Two Round Oblivious Transfer from CDH or LPN

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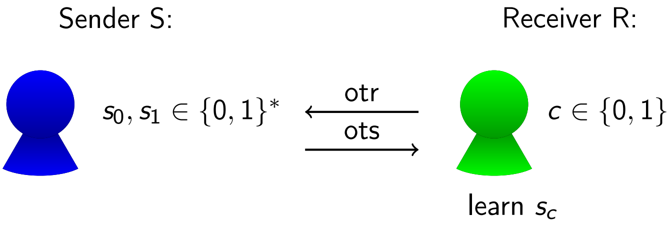
Oblivious Transfer (OT)

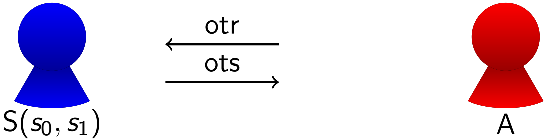


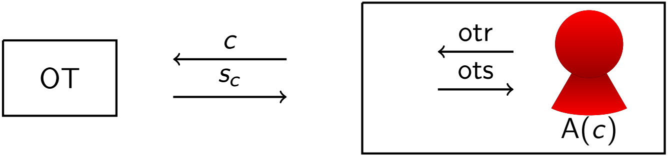
Security

- S does not learn c.
- R does not learn s_{1-c}

Simulation based Security (for Sender S)

For any A,





Security for Receiver R

Simulation based Security

- Same as for Sender
- A′ needs to extract s_0, s_1

Indistinguishability based Security

- weaker than simulation based
- malicious S cannot distinguish R(0) from R(1)

Our Results

Sim. Sender, Ind. Receiver Secure OT ($\tilde{\mathsf{OT}}$) $\Rightarrow$ Sim. Secure OT

- $\tilde{\mathsf{OT}}$ $\Rightarrow$ 2-round ZK
- $\tilde{\mathsf{OT}}$ + 2-round ZK $\Rightarrow$ Sim. Secure OT

CDH or LPN $\Rightarrow$ $\tilde{\mathsf{OT}}$

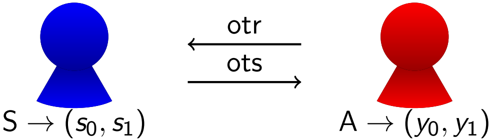
- weaker OT security notions for the sender
- CDH or LPN $\Rightarrow$ weaker notions
- generic transformation from weaker notions to $\tilde{\mathsf{OT}}$

Summary

$\tilde{OT}$ from CDH

1. CDH or LPN $\Rightarrow$ Elementary OT (eOT)
2. Elementary OT $\Rightarrow$ Search OT (sOT)
3. Search OT $\Rightarrow$ Indistinguishable OT (iOT)
4. Indistinguishable OT $\Rightarrow$ $\tilde{OT}$

CDH $\Rightarrow$ eOT $\Rightarrow$ sOT $\Rightarrow$ iOT $\Rightarrow$ $\tilde{\text{OT}}$



Elementary OT Security

$$\Pr[(y_0, y_1) = (s_0, s_1)] \leq \text{negl}$$

CDH ⇒ eOT ⇒ sOT ⇒ iOT ⇒ $\tilde{\text{OT}}$

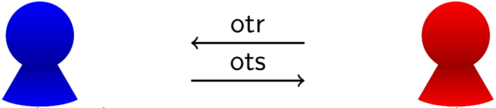
Bellare, Micali [BM90]:

Sender S:	CRS : $(X = g^x)$	Receiver R(c):
		$r \leftarrow \mathbb{Z}_p$
$h_1 = h_0 X$	$\xleftarrow{\text{otr} = h_0}$	$h_0 = g^r X^{-c}$
$s \leftarrow \mathbb{Z}_p$		
$S = g^s$	$\xrightarrow{\text{ots} = S}$	
		output S^r

Correctness and Security

- $s_c = h_c^s = (h_0 X^c)^s = (g^r X^{-c} X^c)^s = S^r$
- $s_{1-c} = h_{1-c}^s = (h_0 X^{1-c})^s = X^{(1-2c)s} S^r$
- computing $s_0 / s_1 = g^{xs}$ solves CDH for challenge X, S

$\mathsf{CDH} \Rightarrow \mathsf{eOT} \Rightarrow \mathsf{sOT} \Rightarrow \mathsf{iOT} \Rightarrow \tilde{\mathsf{OT}}$



$\mathsf{S} \to (s_0, s_1)$

$\mathsf{A}_1 \to (\mathsf{st}, \mathsf{otr})$

$\mathsf{A}_2(\mathsf{st}, \mathsf{ots}, w) \to y_w$

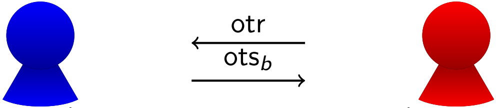
Search OT Security

With $1 - \mathsf{negl}$ probability over $(\mathsf{st}, \mathsf{otr})$,
$\exists w \in \{0, 1\}$ s.t. $\Pr_{\mathsf{ots}}[\mathsf{A}_2(\mathsf{st}, \mathsf{ots}, w) = s_w] \leq \mathsf{negl}$.

