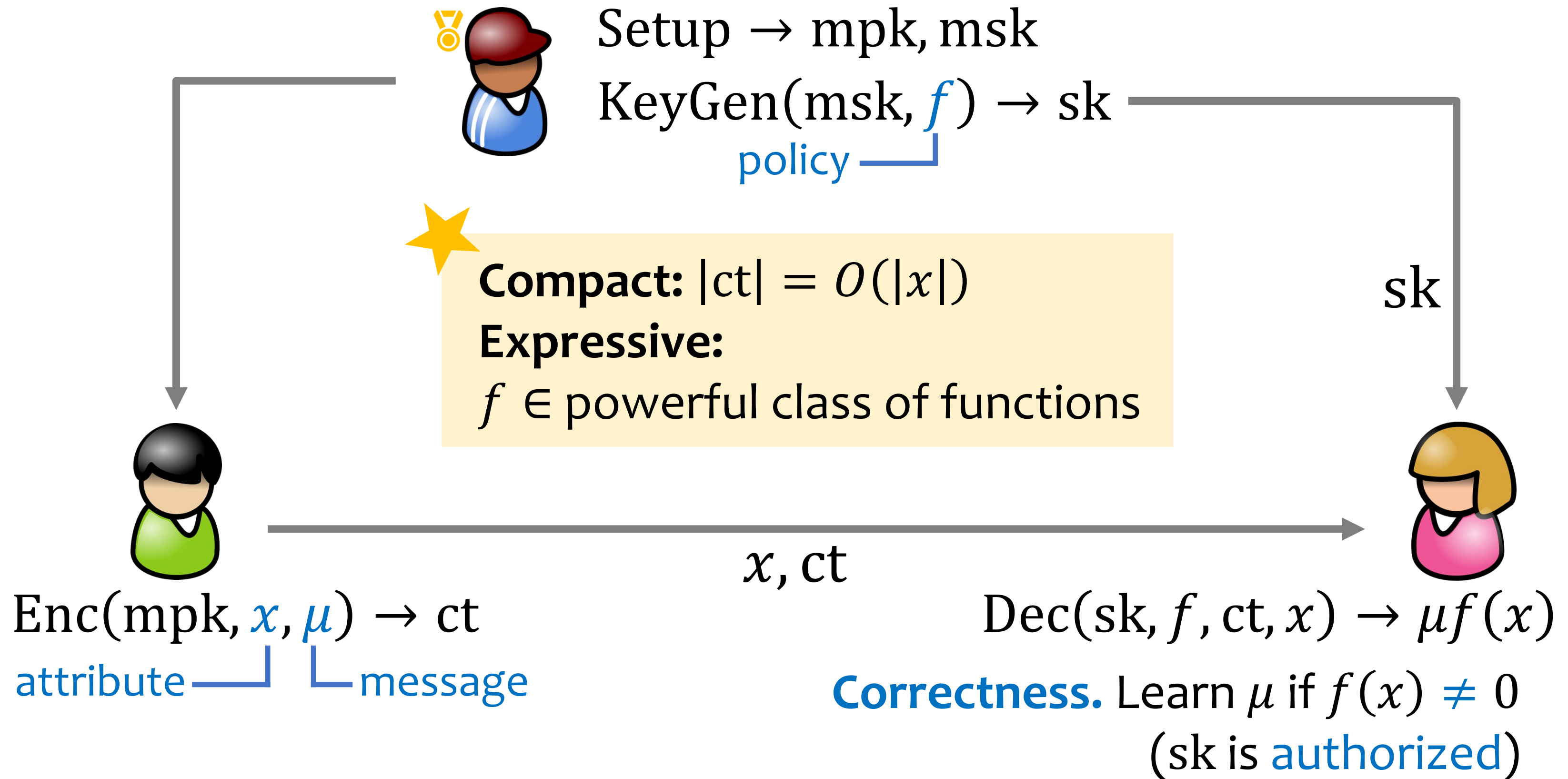


# Compact Adaptively Secure ABE from $k$ -Lin: Beyond NC<sup>1</sup> and Towards NL

Huijia (Rachel) Lin and [Ji Luo](#)

UNIVERSITY *of* WASHINGTON

# Attribute-Based Encryption [SW05]



# Attribute-Based Encryption [SW05]



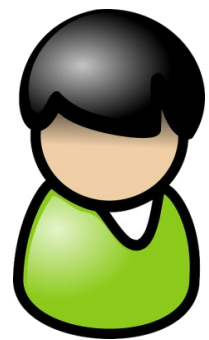
Setup  $\rightarrow$  mpk, msk

KeyGen(msk,  $f_i$ )  $\rightarrow$   $sk_i$

## Collusion Resistance

Message is hidden given arbitrary number of unauthorized keys.

$sk_i$ 's



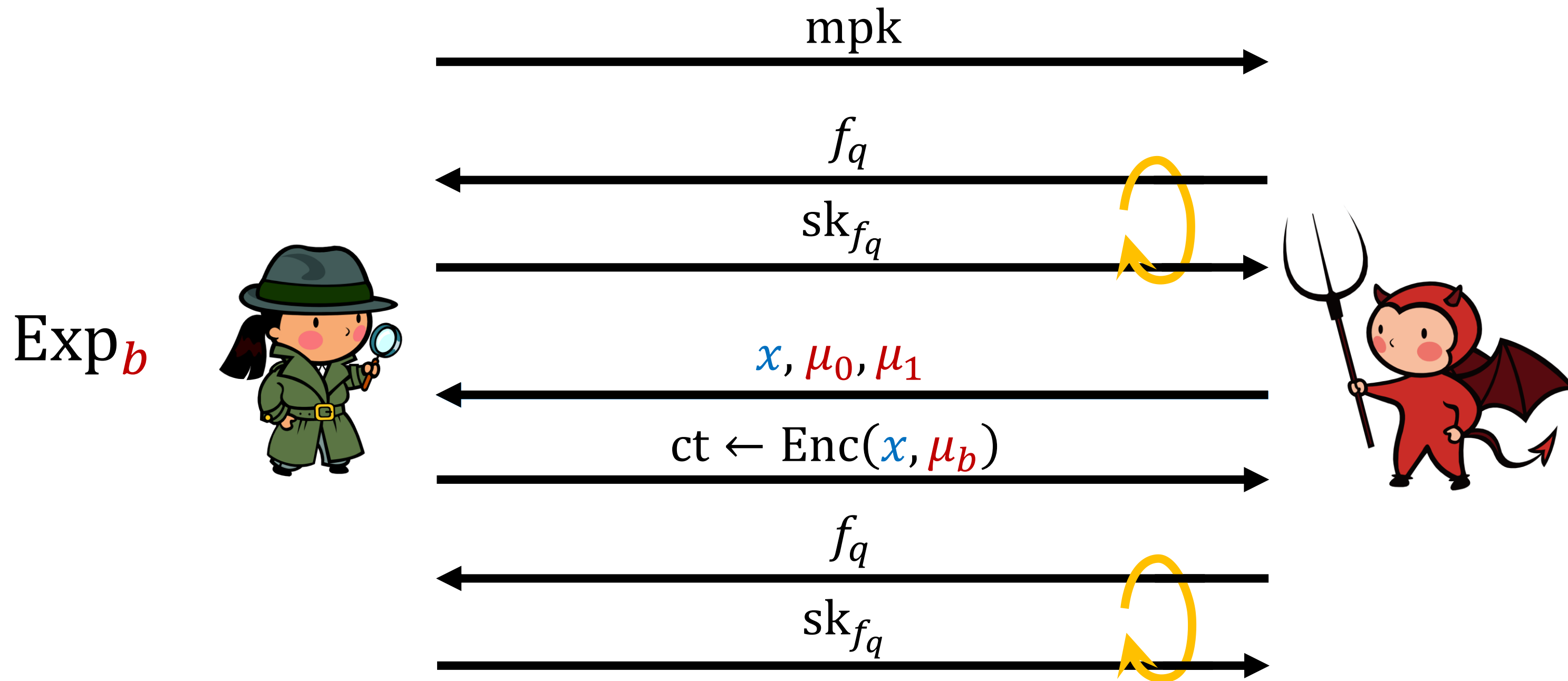
Enc(mp<sub>k</sub>,  $x$ ,  $\mu$ )  $\rightarrow$  ct

