



Unbounded HIBE with Tight Security

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Outline

Unbounded HIBE

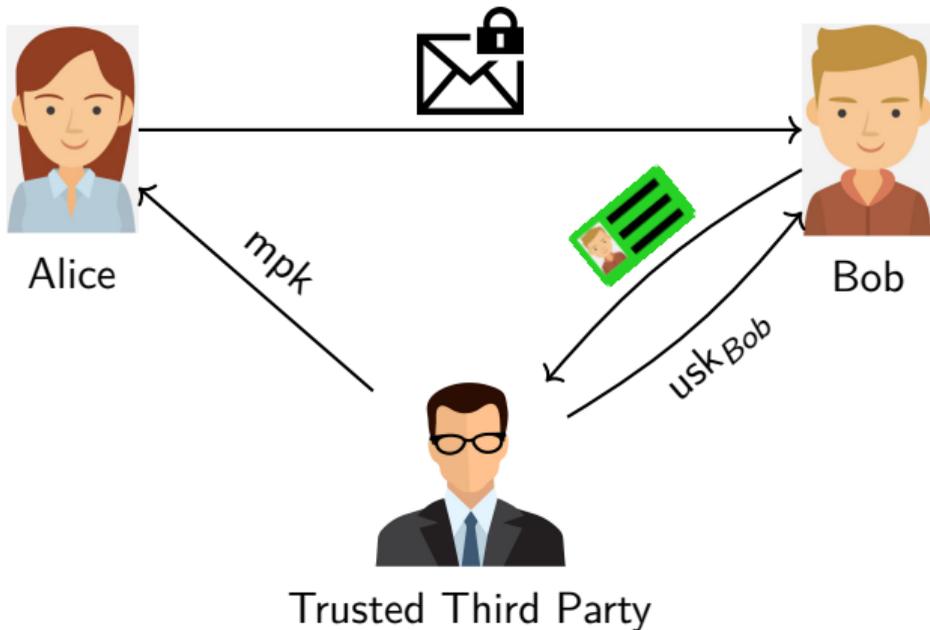
Tight security

Related works

Technical overview

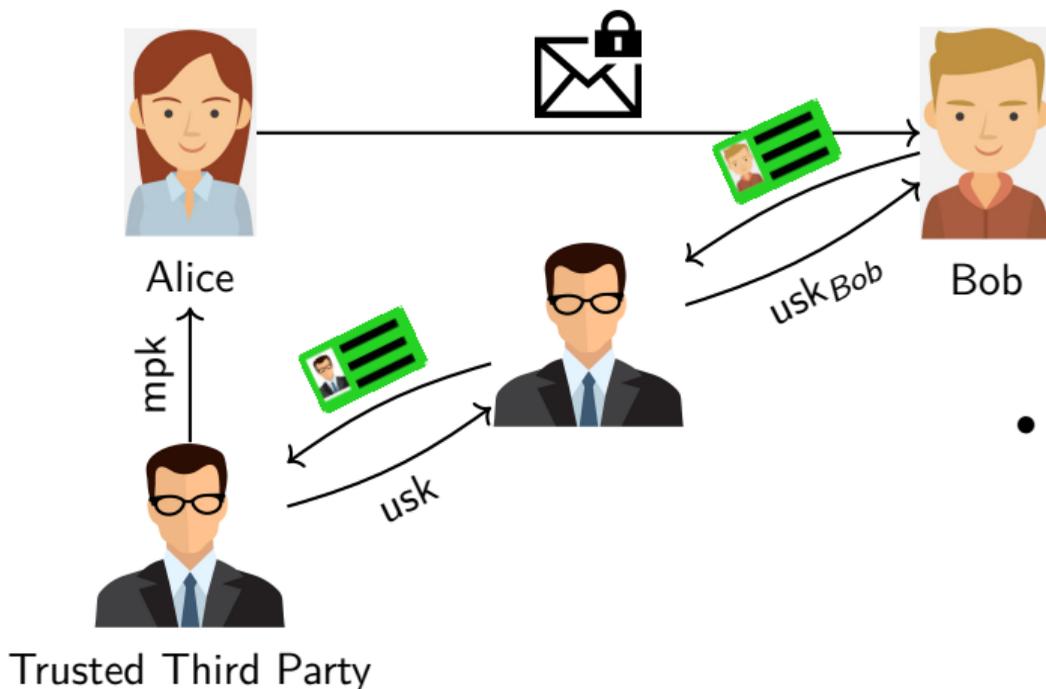
Future work

Identity-based encryption



- Alice needs to obtain only the master public key
- Encryption with identities (e.g. e-mail address)

