

# Always Have a Backup Plan: Fully Secure Synchronous MPC with Asynchronous Fallback

Erica  
Blum

U. Maryland

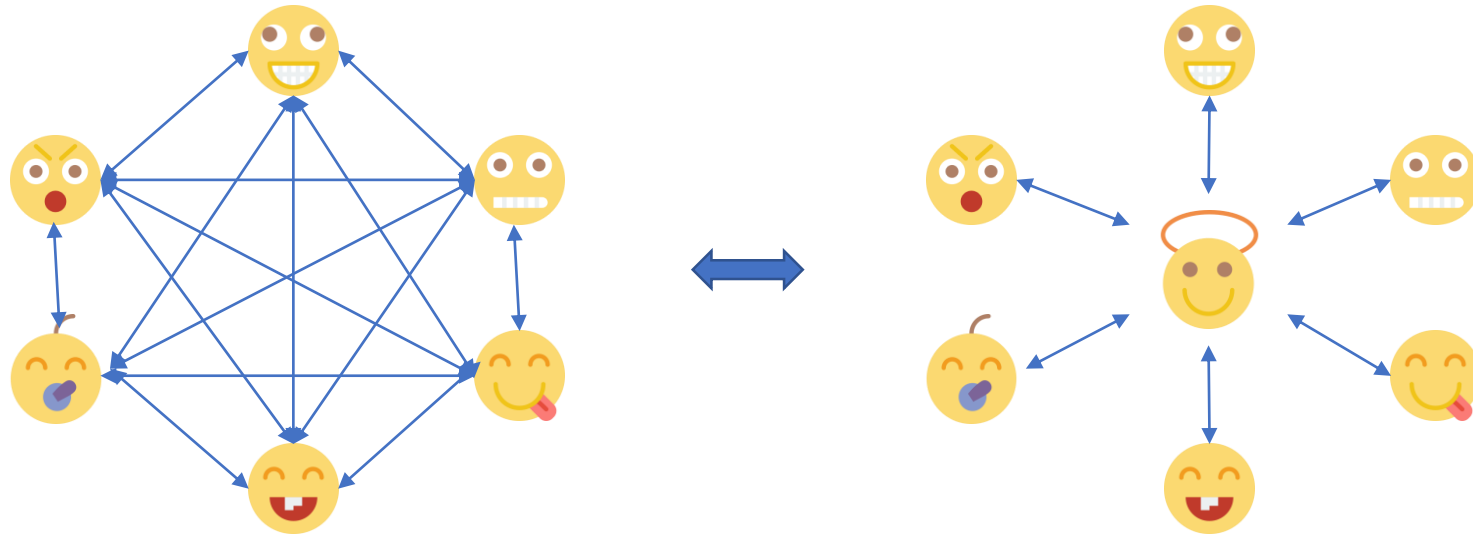
**Chen-Da  
Liu-Zhang**

ETH Zurich

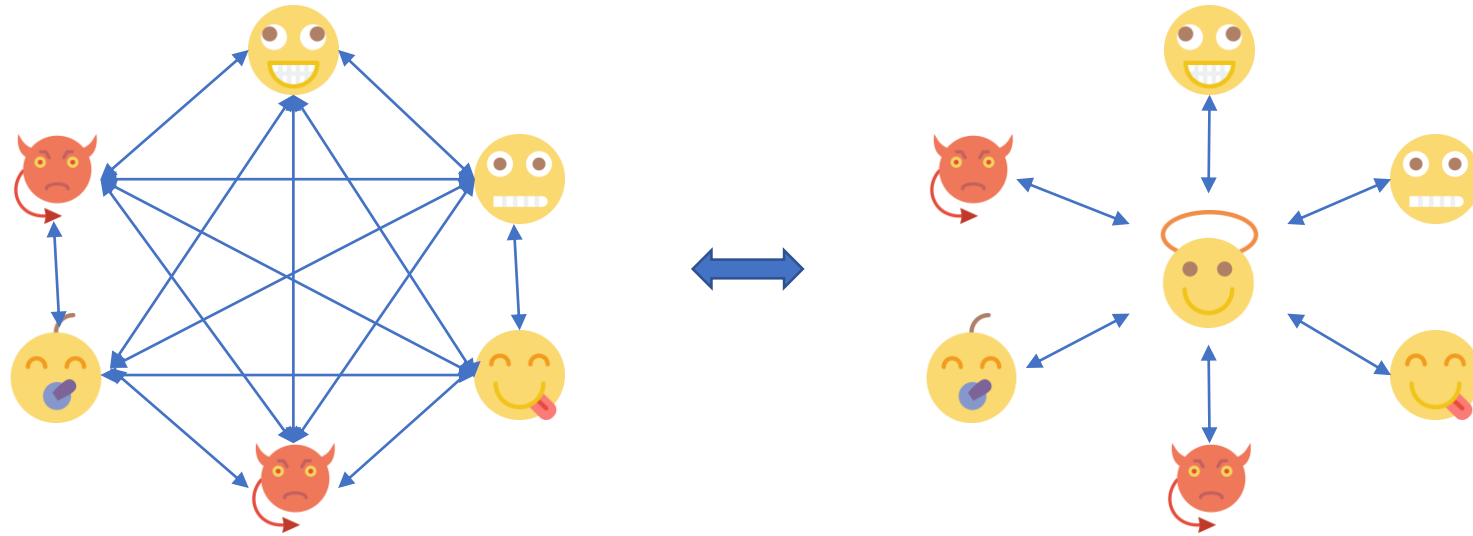
Julian  
Loss

U. Maryland

# Multiparty Computation



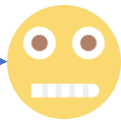
# Multiparty Computation



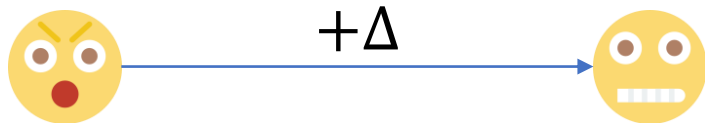
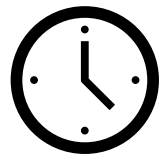
# Synchronous



$+\Delta$

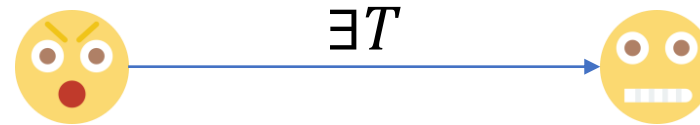


# Synchronous



# Asynchronous

-



# Synchronous

$$t < \frac{n}{2}$$

Input completeness

$$f(x_1, \dots, x_n)$$

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$$t < \frac{n}{3}$$

Up to  $t$  inputs are ignored

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$$f(x_1, x_2, x_3, x_4, \dots, x_{n-1}, x_n)$$





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$$f(x_1, \perp, \perp, x_4, \dots, x_{n-1}, \perp)$$



Is there an MPC that achieves guarantees in both regimes?