$x$ , ct



**Security.** Hide  $\mu$  if  $f_i(x) = 0$  for all  $i$   
( $sk_i$ 's are unauthorized)

# Adaptive IND-CPA Security

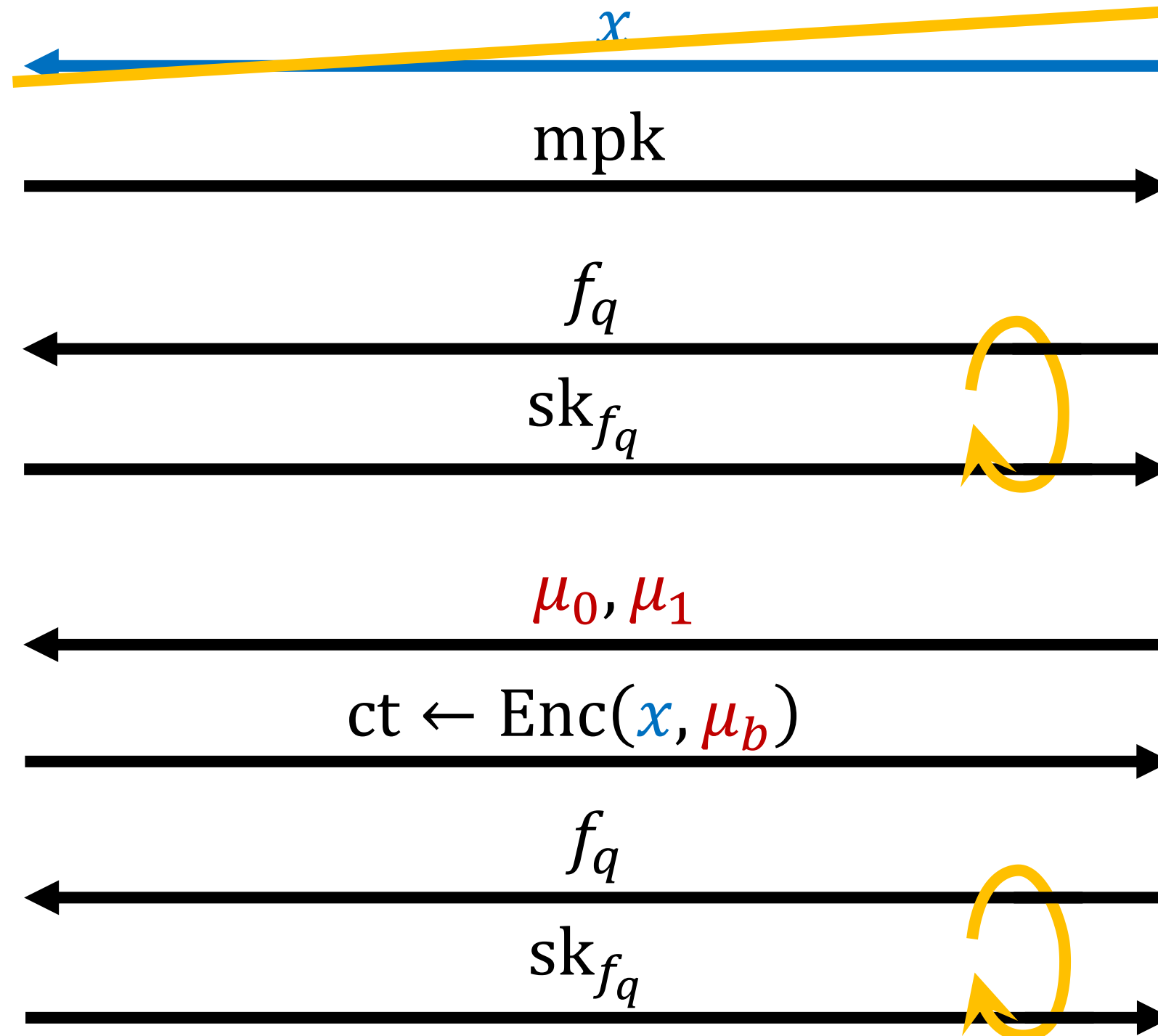


if for all queried keys  $f_q(x) = 0$ , then  $\text{Exp}_0 \approx \text{Exp}_1$

# ~~(Weaker) Selective~~ IND-CPA Security

★ Adaptive Security

Exp<sub>b</sub>



if for all queried keys  $f_q(x) = 0$ , then  $\text{Exp}_0 \approx \text{Exp}_1$

# Challenging to have it all

- ★ Compactness:  $|ct| = O(|x|)$
- ★ Adaptive Security
- ★ Standard Assumptions

NC<sup>1</sup> and ABP  
are **non-uniform**:  
Each sk works with  
attribute of **fixed** length.

*not flexible*

Goal. Have it ALL for expressive classes of policies.

Previously, the largest class was NC<sup>1</sup> [KW19].

Contribution 1. Extend to **ABP**.

Arithmetic **B**ranching **P**rograms  $\supseteq$  NC<sup>1</sup>, **arithmetic** computation over  $\mathbb{Z}_p$ .

# Challenging to have it all

- ★ Compactness:  $|ct| = O(|x|)$
- ★ Adaptive Security
- ★ Standard Assumptions

**flexible**

ABE for **uniform** computation:  
Each  $sk$  works with attribute of **any** length.

**Contribution 2. DFA, NFA** (regular languages)

the **first** ABE for uniform computation with all above

**L, NL** \* (log-space Turing machines)

\* relaxed compactness

# Related Works: Non-Uniform Model

**NOT compact**

[LOSTW10] for MSP

**NOT adaptive**

[GPSW06] for MSP  
[GVW13, BGGHNSVV14]  
for  $P/\text{poly}$

**NON-standard  
assumptions**

[LW12] for MSP  
 $q$ -type assumption

---

**all-in-one: compact, adaptive, standard assumptions**

[KW19] for  $\text{NC}^1$

this work for ABP  $\Leftarrow k\text{-Lin}$  in pairing groups

concurrent [GW20] for BP



# Related Works: Uniform Model

NOT compact

or

NOT adaptive

or

NON-standard assumptions

[Wat12, Att14, AMY19, GWW19] for DFA

concurrent [GW20] for NFA

---

**all-in-one: compact, adaptive, standard assumptions**

→ this work for DFA, NFA

concurrent [GW20] for DFA

*k*-Lin

---

**beyond finite automata**

[AS16] for P (FE, based on **iO**)

→ this work for L, NL  $|ct| = O(|x|TS^S)$  (relaxed compactness)  
 $|sk| = O(|TM|)$

# New General Framework

computational tool

Inner-Product

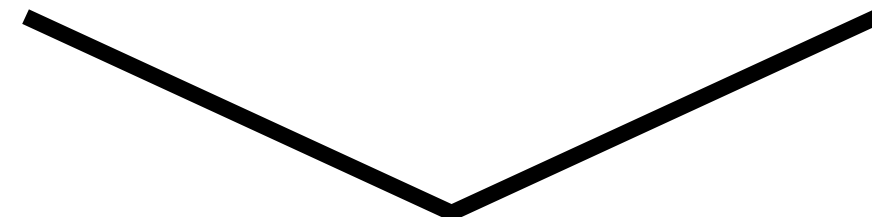
Functional Encryption

information-theoretic tool

Arithmetic Key

Garbling Scheme

special randomized encoding



1-ABE

=

1-key

1-ciphertext

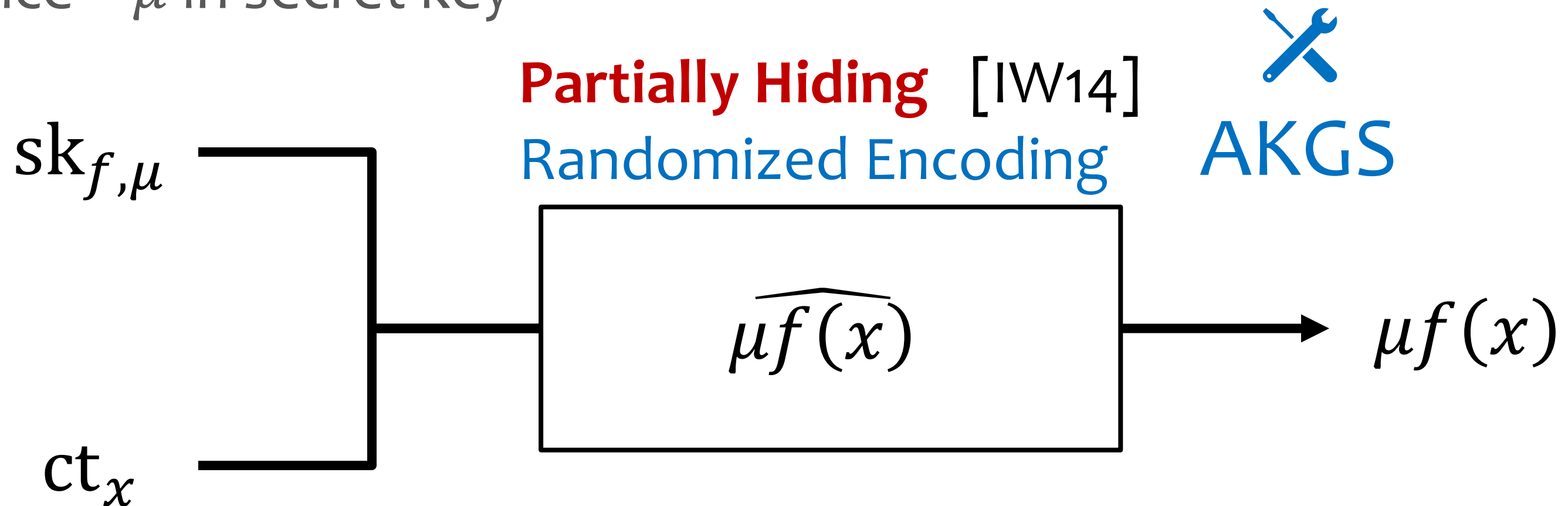
secret-key



ABE

# 1-ABE via AKGS and IPFE

convenience –  $\mu$  in secret key



use  $\mu$  as one-time pad

Secure:  $\widehat{\mu f(x)}$  hides  $\mu$  beyond  $\mu f(x)$ .

