# CCA-Secure (Puncturable) KEMs from Encryption With Non-Negligible Decryption Errors

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- Main Definitions and Models
- Decryption Error
- Our Compiler
- Evaluation
- Further Results & Conclusions

## Main Definition and Models

## Confidentiality = Indistinguishability



Public key encryption allows two parties to communicate securely even when no prior secret shared key is available to them.

It is extremely useful for establishing secure communications over the Internet: e.g., the TLS protocol.

#### Definition

We say that a public-key encryption (PKE) scheme  $\Pi = ($ KeyGen, Enc, Dec) is perfectly correct if the following holds:

for every message M ∈ M, for every pair (pk, sk) generated by KeyGen on input λ, and all possible coin tosses of Enc and Dec, it should hold that Dec(sk, Enc(pk, M)) = M.

Analogous definition for key encapsulation mechanisms (KEMs).





## Security Games: IND-CPA







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#### Security Games: IND-CCA



#### Definition

The adversary advantage in game  $\mathbf{x} \in \{\text{IND-CPA}, \text{IND-CCA}\}$ , is:

$$Adv^{x}(\mathcal{A}) = \left| \Pr[\mathcal{A} \text{ wins in the } x \text{ game}] - \frac{1}{2} \right|$$

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Such an attack enables an adversary to completely recover the original message for any ciphertext of its choice!

## **Decryption Error**

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When the ability to correctly decrypt valid ciphertexts is dependent on the secret key, the result of the decryption process can leak information about the secret key (e.g., [BS20] and [DRV20]).

Decryption error can be naively decreased by increasing the parameters of the PKE.

- security (e.g., different ways of sampling error in lattice-based PKEs),
- efficiency,
  - size (e.g., public-key, ciphertext),
  - runtime,
- decryption error.

#### Definition

A PKE  $\Pi$  is DNR- $\delta(\cdot)$ -correct if we have that

 $\Pr[\operatorname{Dec}(\operatorname{sk}, \operatorname{Enc}(\operatorname{pk}, M)) \neq M] \leq \delta(\lambda),$ 

where the probability is taken over the choice of keypairs  $(pk, sk) \leftarrow KeyGen(\lambda)$ ,  $M \in \mathcal{M}$ , and over the random coins of Enc and Dec.

#### Definition

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$$\mathbb{E}\left[\max_{\mathsf{M}\in\mathcal{M}}\mathsf{Pr}[\mathsf{C}\leftarrow\mathsf{Enc}(\mathsf{pk},\mathsf{M}):\mathsf{Dec}(\mathsf{sk},\mathsf{C})\neq\mathsf{M}]\right]\leq\delta(\lambda),$$

where the expected value is taken over  $(pk, sk) \leftarrow KeyGen(\lambda)$ .

Taking the maximum over all possible messages we obtain an upper-bound for the "decryption error" of any single message.

Our Compiler

### Main Idea

- start from a IND-CPA secure PKE with non-negligible correctness error (e.g., 128 bit security level, error  $> 2^{-128}$ ):
  - easier to construct,
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• transform it into an IND-CCA secure PKE/KEM, preserving the negligible correctness error.

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- · majority vote is needed to decide which message obtain is the correct one,
- $\cdot\,$  even if the underlying PKE is IND-CCA secure, the so obtained PKE' is not.

δ	δ'(2)	$\delta'(3)$	$\delta'(4)$
<b>2</b> <sup>-32</sup>	$pprox$ 2 $^{-32}$	$pprox$ 2 $^{-63}$	$pprox$ 2 $^{-94}$
$2^{-64}$	$pprox$ 2 $^{-64}$	$pprox 2^{-127}$	$pprox 2^{-190}$
2 <sup>-96</sup>	$pprox 2^{-96}$	$pprox 2^{-191}$	$pprox 2^{-284}$

**Table 1:** Estimation of the correctness error for the direct product compilers.  $\delta'(\ell)$  denotes the correctness error for  $\ell$  ciphertexts.

 rPKE
 FO transform
 KEM

 IND-CPA
 IND-CCA
 IND-CCA

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The FO transform requires negligible correctness error of the underlying PKE.

