The Summation-Truncation Hybrid: Reusing Discarded Bits for Free

Aldo Gunsing and Bart Mennink

Crypto 2020

PRP vs. PRF

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- A lot of modes only use the forward direction, not making use of the invertibility
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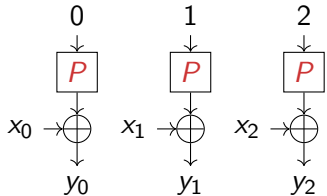
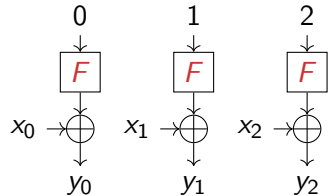
- Prominent example: CTR mode



$n/2$-bit security



n-bit-security

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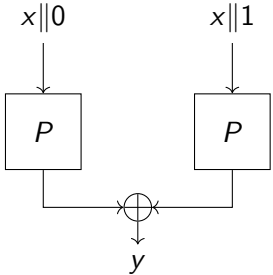
- However, we have little understanding in how to design one
- Alternatively, we can design a PRP-to-PRF conversion
  - PRP-PRF switch: PRP behaves like a PRF up to the birthday bound
  - Conversions like summation and truncation achieve beyond birthday bound security

Summation and Truncation



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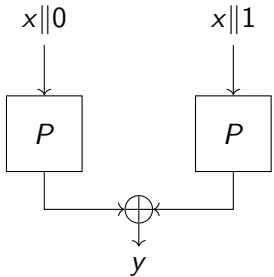

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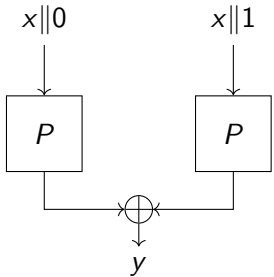

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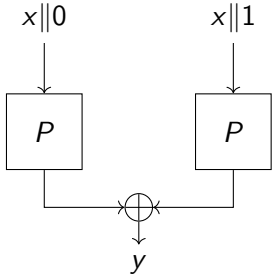

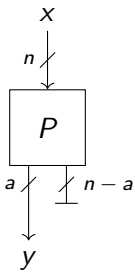

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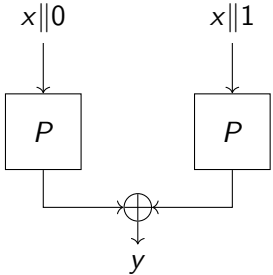

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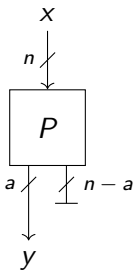

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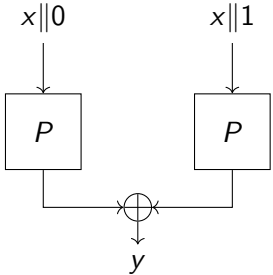

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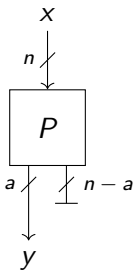

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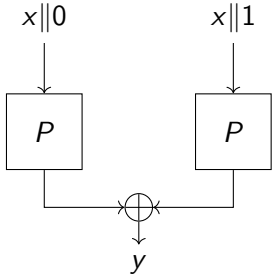

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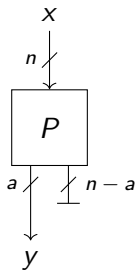

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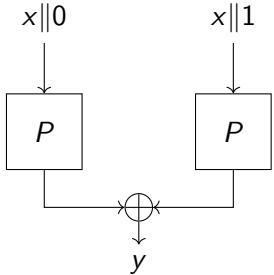

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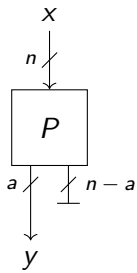

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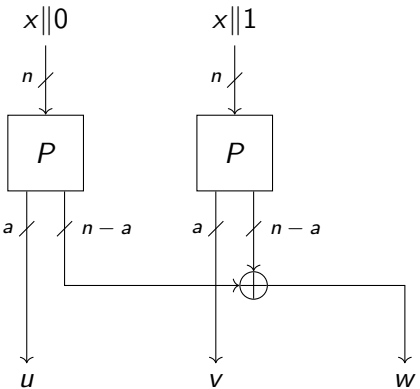