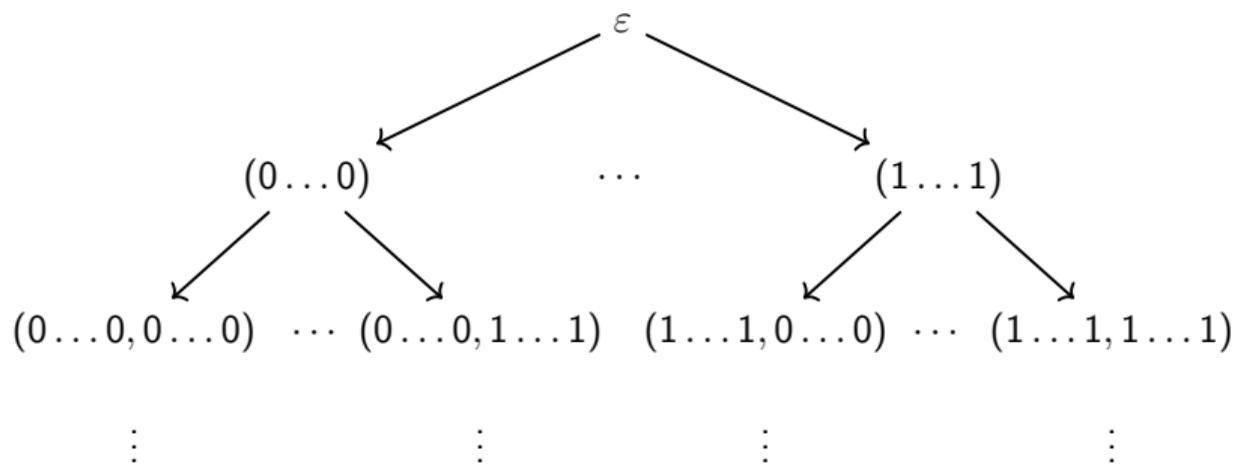
Hierarchical Identity-based encryption



- Hierarchy of key generators

Key delegation

Identities have the form (id_1, \dots, id_p) .



- Each user can generate keys for its children

(Un)bounded HIBE

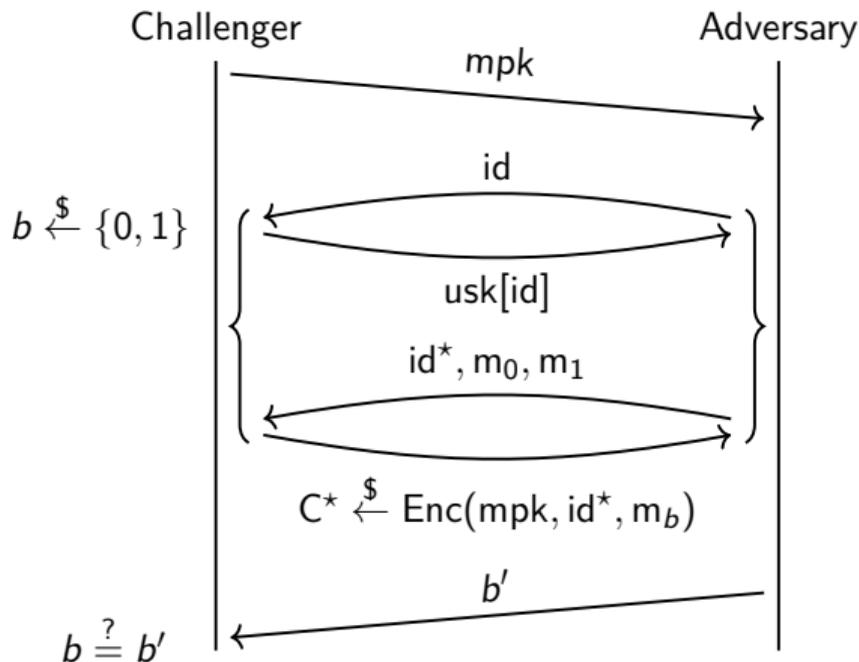
(Bounded) HIBE: Hierarchy depth L is fixed in advance

- mpk grows with L
- usks have a delegation term

Unbounded HIBE: No limit on the hierarchy depth

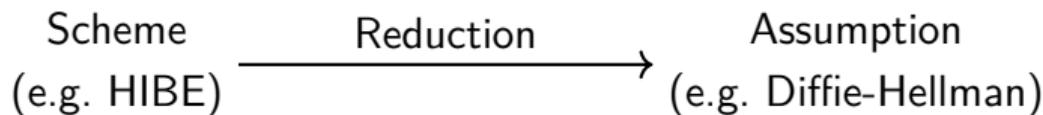
- More flexibility
- Better scalability

Security game (IND-HID-CPA)



- The adversary must not ask user secret keys for prefixes of challenge identities (id^*).
- IND-HID-CCA is easy once you have IND-HID-CPA.
- Master-key KDM-security can be achieved with [GGH20].