Elementary OT $\Rightarrow$ Search OT

$\Pr_{\mathsf{ots}}[\mathsf{A}_2(\mathsf{st}, \mathsf{ots}, w) = s_w] > \frac{3}{4} \Rightarrow$
$\Pr_{\mathsf{ots}}[\forall w, \mathsf{A}_2(\mathsf{st}, \mathsf{ots}, w) = s_w] > \mathsf{negl}$.
Solution: Amplify hardness (Canetti, Halevi, Steiner [CHS05])

CDH $\Rightarrow$ eOT $\Rightarrow$ <u>sOT $\Rightarrow$ iOT</u> $\Rightarrow$ $\widetilde{\text{OT}}$



$\mathsf{S}(\mathsf{otr}, m_0, m_1) \rightarrow \mathsf{ots}_0$
$\mathsf{S}(\mathsf{otr}, m_{1-w}, \mathsf{uniform}) \rightarrow \mathsf{ots}_1$

$\mathsf{A}_1 \rightarrow (\mathsf{st}, \mathsf{otr}, m_0, m_1)$
$\mathsf{A}_2(\mathsf{st}, \mathsf{ots}_b) \rightarrow b'$

Indistinguishable OT Security

With $1 - \mathsf{negl}$ probability over $(\mathsf{st}, \mathsf{otr})$, $\exists w \in \{0, 1\}$ s.t.
$|\Pr_{\mathsf{ots}}[\mathsf{A}_2(\mathsf{st}, \mathsf{ots}_0) = 1] - \Pr_{\mathsf{ots}}[\mathsf{A}_2(\mathsf{st}, \mathsf{ots}_1) = 1]| \leq \mathsf{negl}$.

Search OT $\Rightarrow$ Indistinguishable OT

Goldreich Levin hardcore predicates [GL89], hybrid argument.

CDH $\Rightarrow$ eOT $\Rightarrow$ sOT $\Rightarrow$ iOT $\Rightarrow$ $\tilde{\text{OT}}$

$\text{CRS} = (\text{CRS}_{\text{iOT}}, \text{pk})$

Sender $\mathsf{S}(m_0, m_1)$:

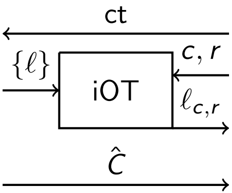
$C[\text{ct}, \text{CRS}, m_0, m_1](c, r)$:

  If $(\text{ct} = \text{Enc}(\text{pk}, c; r))$

  Then output m_c

  Else output $\perp$

$(\hat{C}, \{\ell\}) \leftarrow \text{Garble}(C)$



Receiver $\mathsf{R}(c)$:

$\text{ct} = \text{Enc}(\text{pk}, c; r)$

$m_c = \hat{C}(\ell_{c,r})$

Receiver Ind., Sender Sim. Security

- ct and iOT do not leak c
- Given sk, c can be extracted
- Can iOT and $\hat{C}$ be simulated without m_{1-c}?

Sender's Simulation based Security

Garbled Circuits; Yao [Yao82]

- $\{\ell\}$ and $\hat{C}$ leak m_0 and m_1.
- $\ell_{c,r}$, $\hat{C}$ only leak m_c.

Solution: Use independent $\{\ell\} \setminus \ell_{c,r}$ for $\hat{C}$ and iOT.

Distinguisher Dependent Simulation; Jain, Kalai, Khurana, Rothblum [JKKR17]

- Indistinguishable OT: $\exists w \in \{0,1\}$ s.t. $\ell_w \approx_c$ uniform.
- We test run the adversary to learn $w \in \{0,1\}$.
- In the actual simulation, w is consistent with good probability.
- We can replace $\ell_w \in \{\ell\} \setminus \ell_{c,r}$ with uniform.

Summary

Our Results, eprint.iacr.org/2019/414

1. CDH or LPN $\Rightarrow$ Elementary OT
2. Elementary OT $\Rightarrow$ Search OT
   (Hardness Amplification; Canetti, Halevi, Steiner [CHS05])
3. Search OT $\Rightarrow$ Indistinguishable OT
   (Hardcore Predicates; Goldreich, Levin [GL89])
4. Indistinguishable OT $\Rightarrow$ $\tilde{\text{OT}}$
   (Distinguisher Dependent Simulation; Jain, Kalai, Khurana, Rothblum [JKKR17], Garbled Circuits; Yao [Yao82])
5. $\tilde{\text{OT}}$ + 2-round ZK $\Rightarrow$ Sim. Secure OT
   ($\tilde{\text{OT}}$ $\Rightarrow$ 2-round ZK)