Non-Malleable Secret Sharing against Bounded Joint-Tampering Attacks in the Plain Model

Gianluca Brian

Sapienza University of Rome Rome, Italy Antonio Faonio IMDEA Software Institute Madrid, Spain (Now at EUROCOM)

Maciej Obremski National University of Singapore Singapore, Singapore

Mark Simkin Aarhus University Aarhus, Denmark Daniele Venturi Sapienza University of Rome Rome, Italy

CRYPTO 2020 Online version





Unauthorized

Authorized

• Access structure: t-out-of-n



- Access structure: t-out-of-n
- Correctness: at least t parties are able to reconstruct the secret.





- Access structure: t-out-of-n
- Correctness: at least t parties are able to reconstruct the secret.
- Privacy: less than t parties should not be able to learn any information about the secret.











Side channel attacks: partial information from all the shares may reveal some information about the message!

SECURITY BREACH!



Side channel attacks: partial information from all the shares may reveal some information about the message! **Tampering attacks:** *m*['] may be related to *m*!

SECURITY BREACH!!!



Side channel attacks: partial information from all the shares may reveal some information about the message! **Tampering attacks:** *m*['] may be related to *m*!

SECURITY BREACH!!!

Leakage Resilient Secret Sharing [KMS18]: A reveals nothing about m for a restricted family \mathcal{G} .



Side channel attacks: partial information from all the shares may reveal some information about the message! **Tampering attacks:** *m*['] may be related to *m*!

SECURITY BREACH!!!

Leakage Resilient Secret Sharing [KMS18] : A reveals nothing about *m* for a restricted family \mathcal{G} . **Non-Malleable Secret Sharing** [GK18] : *m'* is unrelated to *m* for a restricted family \mathcal{F} .



Side channel attacks: partial information from all the shares may reveal some information about the message! **Tampering attacks:** *m*['] may be related to *m*!

SECURITY BREACH!!!

Leakage Resilient Secret Sharing [KMS18]: A reveals nothing about *m* for a restricted family \mathcal{G} . **Non-Malleable Secret Sharing** [GK18]: *m'* is unrelated to *m* for a restricted family \mathcal{F} . **Leakage-resilient non-malleability:** the best of both worlds.



Side channel attacks: partial information from all the shares may reveal some information about the message! **Tampering attacks:** *m*['] may be related to *m*!

SECURITY BREACH!!!

Leakage Resilient Secret Sharing [KMS18] : A reveals nothing about *m* for a restricted family \mathcal{G} . **Non-Malleable Secret Sharing** [GK18] : *m'* is unrelated to *m* for a restricted family \mathcal{F} . **Leakage-resilient non-malleability**: the best of both worlds.

Limitations: Impossible for arbitrary families \mathcal{G} and \mathcal{F} .

Our model

• Joint leakage and tampering (selective partitioning, semi-adaptive partitioning).

Our model

- Joint leakage and tampering (selective partitioning, semi-adaptive partitioning).
- $\bullet\,$ Bounded leakage: the total leakage amounts to at most ℓ bits.

Our model

- Joint leakage and tampering (selective partitioning, semi-adaptive partitioning).
- Bounded leakage: the total leakage amounts to at most ℓ bits.

Selective partitioning

• Any one-time statistically non-malleable secret sharing scheme is also leakage resilient.

Our model

- Joint leakage and tampering (selective partitioning, semi-adaptive partitioning).
- Bounded leakage: the total leakage amounts to at most ℓ bits.

Selective partitioning

- Any one-time statistically non-malleable secret sharing scheme is also leakage resilient.
- Corollary: lower bounds for the size of the shares of non-malleable secret sharing schemes using [NS20].

Our model

- Joint leakage and tampering (selective partitioning, semi-adaptive partitioning).
- Bounded leakage: the total leakage amounts to at most ℓ bits.

