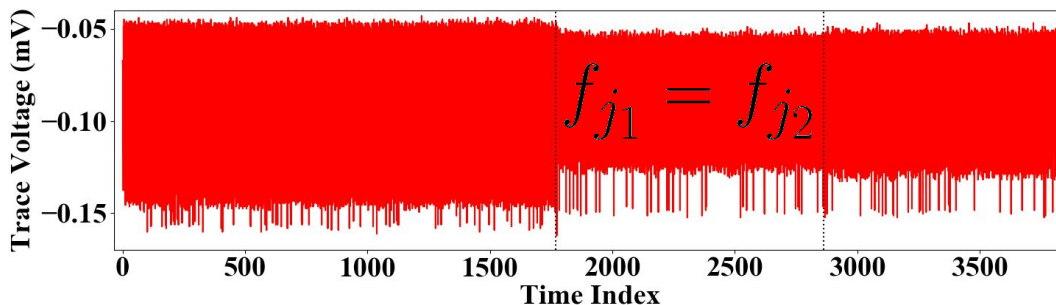
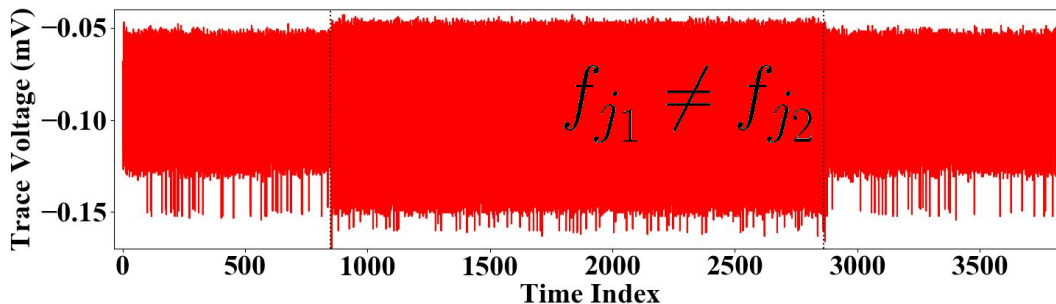


CISPA on Countermeasure 1

- ❖ Two Stages: nonzero f_j identification + clustering
- ❖ The First Stage: continuous? discontinuous?
 - similar to CISPA on Countermeasure 2
 - Zero or Nonzero: output coefficient \rightarrow private-key coefficient

CISPA on Countermeasure 1

- ❖ The Second Stage: $c = c_0 x^{p-1-j_1} + c_0 x^{p-1-j_2}$
 - knowing $f_{j_1} \neq 0$ and $f_{j_2} \neq 0$ + observing the e_{p-1} calculation



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Two Optimizations and One Countermeasure

❖ Optimized Product Scanning

- Modular Reduction: per multiply-and-accumulate → per e_i calculation
- SMLABB → SMLADX: two multiply-and-accumulates per instruction
- 4.4x faster / immune to OTA / still subject to HIDCPA and CISPA

Two Optimizations and One Countermeasure

❖ Optimized Product Scanning

- Modular Reduction: per multiply-and-accumulate → per e_i calculation
- SMLABB → SMLADX: two multiply-and-accumulates per instruction
- 4.4x faster / immune to OTA / still subject to HIDCPA and CISPA

❖ First-Order Masking: both inputs masked

- If the ciphertext not masked: horizontal CPA
- If the private key not masked: SPA or profiling attacks (potentially)

Conclusion

- ❖ Single-Trace Power Analysis on the Product Scanning Method
 - applicable to **NTRU Prime Decap/Encap/KeyGen**
 - targeting the reference/protected/optimized implementations
 - with short observation span, few template traces, or the naked eye

Conclusion

- ❖ Single-Trace Power Analysis on the Product Scanning Method
 - applicable to **NTRU Prime Decap/Encap/KeyGen**
 - targeting the reference/protected/optimized implementations
 - with short observation span, few template traces, or the naked eye

- ❖ Potential Applications
 - other ideal-lattice-based cryptosystems with
 - private/session-key coefficients from **a small set of possibilities**
 - multi-level Karatsuba ending with the product scanning method

Q & A