We compute  $\ell$  independent encryptions of the same message M under the same public key **pk** using randomness **G**(M, i),  $i \in [\ell]$ , where **G** is a RO (random oracle).
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During the decryption correctness of the message can be checked via the de-randomization: it allows us to control if the ciphertext was modified and to get rid of the majority vote.

### The Transformation T\* (2/2)

$\Pi'.Enc(pk,M)$	$\Pi'.Dec(sk,C)$
for $i = 1, \ldots, \ell$ do	$\texttt{res} \leftarrow \bot, \texttt{check} \leftarrow \bot$
$C_i := \Pi.Enc(pk, M; G(M, i))$	for $i = 1, \dots, \ell$ do
$C := (C_1, \ldots, C_\ell)$	$\texttt{res}[i] := \varPi.Dec(sk, C_i)$
$\mathbf{return}\ C$	for $i \in [\ell]$ s.t. $\operatorname{res}[i] \neq \bot$ do
	if $\forall j \in [\ell] : C_j = \Pi.Enc(pk, res[i], G(res[i], j))$
	$\texttt{check} \leftarrow i$
	$\texttt{if check} \neq \bot$
	return res[check]
	$\mathbf{return} \perp$

To the resulting PKE  $\Pi'$  we can then directly apply the transformation  $U^{\perp}$  from the modular analysis of the FO transform [HHK17], to obtain an IND-CCA secure KEM with negligible correctness error in the (Q)ROM.

	pk	<i>C</i>	KeyGen	Enc	Dec
C <sub>p,y</sub>	1 (r) / $\ell$ (d)	$\ell$	1 (r) / ℓ (d)	l	$\ell$
C <sup>*</sup> <sub>p,d</sub> T*	$\ell'$	$\ell'$	$\ell'$	$\ell'$	$\ell'$
T*	1	$\ell'$	1	$\ell'$	$\ell^{\prime 2}$ / $\ell^{\prime}$ ( $\perp$ )

Table 2: Comparison of the runtime and bandwidth overheads of  $C_{p,y}$ ,  $y \in \{r, d\}$ , with  $\ell$  ciphertexts and  $T^*$  and  $C^*_{p,d}$  with  $\ell'$  ciphertexts such that  $\ell \ge \ell' + 1$ .

### Evaluation

#### NIST Post-Quantum Competition

- Important competitions for cryptographic schemes in the past: AES, SHA-1, SHA-3;
- Now running a Post-Quantum Cryptography Standardization project: Signatures and PKE/KEMs.



How does increasing from 1 to  $\ell$  ciphertexts compare to increasing the parameters at comparable resulting decryption errors for (existing) round-2 submissions in the NIST PQC?

N.B. Our work took place before Round-3 started.

## Code-based PKEs/KEMs

Encryption/KEMs	assumption	problem
Classic McEliece	codes	Goppa
NTS-KEM	codes	Goppa
BIKE	codes	short Hamming
HQC	codes	short Hamming
LEDAcrypt	codes	short Hamming
ROLLO	codes	low rank
RQC	codes	low rank

Encryption/KEMs	assumption	problem
Classic McEliece NTS-KEM	codes codes	Goppa Goppa
BIKE <sup>1</sup>	codes	short Hamming
HQC	codes	short Hamming
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ROLLO	codes	low rank
RQC	codes	low rank

<sup>1</sup>BIKE is a Round-3 Alternate Candidate.