Selective partitioning

- Any one-time statistically non-malleable secret sharing scheme is also leakage resilient.
- Corollary: lower bounds for the size of the shares of non-malleable secret sharing schemes using [NS20].

Semi-adaptive partitioning

 We construct a one-time non-malleable secret-sharing scheme against joint leakage and tampering under semi-adaptive partitioning.

Our model

- Joint leakage and tampering (selective partitioning, semi-adaptive partitioning).
- Bounded leakage: the total leakage amounts to at most ℓ bits.

Selective partitioning

- Any one-time statistically non-malleable secret sharing scheme is also leakage resilient.
- Corollary: lower bounds for the size of the shares of non-malleable secret sharing schemes using [NS20].

Semi-adaptive partitioning

 We construct a one-time non-malleable secret-sharing scheme against joint leakage and tampering under semi-adaptive partitioning.

Both settings

• Corollary: construction of a p-time non-malleable secret sharing scheme from known techniques [OPVV18, BFV19].



S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 \cdots	s_1	s ₂	s 3	S 4	S 5	<i>s</i> ₆	S 7	<i>S</i> 8	S 9		S
--	-------	-----------------------	------------	------------	------------	-----------------------	------------	------------	------------	--	---

















Any one-time $\epsilon/2^{\ell}$ -non-malleable secret sharing scheme is also a ℓ -bounded leakage resilient one-time ϵ -non-malleable secret sharing scheme.





Any one-time $\epsilon/2^{\ell}$ -non-malleable secret sharing scheme is also a ℓ -bounded leakage resilient one-time ϵ -non-malleable secret sharing scheme.



Any one-time $\epsilon/2^{\ell}$ -non-malleable secret sharing scheme is also a ℓ -bounded leakage resilient one-time ϵ -non-malleable secret sharing scheme.



Any one-time $\epsilon/2^{\ell}$ -non-malleable secret sharing scheme is also a ℓ -bounded leakage resilient one-time ϵ -non-malleable secret sharing scheme.



Any one-time $\epsilon/2^{\ell}$ -non-malleable secret sharing scheme is also a ℓ -bounded leakage resilient one-time ϵ -non-malleable secret sharing scheme.



Any one-time $\epsilon/2^{\ell}$ -non-malleable secret sharing scheme is also a ℓ -bounded leakage resilient one-time ϵ -non-malleable secret sharing scheme.









Any one-time $\epsilon/2^{\ell}$ -non-malleable secret sharing scheme is also a ℓ -bounded leakage resilient one-time ϵ -non-malleable secret sharing scheme.



Any one-time $\epsilon/2^{\ell}$ -non-malleable secret sharing scheme is also a ℓ -bounded leakage resilient one-time ϵ -non-malleable secret sharing scheme.



Security against **semi-**adaptive partitioning

S_1 S_2 S_3 S_4 S_5 S_6 S_7 S_8 S_9 \cdots S_n	S	s ₁	S 2	S 3	S 4	S 5	S 6	S7	S 8	S 9		Sn
--	---	----------------	------------	------------	------------	------------	------------	----	------------	------------	--	----




Security against **semi-**adaptive partitioning



- The attacker only tampers within partitions whose subsets do not **partially** overlap with subsets belonging to leakage partitions.
- Much easier to achieve.

Construction inspired by [GK18]

Our *t*-out-of-*n* semi-adaptive leakage-resilient non-malleable secret sharing

Construction inspired by [GK18]



Building blocks:

• NMC: a 2-out-of-2 one-time non-malleable secret sharing scheme (i.e. a non malleable code);

Our t-out-of-n semi-adaptive leakage-resilient non-malleable secret sharing

Construction inspired by [GK18]



Building blocks:

- NMC: a 2-out-of-2 one-time non-malleable secret sharing scheme (i.e. a non malleable code);
- Share_L: a joint-leakage resilient *t*-out-of-*n* secret sharing scheme;
- Share_R: a joint-leakage resilient k'-out-of-n secret sharing scheme, where $k' \approx \sqrt{t}$.

