Quantum-secure message authentication via blind-unforgeability

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UCONN



JOINT CENTER FOR Quantum Information and Computer Science Introduction





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Nowadays: digital signature schemes, message authentication codes (MACs).













Security: UF-CMA

Definition: Unforgeability under chosen message attacks (UF-CMA)

A message authentication code is secure, if no successful forger exists:



Success: *i*) $m^* \neq m_i$ for all i = 1,...,q*ii*) $\operatorname{Mac}_k(m^*) = t^*$

Stronger security model: quantum oracle access to Mac_k :

 $|m\rangle |t\rangle \mapsto |m\rangle |t \oplus \operatorname{Mac}_k(m)\rangle$

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i) Query $|m_1\rangle = \sum_{m \in \{0,1\}^n} |m\rangle |0\rangle$ to obtain $\sum_{m \in \{0,1\}^n} |m\rangle |\operatorname{Mac}_k(m)\rangle$ ii) Measure in the computational basis to obtain $(m, \operatorname{Mac}_k(m))$ for random miii) Output $(m, \operatorname{Mac}_k(m))$

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UF-CMA doesn't make sense anymore...

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We should be worried about:

key k specifies a random periodic function f_k with period p_k $Mac_k(p_k) = 0$, and $Mac_k(x) = f_k(x) \ \forall x \neq p_k$

i) run period finding (a subroutine of Shor's algorithm) to find p_k ii) output $(p_k\!,\!0)$

Quantum problems



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- No-cloning principle: can't keep a transcript
- Measurement causes disturbance!

Results

Our results

- We study unforgeability under **quantum** chosen message attacks
- We propose a new security definition: **blind unforgeability (BU)**
- We exhibit a MAC that is secure under a previous definition by Boneh and Zhandry (Eurocrypt 2013) but clearly broken, and BU-insecure
- We characterize BU
 - It implies the previous definition
 - Random functions, Lamport signatures are BU secure
 - Hash-and-Mac/Hash-and-Sign preserves BU security for appropriate hash functions

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Ask q + 1 forgeries for q queries!



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Has some nice properties:

- Equivalent to **UF-CMA** for classical oracle
- A random oracle is BZ-unforgeable (BZ '13)





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One-time Mac that's BZ secure, GYZ (Garg, Yuen&Zhandry, Crypto '17) insecure, assuming iO (Zhandry, Eurocrypt '19)

A MAC that unconditionally "breaks" Boneh-Zhandry:

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$$m = \begin{cases} b \longrightarrow \\ x \longrightarrow \end{cases} f_b^0(x) \longrightarrow f_b^0(x) \\ f_b^1(x) \end{pmatrix} \xrightarrow{f_b^0(x)} f_b^0(x) = \hat{f}_0^0(x \mod p) \text{ for random } p, f_0^1 = \hat{f}_0^1 \\ f_1^0(x) = \begin{cases} 0^n & x = p \\ \hat{f}_1^0(x) & \text{else} \end{cases}, f_1^1 \equiv 0^n \end{cases}$$

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Key step: ignorance is necessary

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More formally: for Mac_k

- 1. Select $B_{\varepsilon} \subset \{0,1\}^n$ by putting every $m \in B_{\varepsilon}$ independently with probability ε ;
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A MAC Mac_k is blind-unforgeable if for every adversary \mathscr{A} with a quantum oracle for $B_{\epsilon}\operatorname{Mac}_k$,

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- classifies the examples we have seen thus far correctly.

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Check, e.g., for random functions:

- if oracle is blinded...
- ... Mac_k(m) for blinded m is independent of post-query state,
- this adversary fails.



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Random periodic function
shielded by a random functionb = 0Random function
punctured at the periodb = 1

Check, say for $\varepsilon = 0.0001$,

- oracle is blinded only on few random inputs...
- ...post-query state won't change too much;
- (1p,0) is blinded with *independent* probability ε;
- so this adversary succeeds!

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Tools:

- A simulation lemma that relates an adversary's performance in the blinded and unblinded cases
- > Zhandry's superposition representation of quantum random oracles

Summary, open questions

Summary:

- We exhibit a MAC that is secure according to a definition by Boneh and Zhandry but allows for an intuitive forgery attack.
- We propose a replacement definition: Blind Unforgeability
- Blind unforgeability has a lot of nice properties and classifies all known examples correctly.

Open questions:

- The security game for blind unforgeability is not natural. Can this be fixed?
- Are popular schemes (MACs and DSS) blind-unforgeable? We only have NMAC, HMAC and Lamport in the QROM for now...