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$$n/3 \leq t_s < n/2$$

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$$t_a < n/3$$

Up to  $L$  inputs are ignored

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Asynchronous

$$t_a < n/3$$

Up to  $L$  inputs are ignored

YES! If and only if  $t_a + 2t_s < n$  and  $L \geq t_s$

# Related Work

[BKL19],[BKL20]: Trade-offs for Consensus primitives.

$t_s$ -secure (resp.  $t_a$ ) under a synchronous (resp. asynch.) network  
for any  $t_a + 2t_s < n$

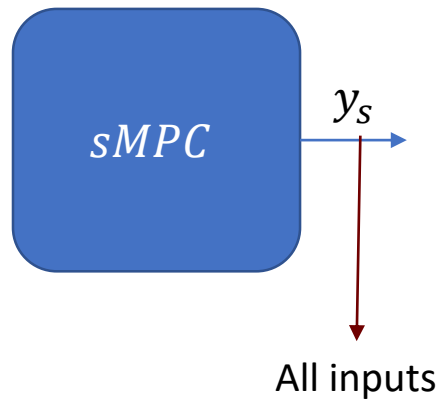
[HNP05]: Asynchronous MPC for  $t < \frac{n}{3}$  with input completeness when the  
network is synchronous

Feasibility

# Feasibility

## Core Primitive

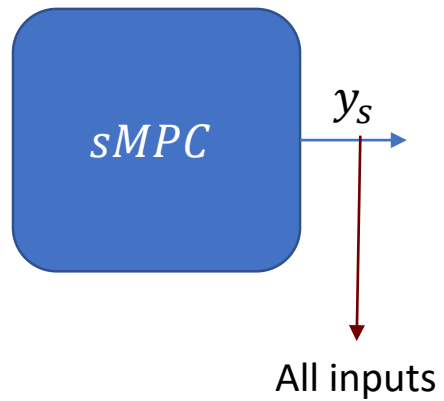
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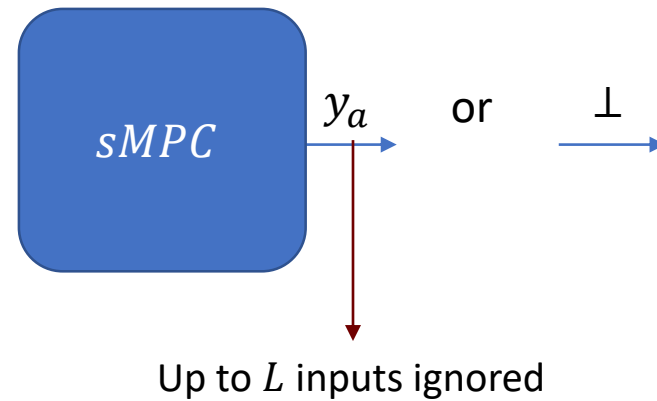
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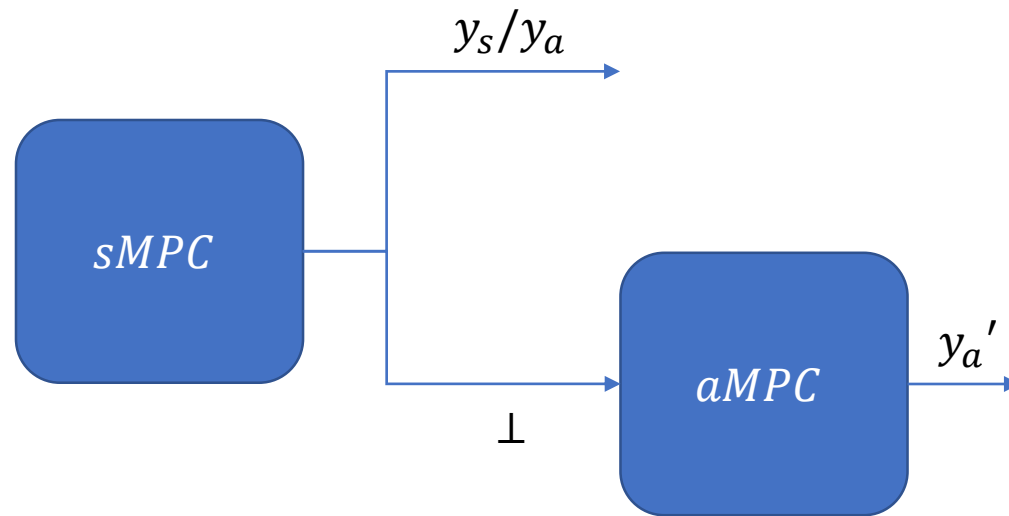
Asynchronous