Our t-out-of-n semi-adaptive leakage-resilient non-malleable secret sharing

Construction inspired by [GK18]



Building blocks:

- NMC: a 2-out-of-2 one-time non-malleable secret sharing scheme (i.e. a non malleable code);
- Share_L: a joint-leakage resilient *t*-out-of-*n* secret sharing scheme;
- Share_R: a joint-leakage resilient k'-out-of-n secret sharing scheme, where $k' \approx \sqrt{t}$.
- Security proof inspired by [KMS18]
- We extend their result obtaining security against joint tampering with k' 1 shares (instead of independent tampering).





• Split the tampering set into two subsets \mathcal{T}_0 and \mathcal{T}_1 such that $|\mathcal{T}_0| \geq$ threshold of Share_R.



- Split the tampering set into two subsets \mathcal{T}_0 and \mathcal{T}_1 such that $|\mathcal{T}_0| \geq$ threshold of Share_R.
- Hybrid 1: before tampering, replace the left shares within T_1 with valid and consistent shares of the same secret.



- Split the tampering set into two subsets \mathcal{T}_0 and \mathcal{T}_1 such that $|\mathcal{T}_0| \geq$ threshold of Share_R.
- Hybrid 1: before tampering, replace the left shares within T_1 with valid and consistent shares of the same secret.
 - Since we put the limitation of the **semi**-adaptive partitioning, the two subsets of shares T_0 and T_1 are unrelated each other even conditioning on the leakage.



• Split the tampering set into two subsets \mathcal{T}_0 and \mathcal{T}_1 such that $|\mathcal{T}_0| \geq$ threshold of Share_R.

- Since we put the limitation of the **semi**-adaptive partitioning, the two subsets of shares T_0 and T_1 are unrelated each other even conditioning on the leakage.
- This is because of each subset of each leakage partition containing only shares that are within at most one subset of the tampering partition.



• Split the tampering set into two subsets \mathcal{T}_0 and \mathcal{T}_1 such that $|\mathcal{T}_0| \geq$ threshold of Share_R.

- Since we put the limitation of the **semi**-adaptive partitioning, the two subsets of shares T_0 and T_1 are unrelated each other even conditioning on the leakage.
- This is because of each subset of each leakage partition containing only shares that are within at most one subset of the tampering partition.
- Hybrid 2: replace all the left shares with shares of an unrelated value \hat{s}_{L} .



• Split the tampering set into two subsets \mathcal{T}_0 and \mathcal{T}_1 such that $|\mathcal{T}_0| \geq$ threshold of Share_R.

- Since we put the limitation of the **semi**-adaptive partitioning, the two subsets of shares T_0 and T_1 are unrelated each other even conditioning on the leakage.
- This is because of each subset of each leakage partition containing only shares that are within at most one subset of the tampering partition.
- **Hybrid 2:** replace all the left shares with shares of an unrelated value \hat{s}_{L} .
- Hybrid 3-4: the same as in Hybrid 1-2, but on the right shares.



• Split the tampering set into two subsets \mathcal{T}_0 and \mathcal{T}_1 such that $|\mathcal{T}_0| \geq$ threshold of Share_R.

- Since we put the limitation of the **semi**-adaptive partitioning, the two subsets of shares T_0 and T_1 are unrelated each other even conditioning on the leakage.
- This is because of each subset of each leakage partition containing only shares that are within at most one subset of the tampering partition.
- **Hybrid 2:** replace all the left shares with shares of an unrelated value \hat{s}_{L} .
- Hybrid 3-4: the same as in Hybrid 1-2, but on the right shares.



• Split the tampering set into two subsets \mathcal{T}_0 and \mathcal{T}_1 such that $|\mathcal{T}_0| \geq$ threshold of Share_R.

