

Statistical Zaps and New Oblivious Transfer Protocols

Vipul Goyal

Carnegie Mellon
University

Abhishek Jain

Johns Hopkins
University

Zhengzhong Jin

**Johns Hopkins
University**

Giulio Malavolta

Carnegie Mellon University
University of California,
Berkeley

Statistical Security in 2-party Protocols

Statistical Security in 2-party Protocols

- **Everlasting security** Computational unbounded adversary can't break.
- **Hard to achieve**
 - Impossible for *both* parties to achieve for general functionalities
- **Focus of this work:** One-side Statistical Security
 - Interactive Proof Systems: Statistical Privacy for Prover
 - Oblivious Transfer: Statistical Privacy for Receiver

Statistical Security in 2-party Protocols

- **Everlasting security** Computational unbounded adversary can't break.
- **Hard to achieve**
 - Impossible for *both* parties to achieve for general functionalities
- **Focus of this work:** One-side Statistical Security
 - Interactive Proof Systems: Statistical Privacy for Prover
 - Oblivious Transfer: Statistical Privacy for Receiver

Interactive Proof System



Prover

ω : witness

$x \in L$



Verifier

Interactive Proof System



Prover

ω : witness

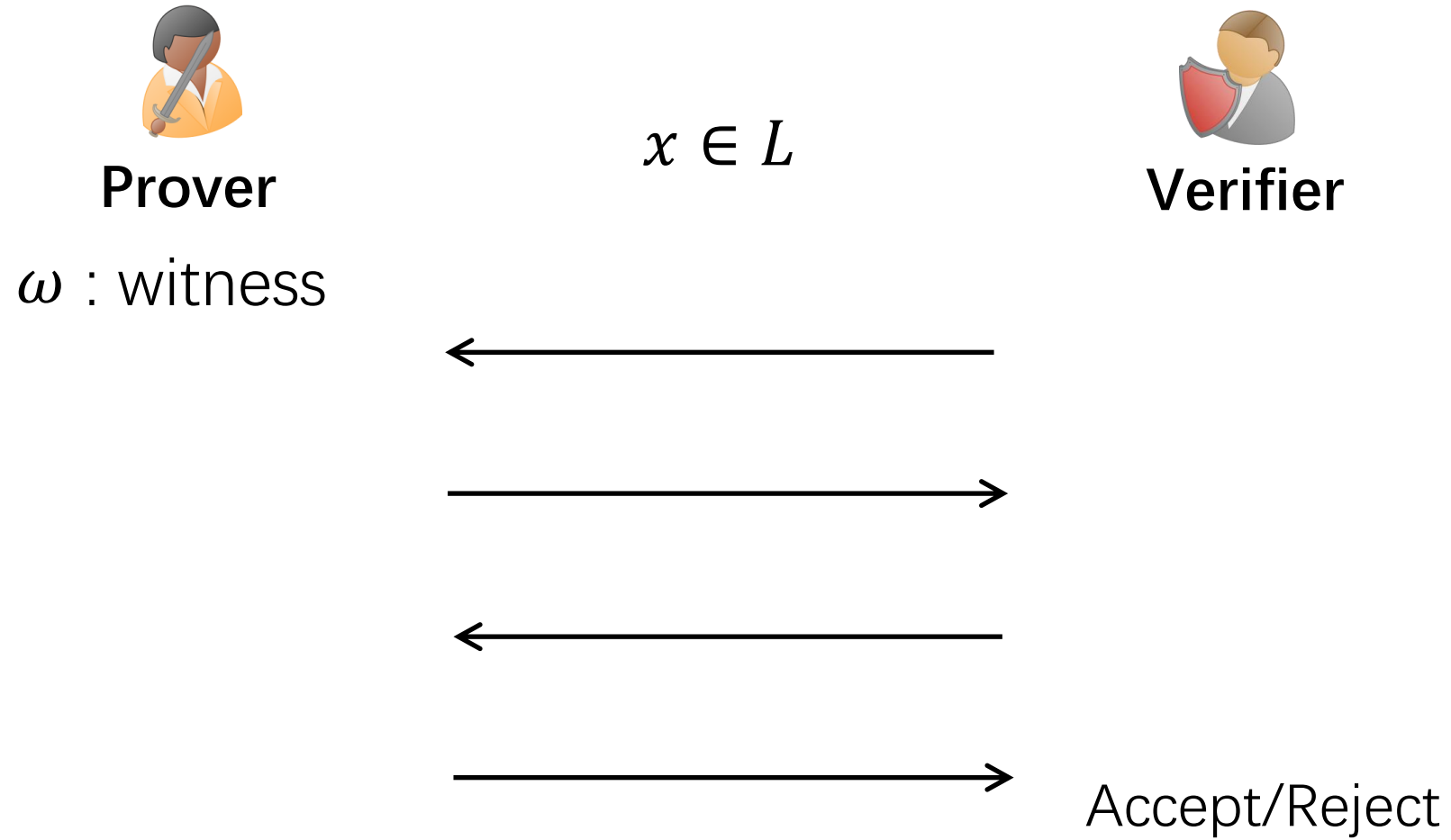
$x \in L$



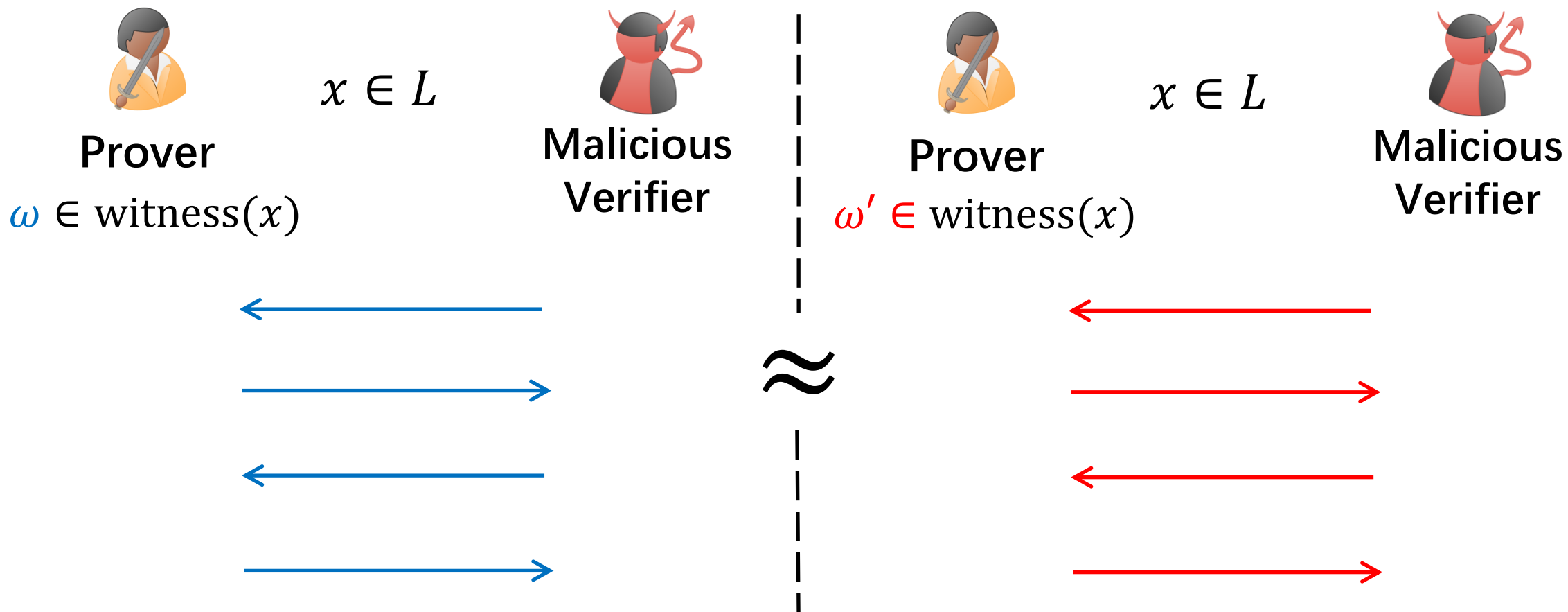
Verifier



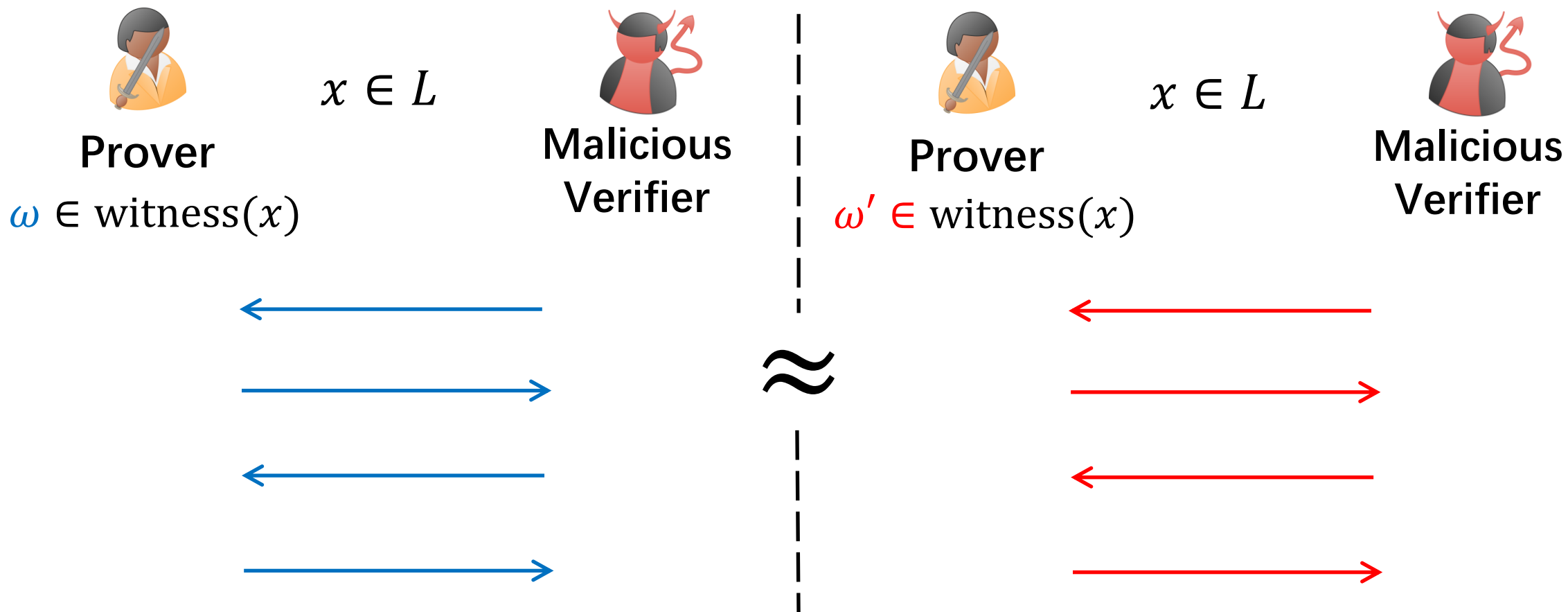
Interactive Proof System



Witness Indistinguishability (WI)



Witness Indistinguishability (WI)



- Unlike zero-knowledge, WI can be achieved in 2-round

Zaps: 2-round Public-Coin WI [DN00]



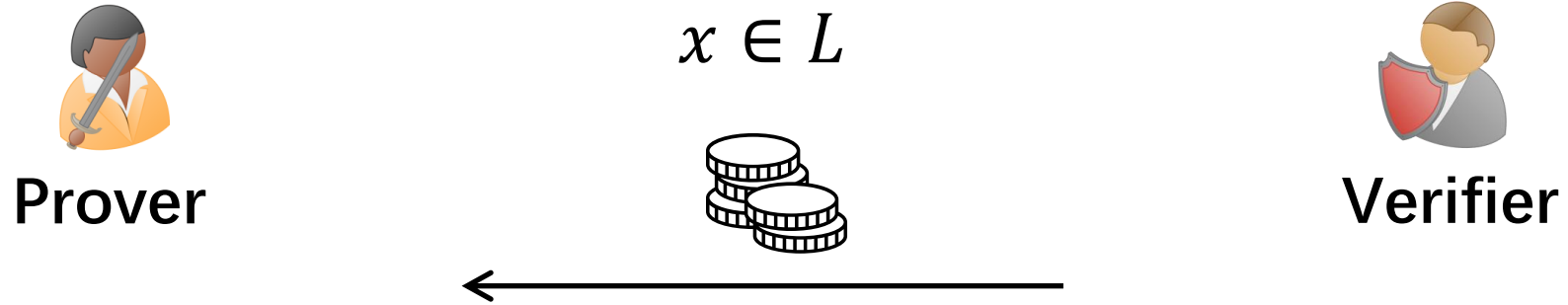
Prover

$x \in L$

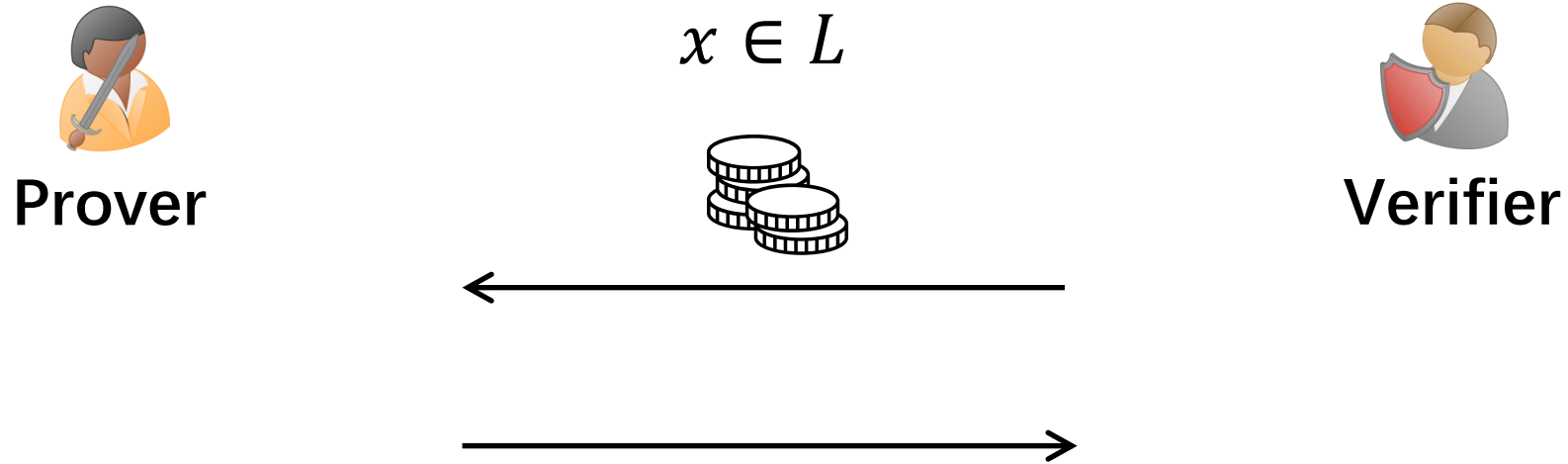


Verifier

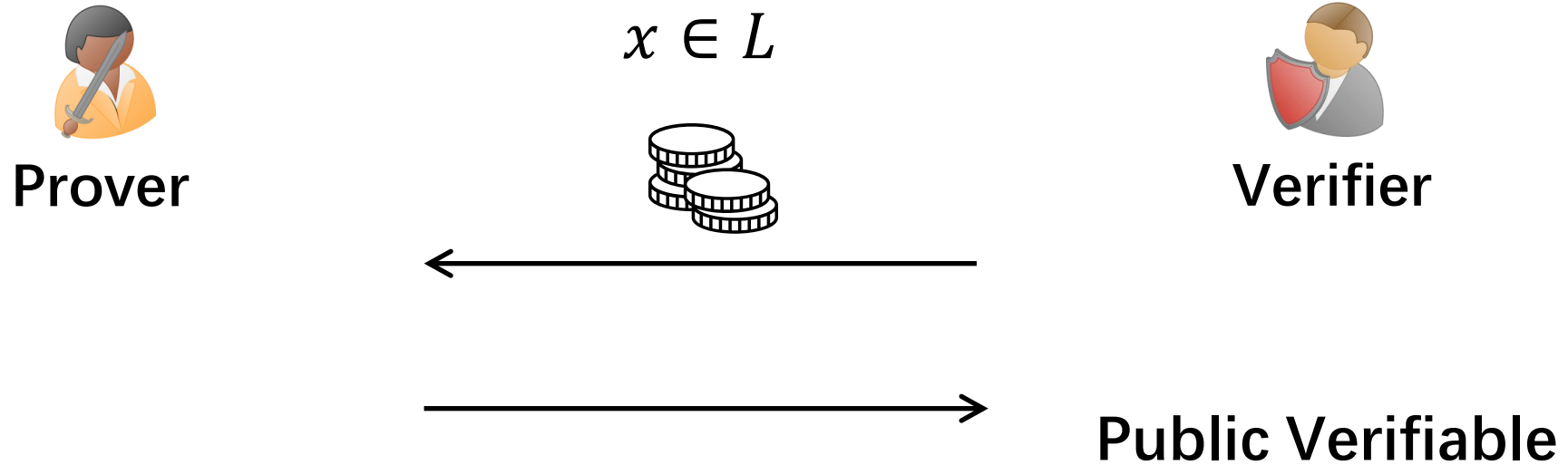
Zaps: 2-round Public-Coin WI [DN00]