It does **not** hide  $f, x$ .

 compute using IPFE  $\Rightarrow$

Simple: RE is **linear** in  $x$ .

# Arithmetic Key Garbling Scheme

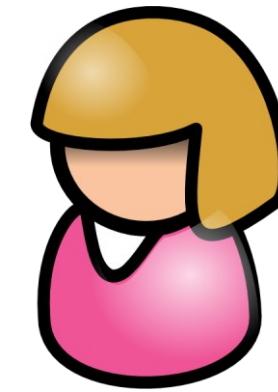
1. Label functions:  $L_1, \dots, L_m \leftarrow \text{Garble}(f, \mu; r)$
2. Garblings:  $\ell_1, \dots, \ell_m = L_1(x), \dots, L_m(x)$

a.k.a. "labels"

$$f: \mathbb{Z}_p^n \rightarrow \mathbb{Z}_p$$
$$x \in \mathbb{Z}_p^n$$



$$f, x, \ell_1, \dots, \ell_m$$

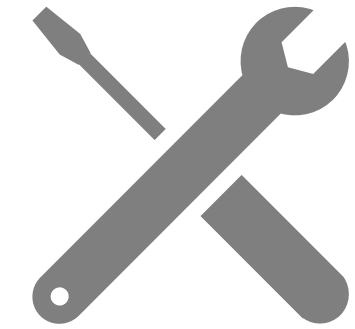


$$\text{Eval}(f, x, \ell_1, \dots, \ell_m) = \mu f(x)$$

**Security (partial hiding).**

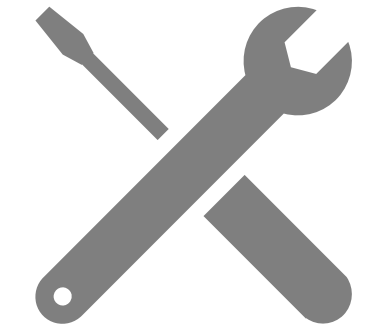
$$\text{Sim}(\underline{f}, x, \mu f(x)) \rightarrow \ell_1, \dots, \ell_m$$

not hidden



# Arithmetic Key Garbling Scheme

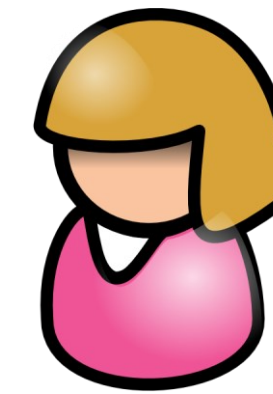
1. Label functions:  $L_1, \dots, L_m \leftarrow \text{Garble}(f, \mu; r)$
2. Garblings:  $\ell_1, \dots, \ell_m = L_1(x), \dots, L_m(x)$



$$f: \mathbb{Z}_p^n \rightarrow \mathbb{Z}_p$$
$$x \in \mathbb{Z}_p^n$$



$$f, x, \ell_1, \dots, \ell_m$$



$$\text{Eval}(f, x, \ell_1, \dots, \ell_m) = \mu f(x)$$

## Linearity.

1.  $L_1, \dots, L_m$  are linear in  $x$ :  $L_j(x) = \langle L_j, x \rangle$
2. coefficients of  $L_1, \dots, L_m$  are linear in  $\mu, r$
3. Eval is linear in  $\ell_1, \dots, \ell_m$

thanks to  
partial hiding

# Inner-Product Functional Encryption

$$\begin{array}{l} \text{isk} \leftarrow \text{KeyGen}(\text{msk}, \mathbf{v}) \\ \text{ict} \leftarrow \text{Enc}(\text{msk}, \mathbf{u}) \end{array} \xrightarrow{\text{Dec}} \langle \mathbf{u}, \mathbf{v} \rangle$$



## Function-Hiding Property

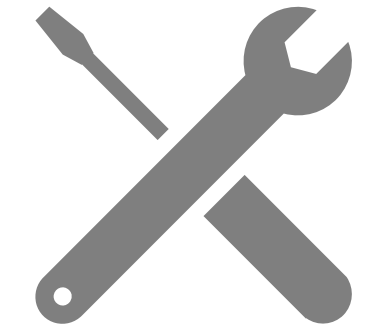
$$\left\{ \begin{array}{l} \text{isk}(\mathbf{v}_1) \quad \text{isk}(\mathbf{v}_2) \quad \cdots \quad \text{isk}(\mathbf{v}_I) \\ \text{isk}(\mathbf{u}_1) \quad \text{ict}(\mathbf{u}_2) \quad \cdots \quad \text{ict}(\mathbf{u}_J) \end{array} \right\}$$

**Adaptive Security:**  
isk/ict can interleave.

if  $\langle \mathbf{u}_i, \mathbf{v}_j \rangle = \langle \mathbf{u}'_i, \mathbf{v}'_j \rangle$  for all  $i, j$   $\approx$   $\left\{ \begin{array}{l} \text{isk}(\mathbf{v}'_1) \quad \text{isk}(\mathbf{v}'_2) \quad \cdots \quad \text{isk}(\mathbf{v}'_I) \\ \text{isk}(\mathbf{u}'_1) \quad \text{ict}(\mathbf{u}'_2) \quad \cdots \quad \text{ict}(\mathbf{u}'_J) \end{array} \right\}$

# Pairing-Based IPFE [ALS16, LV16]

$$\begin{array}{l} \llbracket \text{isk} \rrbracket_2 \leftarrow \text{KeyGen}(\text{msk}, \llbracket \mathbf{v} \rrbracket_2) \\ \llbracket \text{ict} \rrbracket_1 \leftarrow \text{Enc}(\text{msk}, \llbracket \mathbf{u} \rrbracket_1) \end{array} \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \text{Dec} \\ = \text{pairing} \end{array} \rightarrow \llbracket \langle \mathbf{u}, \mathbf{v} \rangle \rrbracket_{\mathbb{T}}$$



## Asymmetric Pairing Groups

$$\begin{array}{l} G_1: \llbracket a \rrbracket_1 = g_1^a \\ G_2: \llbracket b \rrbracket_2 = g_2^b \end{array} \begin{array}{c} \diagup \\ \diagdown \end{array} \begin{array}{c} \text{pairing} \\ \text{operation} \end{array} \rightarrow \llbracket ab \rrbracket_{\mathbb{T}} = g_{\mathbb{T}}^{ab} \in G_{\mathbb{T}}$$

# 1-ABE via AKGS and IPFE

$$L_1, \dots, L_m \leftarrow \text{Garble}(f, \mu)$$

$$\text{sk}_{f, \mu} = \{\text{isk}(L_j)\}_{j \in [m]}$$

$$\text{ct}_x = \text{ict}(x)$$

labels in the exponent

IPFE  
Dec

$$\llbracket \ell_j = L_j(x) \rrbracket_{\mathbb{T}}$$

Eval ✓ linear

$$\llbracket \mu f(x) \rrbracket_{\mathbb{T}}$$

## Intuitions for Security.

- IPFE  $\implies$  only  $\ell_j$ 's are revealed
- AKGS  $\implies$  only  $\mu f(x)$  is revealed



