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SILVER: Statistical Independence and Leakage Verification

26th Annual International Conference on the Theory and Application of Cryptology and Information Security, ASIACRYPT 2020

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Motivation

Problem

- **Fragmented view on security notions of masked implementations**
- **Tool with a wide feature range: maskVerif**
  - **Language-based →not in direct conformity with security notions**

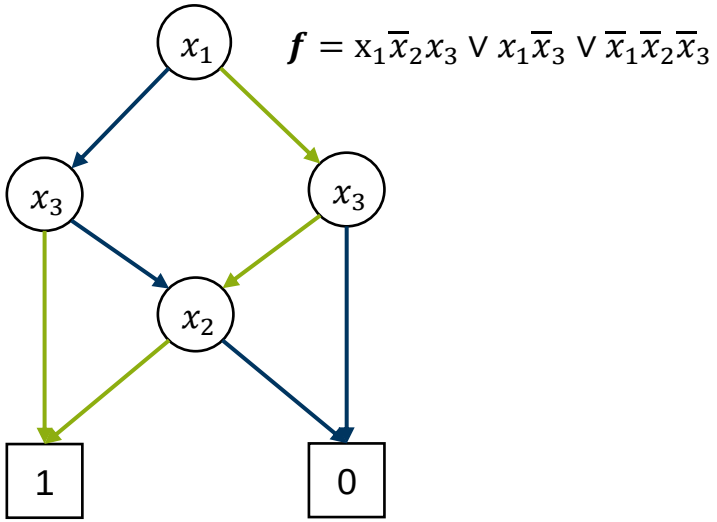
Goal

- **Building a sound and complete verification tool**

Background

ROBDD

- ***Reduced Ordered*** **Binary Decision Diagram**
- **Ordered: Every path from root to leaf traverses every variable only once and in the same order**
- **Reduced: No isomorphic subtrees and no redundant nodes**

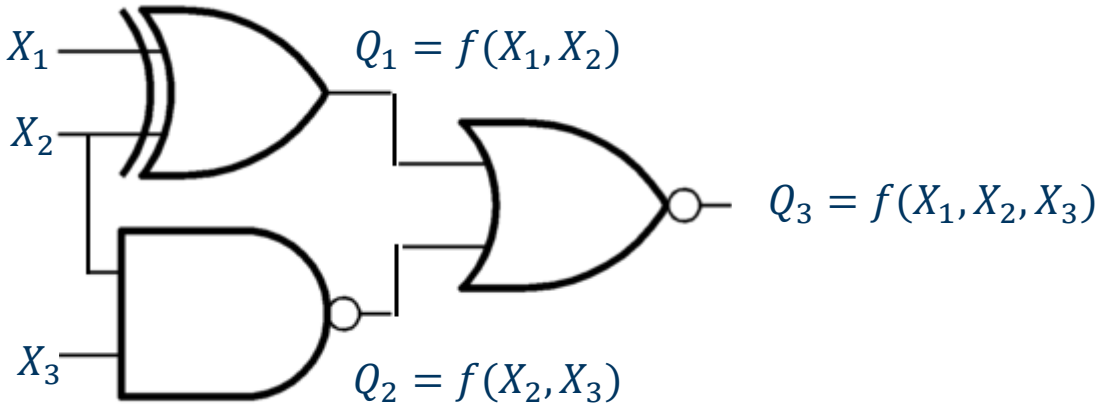


Canonical representation of a boolean function $f: F_2^n \rightarrow F_2$

Background

ROBDD

- **Wires of a circuit can be seen as binary random variables**
- **These wires are functions of the input**
- **Can be efficiently represented as ROBDD**



Background

Statistical Independence

- **Two Events A, B:** $\boldsymbol{Pr}[\boldsymbol{A} \wedge \mathbf{B}] = \mathbf{Pr}[\mathbf{A}] \times \boldsymbol{Pr}[\boldsymbol{B}]$
- **Two Random Variables:** $\boldsymbol{Pr}[\boldsymbol{X} = \boldsymbol{x} \wedge \boldsymbol{Y} = \boldsymbol{y}] = \mathbf{Pr}[\boldsymbol{X} = \boldsymbol{x}] \times \boldsymbol{Pr}[\boldsymbol{Y} = \boldsymbol{y}] \; \forall \boldsymbol{x}, \boldsymbol{y}$