- Since we put the limitation of the **semi**-adaptive partitioning, the two subsets of shares T_0 and T_1 are unrelated each other even conditioning on the leakage.
- This is because of each subset of each leakage partition containing only shares that are within at most one subset of the tampering partition.
- Hybrid 2: replace all the left shares with shares of an unrelated value \hat{s}_{L} .
- Hybrid 3-4: the same as in Hybrid 1-2, but on the right shares.
- Now we can safely reduce to non-malleability of the non-malleable code.

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

• Sample random coins *r* for Com.

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share*(m):Algorithm $\operatorname{Rec}^*((s_i^*)_{i \in \mathcal{I}})$:• Sample random coins r for Com.• Parse each $s_i^* = (\operatorname{com}_i, s_i)$.• Compute com \leftarrow Com(m; r).• Parse each $s_i^* = (\operatorname{com}_i, s_i)$.• Share $(s_1, \ldots, s_n) \leftarrow$ \$Share(m||r).• Parse each $s_i^* = (\operatorname{com}_i, s_i)$.• Output (s_1^*, \ldots, s_n^*) .

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.
- Reconstruct $m || r \leftarrow \text{Rec}((s_i)_{i \in \mathcal{I}})$.

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.
- Reconstruct $m || r \leftarrow \text{Rec}((s_i)_{i \in \mathcal{I}})$.
- Check that (m, r) is a valid opening for com.

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.
- Reconstruct $m || r \leftarrow \text{Rec}((s_i)_{i \in \mathcal{I}})$.
- Check that (m, r) is a valid opening for com.
- If everything is OK, output m; otherwise, output \perp .

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

Key ideas (very similar to [BFV19])

• By induction over the number of queries.

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.
- Reconstruct $m || r \leftarrow \text{Rec}((s_i)_{i \in \mathcal{I}})$.
- Check that (m, r) is a valid opening for com.
- If everything is OK, output m; otherwise, output \perp .

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ \$Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

Key ideas (very similar to [BFV19])

- By induction over the number of queries.
- Simulate tampering with leakage: obtain the mauled commitment and then extract the respective secret message.

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.
- Reconstruct $m || r \leftarrow \text{Rec}((s_i)_{i \in \mathcal{I}})$.
- Check that (m, r) is a valid opening for com.
- If everything is OK, output m; otherwise, output \perp .

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

Key ideas (very similar to [BFV19])

- By induction over the number of queries.
- Simulate tampering with leakage: obtain the mauled commitment and then extract the respective secret message.
- Check if everything is correct with the last tampering query.

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.
- Reconstruct $m || r \leftarrow \text{Rec}((s_i)_{i \in \mathcal{I}})$.
- Check that (m, r) is a valid opening for com.
- If everything is OK, output m; otherwise, output \perp .

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

Algorithm $\operatorname{Rec}^*((s_i^*)_{i \in \mathcal{I}})$:

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.
- Reconstruct $m || r \leftarrow \text{Rec}((s_i)_{i \in \mathcal{I}})$.
- Check that (m, r) is a valid opening for com.
- If everything is OK, output m; otherwise, output \perp .

Key ideas (very similar to [BFV19])

- By induction over the number of queries.
- Simulate tampering with leakage: obtain the mauled commitment and then extract the respective secret message.
- Check if everything is correct with the last tampering query.
- Commitment scheme \Longrightarrow computational setting.

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ \$Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

Algorithm $\operatorname{Rec}^*((s_i^*)_{i \in \mathcal{I}})$:

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.
- Reconstruct $m || r \leftarrow \text{Rec}((s_i)_{i \in \mathcal{I}})$.
- Check that (m, r) is a valid opening for com.
- If everything is OK, output m; otherwise, output \perp .

Key ideas (very similar to [BFV19])

- By induction over the number of queries.
- Simulate tampering with leakage: obtain the mauled commitment and then extract the respective secret message.
- Check if everything is correct with the last tampering query.
- Commitment scheme \Longrightarrow computational setting.
- Bounded leakage \implies *p*-time non-malleability (instead of continuously).