# Selective Security of 1-ABE

Real World

$\xleftarrow{x}$  s.t.  $f(x) = 0$

$sk_{f,\mu} \quad \{ isk(L_j, 0) \}$

$ct_x \quad ict(x, 0)$

Next step: hardwire labels in secret key

want.  $\mu$  is hidden

$L_1, \dots, L_m \leftarrow \text{Garble}(f, \mu)$

$\ell_j = L_j(x)$

# Hardwire Labels in Secret Key via IPFE

Next step: simulate labels

$\xleftarrow{x}$  s.t.  $f(x) = 0$

want.  $\mu$  is hidden

$sk_{f,\mu} \quad \{ isk ( 0 \quad \ell_j ) \}$

$L_1, \dots, L_m \leftarrow \text{Garble}(f, \mu)$

$\ell_j = L_j(x)$

$ct_x \quad ict ( x \quad 1 )$

# Simulate Labels via AKGS

$\xleftarrow{x}$  s.t.  $f(x) = 0$

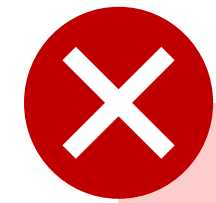
want.  $\mu$  is hidden ✓

$sk_{f,\mu} \quad \{ isk(0, \ell_j) \}$

$\ell_1, \dots, \ell_m \leftarrow \text{Sim}(f, x, \underset{0}{=} \mu f(x))$

$ct_x \quad ict(x, 1)$

# Adaptive Security?



need  $x$  to simulate

$$\text{sk}_{f,\mu} = \{ \text{isk} ( 0, \ell_j ) \} \quad \ell_1, \dots, \ell_m \leftarrow \text{Sim}(f, \underset{0}{x}, \mu f(x))$$

$\xleftarrow{x}$  s.t.  $f(x) = 0$

$$\text{ct}_x = \text{ict} ( x, 1 )$$

**Idea.** Rely on **special structure** of simulator.

# Special Simulation Structure

## Real Garbling

$\ell_1, \dots, \ell_m$  are uniformly random subject to correctness:

$$\text{Eval}(f, x, \ell_1, \dots, \ell_m) = \mu f(x).$$

 linear constraint

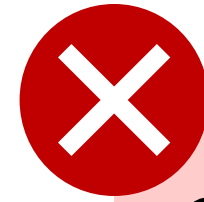
## Simulator

1. Draw  $\ell_2, \dots, \ell_m \leftarrow \mathbb{Z}_p$ . ☺ independent of  $x$

2. Find unique  $\ell_1$  s.t. evaluation is correct.

☺ only one label depends on  $x$

# Simulation for Adaptive Security



equation depends on  $x$

$sk_{f,\mu}$	$isk ( 0 \ell_1 )$	find $\ell_1$ s.t. $Eval(f, x, \dots) = \mu f(x) \stackrel{!}{=} 0$
	$isk ( 0 \ell_2 )$	$\ell_2 \leftarrow \mathbb{Z}_p$
	$\vdots$	$\vdots$
	$isk ( 0 \ell_j )$	$\ell_j \leftarrow \mathbb{Z}_p$
	$\vdots$	$\vdots$
	$\xleftarrow{x}$	s.t. $f(x) = 0$
$ct_x$	$ict ( x 1 )$	

**Idea. Put  $\ell_1$  in ciphertext**

# Simulation for Adaptive Security

**valid simulation strategy**

$sk_{f,\mu}$      $isk ( 0 \quad 1 \quad 0 )$   
 $isk ( 0 \quad 0 \quad \ell_2 )$   
 $\vdots$   
 $isk ( 0 \quad 0 \quad \ell_j )$   
 $\vdots$

$\ell_2 \leftarrow \mathbb{Z}_p$

$\vdots$

$\ell_j \leftarrow \mathbb{Z}_p$

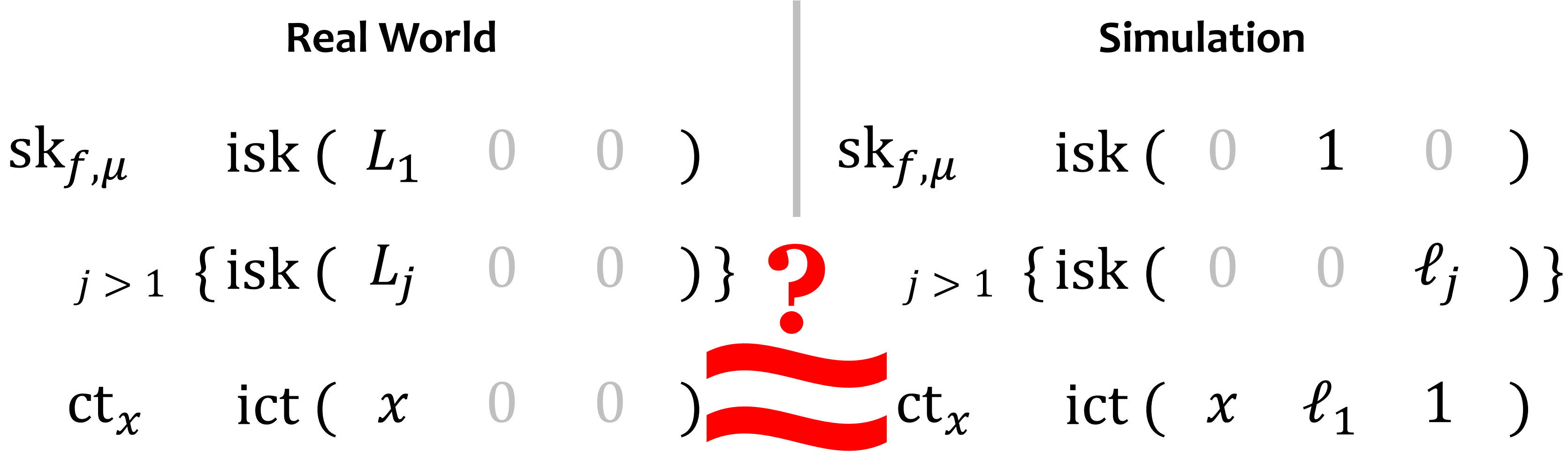
$\vdots$

$\xleftarrow{x} \text{ s.t. } f(x) = 0$

find  $\ell_1$  s.t.  $\text{Eval}(f, x, \dots) = 0$

$ct_x$      $ict ( x \quad \ell_1 \quad 1 )$

# Real World vs. Simulation



need same labels to use IPFE

$L_1, \dots, L_m \leftarrow \text{Garble}(f, \mu)$   
 $\ell_1, \dots, \ell_m = L_1(x), \dots, L_m(x)$

honestly generated labels

$\ell_2, \dots, \ell_m \leftarrow \mathbb{Z}_p$   
 find  $\ell_1$  s.t.  $\text{Eval}(\dots) = \mu f(x) = 0$

simulated labels



same distribution of labels



# Bridging the Gap: Piecewise Security

$$L_1, \dots, L_m \leftarrow \text{Garble}(f, \mu)$$

**Labels** are **marginally random** given **subsequent label functions**.

for  $j > 1$  and all  $x$ :

$$(L_j(x), L_{j+1}, \dots, L_m) \equiv (\$, L_{j+1}, \dots, L_m)$$

piecewise  
security

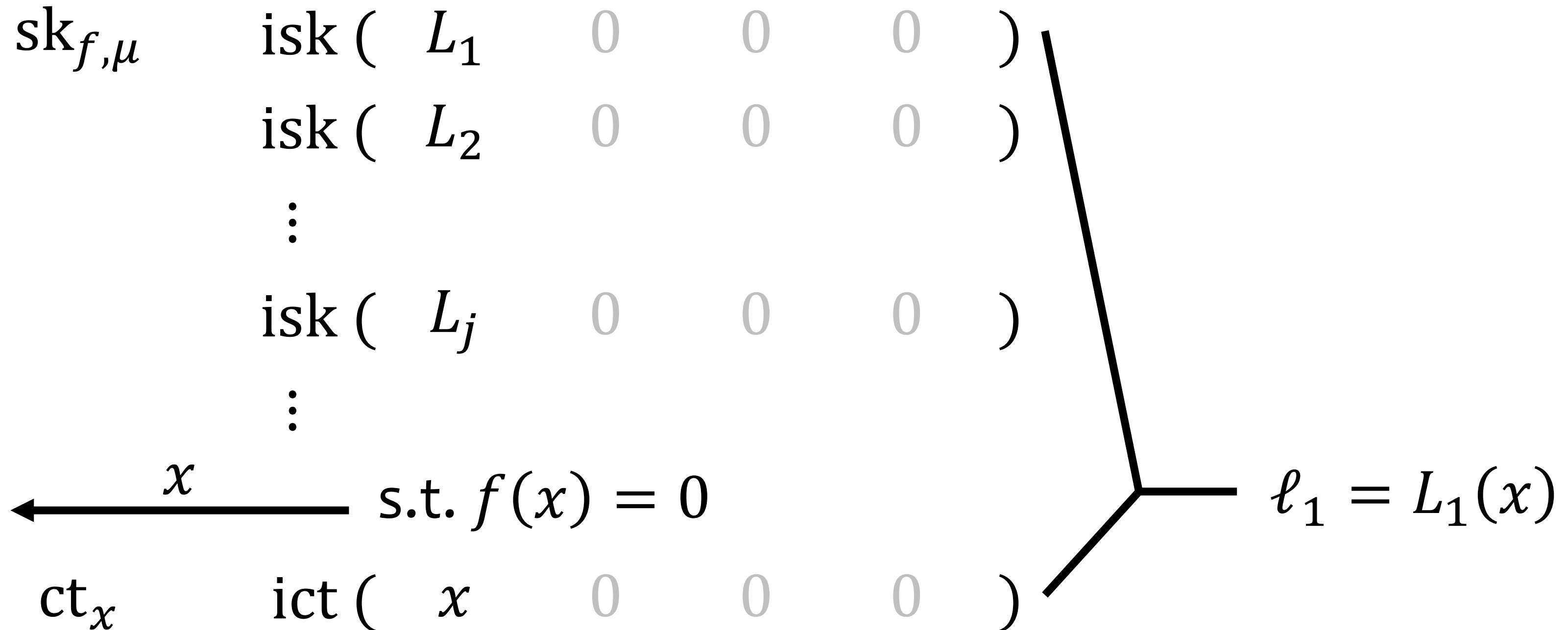
$\ell_1$  is uniquely determined by  $\text{Eval}(\dots) = \mu f(x)$ .