Background

Statistical Independence of Two Binary Random Variables

- **One can show that for $\boldsymbol{X} \in \mathbb{F}_2, \boldsymbol{Y} \in \mathbb{F}_2$:**
  - $\boldsymbol{X}$ and $\boldsymbol{Y}$ are stat. independent $\Leftrightarrow \Pr[X = 1 \wedge Y = 1] = \Pr[X = 1] \times \Pr[Y = 1]$
- **Simplifies verification to check of one event combination instead of four**

Background

Statistical Independence of Joint Distributions over Binary Random Variables

- **We showed that for $\boldsymbol{X} \in \mathbb{F}_2^{\boldsymbol{n}}$, $\boldsymbol{Y} \in \mathbb{F}_2^{\boldsymbol{m}}$:**
  - $\boldsymbol{X}$ and $\boldsymbol{Y}$ are stat. independent $\Leftrightarrow \forall\, \boldsymbol{X}' \subseteq \mathbf{X}, \mathbf{Y}' \subseteq \mathbf{Y}$: $\Pr[\boldsymbol{X}' = (1, \dots, 1) \wedge \boldsymbol{Y}' = (1, \dots, 1)] = \Pr[\boldsymbol{X}' = (1, \dots, 1)] \times \Pr[\boldsymbol{Y}' = (1, \dots, 1)]$
  - Example: $\boldsymbol{X} = (X_1, X_2), \boldsymbol{Y} = (Y_1, Y_2)$
    - $\Pr[X_i = 1 \wedge Y_j = 1] \stackrel{?}{=} \Pr[X_i = 1] \times \Pr[Y_j = 1]$
    - $\Pr[X_i = 1 \wedge \boldsymbol{Y} = (1,1)] \stackrel{?}{=} \Pr[X_i = 1] \times \Pr[\boldsymbol{Y} = (1,1)]$
    - $\Pr[Y_i = 1 \wedge \boldsymbol{X} = (1,1)] \stackrel{?}{=} \Pr[Y_i = 1] \times \Pr[\boldsymbol{X} = (1,1)]$
    - $\Pr[\boldsymbol{Y} = (1,1) \wedge \mathbf{X} = (1,1) \stackrel{?}{=} \Pr[\boldsymbol{Y} = (1,1)] \times \Pr[\boldsymbol{X} = (1,1)]$

Background

Reduction to Check of Binary Random Variables

- $\Pr[\boldsymbol{X} = (X_1, X_2, X_2) = (1,1,1)] = \Pr[\underbrace{X_1 X_2 X_2}_{\text{simple binary random variable}} = 1]$

- **We reduced the check of independence for *joint distribution of random variables* to the check of *simple binary random variables***
  - *Can be efficiently realized utilizing ROBDDs*

Application to Formal Verification of Masked Implementation

Approach

- **Redefine common security notions by the means of statistical independence of joint distributions over binary random variables**
- **Check this independence utilizing BDDs**

Application to Formal Verification of Masked Implementation

ISW d-probing security

- **Unshared function with multiple sensitive inputs, for example:** (X, Y, Z)
- **Every set of probes** $\boldsymbol{Q} = (Q_1, Q_2, \ldots, Q_d)$ **on wires of the shared circuit:**
  - $\boldsymbol{Q}$ is independent of (X, Y, Z)

joint distributions of binary random variables ⇒ utilizing ROBDDS

Application to Formal Verification of Masked Implementation

d-Non-Interference

- **Idea: For each set of probes $\boldsymbol{Q} = (Q_1, Q_2, \dots, Q_d)$ on the circuit, their must exists a simulation set $\boldsymbol{S}$ of input shares and restricted to max. $\boldsymbol{d}$ shares per input, s.t. $\boldsymbol{Q}$ is perfectly simulatable using $\boldsymbol{S}$.**