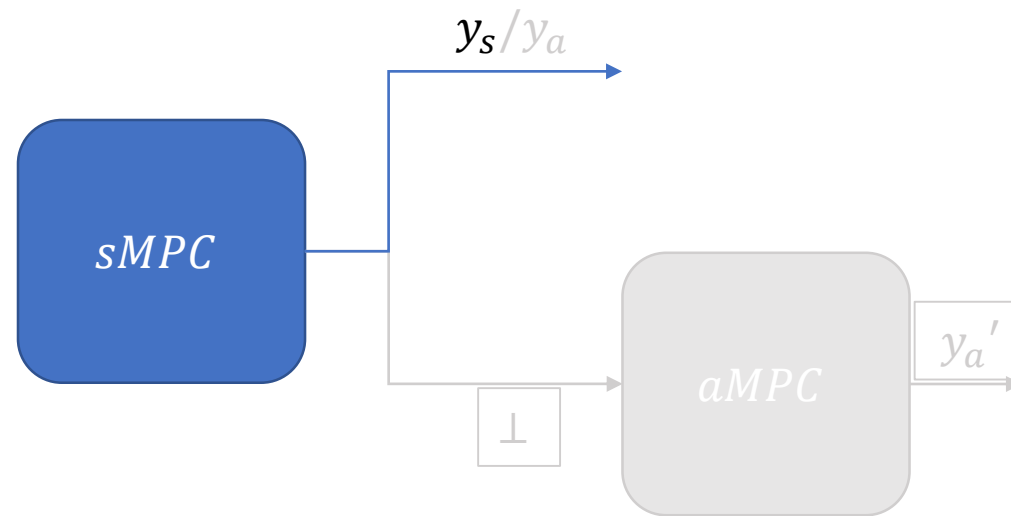
# Feasibility

Compiler



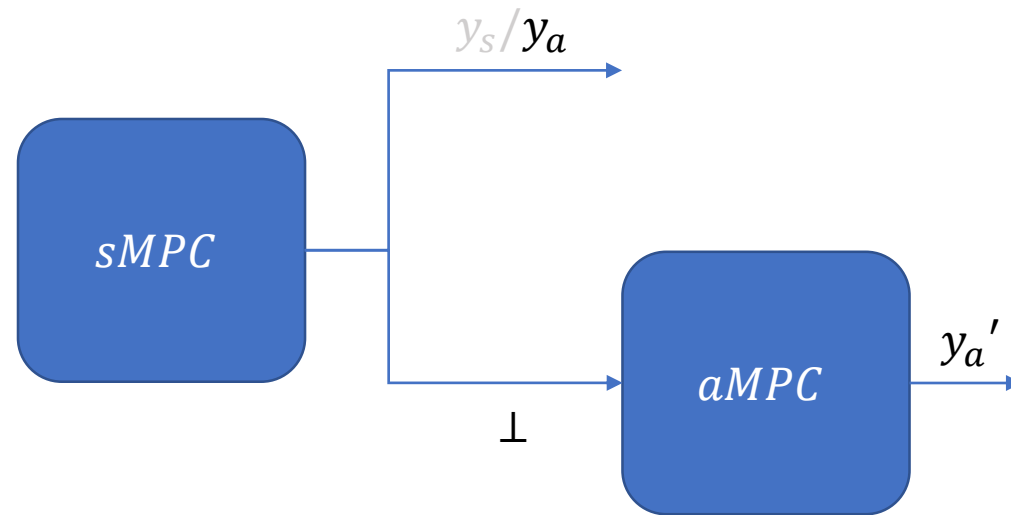
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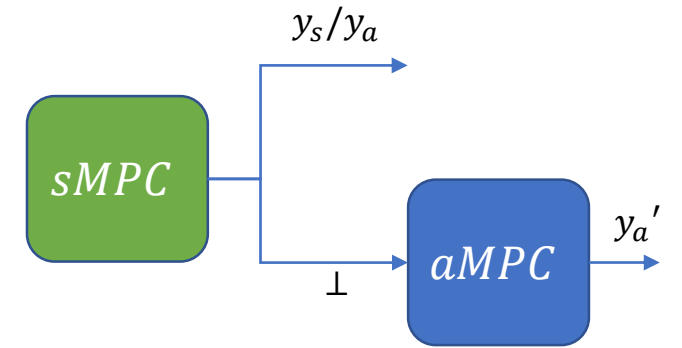
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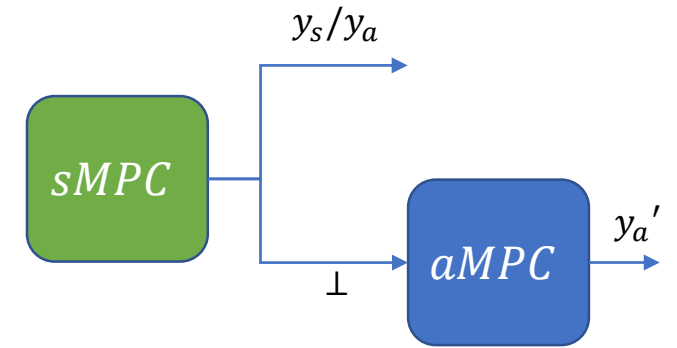
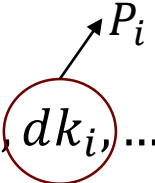
Threshold-Homomorphic Encryption



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## Threshold-Homomorphic Encryption

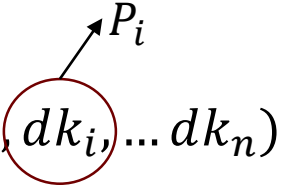
Public key  $ek$   
Secret key  $dk = (dk_1, dk_2, \dots, dk_i, \dots, dk_n)$



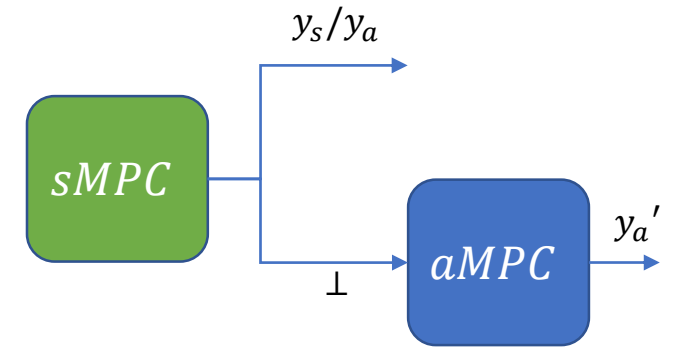
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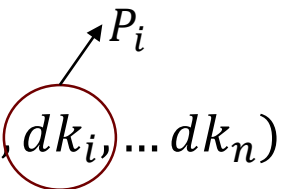
$$[x_i] = Enc_{ek}(x_i)$$