We show that AKGS for ABP [IW14] is piecewise secure.

# Adaptive Security of 1-ABE

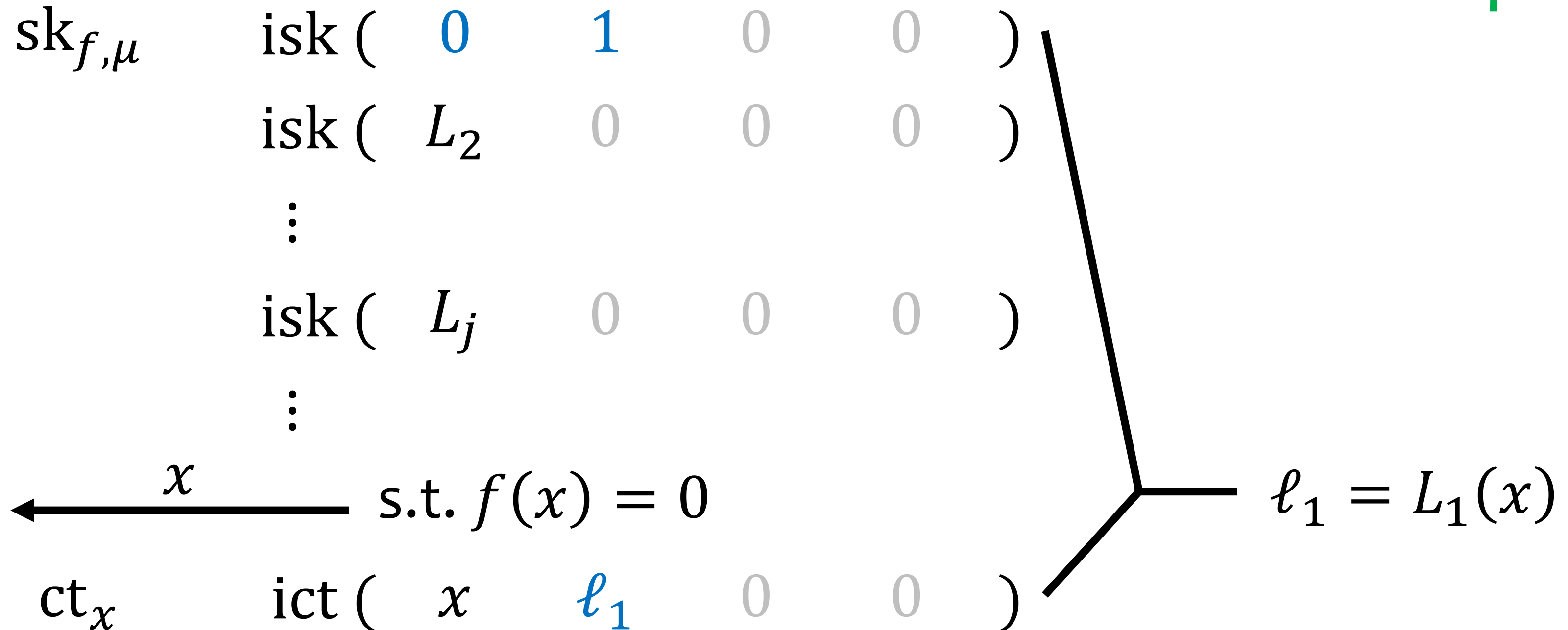
Real World

Next step: hardwire  $\ell_1$  in ciphertext

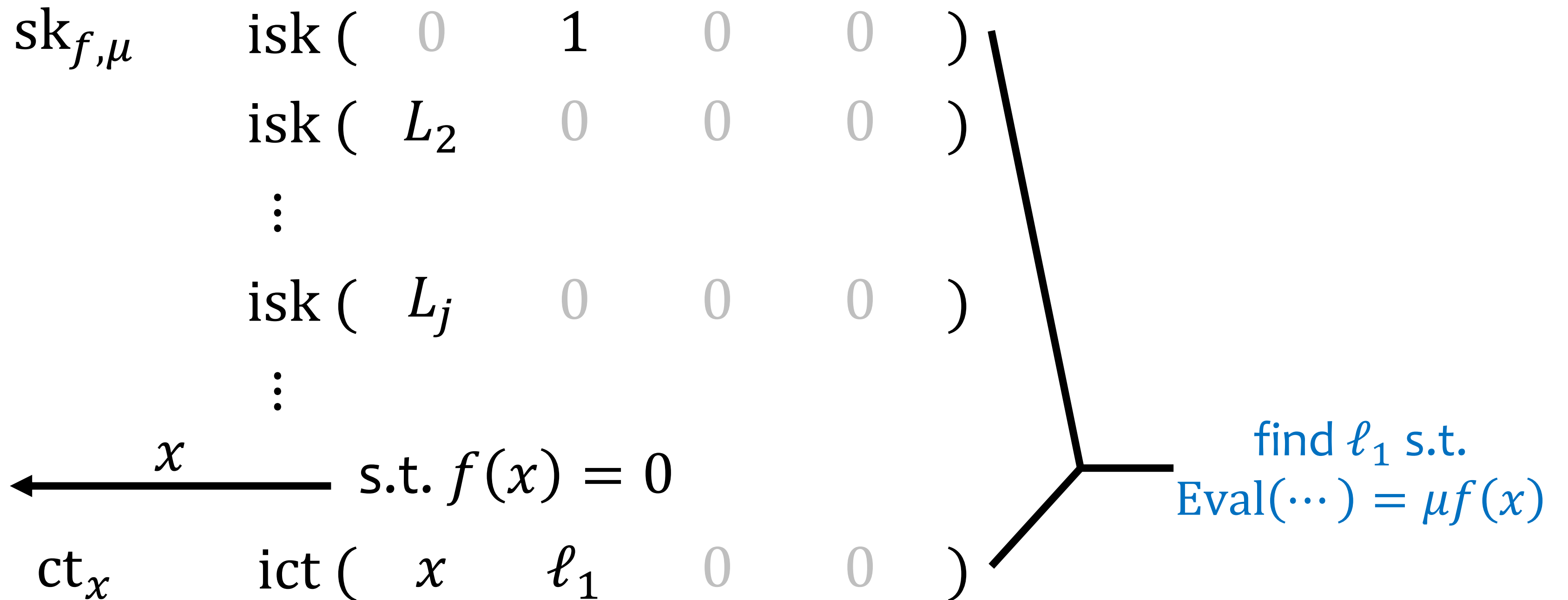


# Hardwire $\ell_1$ in Ciphertext via IPFE

Next step: find unique  $\ell_1$  from correctness equation