Known techniques [OPVV18, BFV19], building blocks: Share, Com.

Algorithm Share^{*}(*m*):

- Sample random coins *r* for Com.
- Compute com \leftarrow Com(m; r).
- Share $(s_1, \ldots, s_n) \leftarrow$ Share(m||r).
- Let, for all $i \in [n]$, $s_i^* = (\text{com}, s_i)$.
- Output $(s_1^*, ..., s_n^*)$.

Algorithm $\operatorname{Rec}^*((s_i^*)_{i \in \mathcal{I}})$:

- Parse each $s_i^* = (com_i, s_i)$.
- Check if all the com are all the same.
- Reconstruct $m || r \leftarrow \text{Rec}((s_i)_{i \in \mathcal{I}})$.
- Check that (m, r) is a valid opening for com.
- If everything is OK, output m; otherwise, output \perp .

Key ideas (very similar to [BFV19])

- By induction over the number of queries.
- Simulate tampering with leakage: obtain the mauled commitment and then extract the respective secret message.
- Check if everything is correct with the last tampering query.
- Bounded leakage \implies *p*-time non-malleability (instead of continuously).
- Security against joint tampering.

Our results

• We prove that a non-malleable secret sharing scheme is also leakage resilient.

Our results

- We prove that a non-malleable secret sharing scheme is also leakage resilient.
- We give a construction of a leakage-resilient non-malleable secret sharing scheme against **semi-**adaptive partitioning.

Our results

- We prove that a non-malleable secret sharing scheme is also leakage resilient.
- We give a construction of a leakage-resilient non-malleable secret sharing scheme against **semi-**adaptive partitioning.
- Corollary: lower bounds on the size of the shares of a non-malleable secret sharing scheme.

Our results

- We prove that a non-malleable secret sharing scheme is also leakage resilient.
- We give a construction of a leakage-resilient non-malleable secret sharing scheme against **semi-**adaptive partitioning.
- Corollary: lower bounds on the size of the shares of a non-malleable secret sharing scheme.
- **Corollary:** construction of a *p*-time non-malleable secret sharing scheme.

Our results

- We prove that a non-malleable secret sharing scheme is also leakage resilient.
- We give a construction of a leakage-resilient non-malleable secret sharing scheme against **semi-**adaptive partitioning.
- Corollary: lower bounds on the size of the shares of a non-malleable secret sharing scheme.
- **Corollary:** construction of a *p*-time non-malleable secret sharing scheme.

Open problems / Work in Progress

- Actually, we already have some preliminary work in progress...
- Continuous non-mallebility against joint selective/(semi-)adaptive in the plain model.

Our results

- We prove that a non-malleable secret sharing scheme is also leakage resilient.
- We give a construction of a leakage-resilient non-malleable secret sharing scheme against **semi-**adaptive partitioning.
- Corollary: lower bounds on the size of the shares of a non-malleable secret sharing scheme.
- **Corollary:** construction of a *p*-time non-malleable secret sharing scheme.

Open problems / Work in Progress

- Actually, we already have some preliminary work in progress...
- Continuous non-mallebility against joint selective/(semi-)adaptive in the plain model.
- Optimal rate, i.e. size of message size of share

Our results

- We prove that a non-malleable secret sharing scheme is also leakage resilient.
- We give a construction of a leakage-resilient non-malleable secret sharing scheme against **semi-**adaptive partitioning.
- Corollary: lower bounds on the size of the shares of a non-malleable secret sharing scheme.
- **Corollary:** construction of a *p*-time non-malleable secret sharing scheme.

Open problems / Work in Progress

- Actually, we already have some preliminary work in progress...
- Continuous non-mallebility against joint selective/(semi-)adaptive in the plain model.
- Optimal rate, i.e. size of message size of share

Thank You!