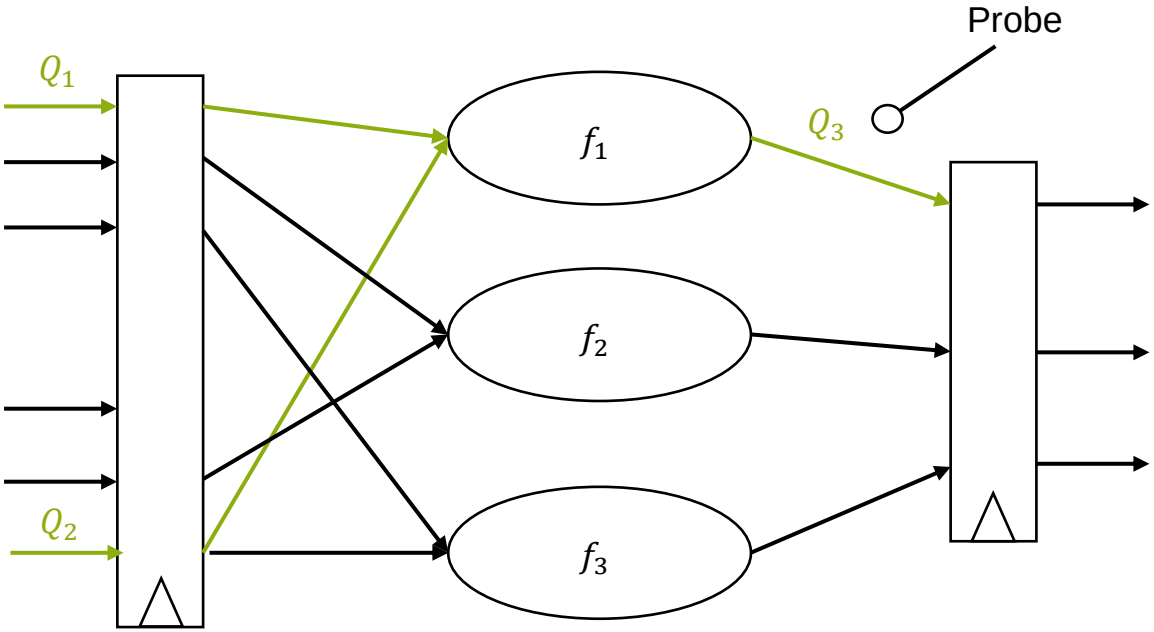
- **Translated into probability distributions:**
  - $\boldsymbol{Pr[Q|S] = Pr[Q|Sh(X)]}$ **where** $\boldsymbol{Sh(X)}$ **consists of all the shared inputs**

- **If all input shares are i.i.d. this simplifies to:**
  - **Independence of** $\boldsymbol{(Q \cup S)}$ **and** $\boldsymbol{Sh(X) \backslash S}$

joint distibutions of binary random variables ⇒ utilizing ROBDDS

Application to Formal Verification of Masked Implementation

Glitch-Extended Probes



$$Q = \{Q_3\} \Rightarrow Q_{extended} = \{Q_1, Q_2\}$$

Application to Formal Verification of Masked Implementation

Uniformity (Single Unshared Output Example, 3 Shares)

- **Uniformity:** $Pr[(Z_1 = z_1, Z_2 = z_2, Z_3 = z_3) \mid Z = z] = p$, $if\ z = z_1 \oplus z_2 \oplus z_3$, **else 0**
- **We showed, that instead we can check:**
  - $(Z_1, Z_2), (Z_1, Z_3), (Z_2, Z_3)$ **are each balanced**
- **Reduces uniformity check to balancedness verification of (joint) output shares**

We generalized this approach even for multiple unshared outputs.

Application to Formal Verification of Masked Implementation

Uniformity

- $Z = (Z_1, Z_2, \dots, Z_m)$ is balanced iff each component function is balanced.

binary random variable ⇒ utilizing ROBDDS

Supported Security Notions

Overview

- **d-probing security**
- **d-Non-Interference (d-NI)**
- **d-Strong-Non-Interference (d-SNI)**
- **d-Probe-Isolating-Non-Interference (d-PINI)**
- **Output Uniformity**

standard (software)
and
glitch-extended (hardware)

Program Flow: SILVER

Overview

- **Netlist is in self-defined format**
- **Result contains for each supported security Notion:**
  - **Security order d**
  - **First failing probe for order (d+1)**

Results

Table of Tested Designs:

Scheme	Pos.†	d	Probing		NI		SNI		PINI		Unif.
			std.	rob.	std.	rob.	std.	rob.	std.	rob.	
Gadgets											
DOM [29]	19	1	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✗[0 ms]	✗[0 ms]	✗[0 ms]	✓[0 ms]
DOM [29]	42	2	✓[3 ms]	✓[4 ms]	✓[6 ms]	✓[19 ms]	✓[8 ms]	✗[0 ms]	✗[0 ms]	✗[0 ms]	✓[0 ms]
DOM [29]	74	3	✓[98 ms]	✓[1.2 s]	✓[2.2 s]	✓[23.7 s]	✓[3.2 s]	✗[0 ms]	✗[0 ms]	✗[0 ms]	✓[0 ms]
DOM SNI [26]	21	1	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✗[0 ms]	✗[0 ms]	✓[0 ms]
DOM SNI [26]	45	2	✓[3 ms]	✓[5 ms]	✓[6 ms]	✓[30 ms]	✓[7 ms]	✓[29 ms]	✗[0 ms]	✗[0 ms]	✓[0 ms]
DOM SNI [26]	78	3	✓[0.1 s]	✓[1.5 s]	✓[2.4 s]	✓[39.4 s]	✓[3.7 s]	✓[39.4 s]	✗[0.0 s]	✗[0.0 s]	✓[0.0 s]
PARA1 [5]	22	1	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✗[0 ms]	✗[0 ms]	✓[0 ms]
PARA2 [5]	45	2	✓[3 ms]	✓[6 ms]	✓[5 ms]	✓[32 ms]	✓[8 ms]	✓[37 ms]	✗[0 ms]	✗[0 ms]	✓[0 ms]
PARA3 [5]	68	3	✓[61 ms]	✓[0.5 s]	✓[1.2 s]	✓[12.1 s]	✗/✓[0.6 s]	✗[0 ms]	✗[0 ms]	✗[0 ms]	✓[0 ms]
PARA3 SNI [5]	82	3	✓[0.2 s]	✓[1.4 s]	✓[2.8 s]	✓[35.5 s]	✓[4.1 s]	✓[40.4 s]	✗[0 ms]	✗[0 ms]	✓[0 ms]
PINI1 [17]	21	1	✓[0 ms]	✗[0 ms]	✓[0 ms]	✗[0 ms]	✓[0 ms]	✗[0 ms]	✓[0 ms]	✗[0 ms]	✓[0 ms]
PINI2 [17]	51	2	✓[7 ms]	✗[0 ms]	✓[10 ms]	✗[0 ms]	✓[12 ms]	✗[0 ms]	✓[22 ms]	✗[0 ms]	✓[0 ms]
HPC1 [16]	22	1	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✗[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]
HPC1 [16]	52	2	✓[5 ms]	✓[7 ms]	✓[7 ms]	✓[23 ms]	✓[9 ms]	✗[0 ms]	✓[16 ms]	✓[46 ms]	✓[0 ms]
HPC2 [16]	32	1	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✗[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]
HPC2 [16]	75	2	✓[6 ms]	✓[12 ms]	✓[11 ms]	✓[37 ms]	✓[13 ms]	✗[0 ms]	✓[19 ms]	✓[61 ms]	✓[0 ms]
ISW SNI REF [26]	26	1	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]
ISW SNI REF [26]	65	2	✓[5 ms]	✓[8 ms]	✓[7 ms]	✓[36 ms]	✓[9 ms]	✓[34 ms]	✓[16 ms]	✓[59 ms]	✓[0 ms]
CMS3 [36]	104	3	✗/✓[0.1 s]	✗/✓[0.3 s]	✗/✓[0.8 s]	✗/✓[2.6 s]	✗/✓[1.3 s]	✗/✓[4.4 s]	✗[0 ms]	✗[0 ms]	✓[0 ms]
UMA2 [36]	81	2	✗/✓[2 ms]	✗/✓[0 ms]	✗/✓[7 ms]	✗/✓[4 ms]	✗/✓[6 ms]	✗/✓[3 ms]	✗[0 ms]	✗[0 ms]	✓[0 ms]
DOM2 DEP‡ [36]	56	2	✓[4 ms]	✗/✓[8 ms]	✓[3 ms]	✗/✓[20 ms]	✓[4 ms]	✗[0 ms]	✓[4 ms]	✗/✓[21 ms]	✓[0 ms]

Results

Table of Tested Designs (continued):