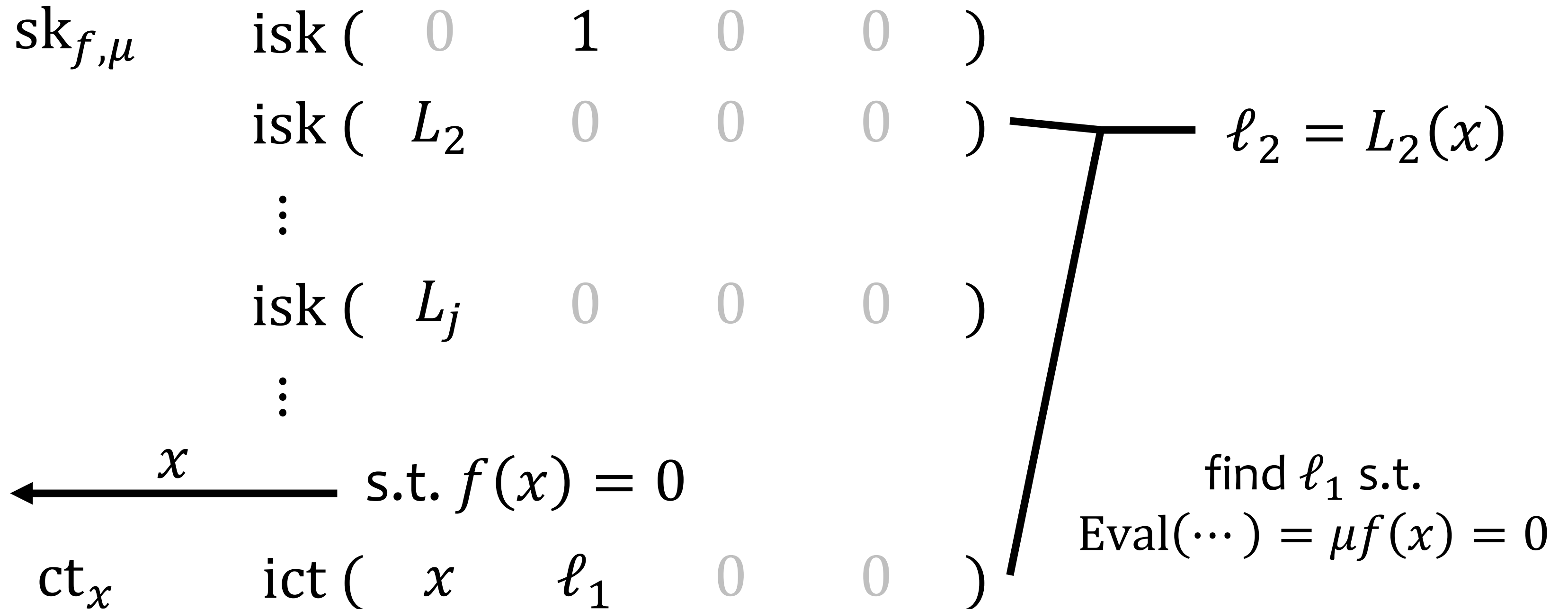


# Find Unique $\ell_1$ via AKGS



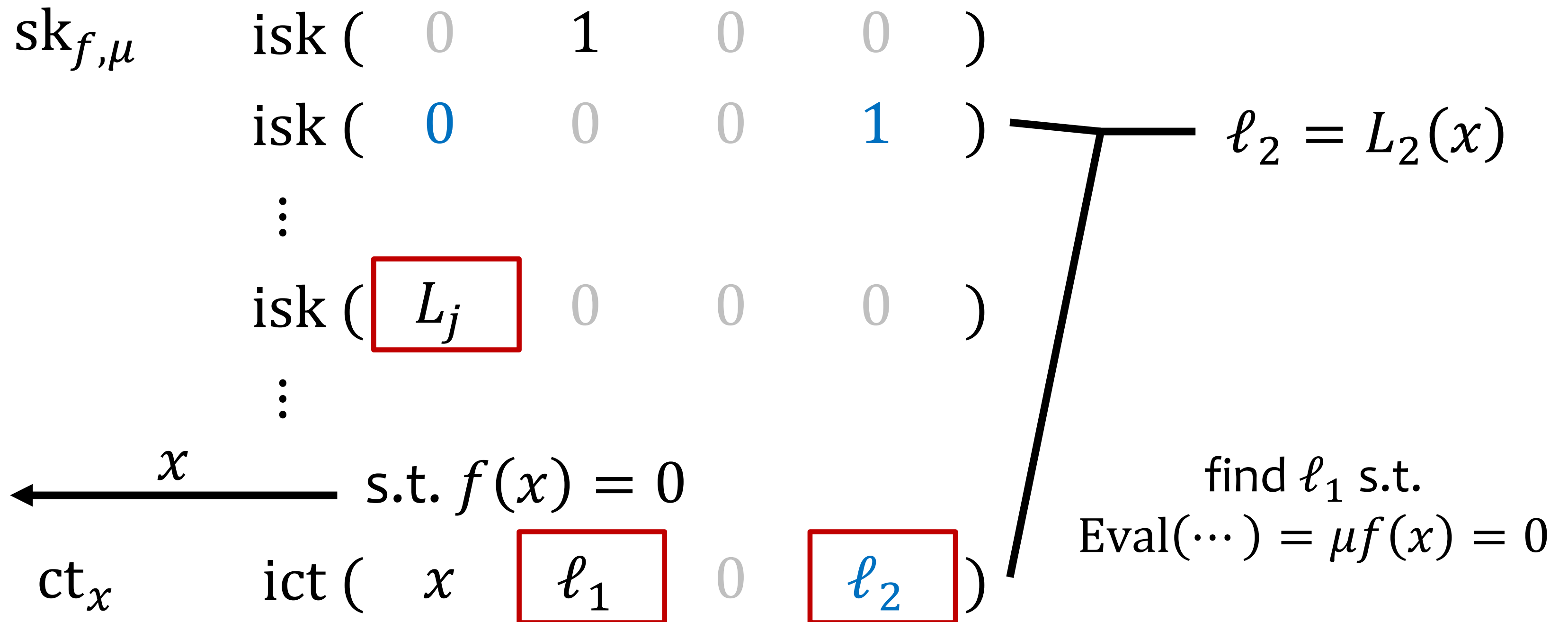
# Goal. Simulate $\ell_2$ as Random

Next step: hardwire  $\ell_2$  in ciphertext



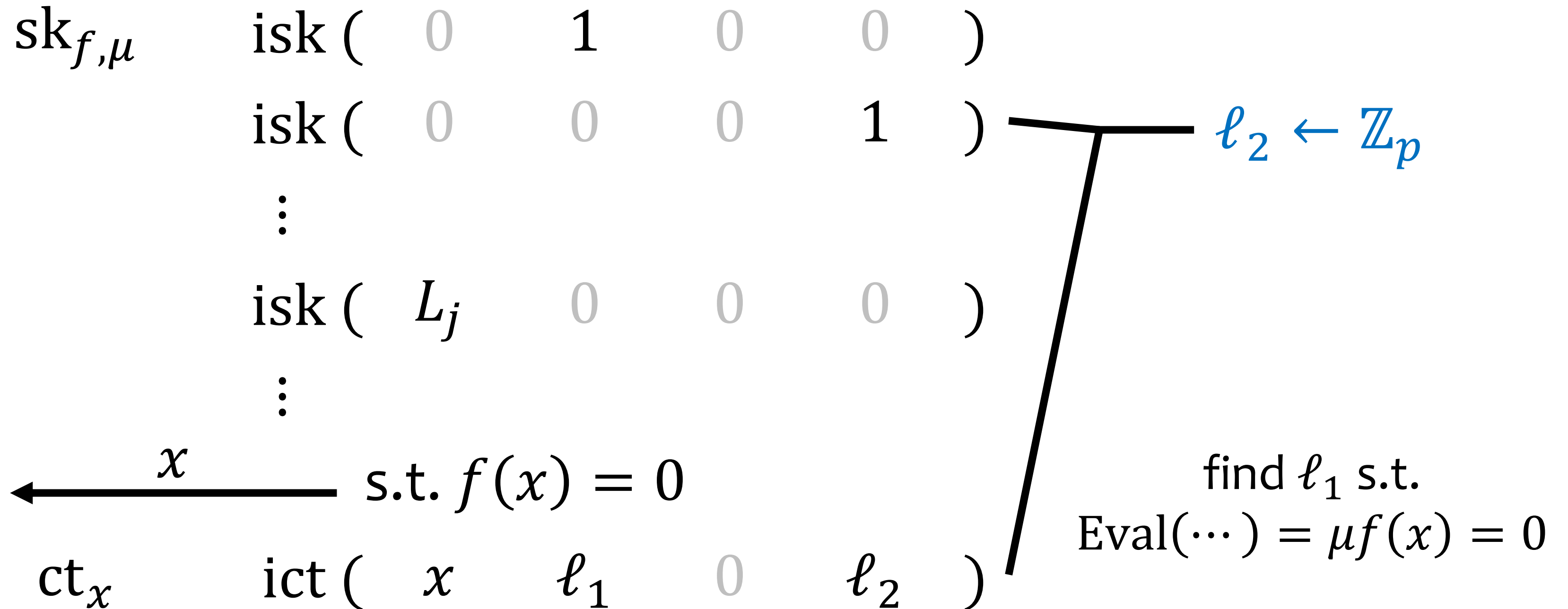
# Hardwire $\ell_2$ in Ciphertext via IPFE