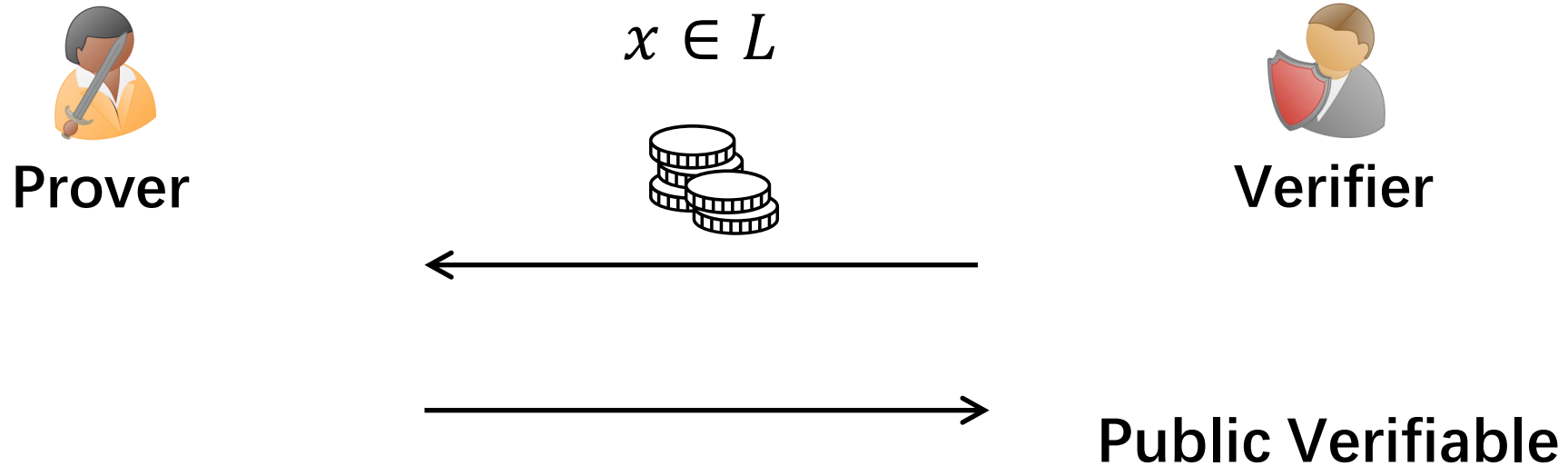
Zaps: 2-round Public-Coin WI [DN00]



Zaps: 2-round Public-Coin WI [DN00]

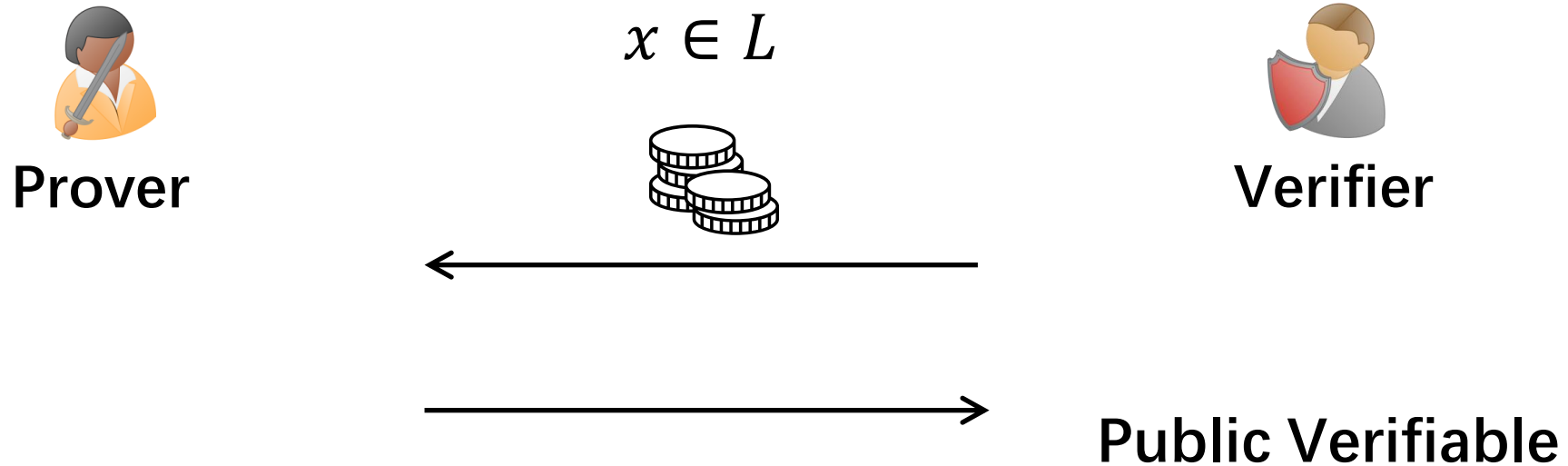


Zaps: 2-round Public-Coin WI [DN00]



Public Coin: Verifier only uses public random coins

Zaps: 2-round Public-Coin WI [DN00]



Public Coin: Verifier only uses public random coins

Many Applications:

- Round-efficient secure multiparty computation [HHPV18]
- Resettable-secure protocols [DGS09]

.....

Previous Works

[DN00] Zaps and NIZK proofs in common random string model are equivalent.

Previous Works

[DN00] Zaps and NIZK proofs in common random string model are equivalent.

NIZKs

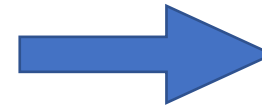
- Quadratic Residuosity Assumption [DMP88]
- Trapdoor permutation [FLS90]
- Decisional Linear Assumption [GOS06]

Previous Works

[DN00] Zaps and NIZK proofs in common random string model are equivalent.

NIZKs

- Quadratic Residuosity Assumption [DMP88]
- Trapdoor permutation [FLS90]
- Decisional Linear Assumption [GOS06]



Previous Works

[DN00] Zaps and NIZK proofs in common random string model are equivalent.

NIZKs

- Quadratic Residuosity Assumption [DMP88]
- Trapdoor permutation [FLS90]
- Decisional Linear Assumption [GOS06]



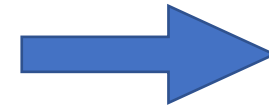
Zaps

Previous Works

[DN00] Zaps and NIZK proofs in common random string model are equivalent.

NIZKs

- Quadratic Residuosity Assumption [DMP88]
- Trapdoor permutation [FLS90]
- Decisional Linear Assumption [GOS06]



Zaps

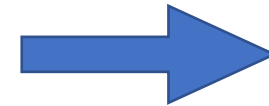
- [BP15] Zaps from Indistinguishability Obfuscation
- Above works are *computational* Zap proofs

Previous Works

[DN00] Zaps and NIZK proofs in common random string model are equivalent.

NIZKs

- Quadratic Residuosity Assumption [DMP88]
- Trapdoor permutation [FLS90]
- Decisional Linear Assumption [GOS06]



Zaps

- [BP15] Zaps from Indistinguishability Obfuscation
- Above works are *computational* Zap proofs

Question (1): Does there exist statistical Zaps?

Question (1): Does there exist statistical Zaps?

Result (1): Statistical Zaps from quasi-poly hard Learning with Errors

Question (1): Does there exist statistical Zaps?

Result (1): Statistical Zaps from quasi-poly hard Learning with Errors

[KKS18] achieves statistical *private-coin* WI.

Oblivious Transfer (OT)



Sender

m_0	m_1
-------	-------



Receiver

$\beta \in \{0,1\}$

Oblivious Transfer (OT)



Sender



Receiver

$\beta \in \{0,1\}$

Oblivious Transfer (OT)



Sender



Receiver

$\beta \in \{0,1\}$



Oblivious Transfer (OT)



Sender



Receiver

$\beta \in \{0,1\}$

Get m_β



Oblivious Transfer (OT)



Sender



Receiver

$\beta \in \{0,1\}$

Get m_β

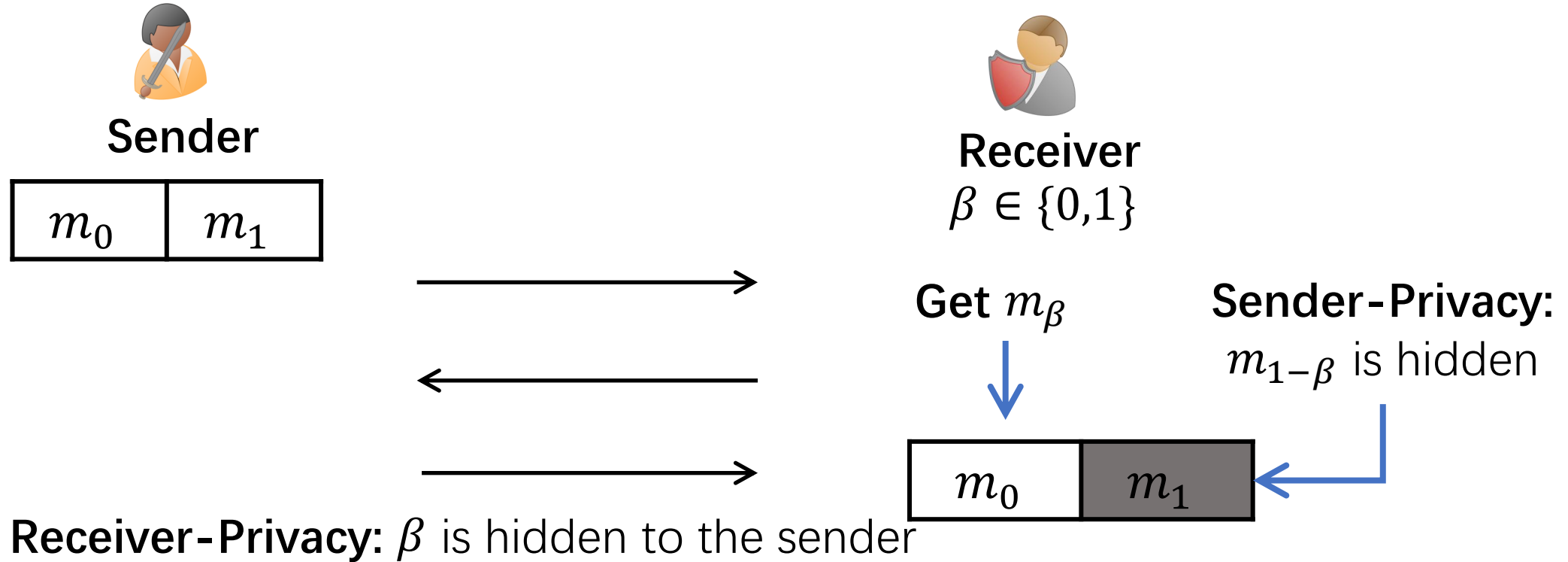


Sender-Privacy:

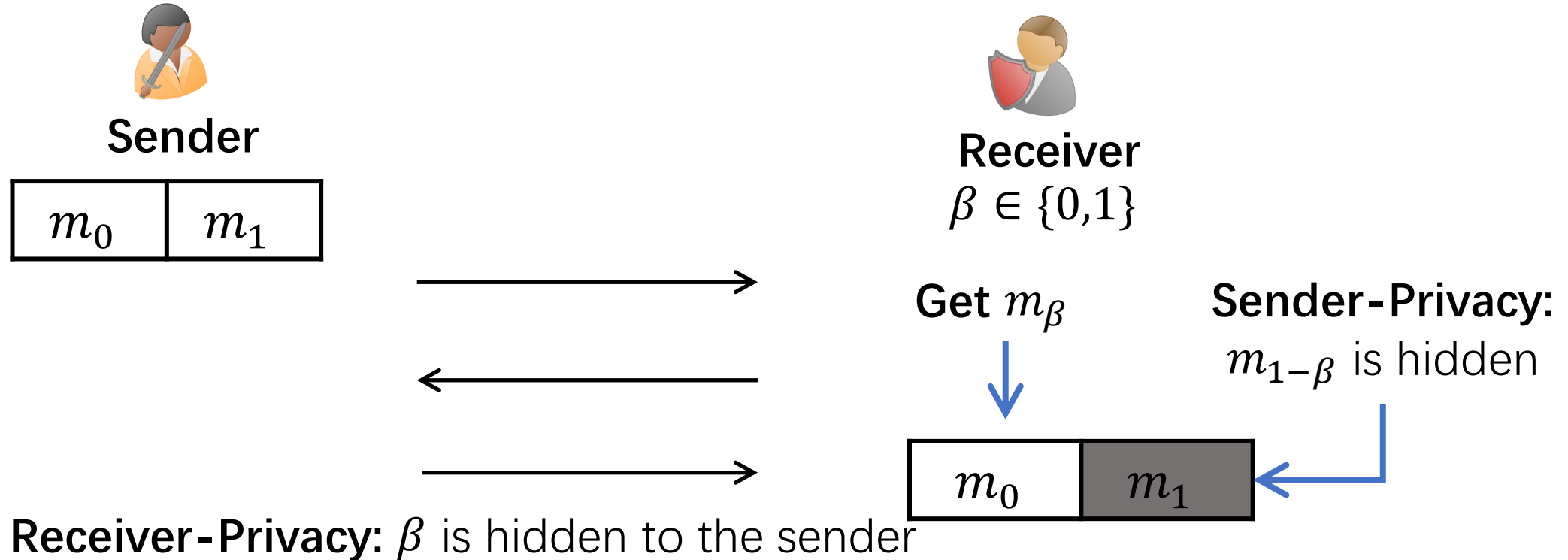
$m_{1-\beta}$ is hidden



Oblivious Transfer (OT)



Oblivious Transfer (OT)



Many Applications:

- Secure multiparty computation [Yao86, GMW87]
- 2-round WI [JKKR17, BGI+17, KKS18]
- Non-malleable commitment [KS17]

Natural Question

2-round statistical sender-private OT in plain model

[NP01, AIR01, Kal05, HK12, BD18]

Natural Question

2-round statistical sender-private OT in plain model

[NP01, AIR01, Kal05, HK12, BD18]

Can we construct **2-round statistical receiver-private** OT?

Natural Question

2-round statistical sender-private OT in plain model

[NP01, AIR01, Kal05, HK12, BD18]

Can we construct **2-round statistical receiver-private** OT?

Impossible!

Natural Question

2-round statistical sender-private OT in plain model

[NP01, AIR01, Kal05, HK12, BD18]

Can we construct **2-round statistical receiver-private** OT?

Impossible!



Sender



**Non-uniform
Malicious Receiver**

Natural Question

2-round statistical **sender-private** OT in plain model

[NP01, AIR01, Kal05, HK12, BD18]

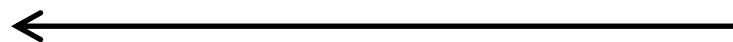
Can we construct 2-round statistical **receiver-private** OT?

Impossible!



Sender

$$\begin{aligned} ot_1 &= OT_1(\beta = 0; r_0) \\ &= OT_1(\beta = 1; r_1) \end{aligned}$$



Non-uniform
Malicious Receiver

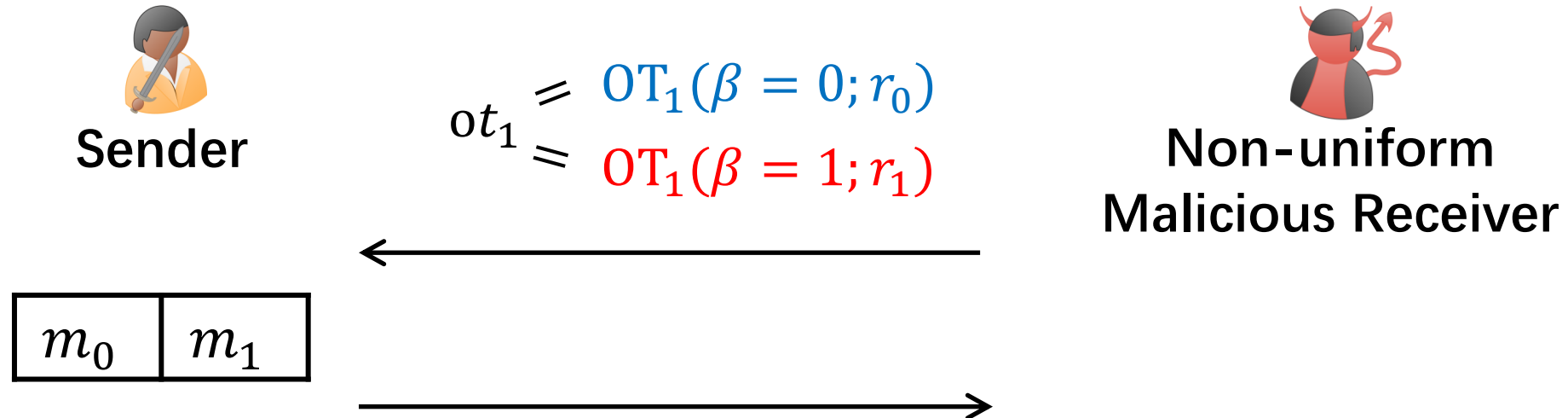
Natural Question

2-round statistical **sender-private** OT in plain model

[NP01, AIR01, Kal05, HK12, BD18]

Can we construct 2-round statistical **receiver-private** OT?

Impossible!



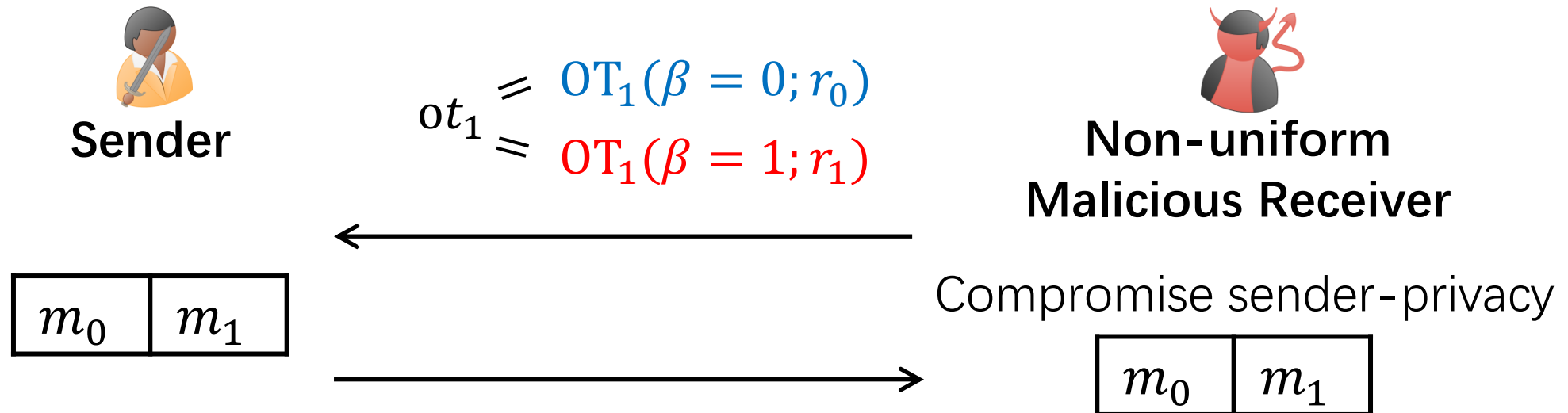
Natural Question

2-round statistical **sender-private** OT in plain model

[NP01, AIR01, Kal05, HK12, BD18]

Can we construct 2-round statistical **receiver-private** OT?