S-boxes											
$\text{PRESENT}_{\text{TI}}$ [40]	177	2	✓[4 ms]	✓[8 ms]	^{1}✗[4 ms]	^{1}✗[0 ms]	^{1}✗[3 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	✓[2 ms]
$\text{PRESENT}_{\text{TI}}$ [25]	377	2	✓[15 ms]	✓[6 ms]	^{1}✗[2 ms]	^{1}✗[0 ms]	^{1}✗[2 ms]	^{1}✗[0 ms]	^{1}✗[1 ms]	^{1}✗[0 ms]	✓[0 ms]
$\text{PRESENT}_{\text{TI}}$ [25]	161	2	^{1}✗[3 ms]	^{1}✗[4 ms]	^{1}✗[32 ms]	^{1}✗[0 ms]	^{1}✗[26 ms]	^{1}✗[0 ms]	^{1}✗[2 ms]	^{1}✗[0 ms]	✗[0 ms]
$\text{PRINCE}_{\text{TI}}$ [37]	150	2	✓[2 ms]	✓[10 ms]	^{1}✗[2 ms]	^{1}✗[0 ms]	^{1}✗[2 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	✓[0 ms]
$\text{PRINCE}_{\text{CMS}}$ [14]	261	1	✓[3 ms]	✓[97 ms]	✓[7 ms]	✓[2.8 s]	✓[9 ms]	^{1}✗[1 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	✓[0 ms]
$\text{SKINNY8}_{\text{TI}}$ [7]	240	2	✓[51.2 s]	✓[2 min]	^{1}✗[2 min]	^{1}✗[2.0 s]	^{1}✗[2 min]	^{1}✗[2.0 s]	^{1}✗[77 ms]	^{1}✗[1.3 s]	✓[29.6 s]
$\text{SKINNY8}_{\text{CMS}}$ [8]	192	1	✓[20 ms]	✓[0.3 s]	^{1}✗[0.3 s]	^{1}✗[17 ms]	^{1}✗[0.3 s]	^{1}✗[15 ms]	^{1}✗[1 ms]	^{1}✗[1 ms]	✓[1 ms]
AES_{DOM} [29]	884	1	✓[3.3 s]	✓[21 min]	^{1}✗[0.8 s]	^{1}✗[0.4 s]	^{1}✗[0.8 s]	^{1}✗[0.4 s]	^{1}✗[0.2 s]	^{1}✗[40 ms]	✓[0.1 s]
AES_{CMS} [19]	938	1	✓[9.4 s]	✓[2.9 h]	^{1}✗[0.9 s]	^{1}✗[0.5 s]	^{1}✗[0.9 s]	^{1}✗[0.5 s]	^{1}✗[0.2 s]	^{1}✗[42 ms]	✓[1.8 s]
Functions											
A_{in} [37]	18	1	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]
A_{m} [37]	20	1	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]
A_{out} [37]	20	1	✓[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	✓[0 ms]	✓[0 ms]	✓[0 ms]
$\mathcal{Q}_{12}^{4}$ [42]	48	1	✓[0 ms]	✓[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	^{1}✗[0 ms]	✓[0 ms]

† Number of possible probe positions, i.e., output wires of gates. ‡ Assuming identical inputs, i.e., $a = b$.

Discussion

Advantage of Avoiding False Negatives

Q_{12}^4:

$$
\begin{aligned}
x_1 &= F_1(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_1 \\
x_2 &= F_2(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_2 \\
y_1 &= G_1(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_1c_1 \oplus b_1 \\
y_2 &= G_2(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_1c_2 \\
y_3 &= G_3(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_2c_1 \oplus b_2 \\
y_4 &= G_4(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_2c_2 \\
z_1 &= H_1(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_1b_1 \oplus a_1c_1 \oplus c_1 \\
z_2 &= H_2(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_1b_2 \oplus a_1c_2 \\
z_3 &= H_3(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_2b_1 \oplus a_2c_1 \\
z_4 &= H_4(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = a_2b_2 \oplus a_2c_2 \oplus c_2 \\
t_1 &= K_1(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = d_1 \\
t_2 &= K_2(\mathbf{a},\mathbf{b},\mathbf{c},\mathbf{d}) = d_2
\end{aligned}
\qquad
\begin{aligned}
\bar{x}_1 &= x_1 \\
\bar{x}_2 &= x_2 \\
\bar{y}_1 &= y_1 \oplus y_2 \\
\bar{y}_2 &= y_3 \oplus y_4 \\
\bar{z}_1 &= z_1 \oplus z_2 \\
\bar{z}_2 &= z_3 \oplus z_4 \\
\bar{t}_1 &= t_1 \\
\bar{t}_2 &= t_2
\end{aligned}
$$

Reparaz, O., Bilgin, B., Nikova, S., Gierlichs, B., Verbauwhede, I.: Consolidating Masking Schemes.