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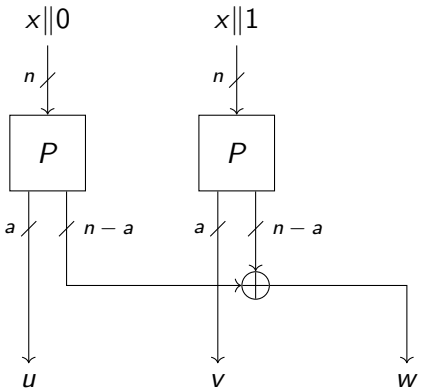

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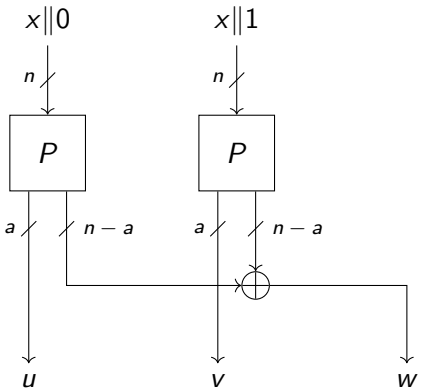

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- But we show that it has equal security!

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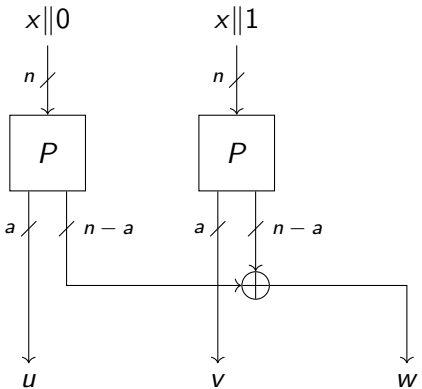
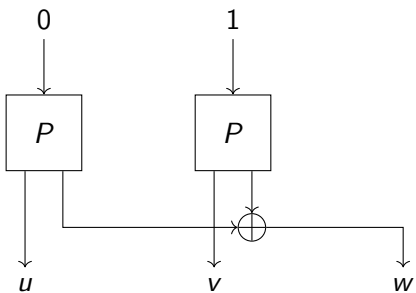


- Instead of discarding bits, we can reuse them by applying summation
- This leads to the Summation-Truncation Hybrid (STH)
- Outputs $n - a$ extra bits compared to truncation
- But we show that it has equal security!
- Identical to summation when $a = 0$

Proof Sketch: Idea

$P \xleftarrow{\$} \mathrm{Perm}[n]$



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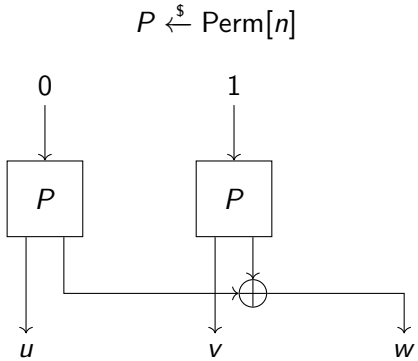

- Try to separate the truncation and summation parts

Proof Sketch: Separating STH

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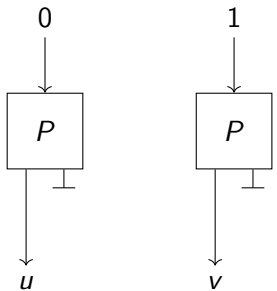

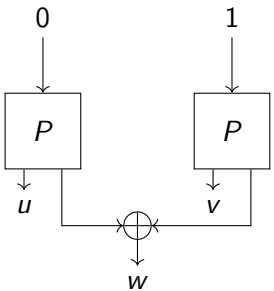

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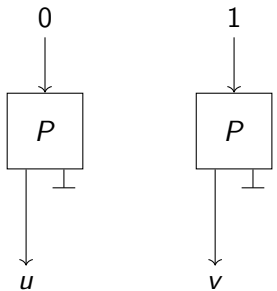