Impossible!



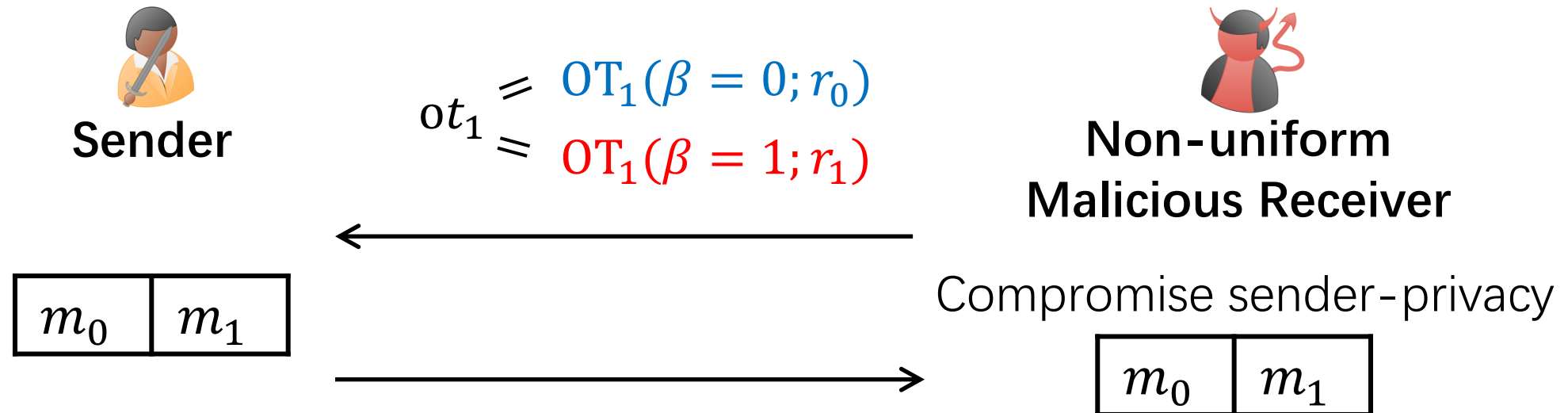
Natural Question

2-round statistical **sender-private** OT in plain model

[NP01, AIR01, Kal05, HK12, BD18]

Can we construct 2-round statistical **receiver-private** OT?

Impossible!



- [KKS18] 3-round protocol from *super-poly* hardness assumptions

Question (2): Based on *polynomial hardness* assumptions, does there exist 3-round statistical receiver-private OT in the plain model?

Question (2): Based on *polynomial hardness* assumptions, does there exist 3-round statistical receiver-private OT in the plain model?

Result (2): 3-round statistical receiver-private OT from poly-hardness

Construction (1): 2-round statistical sender-private OT

Construction (2): Computational Diffie-Hellman assumption

Question (2): Based on *polynomial hardness* assumptions, does there exist 3-round statistical receiver-private OT in the plain model?

Result (2): 3-round statistical receiver-private OT from poly-hardness

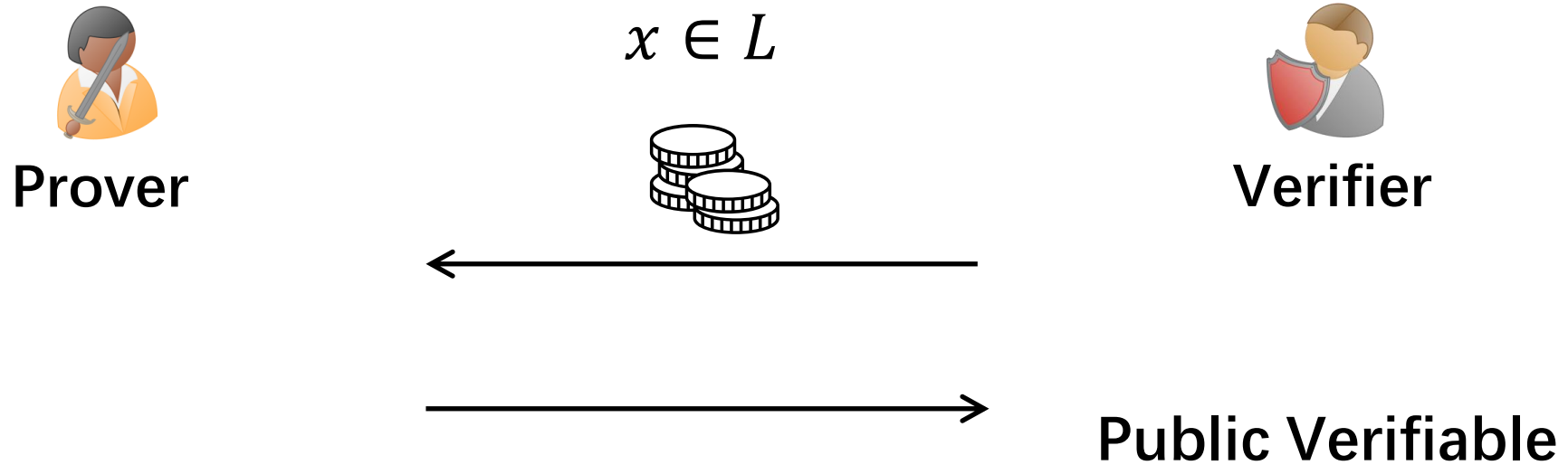
Construction (1): 2-round statistical sender-private OT  OT reversal

Construction (2): Computational Diffie-Hellman assumption

Technical Details

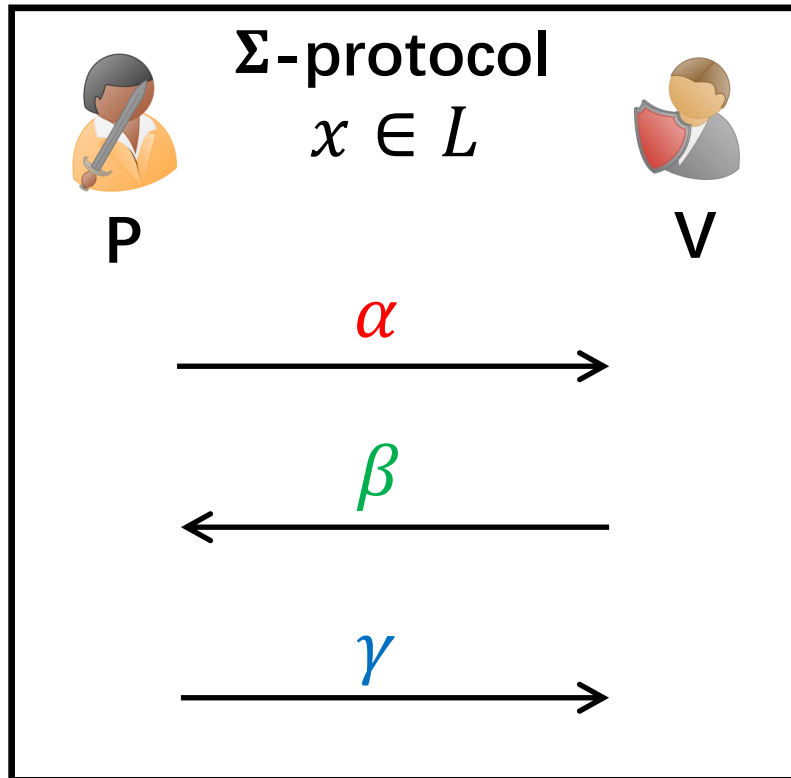
Part I: Statistical Zaps

Statistical Zaps



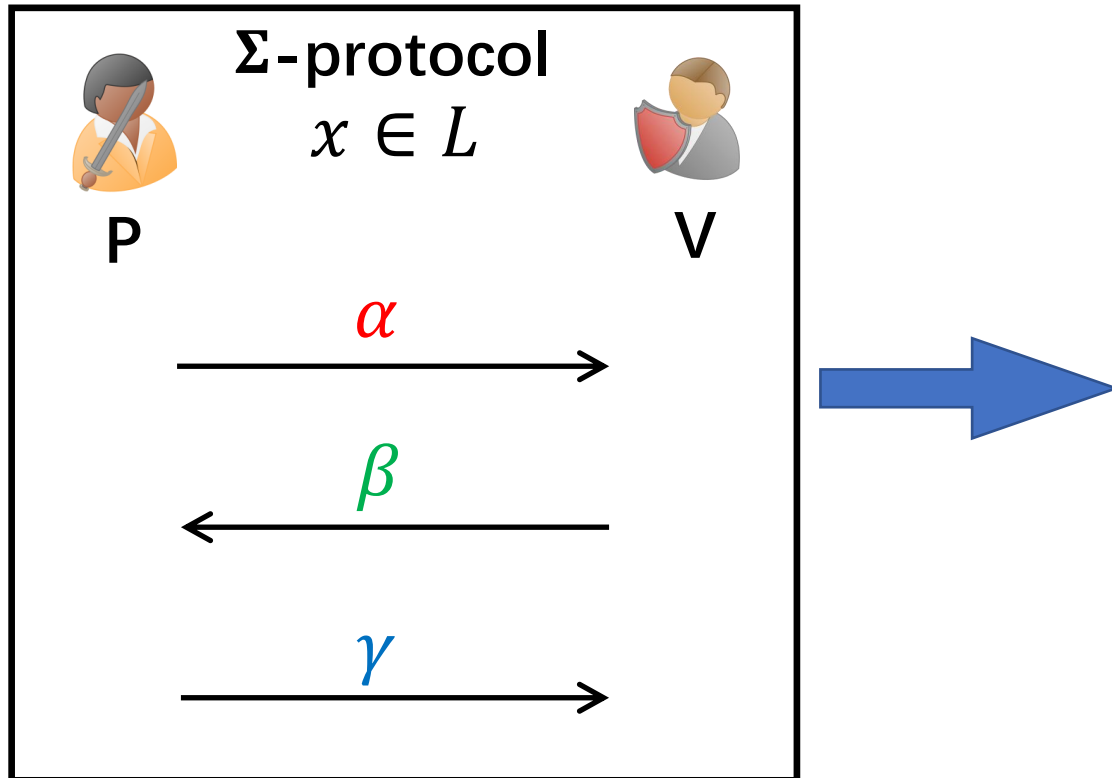
Starting Idea

- Compress a Σ -protocol via a **C**orrelation **I**ntractable **H**ash (CIH) $\{H_k(\cdot)\}_k$
[CGH98, KRR17, CCRR18, HL18, CCH+19, PS19]



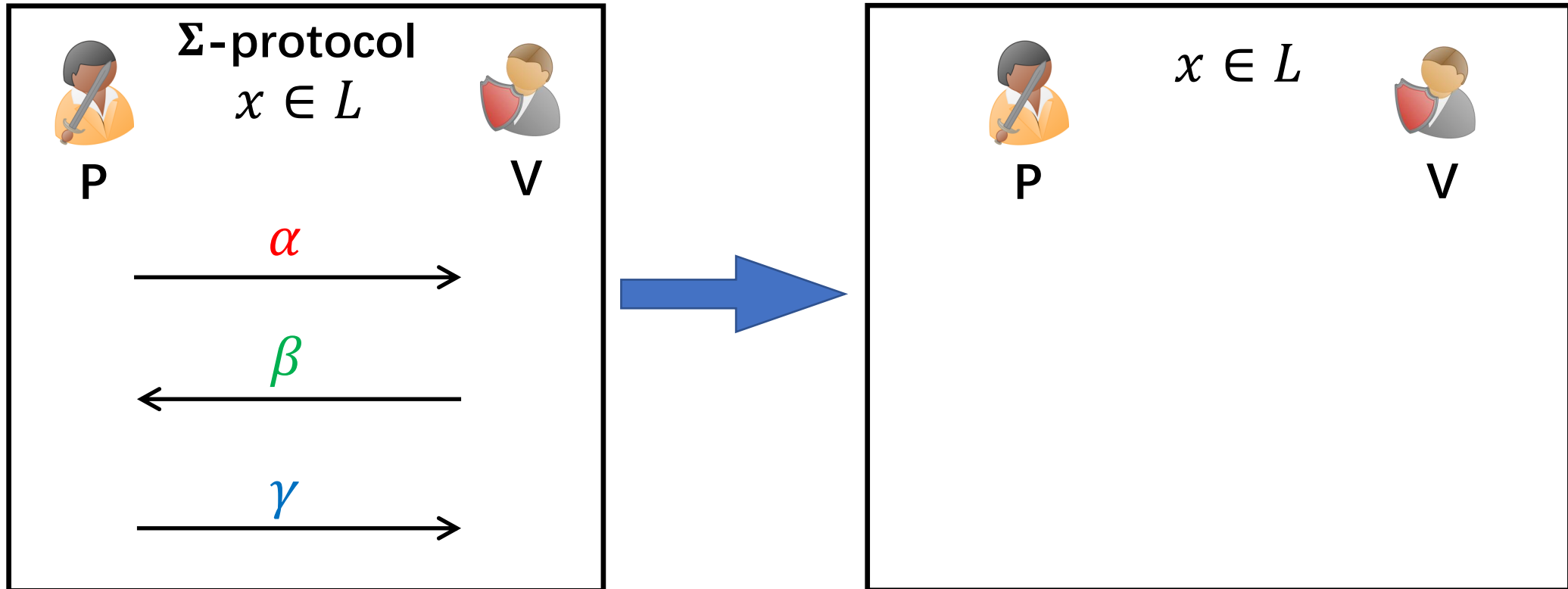
Starting Idea

- Compress a Σ -protocol via a **C**orrelation **I**ntractable **H**ash (CIH) $\{H_k(\cdot)\}_k$
[CGH98, KRR17, CCRR18, HL18, CCH+19, PS19]



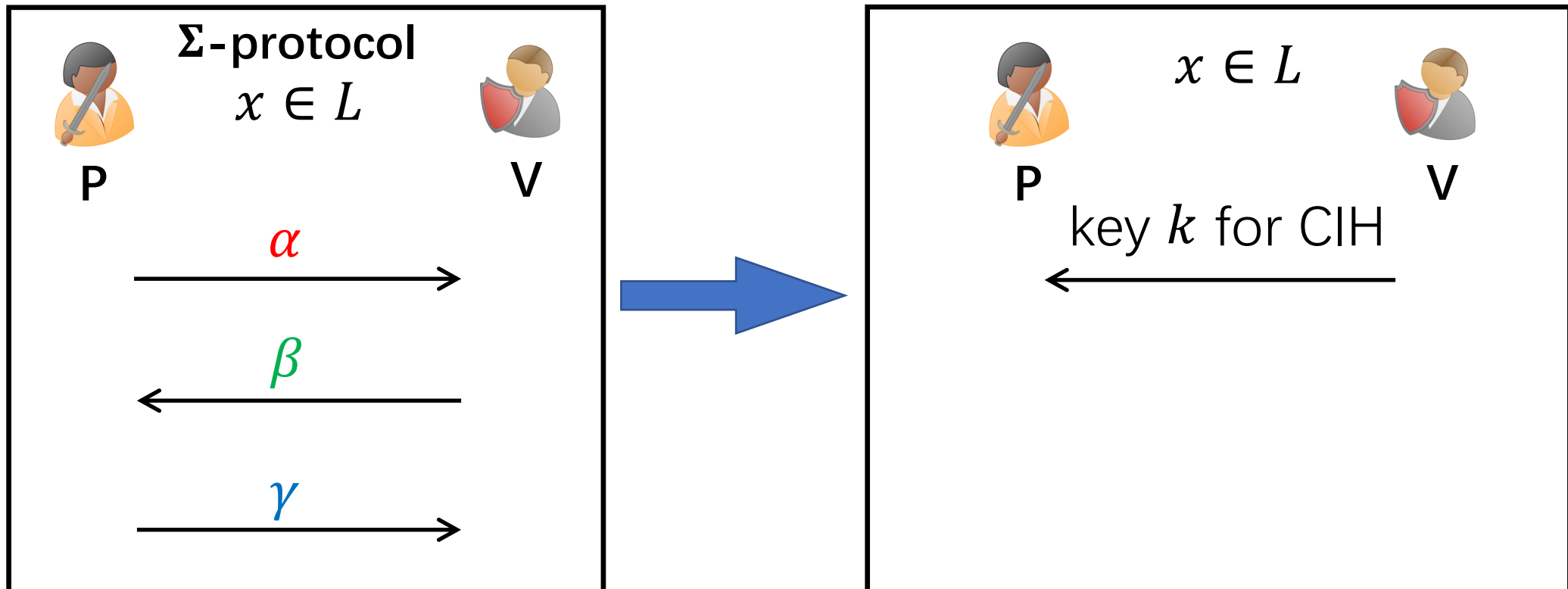
Starting Idea

- Compress a Σ -protocol via a **C**orrelation **I**ntractable **H**ash (CIH) $\{H_k(\cdot)\}_k$
[CGH98, KRR17, CCRR18, HL18, CCH+19, PS19]