Tight security



Can be broken with
probability ε using resources ρ .

Can be broken with
probability ε/ℓ using resources ρ .

Larger security loss requires larger security parameter.

Security loss ℓ can depend on:

- scheme parameters
- λ : the security parameter
- the attacker's resources (e.g. # user secret key queries Q_k
or # challenge ciphertext queries Q_c , hierarchy depth L)

Tight security:

} allowed

} not allowed

History: Unbounded HIBE

Unbounded HIBEs:

[LW11]	CPG	$\mathcal{O}(Q_k L)$ (single-challenge)
[Lew12], [GCTC16]	PPG	$\mathcal{O}(Q_k L)$ (single-challenge)
[OT12] (weakly unbounded)	PPG	$\mathcal{O}(Q_k L^2)$ (single-challenge)
This work	PPG	$\mathcal{O}(\gamma)$ (multi-challenge)

- Q_k : # user secret key queries
- L : maximum hierarchy depth
- γ : Output bit length of a CRHF

History: Tight (H)IBE

Tight IBEs in prime-order pairing groups:

[CW13], [BKP14]	$\mathcal{O}(n)$ (single-challenge)
[AHY15], [GCD ⁺ 16], [GDCC16], [HJP18]	$\mathcal{O}(n)$ (multi-challenge)

- n : Bit-length of the identities

Tight HIBEs in prime-order pairing groups:

[LP19]	$\mathcal{O}(\gamma)$ (single-challenge)
[LP20]	$\mathcal{O}(\gamma L)$ (multi-challenge)

- γ : Output bit length of a CRHF

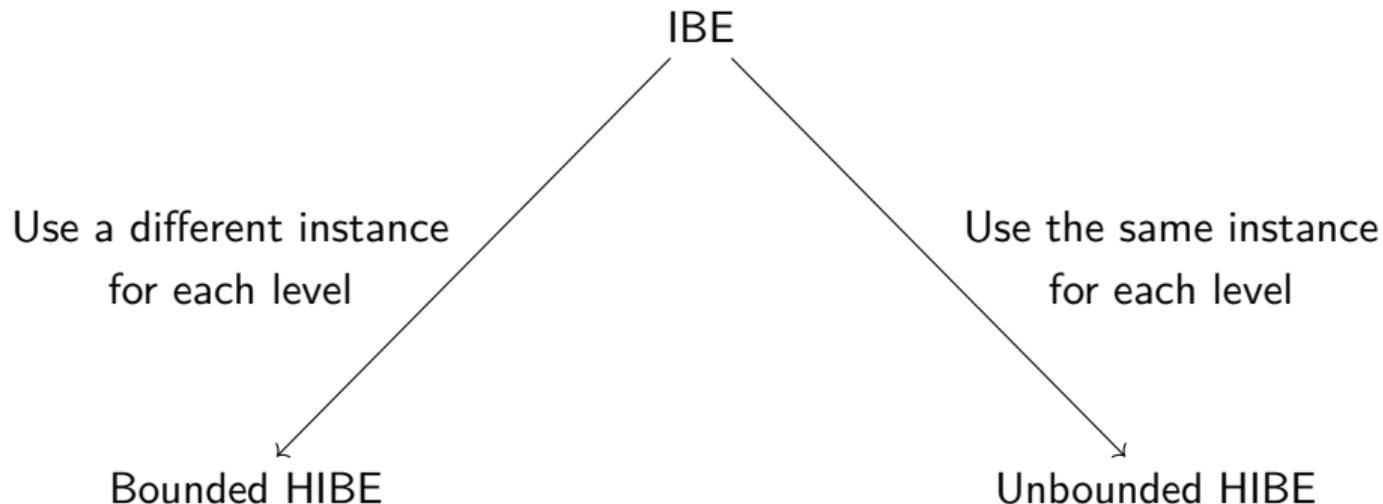
The gap

Scheme	$ \text{mpk} $	$ \text{usk} $	$ C $	Loss	MC	Assumption
[LP20]	$\mathcal{O}(\gamma LL)$	$\mathcal{O}(p)$	$\mathcal{O}(p)$	$\mathcal{O}(\gamma LL)$	✓	SXDH

- L : maximum hierarchy depth
- p : actual hierarchy depth
- γ : bit-length of hashes

- We need to get rid of the factor L in $|\text{mpk}|$ and in the security loss.