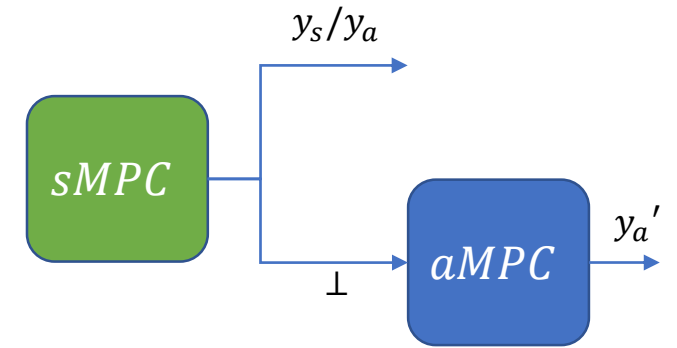
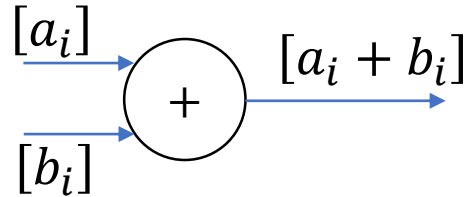
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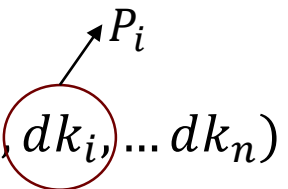
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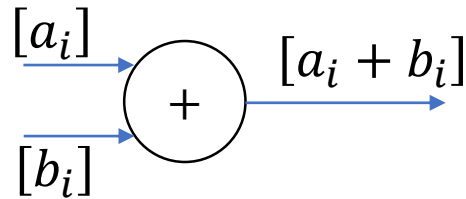
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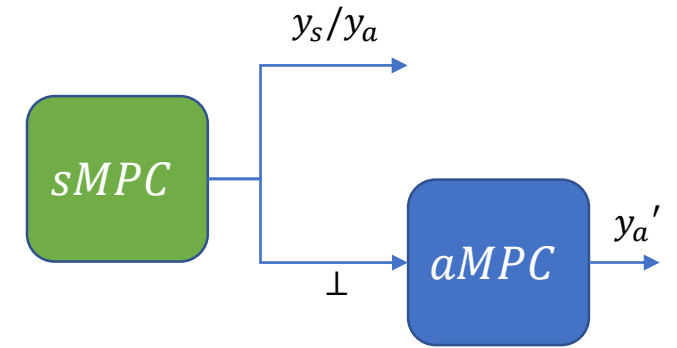
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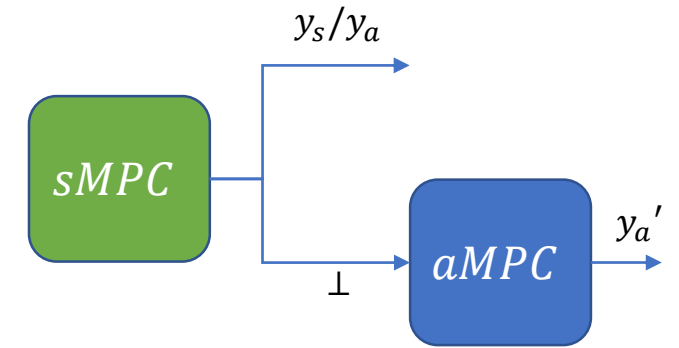


$$d_i = DecShare_{dk_i}([x_i])$$





# Feasibility

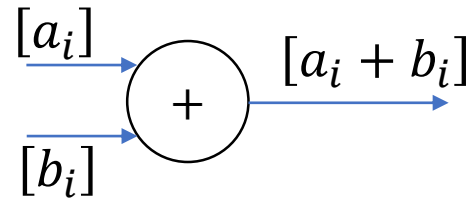


## Threshold-Homomorphic Encryption

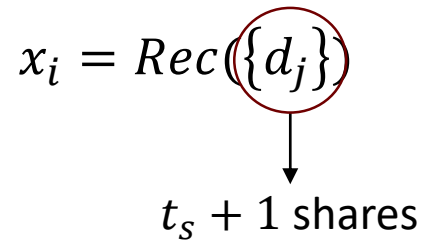
Public key  $ek$   
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*Note: In the original image,  $dk_i$  is circled in red, and an arrow points from this circle to the label  $P_i$ .*

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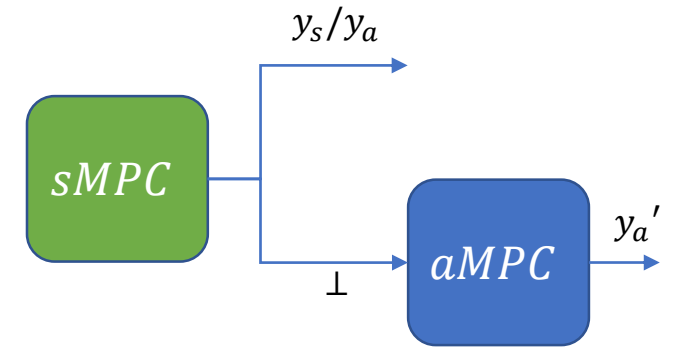


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# Feasibility

[CDN'01] approach

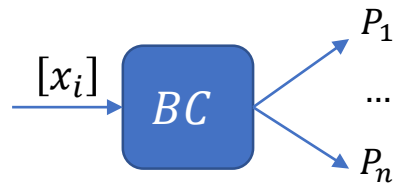


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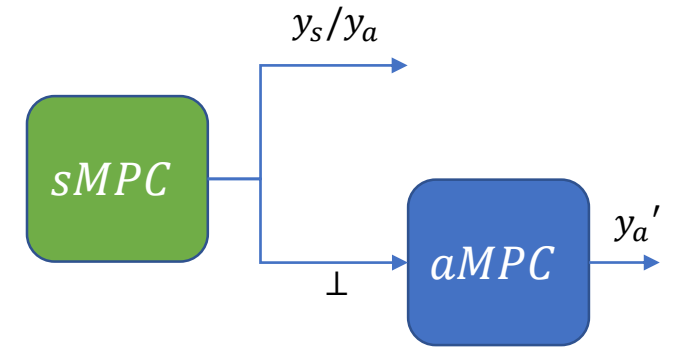
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Input Distribution

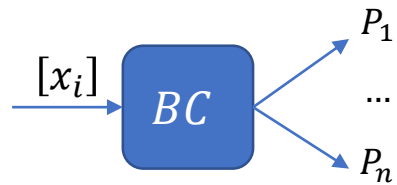


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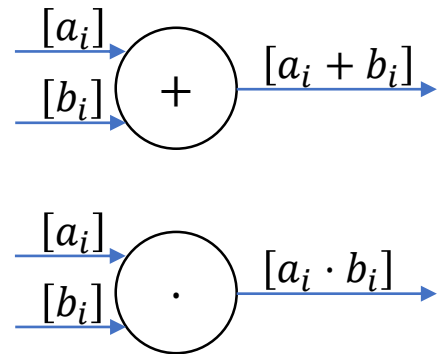
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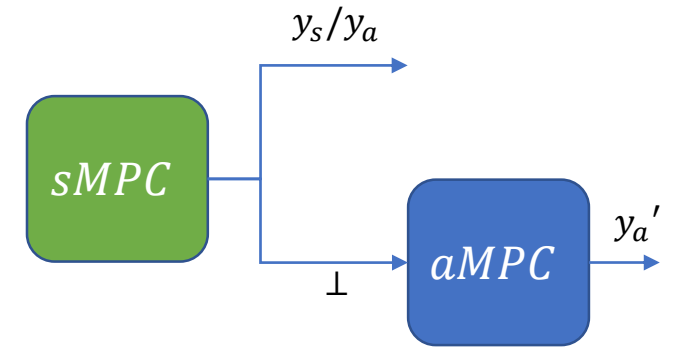
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Input Distribution