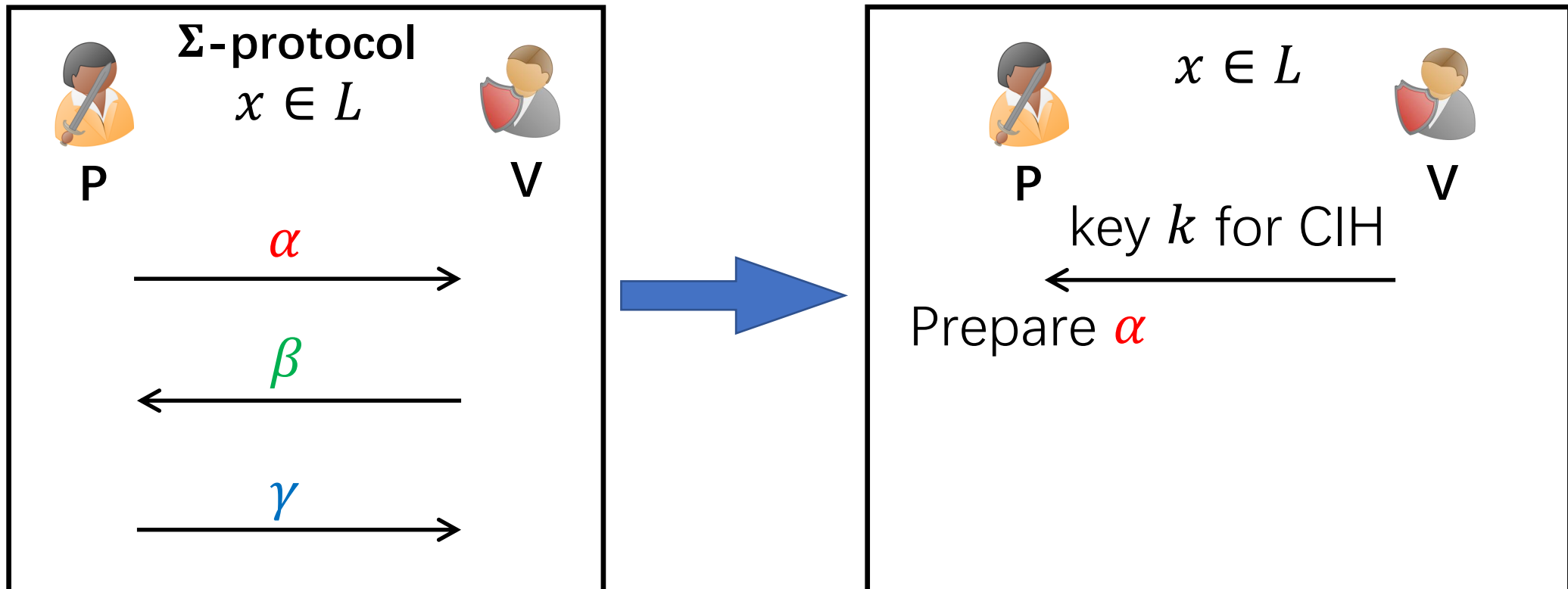
Starting Idea

- Compress a Σ -protocol via a **C**orrelation **I**ntractable **H**ash (CIH) $\{H_k(\cdot)\}_k$ [CGH98, KRR17, CCRR18, HL18, CCH+19, PS19]



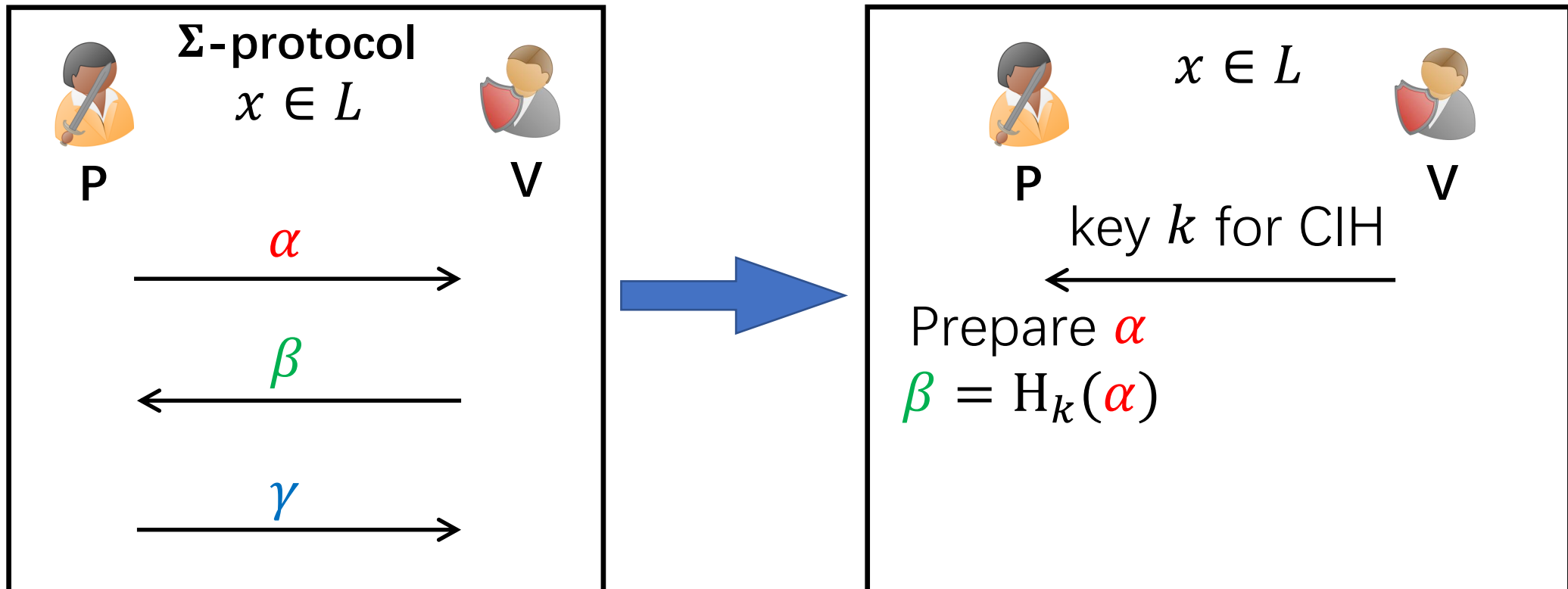
Starting Idea

- Compress a Σ -protocol via a **C**orrelation **I**ntractable **H**ash (CIH) $\{H_k(\cdot)\}_k$ [CGH98, KRR17, CCRR18, HL18, CCH+19, PS19]



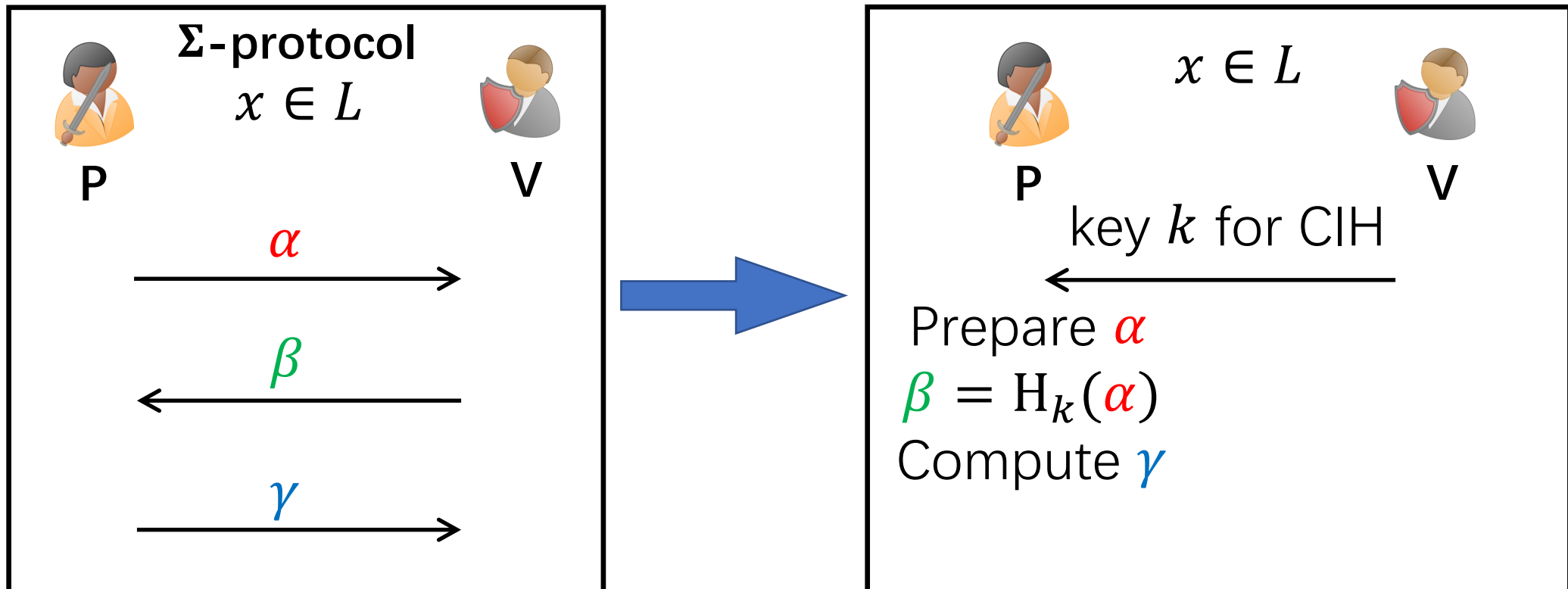
Starting Idea

- Compress a Σ -protocol via a **C**orrelation **I**ntractable **H**ash (CIH) $\{H_k(\cdot)\}_k$ [CGH98, KRR17, CCRR18, HL18, CCH+19, PS19]



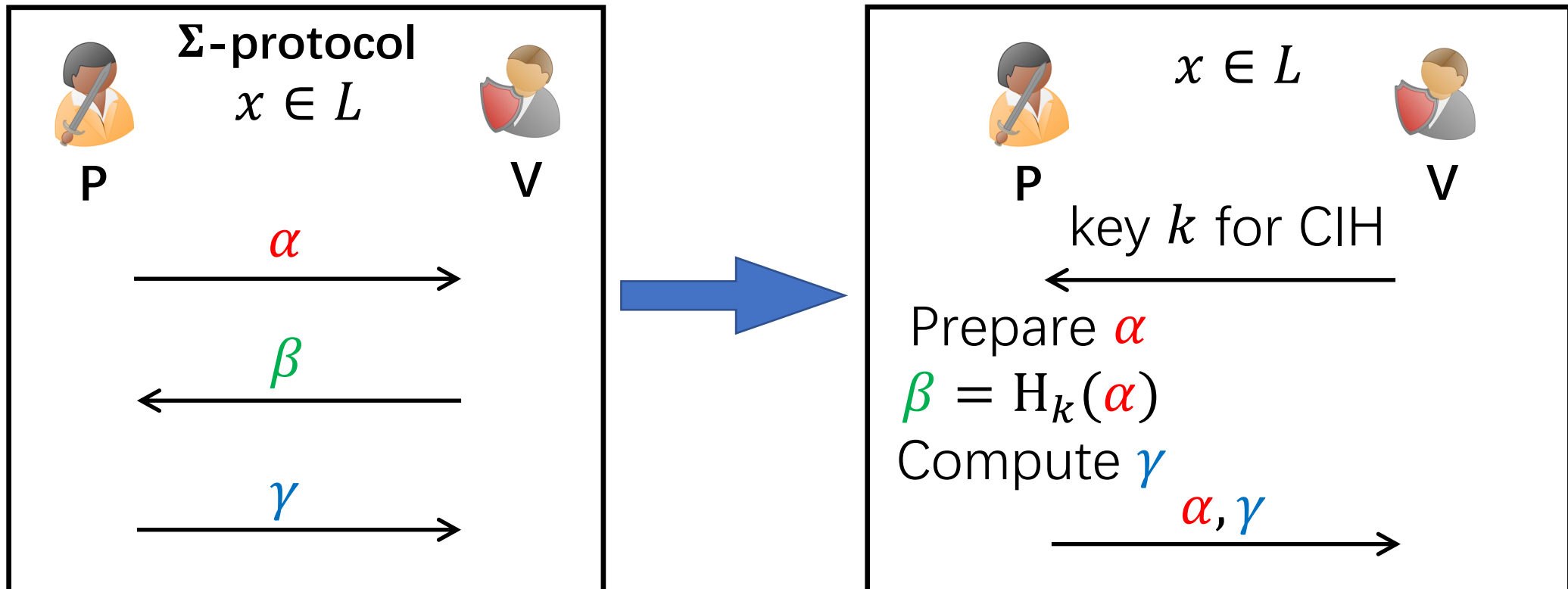
Starting Idea

- Compress a Σ -protocol via a **C**orrelation **I**ntractable **H**ash (CIH) $\{H_k(\cdot)\}_k$ [CGH98, KRR17, CCRR18, HL18, CCH+19, PS19]



Starting Idea

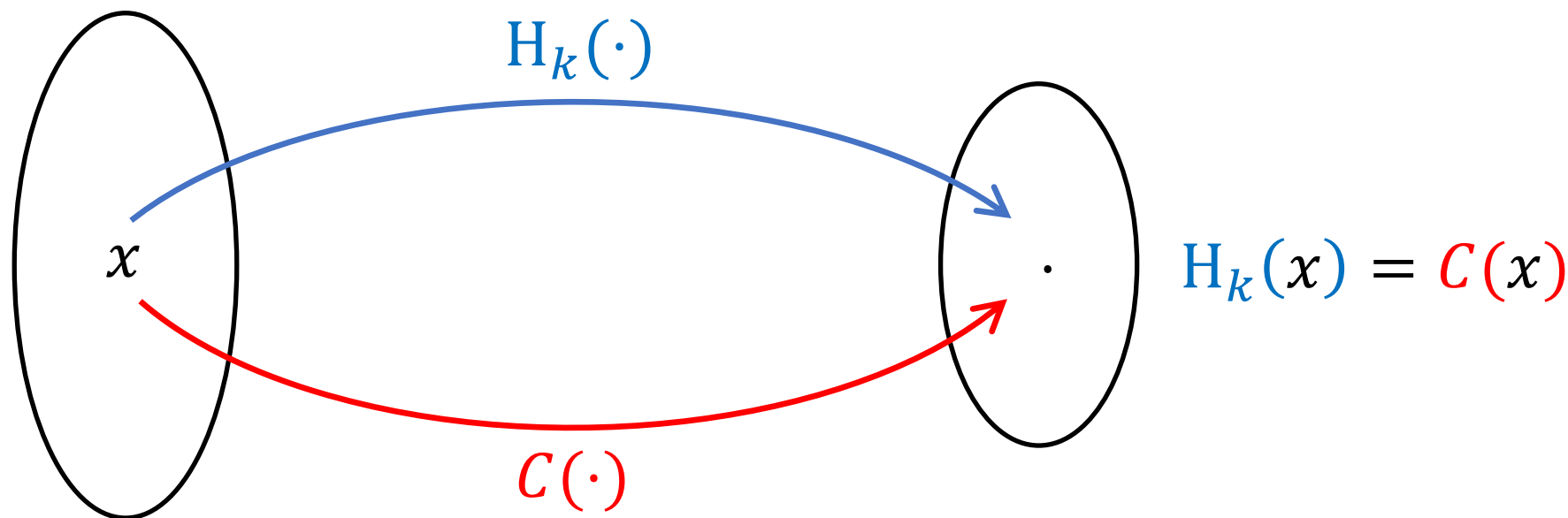
- Compress a Σ -protocol via a **C**orrelation **I**ntractable **H**ash (CIH) $\{H_k(\cdot)\}_k$ [CGH98, KRR17, CCRR18, HL18, CCH+19, PS19]



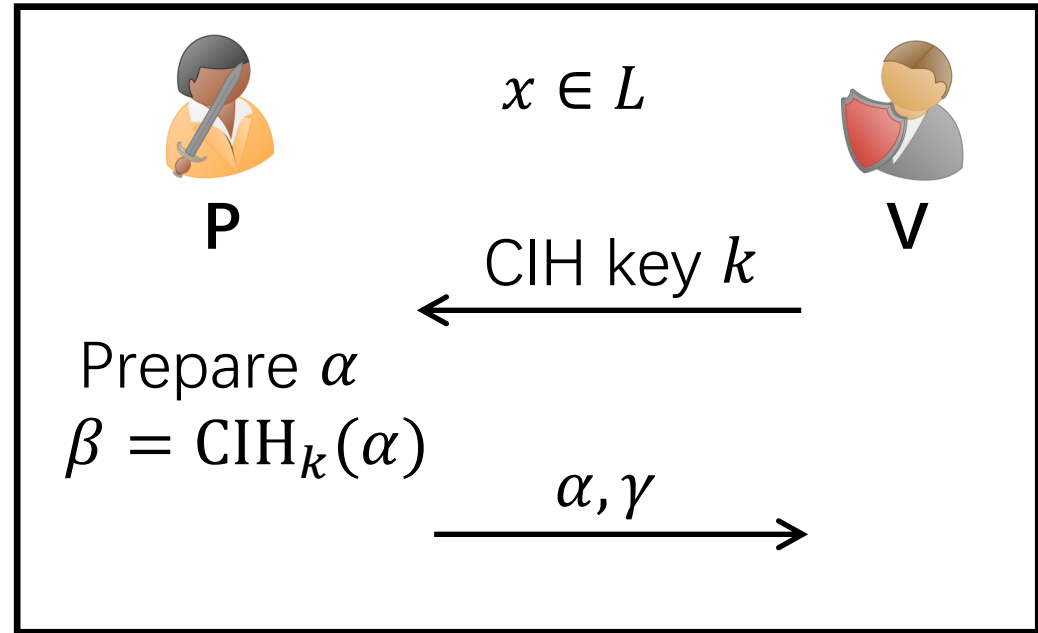
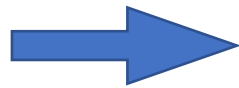
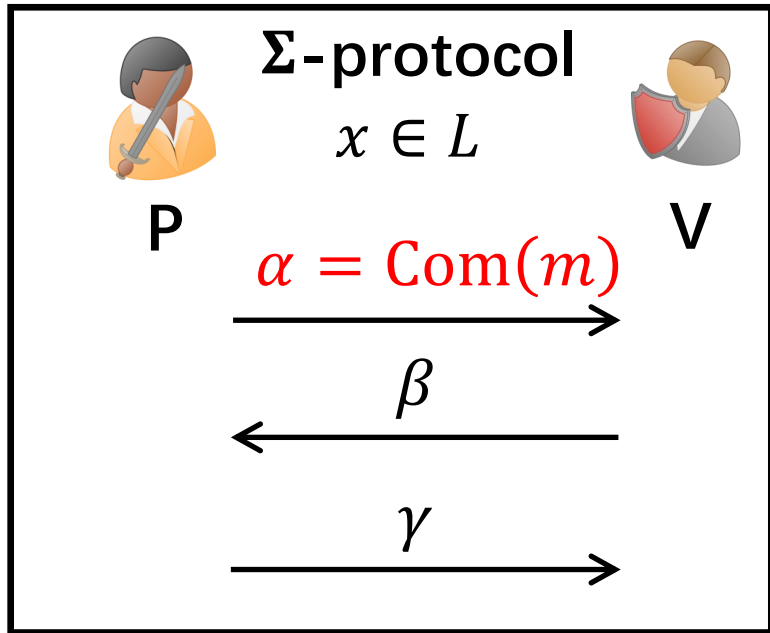
Correlation Intractable Hash (CIH)