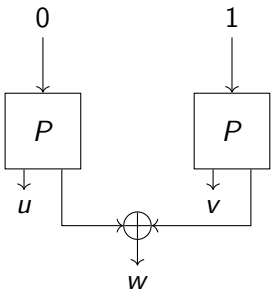

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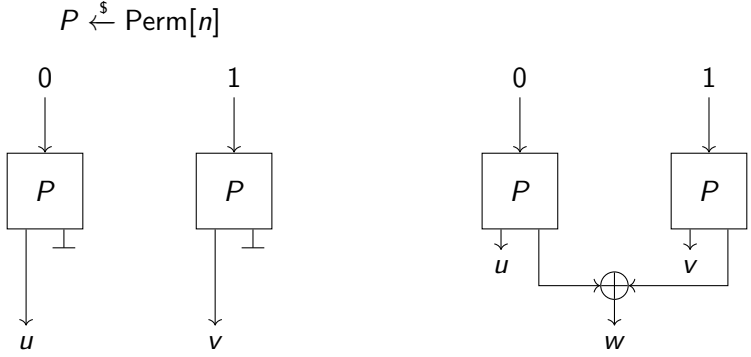
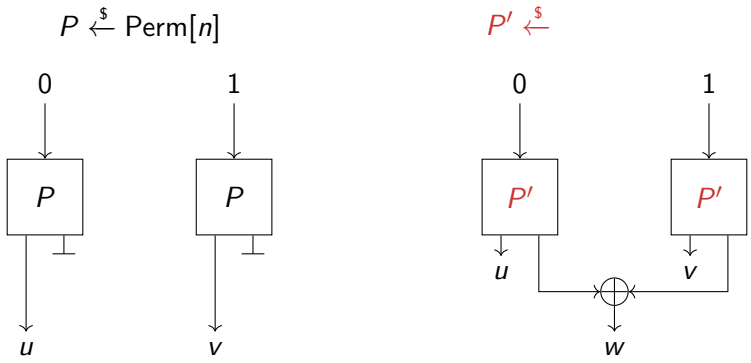


- Just write the two parts separately
- Problem: there is a shared secret P

Proof Sketch: Permutation-Separated STH



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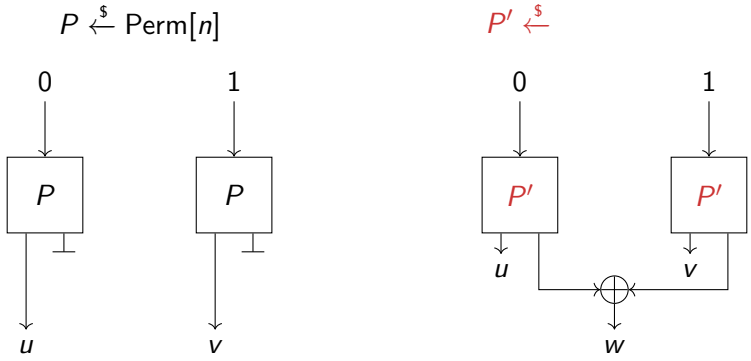

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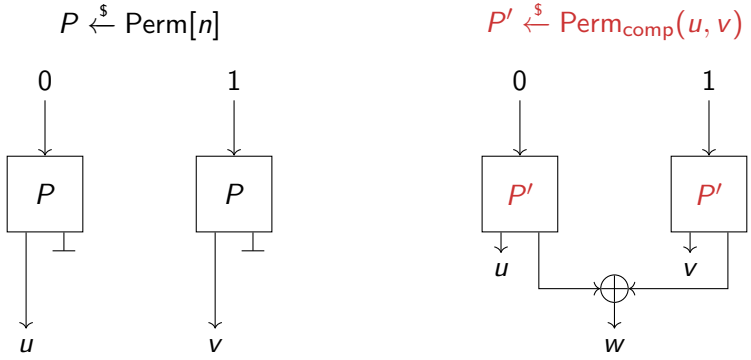

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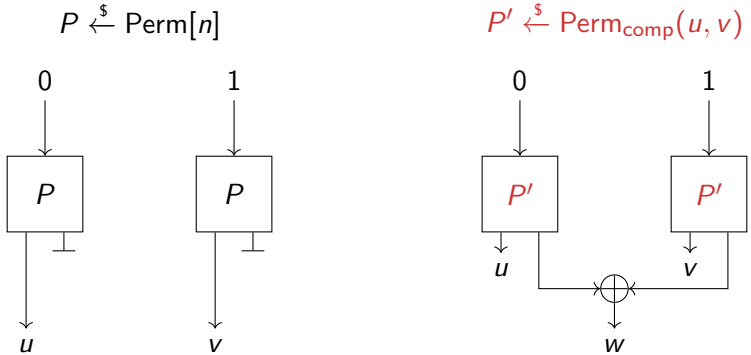
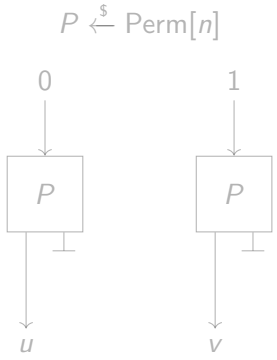
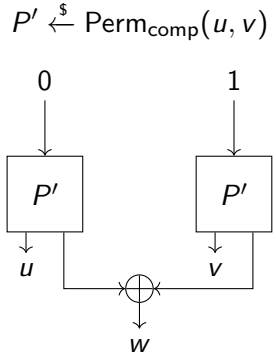