Circuit Evaluation

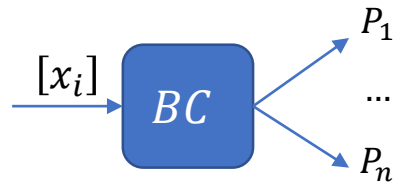


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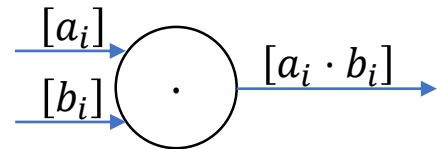
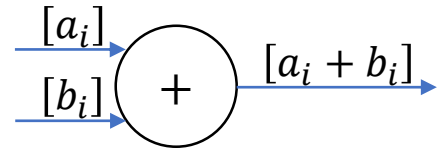
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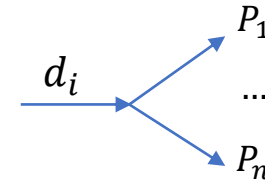
Input Distribution



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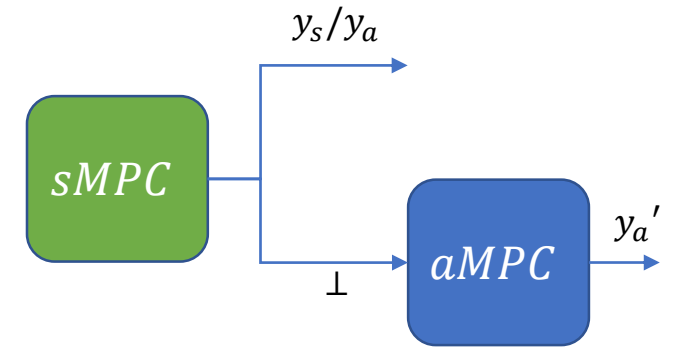
$c$  output ciphertext

$$d_i = DecShare_{dk_i}(c)$$



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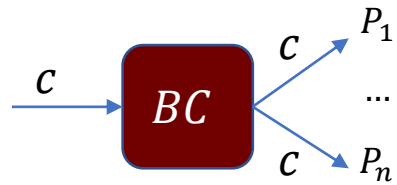
Threshold Decryption



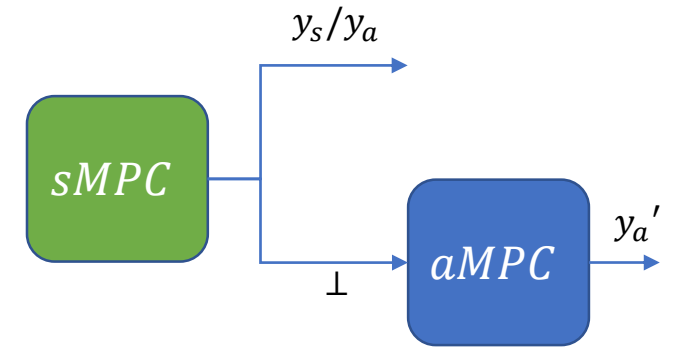
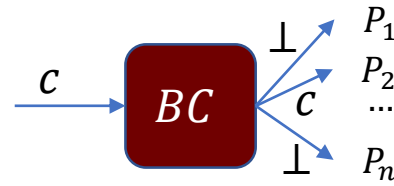
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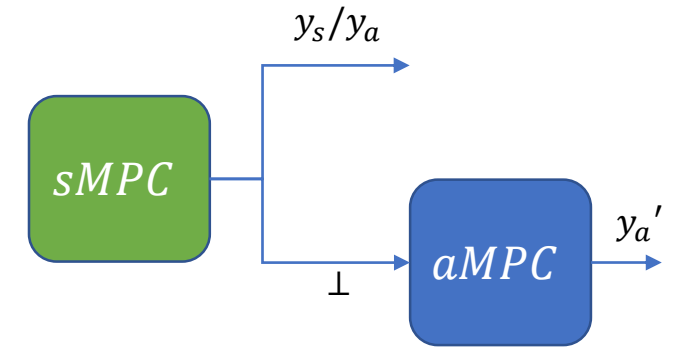
Synchronous



Asynchronous

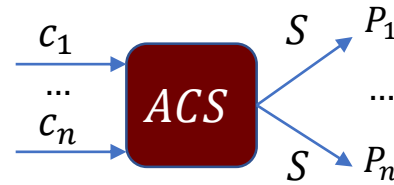
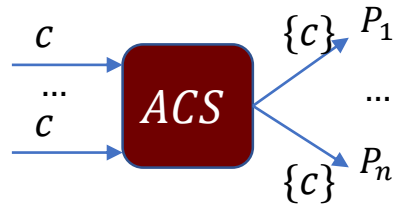
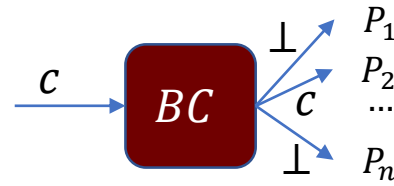
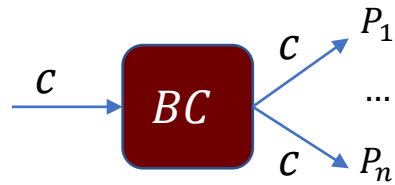


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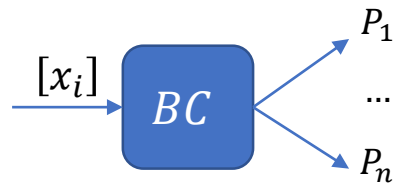
$S$  includes an honest  $P_i$ 's input

# Feasibility

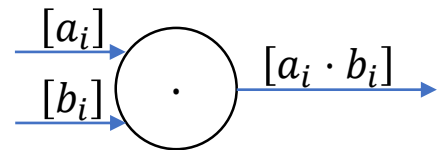
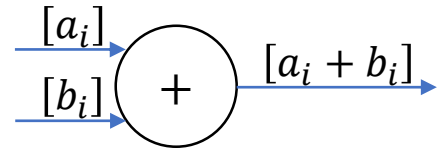
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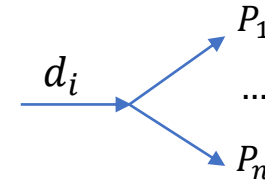
Input Distribution



Circuit Evaluation

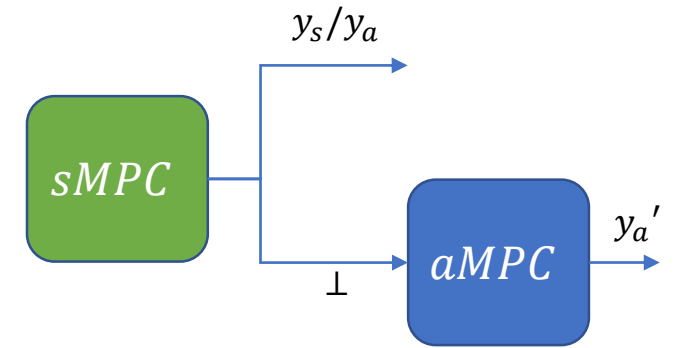
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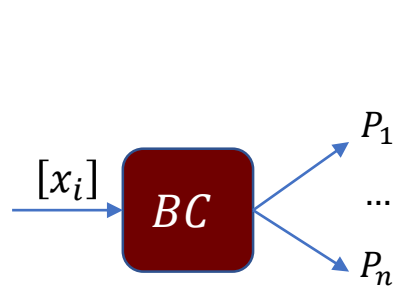