A CIH is a hash function $\{H_k(\cdot)\}_k$:

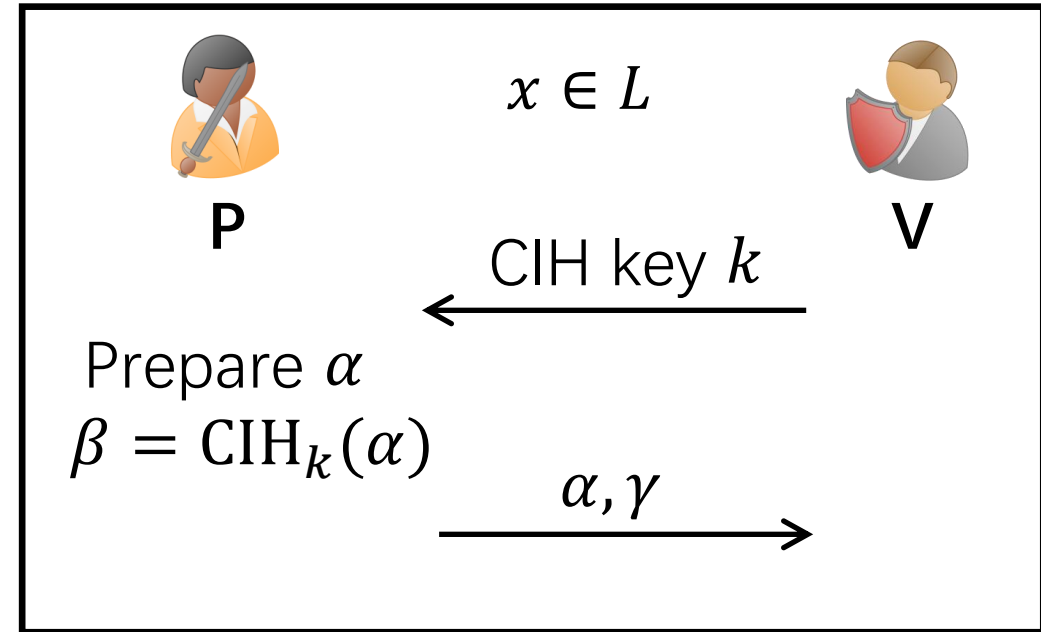
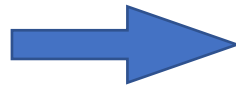
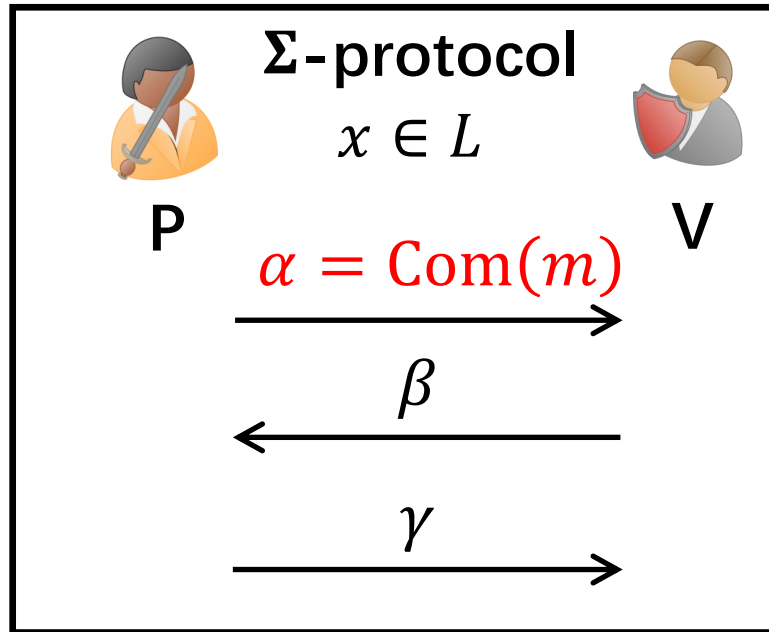
$\forall C$, let $k \leftarrow \{0,1\}^{\text{poly}(\lambda)}$, it's hard to find an x , such that



Idea for Security

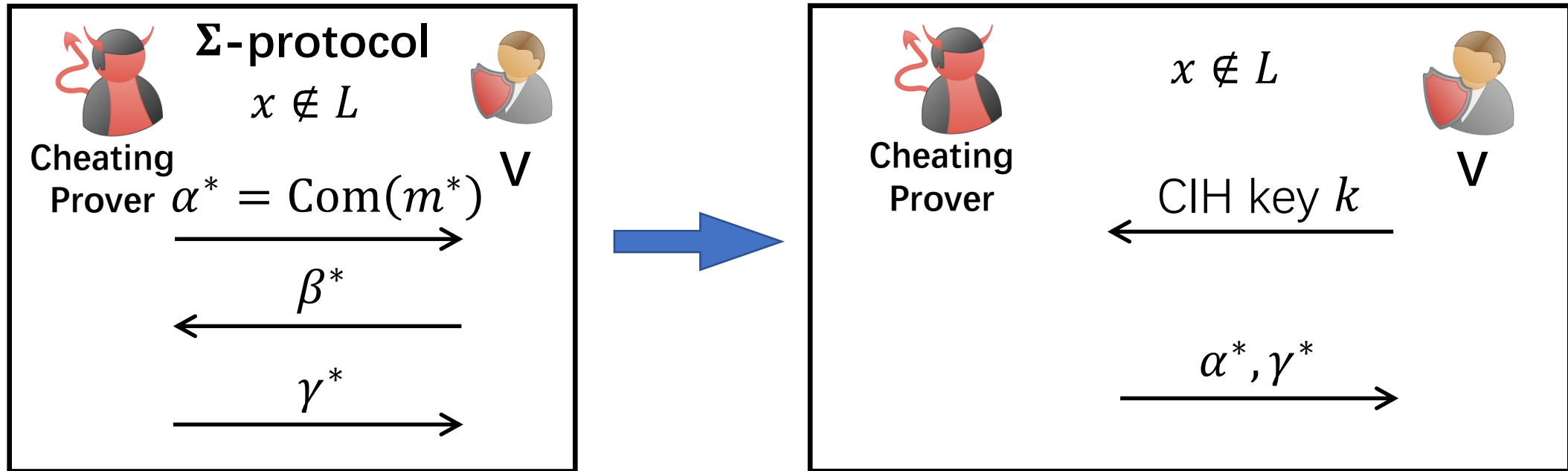


Idea for Security



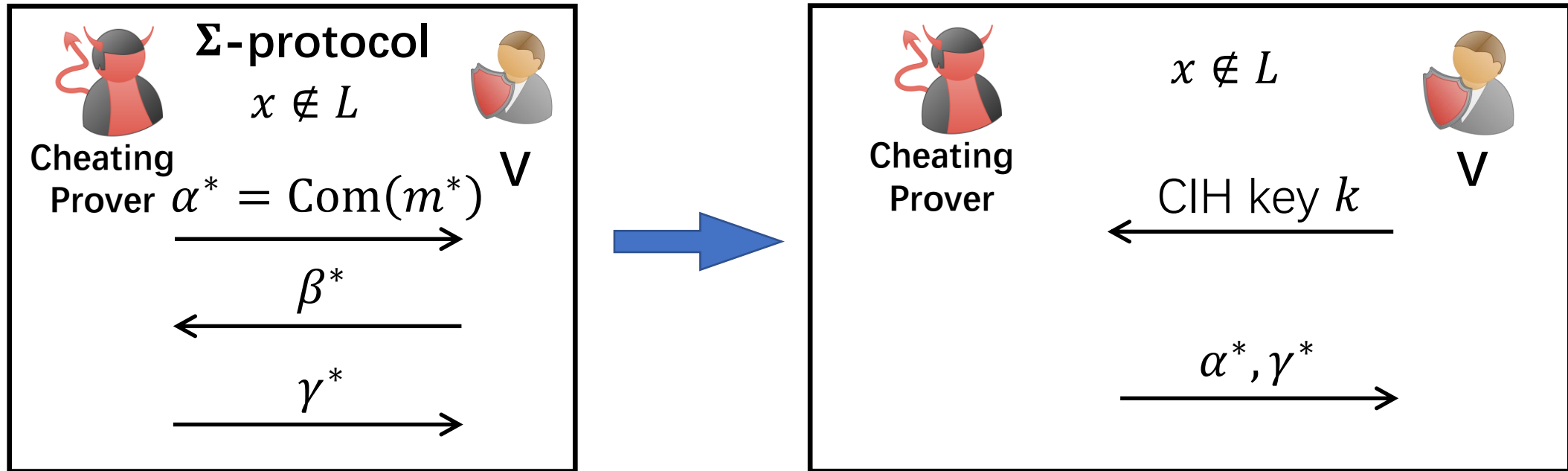
- **WI:** follows from hiding property of the commitment

Idea for Security



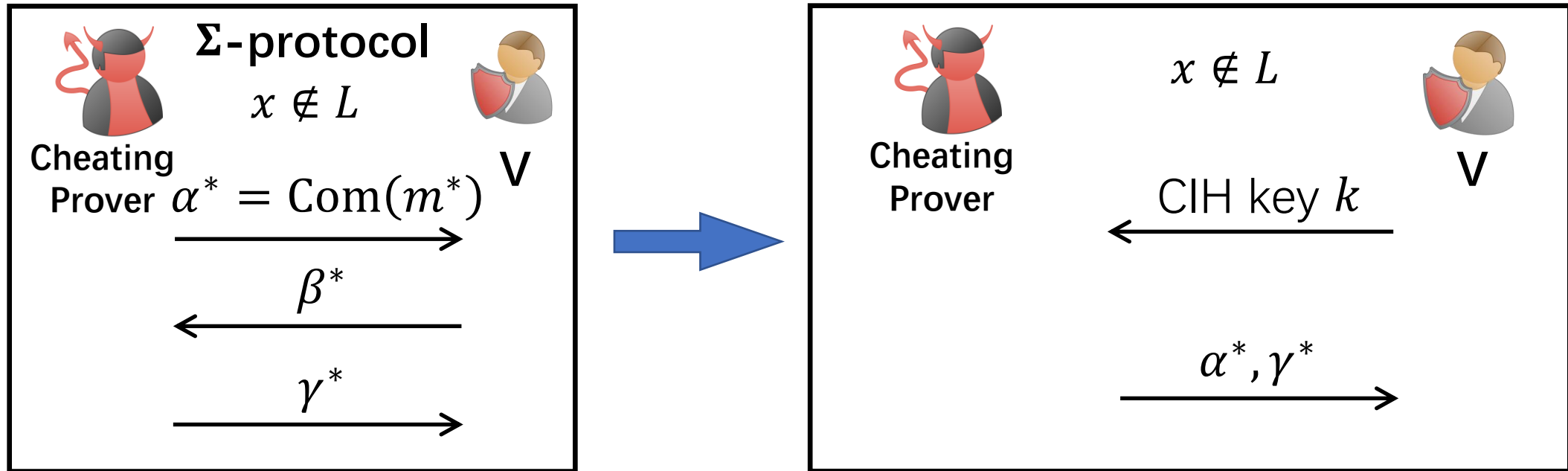
- **Soundness:** Extract m^* from α^* using a trapdoor
 Given m^* , the (only) accepting β^* is efficiently computable
 Verifier accepts $\Rightarrow \beta^* = \text{CIH}_k(\alpha^*) = C(\alpha^*)$
- **Hiding & Extractable commitments** can be built in CRS model
 \Rightarrow Zaps in CRS model

Idea for Security



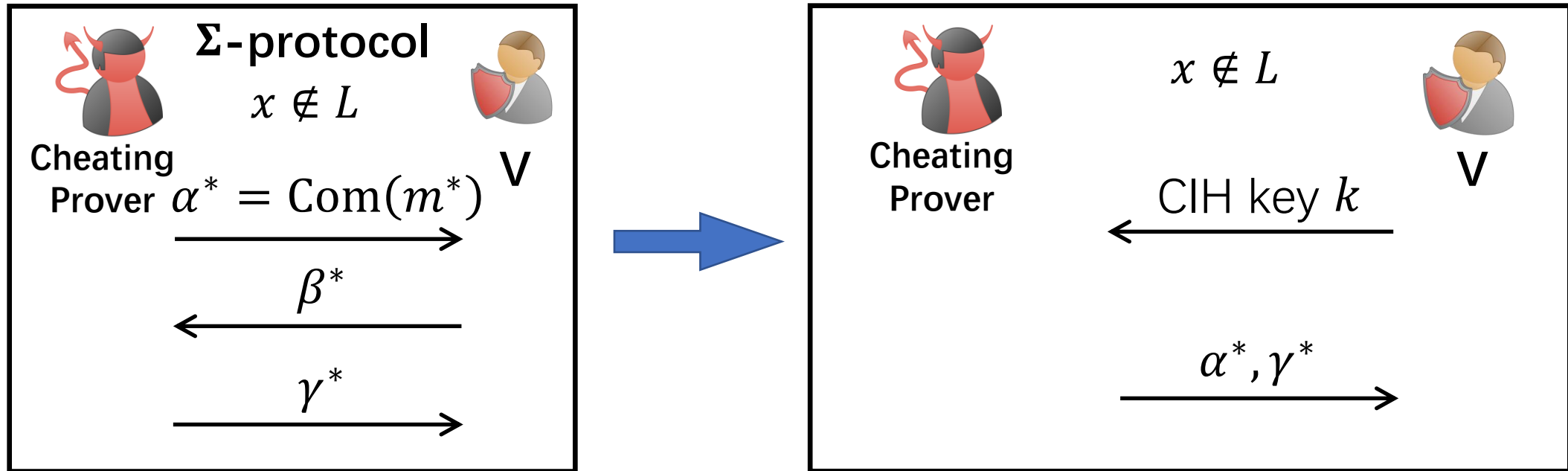
- **Soundness:** Extract m^* from α^* using a trapdoor
 Given m^* , the (only) accepting β^* is efficiently computable
 Verifier accepts $\Rightarrow \beta^* = \text{CIH}_k(\alpha^*) = C(\alpha^*)$
- **Hiding & Extractable commitments** can be built in CRS model
 \Rightarrow Zaps in CRS model

Idea for Security



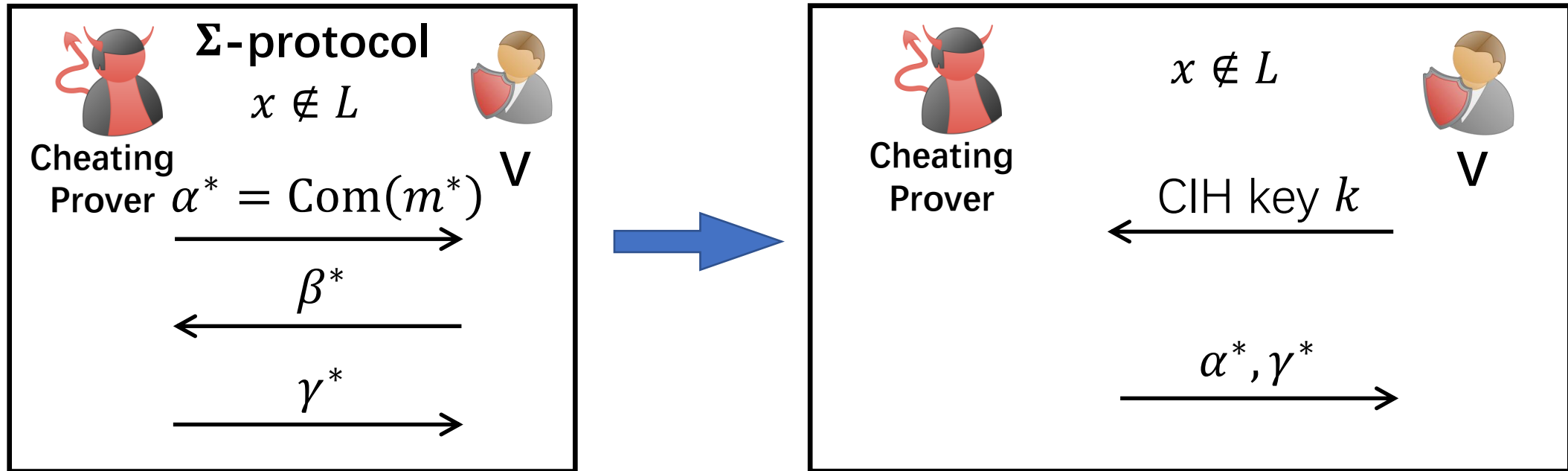
- **Soundness:** Extract m^* from α^* using a trapdoor
 Given m^* , the (only) accepting β^* is efficiently computable }
 Verifier accepts $\Rightarrow \beta^* = \text{CIH}_k(\alpha^*) = C(\alpha^*)$
- **Hiding & Extractable commitments** can be built in CRS model
 \Rightarrow Zaps in CRS model