Next step: replace  $\ell_2$  by random



# Replace $\ell_2$ by Random via AKGS

Next step: put  $\ell_2$  back into secret key



# Put $\ell_2$ Back into Secret Key via IPFE

Goal achieved: simulate  $\ell_2$

Next step: simulate the other labels

$$\text{sk}_{f,\mu} \quad \text{isk} \left( \begin{array}{cccc} 0 & 1 & 0 & 0 \end{array} \right)$$

$$\text{isk} \left( \begin{array}{cccc} 0 & 0 & \ell_2 & 0 \end{array} \right)$$

$\vdots$

$$\text{isk} \left( \begin{array}{cccc} L_j & 0 & 0 & 0 \end{array} \right)$$

$\vdots$

$$\xleftarrow{x} \text{s.t. } f(x) = 0$$

$$\text{ct}_x \quad \text{ict} \left( \begin{array}{cccc} x & \ell_1 & 1 & 0 \end{array} \right)$$

$$\ell_2 \leftarrow \mathbb{Z}_p$$

$$\text{find } \ell_1 \text{ s.t. } \text{Eval}(\dots) = \mu f(x) = 0$$



# Adaptive Security of 1-ABE

## Final Simulation

$\mu$  is hidden ✓

$$\text{sk}_{f,\mu} \quad \text{isk} \left( \begin{array}{cccc} 0 & 1 & 0 & 0 \end{array} \right)$$

$$\text{isk} \left( \begin{array}{cccc} 0 & 0 & \ell_2 & 0 \end{array} \right)$$

⋮

$$\text{isk} \left( \begin{array}{cccc} 0 & 0 & \ell_j & 0 \end{array} \right)$$

⋮

$$\xleftarrow{x} \text{s.t. } f(x) = 0$$

$$\text{ct}_x \quad \text{ict} \left( \begin{array}{cccc} x & \ell_1 & 1 & 0 \end{array} \right)$$

$$\ell_2 \leftarrow \mathbb{Z}_p$$

⋮

$$\ell_j \leftarrow \mathbb{Z}_p$$

⋮

find  $\ell_1$  s.t.

$$\text{Eval}(\dots) = \mu f(x) = 0$$

# Adaptively Secure 1-ABE

multi  $\{ sk \}$   $\{ isk ( L_j ) \}$

**1** ct  $ict ( x )$

uses msk 

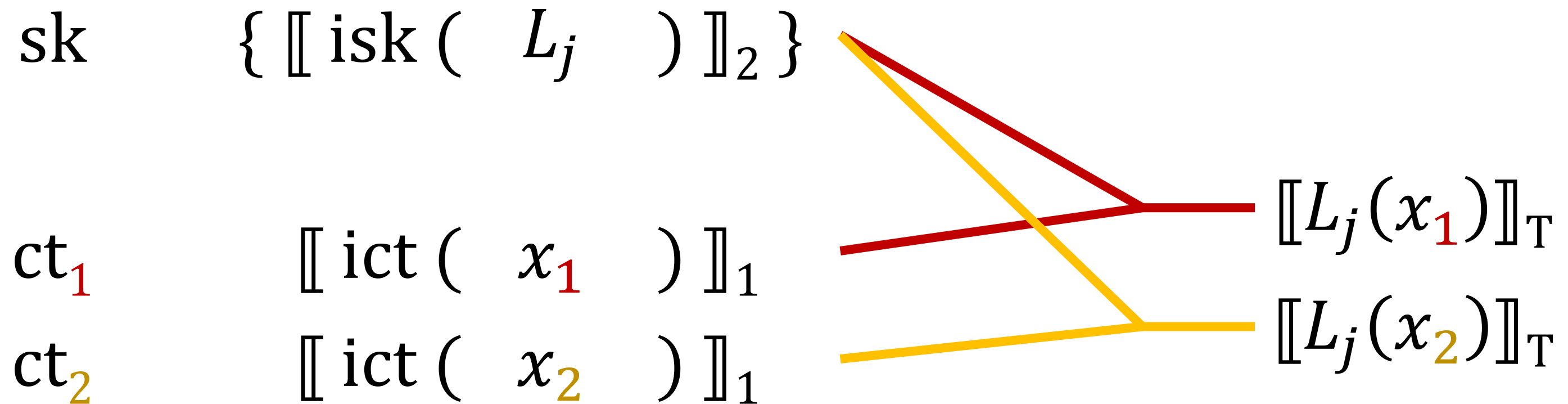
?

multi-ciphertext security

?

make it public-key

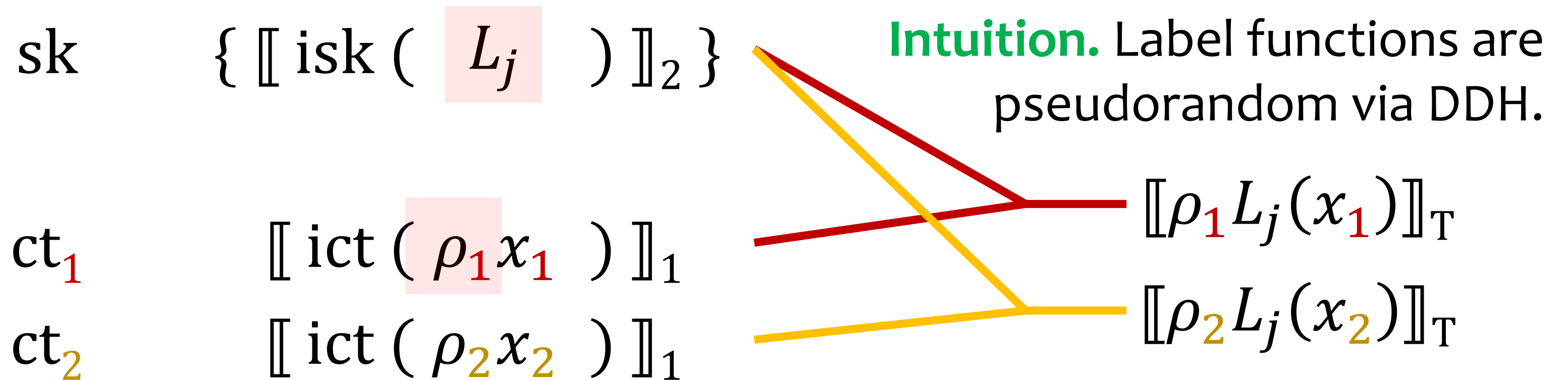
# Multi-Ciphertext Security



**Problem.** Label functions (its randomness) cannot be reused.

**Idea.** Use DDH to rerandomize them.

# Multi-Ciphertext Security



**Problem.**  $\llbracket \rho L_j \rrbracket_T$  is not pseudorandom given  $\llbracket \rho \rrbracket_1, \llbracket L_j \rrbracket_2$ .