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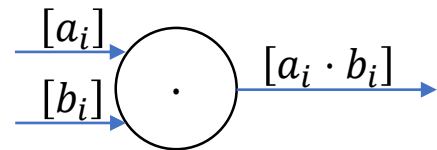
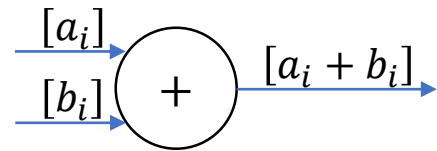
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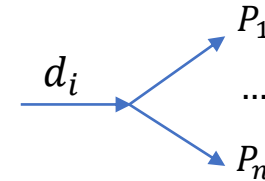
Wait for T time  
Check if there are  
 $n - t_s$  inputs



Circuit Evaluation

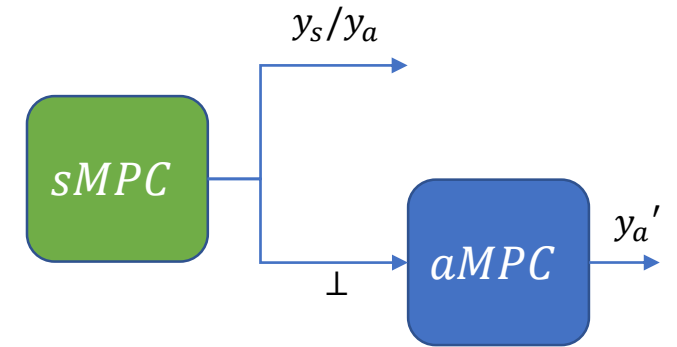
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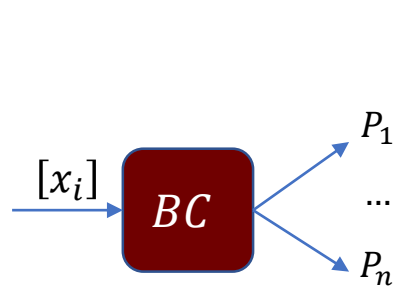


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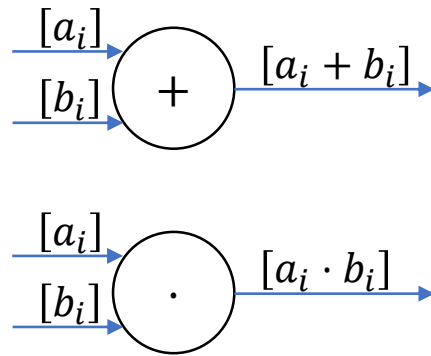
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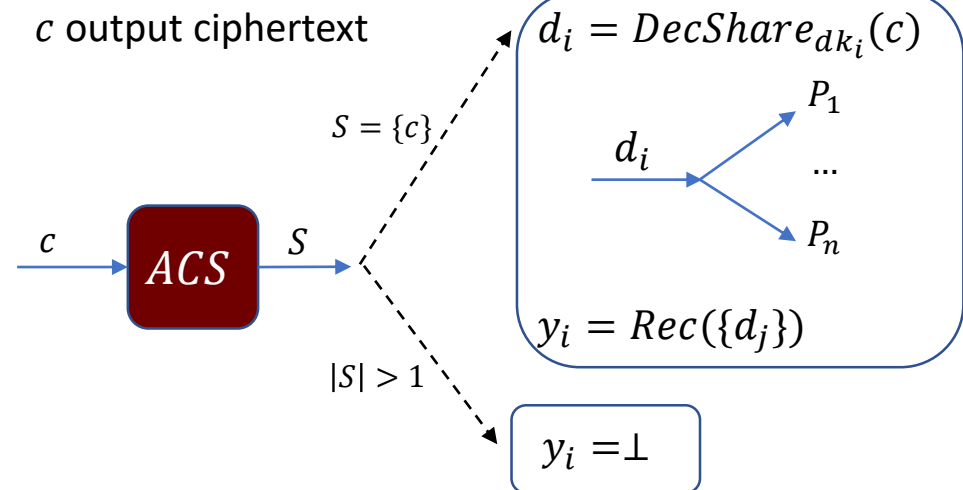


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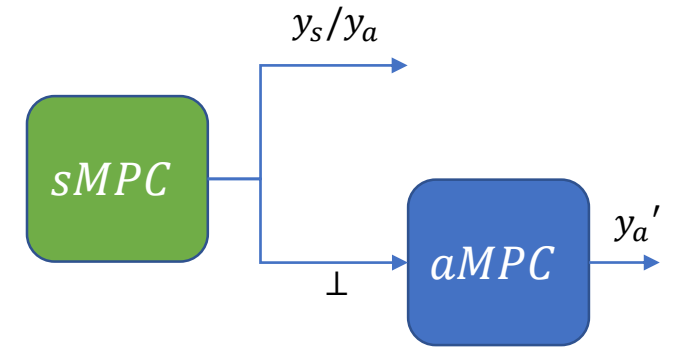


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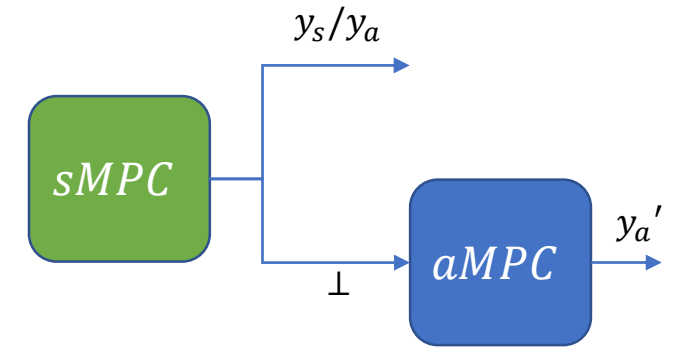