Idea for Security



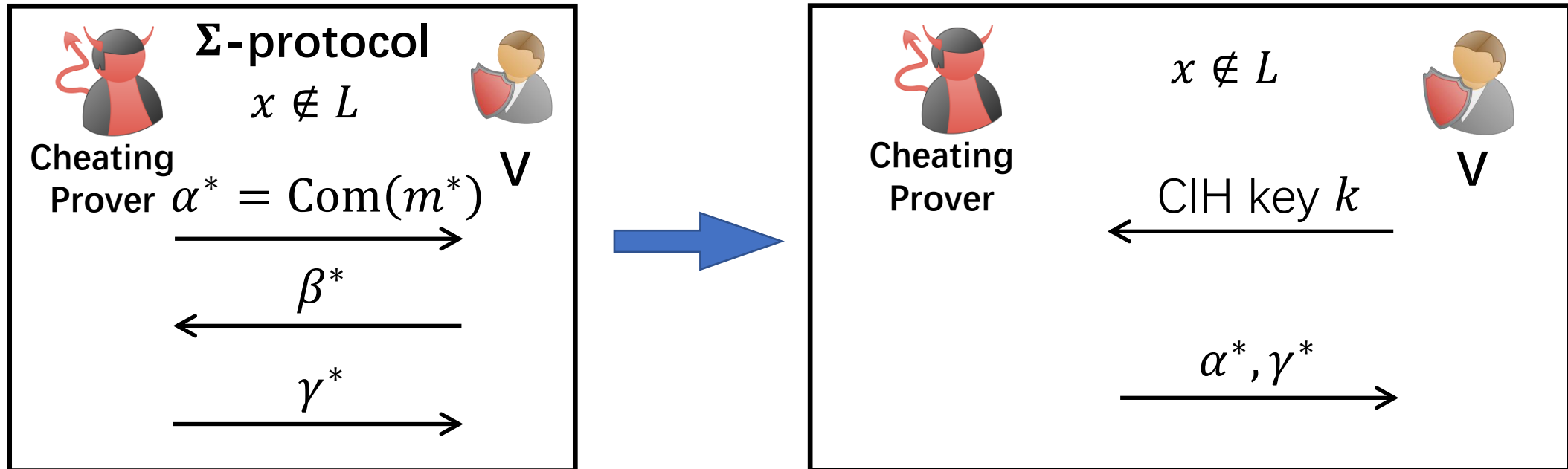
- **Soundness:** Extract m^* from α^* using a trapdoor
 Given m^* , the (only) accepting β^* is efficiently computable } $\beta^* = C(\alpha^*)$
 Verifier accepts $\Rightarrow \beta^* = \text{CIH}_k(\alpha^*) = C(\alpha^*)$
- **Hiding & Extractable commitments** can be built in CRS model
 \Rightarrow Zaps in CRS model

Idea for Security



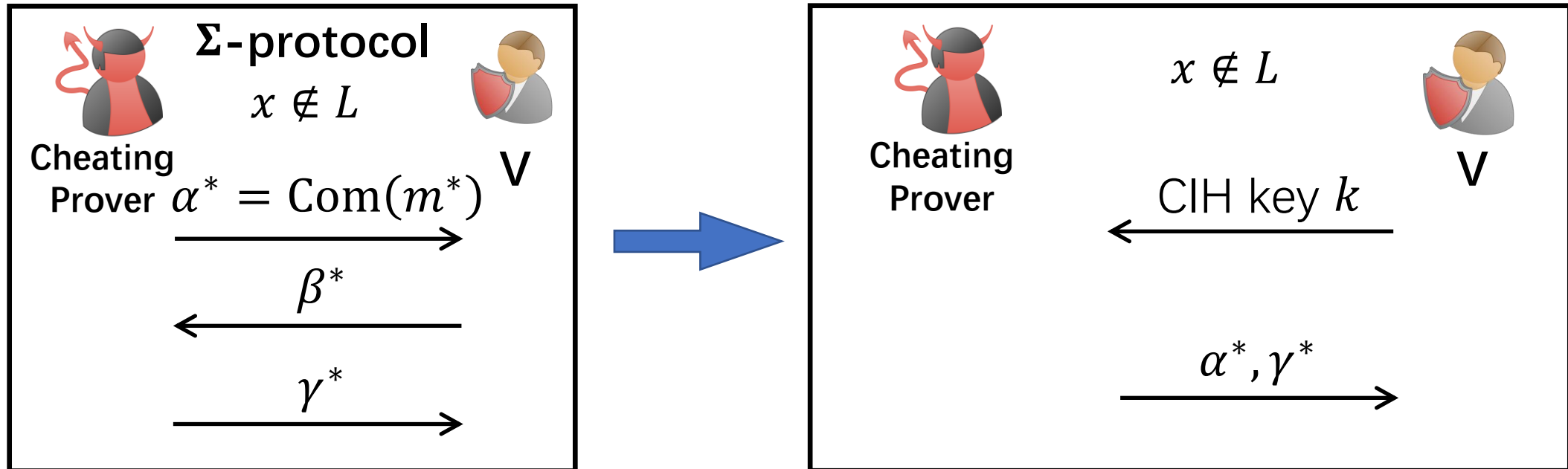
- Soundness:** Extract m^* from α^* using a trapdoor
 Given m^* , the (only) accepting β^* is efficiently computable } $\beta^* = C(\alpha^*)$
 Verifier accepts $\Rightarrow \beta^* = \text{CIH}_k(\alpha^*) = C(\alpha^*)$
- Hiding & Extractable commitments** can be built in CRS model
 \Rightarrow Zaps in CRS model

Idea for Security



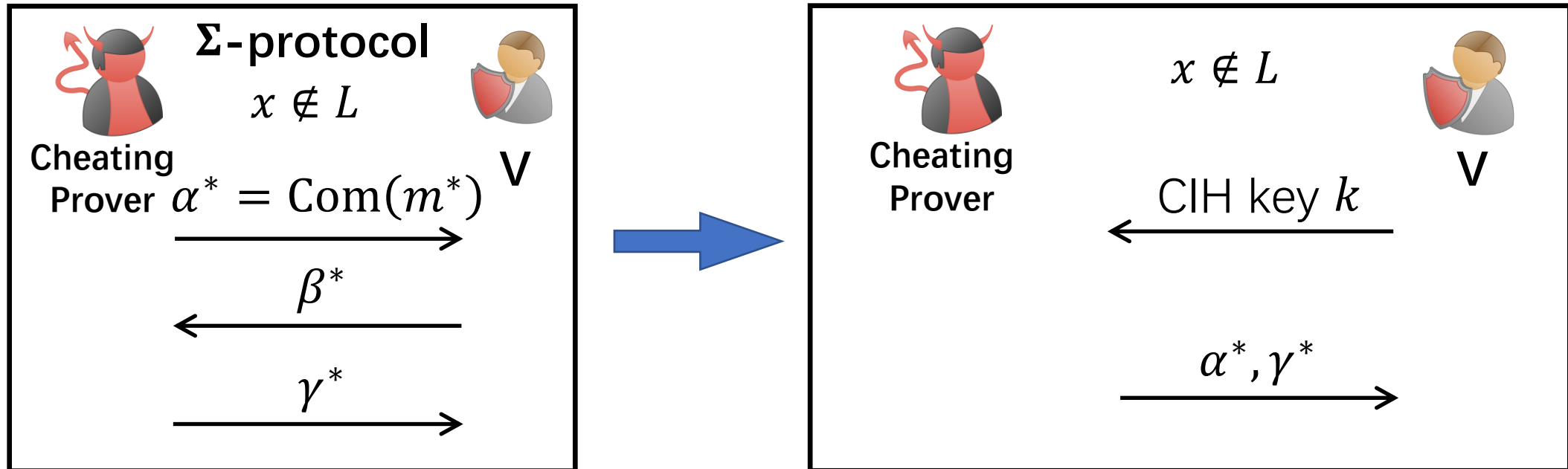
- Soundness:** Extract m^* from α^* using a trapdoor
 Given m^* , the (only) accepting β^* is efficiently computable } $\beta^* = C(\alpha^*)$
 Verifier accepts $\Rightarrow \beta^* = \text{CIH}_k(\alpha^*) = C(\alpha^*)$ **Contradicts CIH!**
- Hiding & Extractable commitments** can be built in CRS model
 \Rightarrow Zaps in CRS model

Idea for Security



- Soundness:** Extract m^* from α^* using a trapdoor
 Given m^* , the (only) accepting β^* is efficiently computable } $\beta^* = C(\alpha^*)$
 Verifier accepts $\Rightarrow \beta^* = \text{CIH}_k(\alpha^*) = C(\alpha^*)$ **Contradicts CIH!**
- Hiding & Extractable commitments** can be built in CRS model
 \Rightarrow Zaps in CRS model

Idea for Security



- Soundness:** Extract m^* from α^* using a trapdoor
 Given m^* , the (only) accepting β^* is efficiently computable } $\beta^* = C(\alpha^*)$
 Verifier accepts $\Rightarrow \beta^* = \text{CIH}_k(\alpha^*) = C(\alpha^*)$ **Contradicts CIH!**
- Hiding & Extractable commitments** can be built in CRS model
 \Rightarrow Zaps in CRS model

Hiding & Extractability in Plain Model

- Use a 2-round statistical sender-private oblivious transfer

Hiding & Extractability in Plain Model

- Use a 2-round statistical sender-private oblivious transfer



P



V

Hiding & Extractability in Plain Model

- Use a 2-round statistical sender-private oblivious transfer



P

Prepare $m, b' \xleftarrow{\$} \{0,1\}$



V

$b \xleftarrow{\$} \{0,1\}$

Hiding & Extractability in Plain Model

- Use a 2-round statistical sender-private oblivious transfer



P

Prepare $m, b' \leftarrow^{\$} \{0,1\}$

Sender



V

$b \leftarrow^{\$} \{0,1\}$

Receiver(b)

Hiding & Extractability in Plain Model

- Use a 2-round statistical sender-private oblivious transfer



P

Prepare $m, b' \leftarrow^{\$} \{0,1\}$

Sender



V

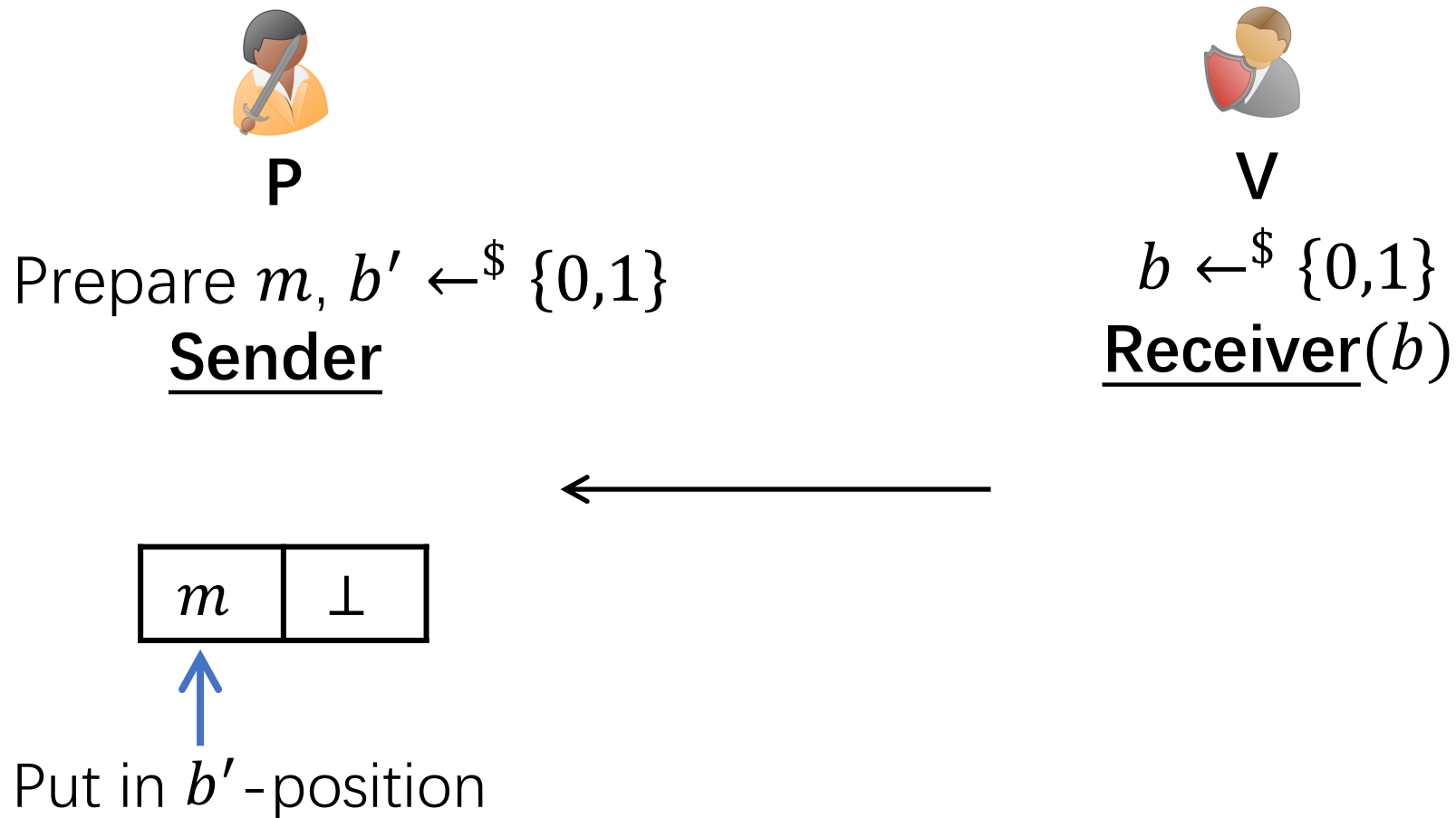
$b \leftarrow^{\$} \{0,1\}$

Receiver(b)