KEM	δ	pk	С	Σ		KeyGen	Encaps	Decaps
<b>O</b> [ROLLO-I-L1,5] ROLLO-II-L1	2 <sup>—150</sup> 2 <sup>—128</sup>	<b>465</b> 1546	2325 1674	2790 3220		0.10 0.69	0.02 /0.10 0.08	0.26 /1.30 0.53
<b>O</b> [ROLLO-I-L3,4] ROLLO-II-L3	2 <sup>-128</sup> 2 <sup>-128</sup>	<b>590</b> 2020	2360 2148	<b>2950</b> 4168		0.13 0.83	<b>0.02</b> / <b>0.08</b>	0.42 /1.68 0.69
<b>O</b> [ROLLO-I-L5,4] ROLLO-II-L5	2 <sup>-168</sup> 2 <sup>-128</sup>	947 2493	7576 2621	8523 5114		<b>0.20</b> 0.79	0.03 /0.12 0.10	0.78 /3.12 0.84
O[BIKE-2-L1,3] BIKE-2-CCA-L1	2 <sup>-147</sup> 2 <sup>-128</sup>	<b>10163</b> 11779	30489 12035	40652 23814		4.79 6.32	0.14 /0.42 0.20	<b>3.29</b> /9.88 4.12

**Table 3:** Sizes (in bytes) and runtimes (in ms and millions of cycles for BIKE), where O denotes the transformed scheme. Runtimes are taken from the optimized implementations, if available, and are only intra-scheme comparable.

## Lattice-based PKEs/KEMs

### Round 2 Submissions (2/2)

Encryption/KEMs	assumption	problem
Crystals-Kyber	lattice	MLWE
Saber	lattice	MLWR
FrodoKEM	lattice	LWE
Round 5	lattice	LWR
LAC	lattice	RLWE
NewHope	lattice	RLWE
Three Bears	lattice	IMLWE
NTRU	lattice	NTRU
NTRUprime	lattice	NTRU

#### Round 2 Submissions (2/2)

Encryption/KEMs	assumption	problem
Crystals-Kyber	lattice	MLWE
Saber	lattice	MLWR
FrodoKEM <sup>2</sup>	lattice	LWE
Round 5	lattice	LWR
LAC	lattice	RLWE
NewHope	lattice	RLWE
Three Bears	lattice	IMLWE
NTRU	lattice	NTRU
NTRUprime	lattice	NTRU

<sup>2</sup>FrodoKEM is a Round-3 Alternate Candidate.

KEM	δ	pk	С	Σ	KeyGen	Encaps	Decaps
<b>O</b> [R5N1-3-PKE-cpa,2]	2 <sup>-130</sup> 2 <sup>-144</sup>	<b>8834</b>	17732	26566	<b>6.69</b>	10.10 /20.20	10.38 /20.75
R5N1-3-KEM-cca		9660	9732	19392	6.78	10.20	10.60
<b>O</b> [FrodoCCS-Rec.,4]	2 <sup>-155</sup>	11280	45152	56432	2.94	3.48/13.94	10.79/43.16
FrodoKEM-640-AES	2 <sup>-138</sup>	9616	9720	19336	1.38	1.86	1.75

**Table 4:** Sizes (in bytes) and runtimes, where O denotes the transformed scheme. FrodoCCS refers to the FrodoKEM version precedent to the NIST competition. Runtimes are taken from the optimized implementations if available.

Further Results & Conclusions

Bloom Filter KEMs:

- recent primitive proposed by Derler et al. [Der+18],
- building block to construct fully forward-secret O-RTT key exchange protocols [Gün+17],
- required perfect decryption of underlying building block (hinders post-quantum instantiations).

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We extended the work generically and showed that one can construct BFKEMs from any IBE and even base it upon IBEs with a (non-)negligible correctness error:

• first post-quantum CCA-secure BFKEM.

- generic way to deal with the error from weaker schemes (i.e., IND-CPA secure ones with non-negligible error) which are easier to design,
- all involved algorithms are easily parallelizable,
- our approach performs well in context of code-based schemes but gives less advantage for lattice-based ones.
- first post-quantum CCA-secure Bloom Filter KEM

- extending analysis to other constructions?
- code- VS lattice-based schemes: why the compiler performs so differently?

# Thank you for your attention!

(full version of the Asiacrypt'20 paper to appear soon on ePrint)



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