- We re-choose the permutation, but keep the same distribution
- $\mathrm{Perm}_{\mathrm{comp}}(u, v)$ is the set of all permutations that give (u, v) as truncation output
- No shared secret, as u and v are public!

Proof Sketch: Isolating Truncation





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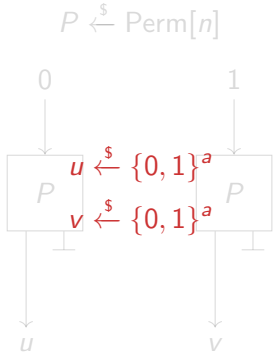

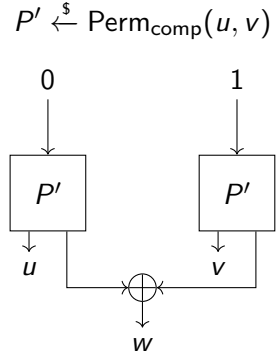

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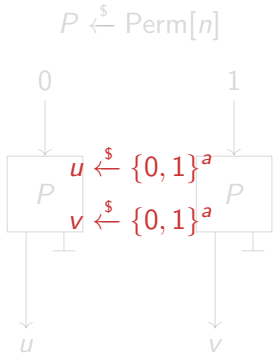

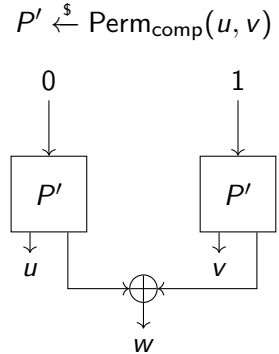

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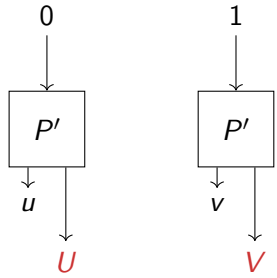
- $\mathrm{Perm}_{\mathrm{comp}}(u, v)$ still well-defined, although u and v are generated differently

Proof Sketch: Summation Modifications

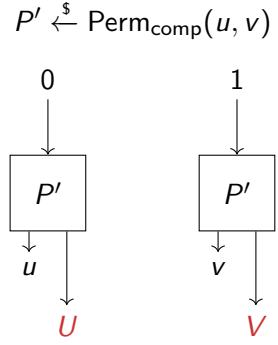
$P' \xleftarrow{\$} \mathsf{Perm}_{\mathsf{comp}}(u, v)$



$w = U \oplus V$

Proof Sketch: Summation Modifications

- Modified summation with subvalues U and V



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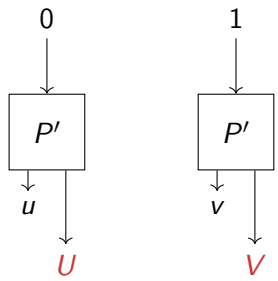

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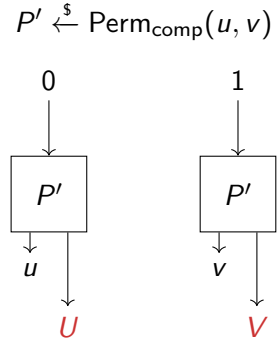

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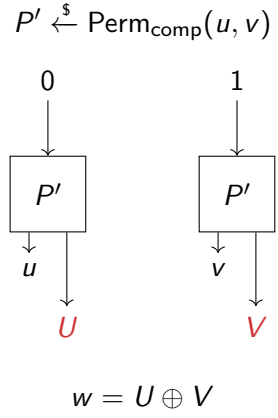
- Modified summation with subvalues U and V
- What are their distributions?
- U has a uniform distribution
- For V it depends:
  - If $v \neq u$, V is uniform from $\{0,1\}^b$
  - If $v = u$, V is uniform from $\{0,1\}^b \setminus \{U\}$