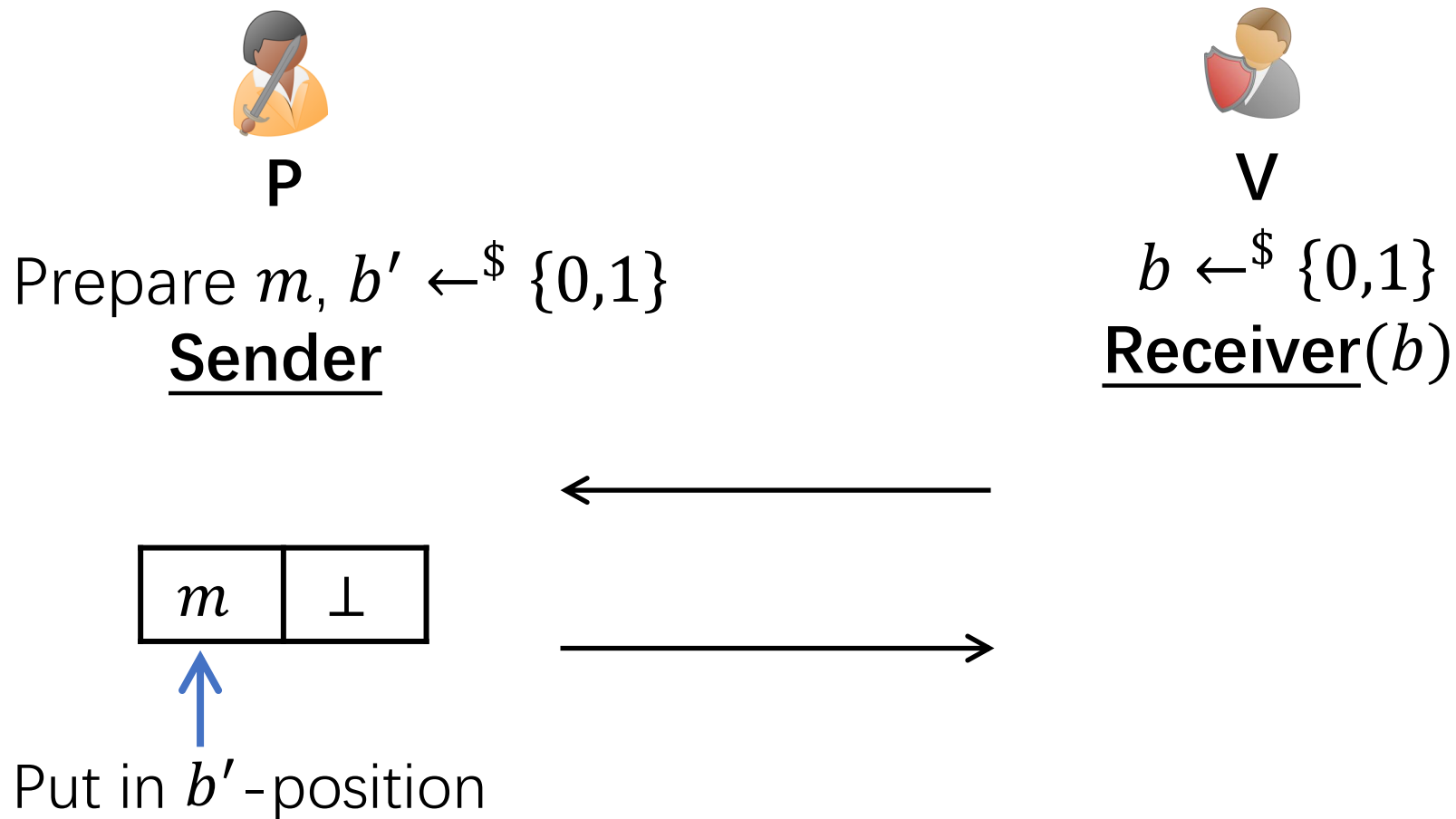
Hiding & Extractability in Plain Model

- Use a 2-round statistical sender-private oblivious transfer



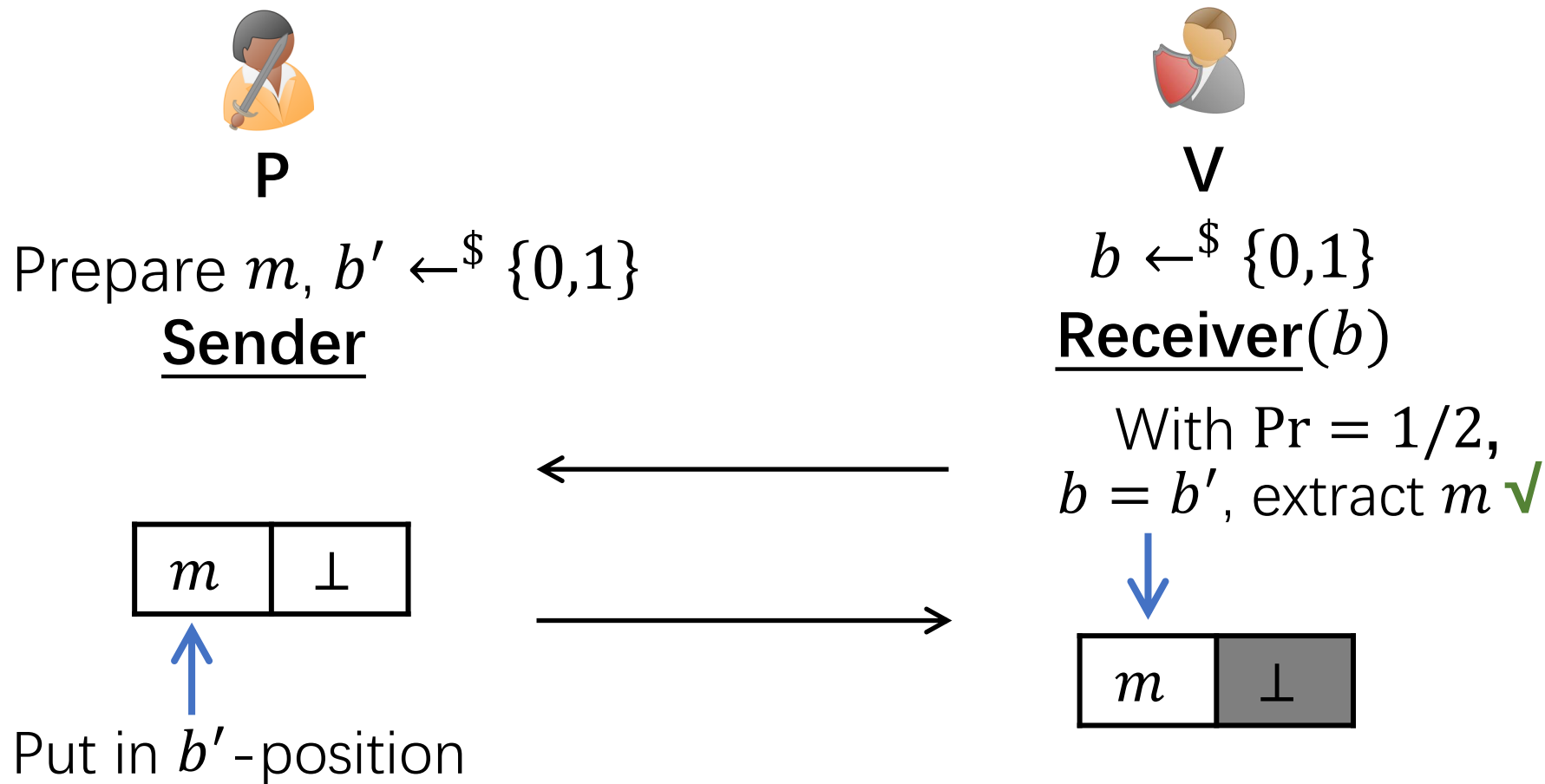
Hiding & Extractability in Plain Model

- Use a 2-round statistical sender-private oblivious transfer



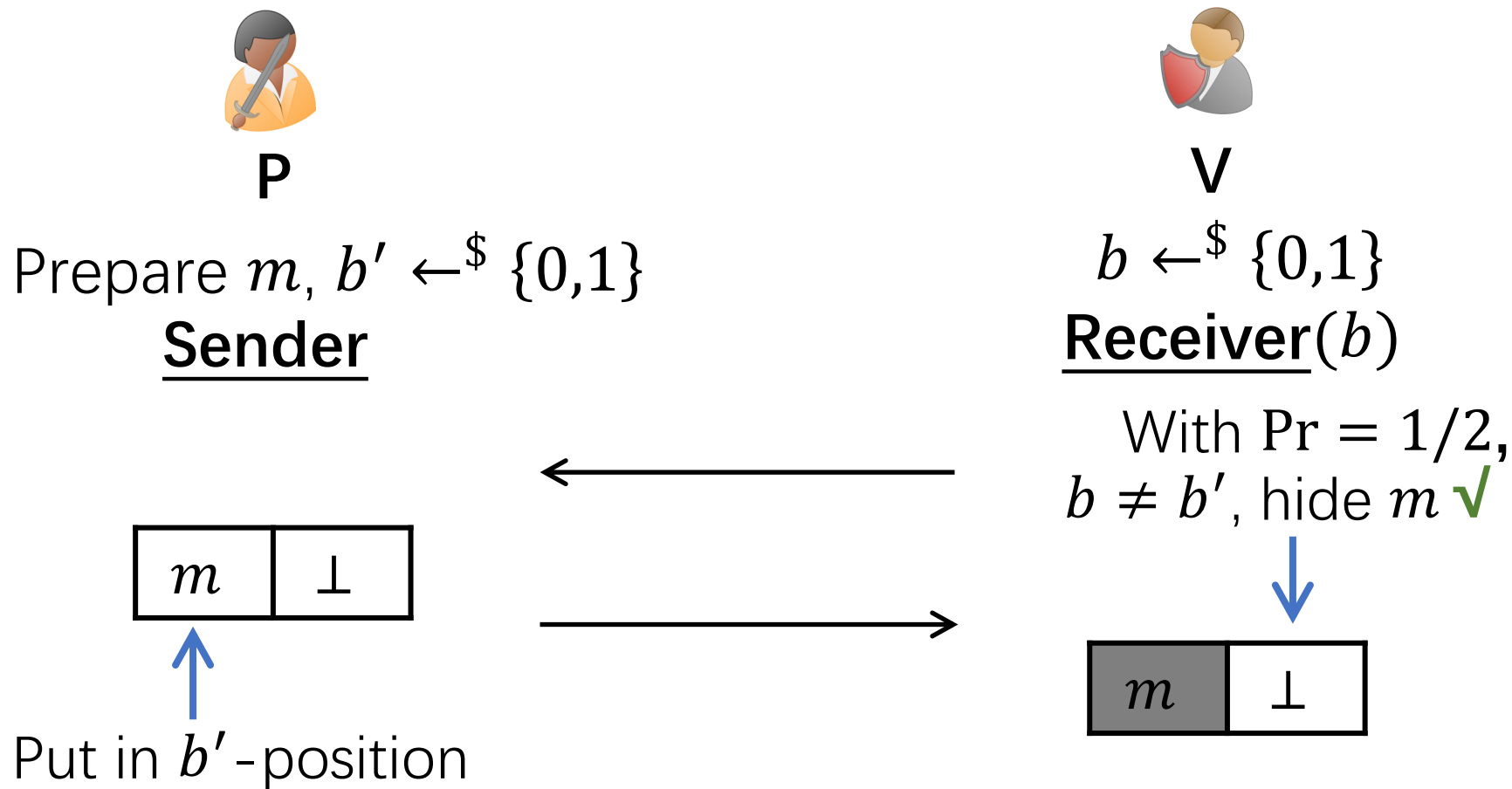
Hiding & Extractability in Plain Model

- Use a 2-round statistical sender-private oblivious transfer

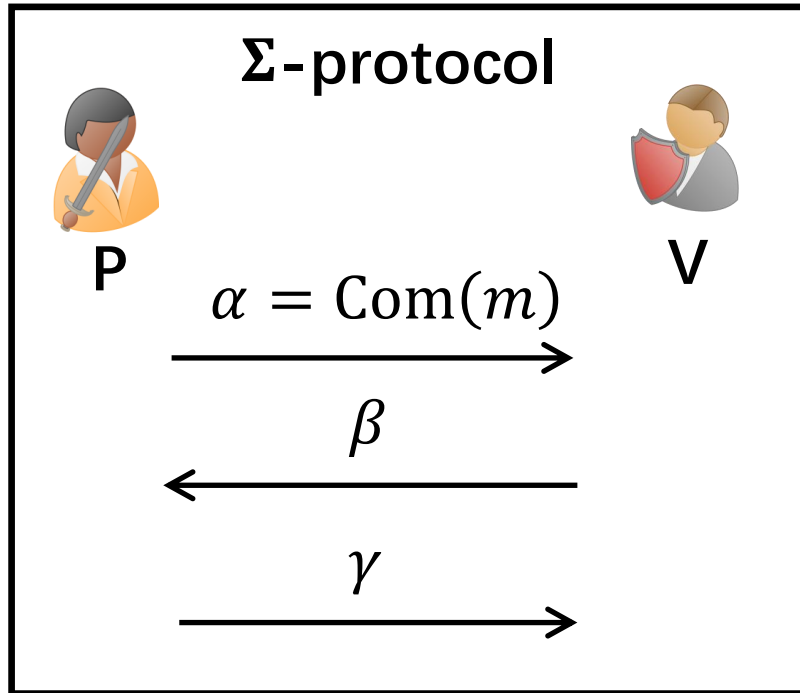


Hiding & Extractability in Plain Model

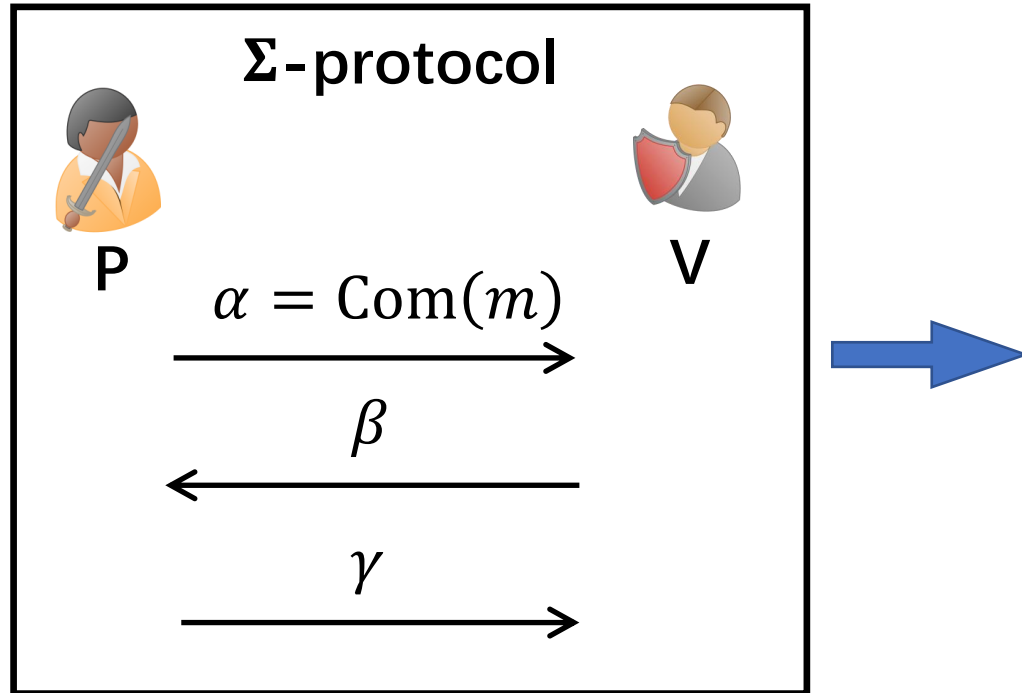
- Use a 2-round statistical sender-private oblivious transfer



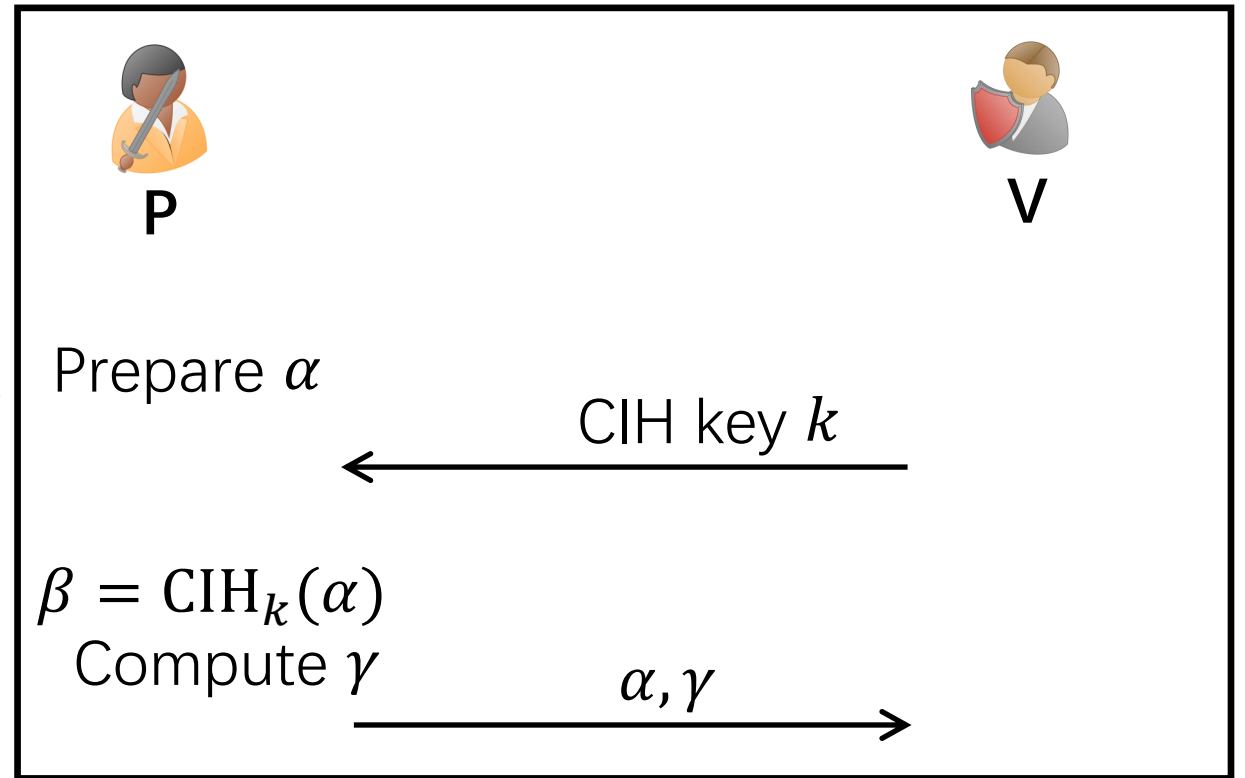
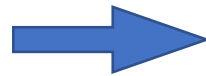
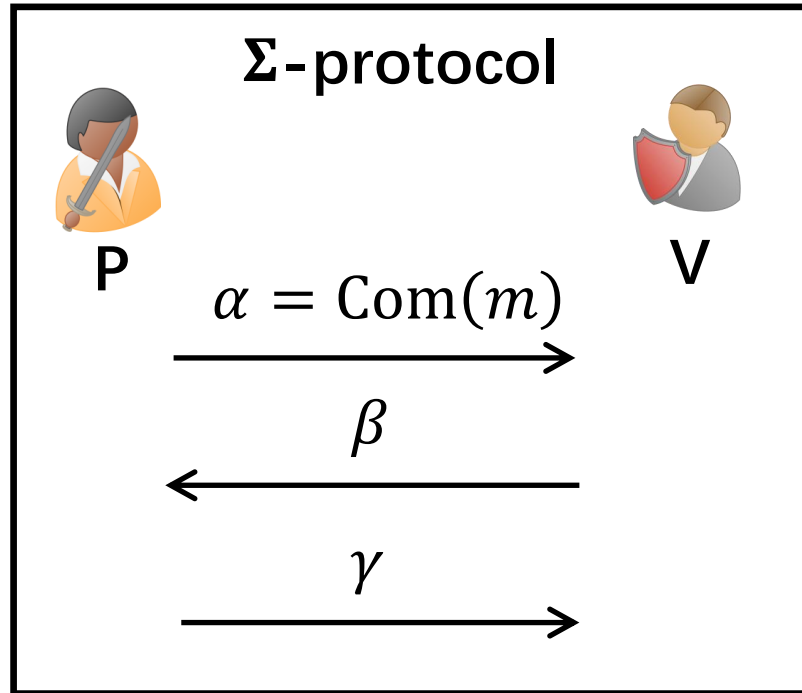
'Weakly Secure' Statistical Zaps