Discussion

Advantages

- **Completeness: SILVER supports all established security and composability notions**
- **Soundness: Due to direct conformity to the security notions, SILVER avoids false negatives and thus avoids introducing unnecessary overhead into the design.**
- **Returning failing probe: Knowing which probe fails to fulfill the security notion allows an easier fix of masked design.**

Disadvantages

- **Due to direct conformity with security notions, SILVER is slower than language-based approaches (like for example maskVerif).**

Demo

Conclusion

With SILVER, we developed an easy-to-use tool for sound and complete verification of the security and composability of masked Implementations.

Additional Information

Tool:

- https://github.com/Chair-for-Security-Engineering/SILVER

ePrint Link:

- https://eprint.iacr.org/2020/634/20200629:134004

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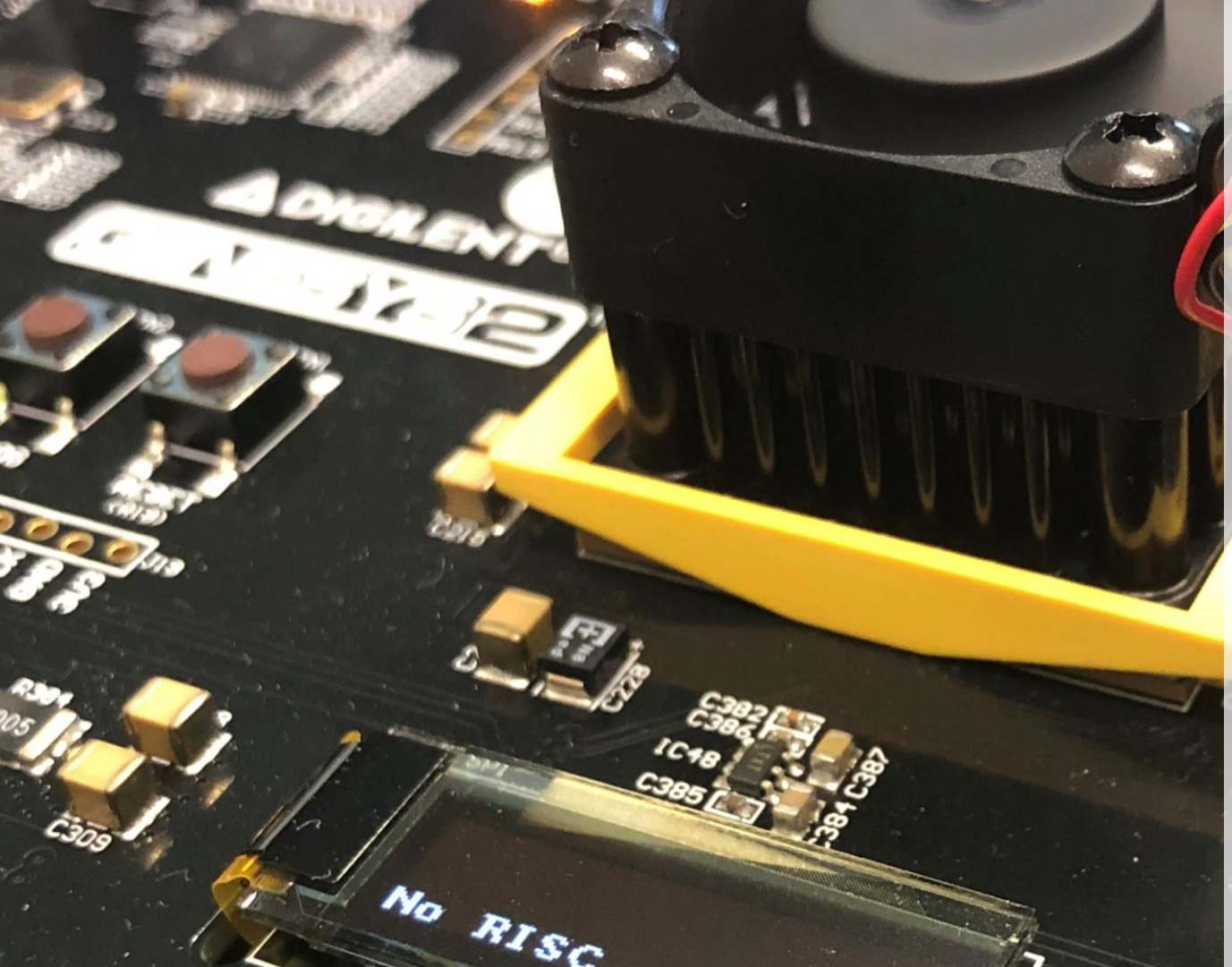

Thank you!

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