High-Level approach



IBE structure

- $usk[id] = \text{Blinding term for id } \mathbf{t} + \text{msk}$

Cancel out if $id = id'$ during decryption

- $ct[id'] = (\text{Dual blinding term for } id' \mathbf{h}, ct \mathbf{h})$

Blinding terms

- Blinding term for x \mathbf{t} · Dual blinding term for x \mathbf{h} = 0
- Blinding term for x \mathbf{t} is pseudorandom
 - If no dual blinding term for x is known
- Blinding terms are publicly sample
 - Dual blinding terms are **not** publicly sample
- Tight realization: Abstraction of [LP20]

Generalisation to Bounded HIBE

- $usk[id_1, \dots, id_p] = \sum_{i=1}^p \left[\text{Level } i \text{ blinding term for } H(id_1 || \dots || id_i) \right] \mathbf{t}_i + \text{msk}$
- Delegation: Blinding terms are publicly sampable
- $ct[id'_1, \dots, id'_{p'}] = \left(\left(\left[\text{Dual level } i \text{ blinding term for } H(id'_1 || \dots || id'_i) \right] \mathbf{h} \right)_{1 \leq i \leq p'}, \left[\text{ct} \right] \mathbf{h} \right)$

Obstacles for Unbounded HIBE:

- mpk has size $\mathcal{O}(nL^2)$
(n : bit-length of the identities)
 - Use a collision-resistant hash function H
- mpk has size $\mathcal{O}(\gamma L)$
 - Can we use the same blinding terms for all levels?
 - ⇒ We need individual randomness on the dual blinding terms

Unbounded HIBE

- $$\text{usk}[id_1, \dots, id_p] = \left(\sum_{i=1}^p \left[\text{Blinding term for "2"} \quad \tilde{\mathbf{t}}_i \right] + \left[\text{msk} \right], \right.$$

$$\left. \left(\left[\text{Blinding term for } H(id_1 || \dots || id_i) \quad \mathbf{t}_i \right] + \left[\text{Blinding term for "1"} \quad \tilde{\mathbf{t}}_i \right] \right)_{1 \leq i \leq p} \right)$$
- $$\text{ct}[id'_1, \dots, id'_{p'}] = \left(\left(\left[\text{Dual blinding term for } H(id'_1 || \dots || id'_i) \quad \mathbf{h}_i \right], \right.$$

$$\left. \left[\text{Dual blinding term for "1"} \quad \mathbf{h}_i \right] + \left[\text{Dual blinding term for "2"} \quad \tilde{\mathbf{h}} \right] \right)_{1 \leq i \leq p'}, \left[\text{ct} \quad \tilde{\mathbf{h}} \right] \right)$$

Comparison of unbounded HIBEs (in prime-order pairing groups)

Scheme	$ \text{mpk} $	$ \text{usk} $	$ C $	Loss	MC	Assumption
[Lew12]	$\mathcal{O}(1)$	$\mathcal{O}(p)$	$\mathcal{O}(p)$	$\mathcal{O}(Q_k)$	✗	2-LIN
[OT12]	$\mathcal{O}(1)$	$\mathcal{O}(p^2 L)$	$\mathcal{O}(p)$	$\mathcal{O}(Q_k L^2)$	✗	2-LIN
[GCTC16]	$\mathcal{O}(1)$	$\mathcal{O}(p)$	$\mathcal{O}(p)$	$\mathcal{O}(Q_k)$	✗	SXDH
Ours	$\mathcal{O}(\gamma)$	$\mathcal{O}(p)$	$\mathcal{O}(p)$	$\mathcal{O}(\gamma)$	✓	SXDH

- L : maximum hierarchy depth
- p : actual hierarchy depth
- γ : bit-length of hashes
- Q_k : # user secret key queries

[GGH20] Tight MC secure (IBE) \implies master-key KDM-secure (IBE)

- Same idea applies here \implies First unbounded HIBE with master-key KDM security

Future work

- Does the “inject-and-pack” strategy work in a more general setting, like predicate encryption?
- Can we have Unbounded HIBE with constant usk/ct size?

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Pictures

Alice, Bob, Trusted Party: freepik.com

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