Proof Sketch: Alternative Description

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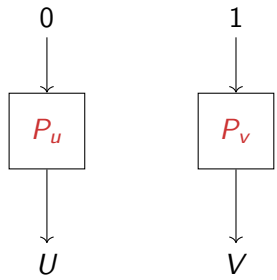

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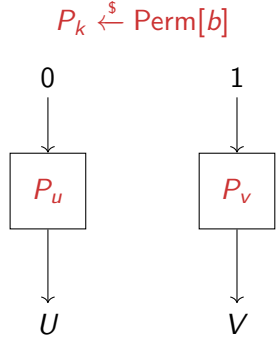

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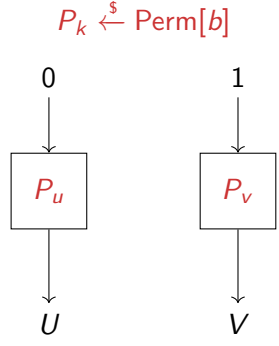

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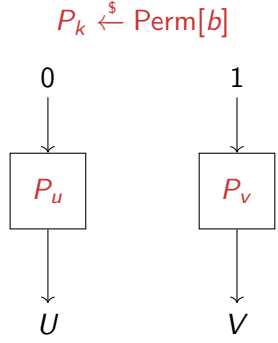

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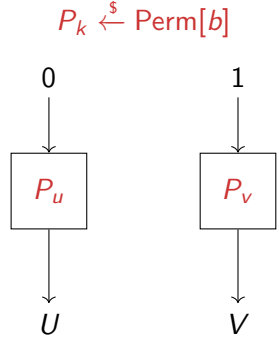
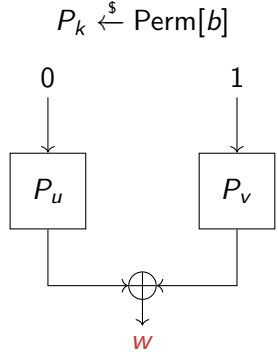


Proof Sketch: Generalized Summation



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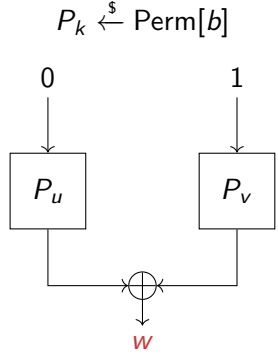

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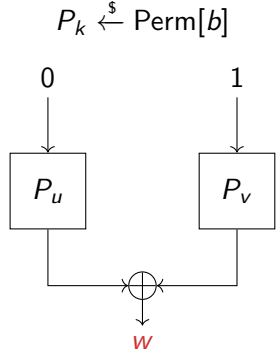

Proof Sketch: Generalized Summation

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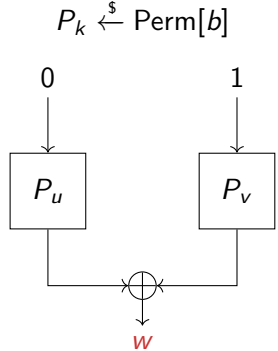
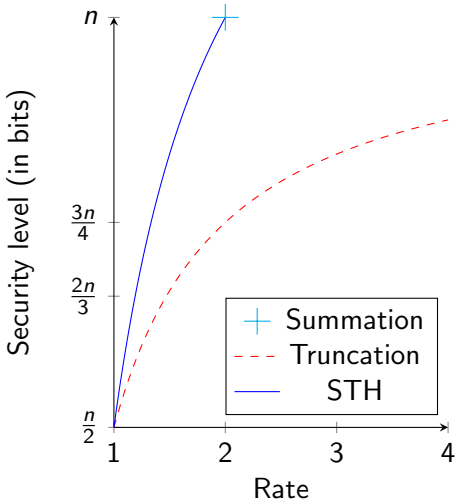
- We modify the proof of summation to cover this variant
- Uses the χ^2-technique



Comparison



Construction	Rate (in/out)	Security level (in bits)
Summation	2	n
Truncation	n/a	$n - a/2$
STH	$2n/(n+a)$	$n - a/2$

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Thank you for your attention!