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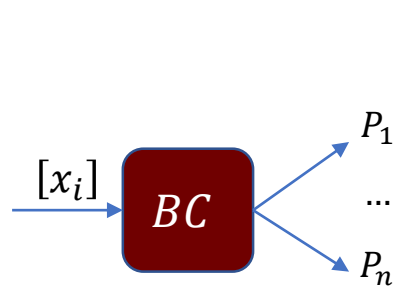
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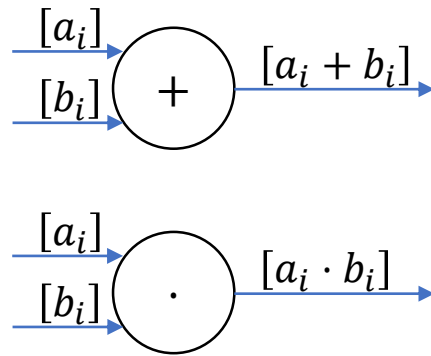


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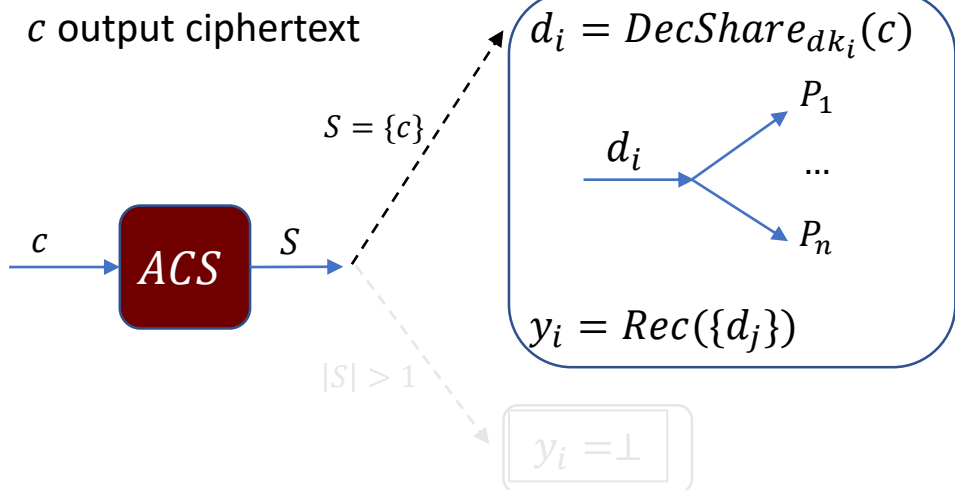


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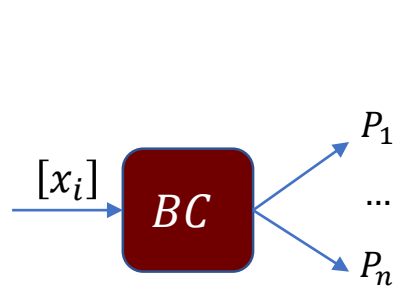
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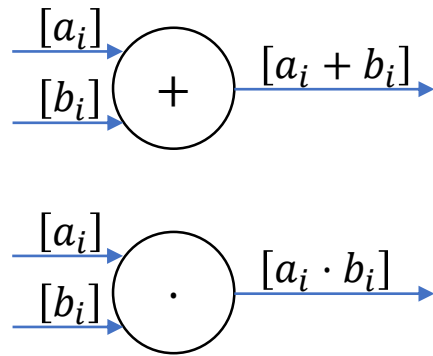
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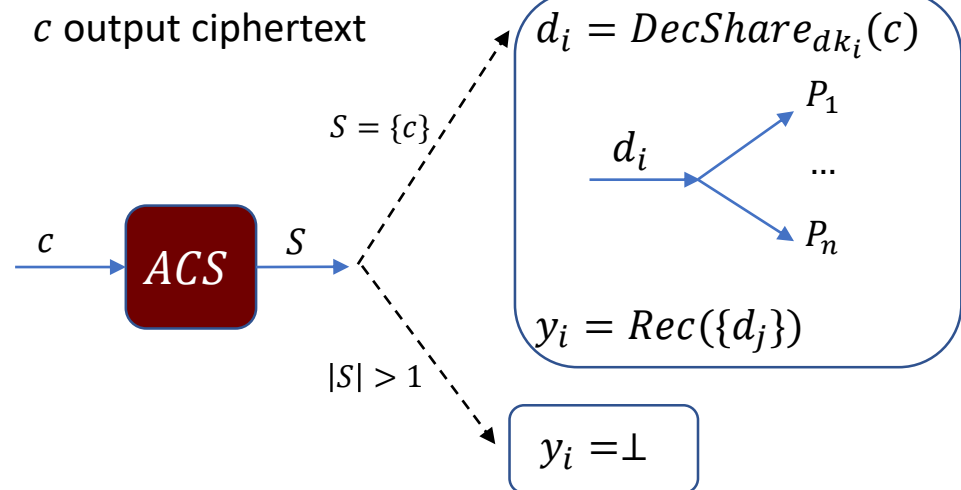


Input Distribution

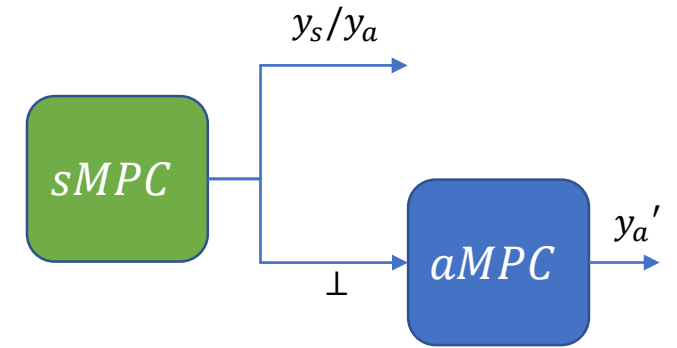


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OR function

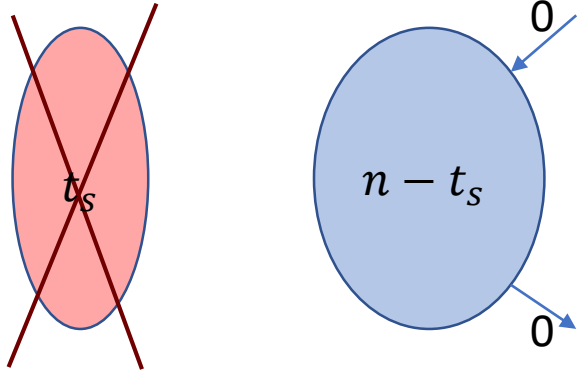


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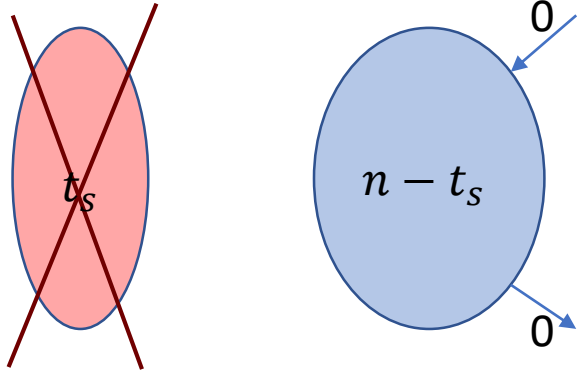
Synchronous