**Idea.** Use IPFE to move  $\rho$  into the same group as  $L_j$ 's, then use DDH.

# Adaptively Secure Secret-Key ABE

multi  $\{ sk \}$   $\{ \llbracket isk ( L_j ) \rrbracket_2 \}$

multi  $\{ ct \}$   $\llbracket ict ( \rho x ) \rrbracket_1$

uses msk



multi-ciphertext security

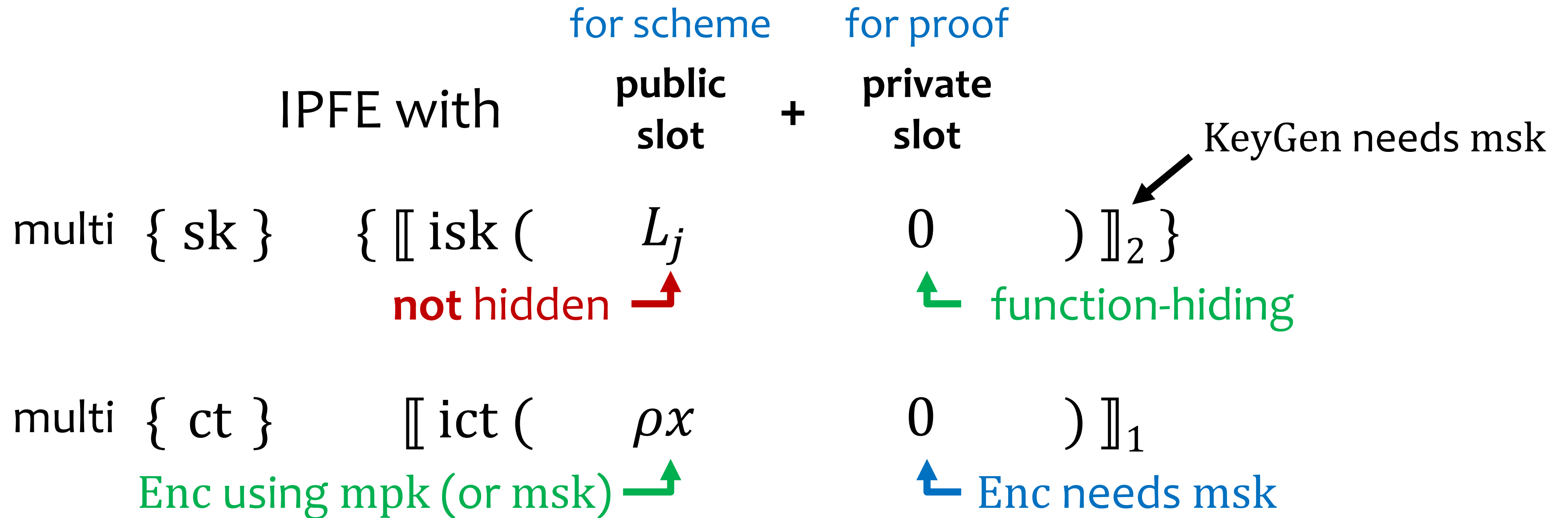


make it public-key

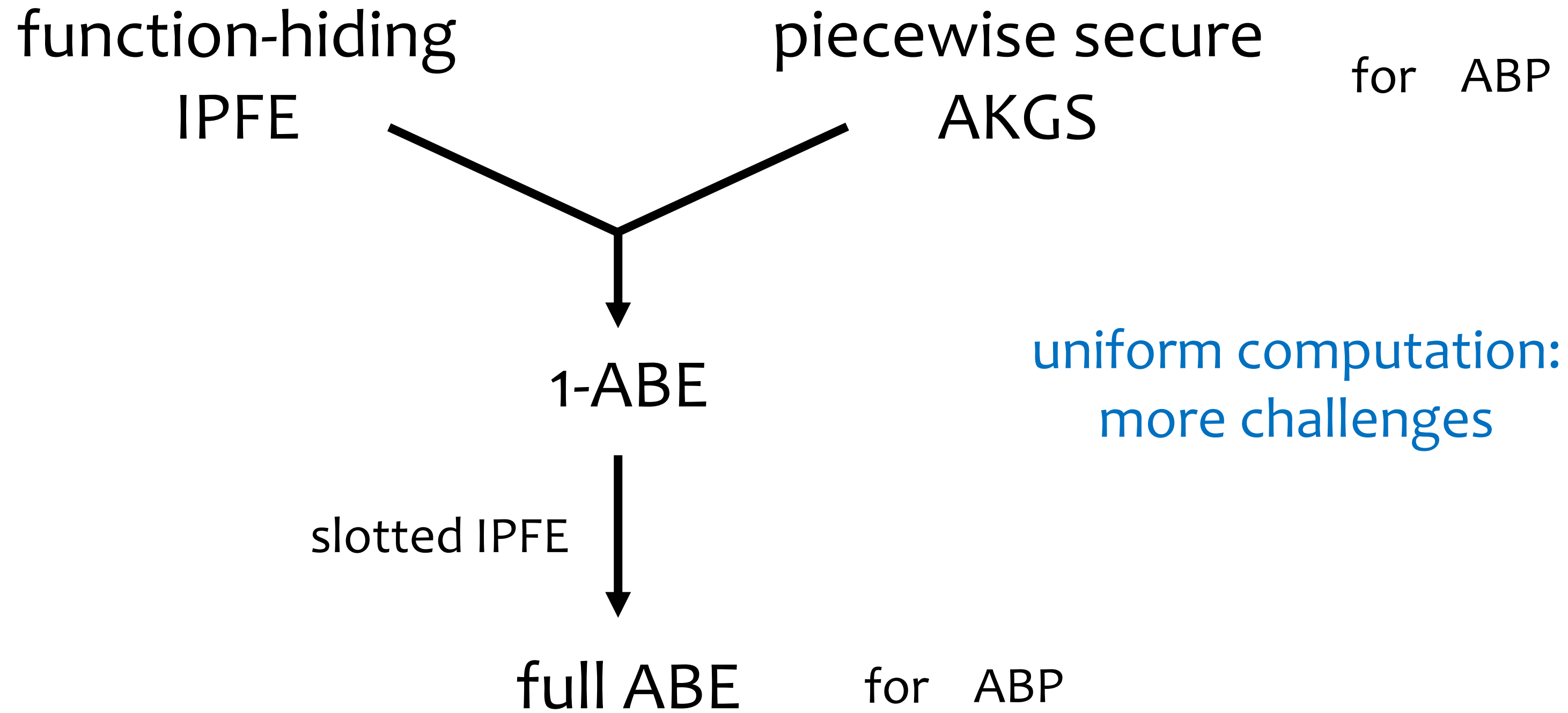
Slotted IPFE



# Public-Key ABE via Slotted IPFE



make it public-key



# Ideas for Uniform Model

DFA/NFA/L/NL = matrix multiplication

✓ piecewise secure AKGS for each input length

✗ **unique challenge:**

$$\#[\ell, r] \propto |x| T S 2^S |\mathbf{TM}| \quad (\text{or } |x| \cdot |\mathbf{TM}| \text{ for DFA/NFA})$$

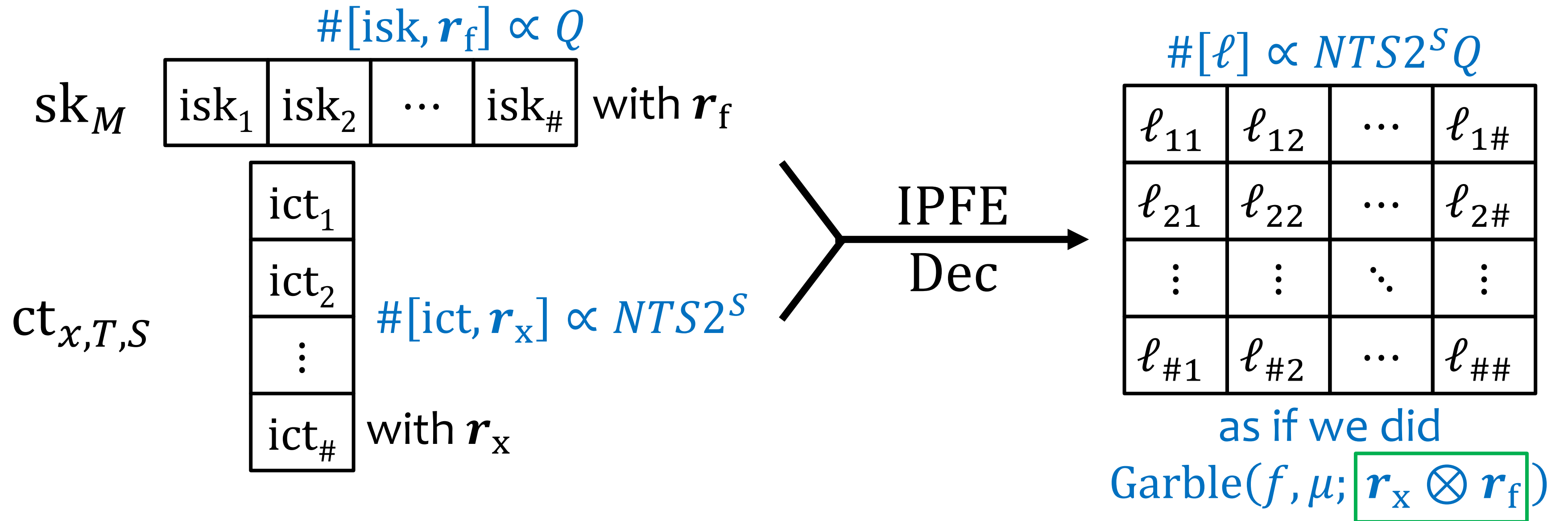
$$|ct| \propto |x| T S 2^S$$

$$|sk| \propto |\mathbf{TM}|$$

Neither sk nor ct can fit all label functions / labels!



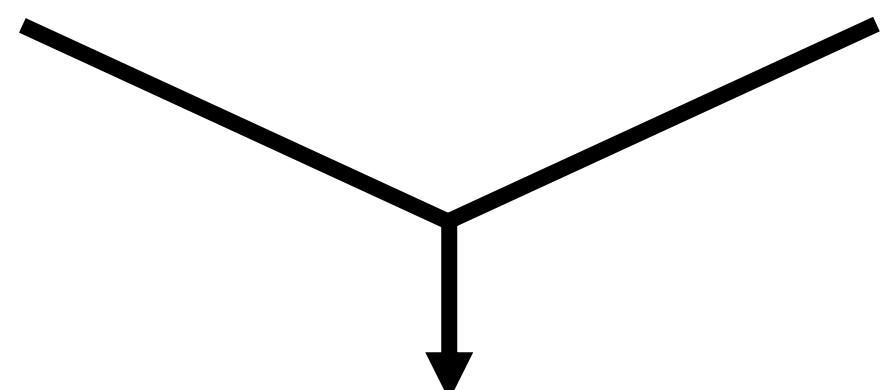
# Tensoring for Expansion



**Intuition.**  $[[\mathbf{r}_x \otimes \mathbf{r}_f]]_T \stackrel{\text{DDH}}{\approx} [[\mathbf{r}]]_T$

function-hiding  
IPFE

piecewise secure  
AKGS



1-ABE

slotted IPFE

full ABE

for

ABP  
DFA/NFA  
L/NL

*Thank you!*

[ia.cr/2020/318](https://ia.cr/2020/318)