# Impossibility

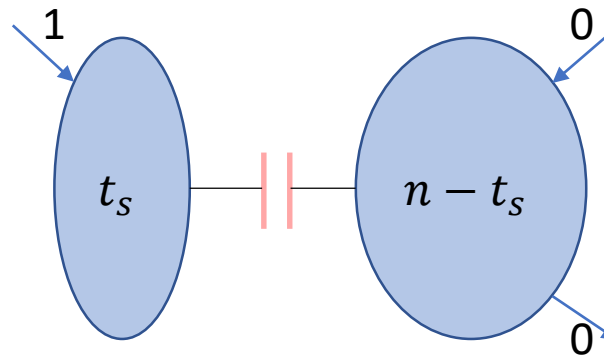
YES! If and only if  $t_a + 2t_s < n$  and  $L \geq t_s$

$L < t_s$  is impossible:  $t_s$ -security implies  $L \geq t_s$

OR function



Synchronous



Asynchronous

# Impossibility

YES! If and only if  $t_a + 2t_s < n$  and  $L \geq t_s$

$t_a + 2t_s \geq n$  is impossible (even when all inputs are ignored) [BKL19]

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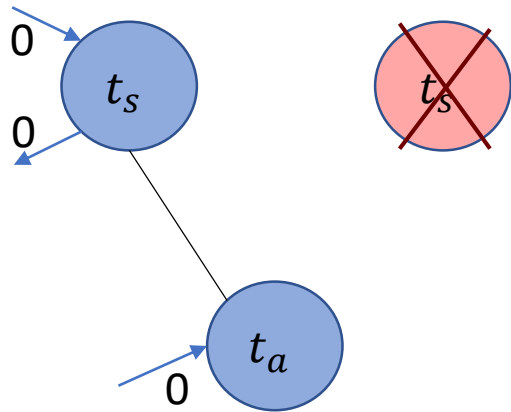
Majority function

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Majority function



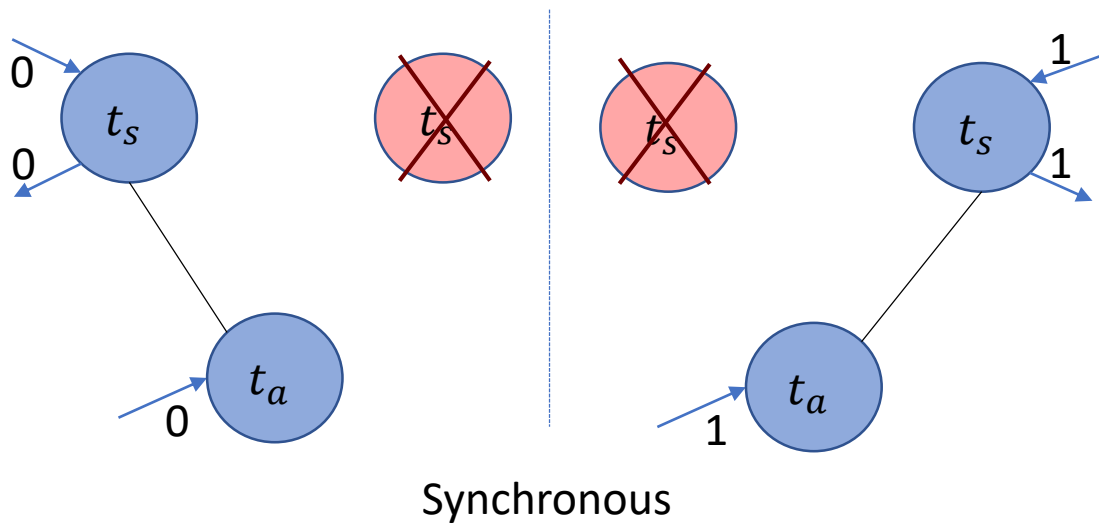
Synchronous

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Majority function

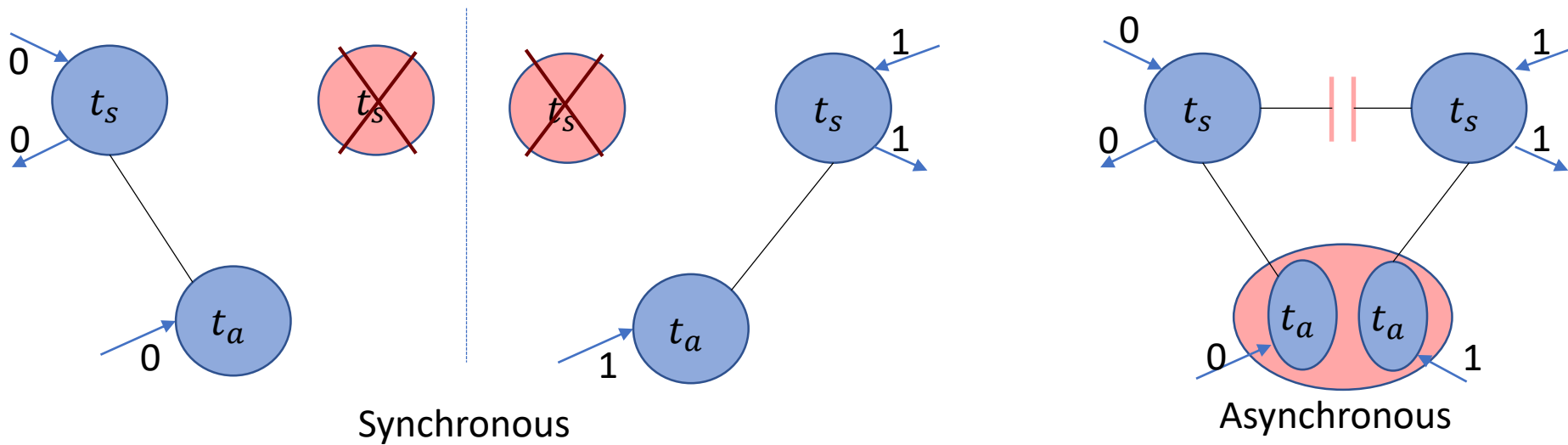


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Majority function



# References and Credits

ETH Zurich, Department of Computer Science  
[lichen@inf.ethz.ch](mailto:lichen@inf.ethz.ch)

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