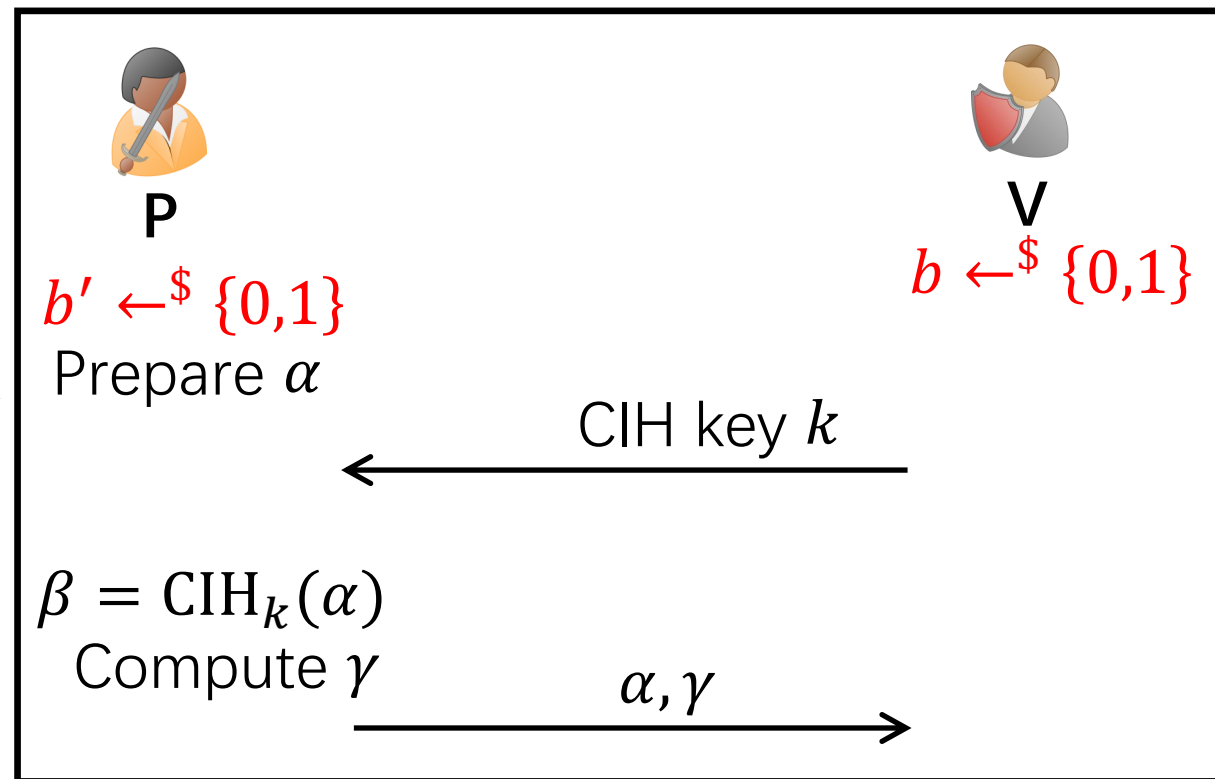
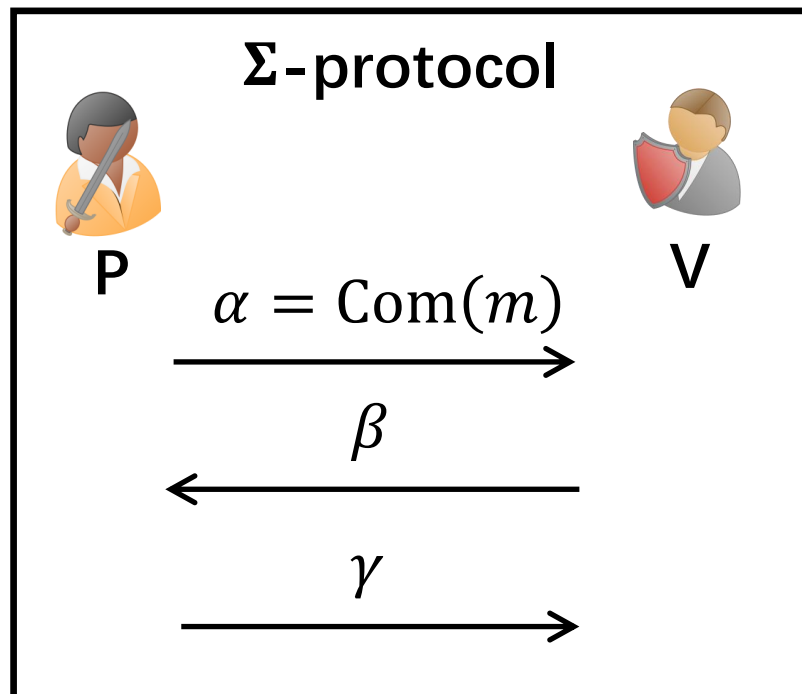
'Weakly Secure' Statistical Zaps



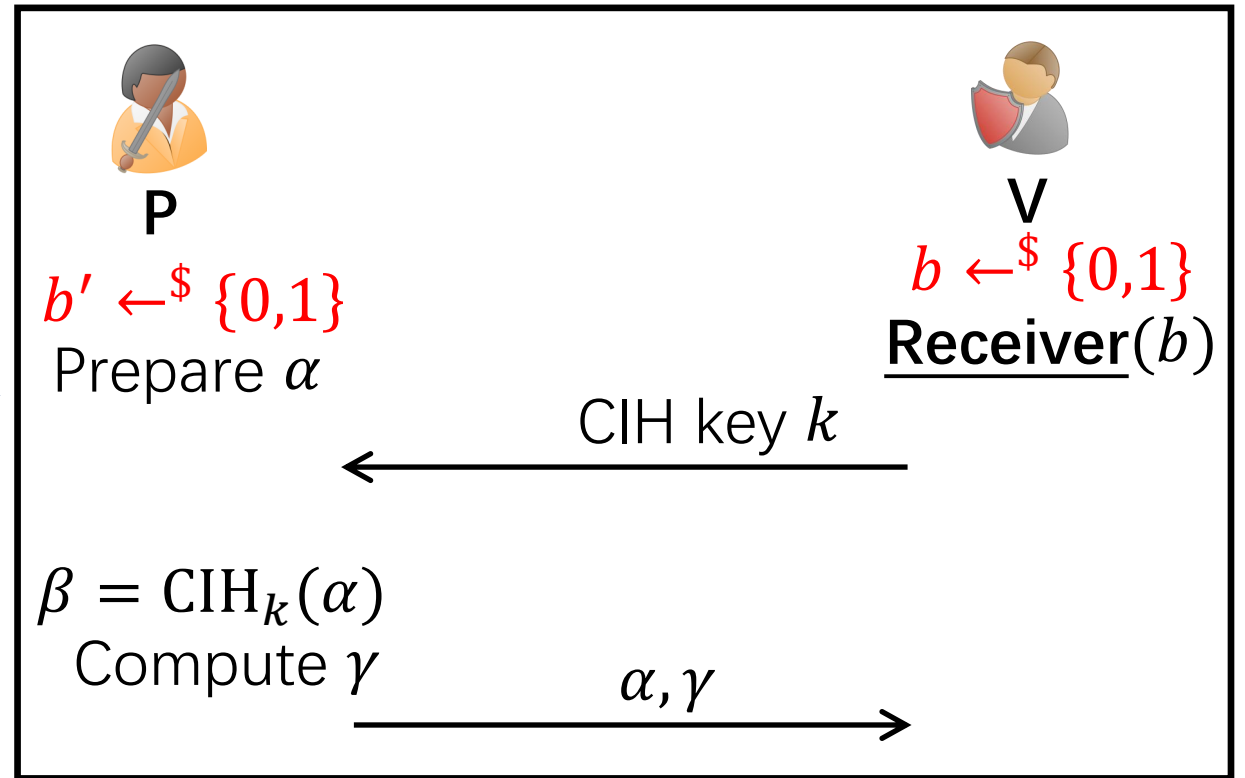
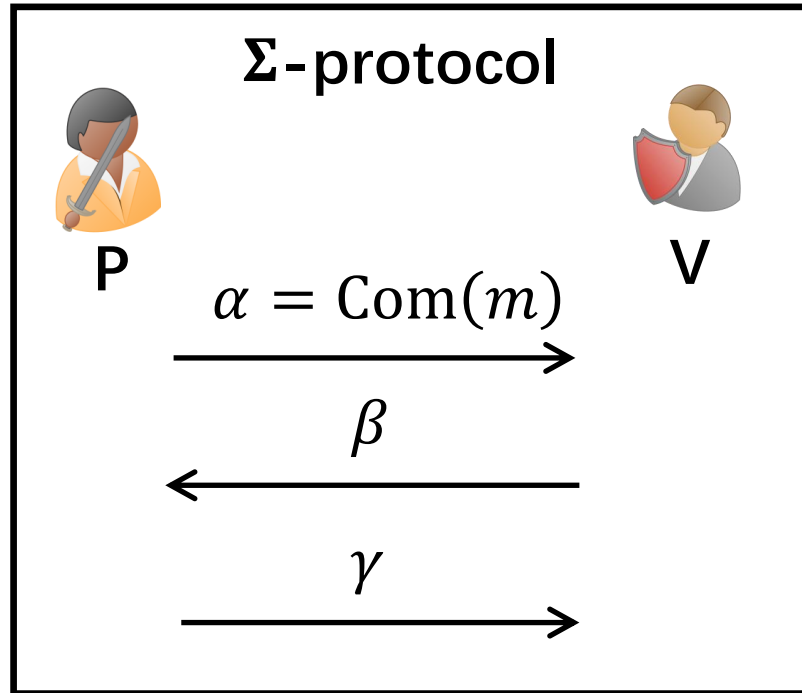
'Weakly Secure' Statistical Zaps



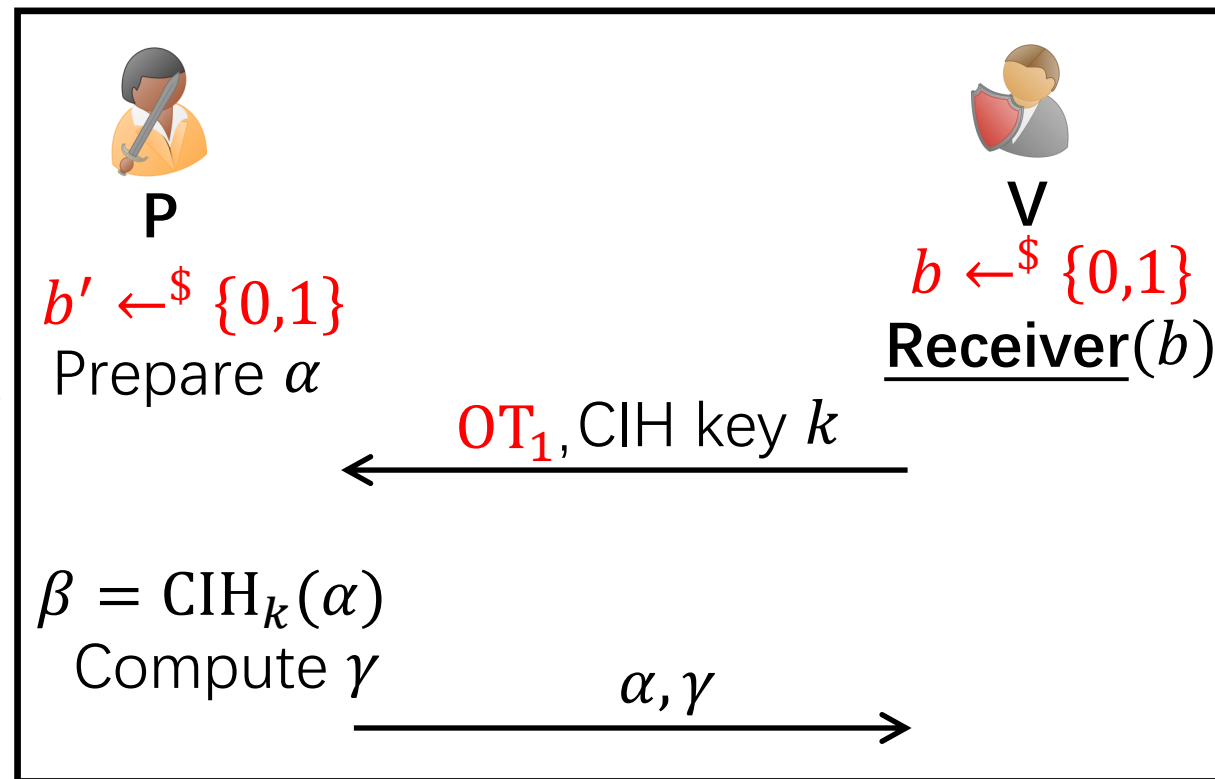
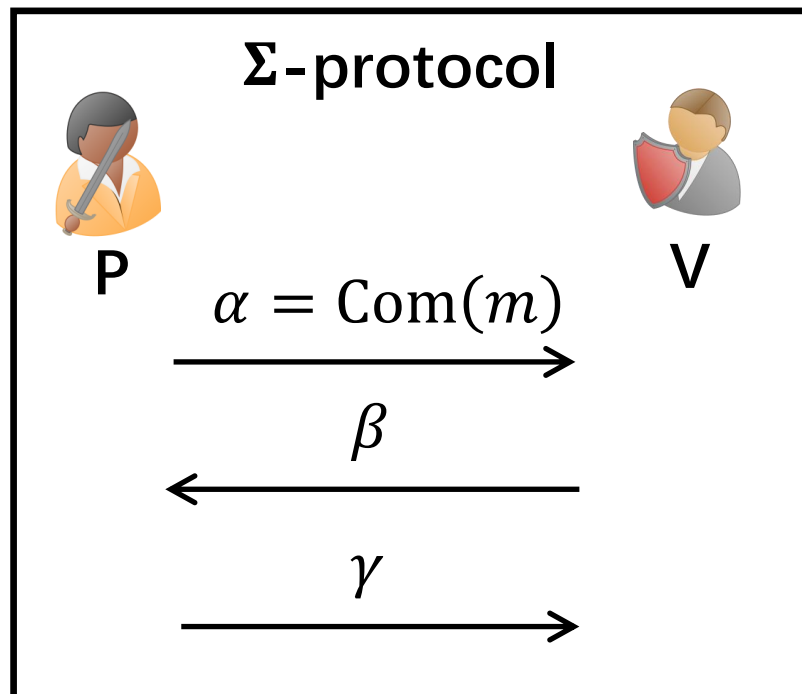
'Weakly Secure' Statistical Zaps



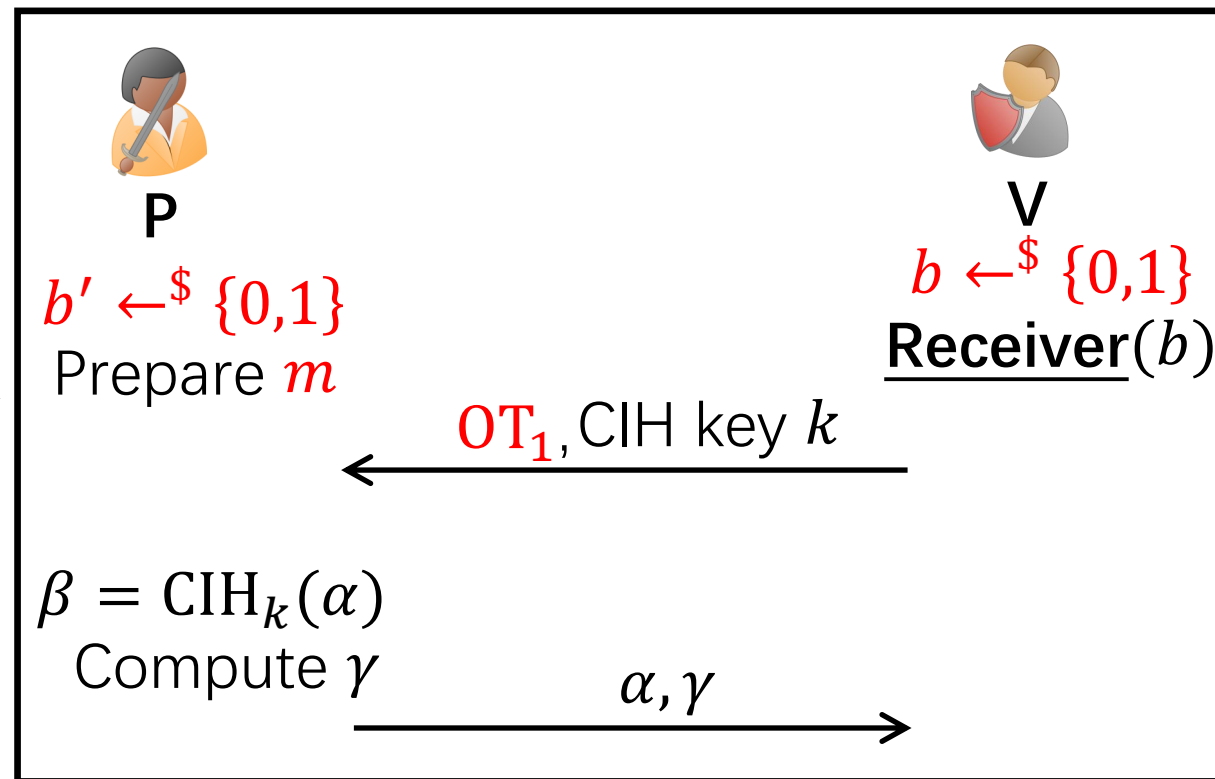
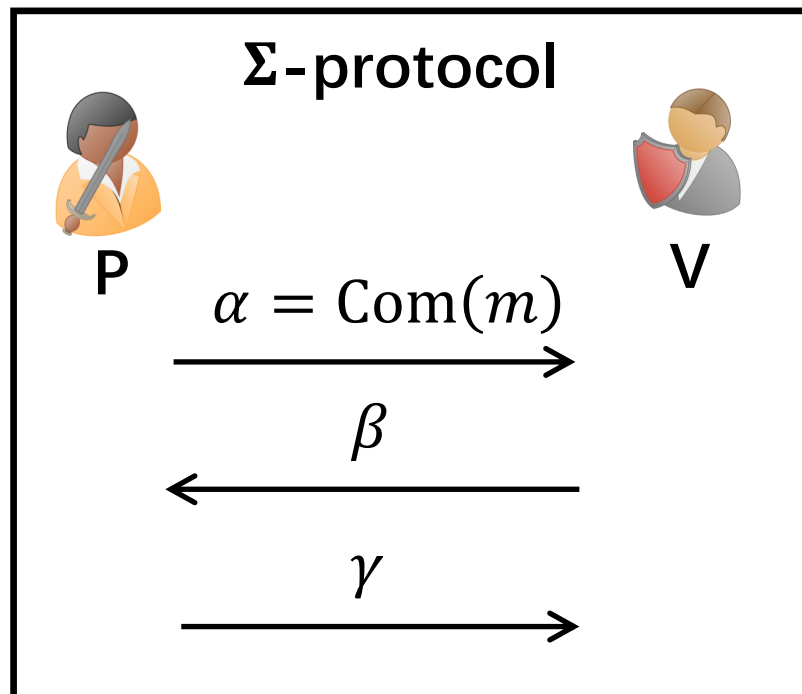
'Weakly Secure' Statistical Zaps



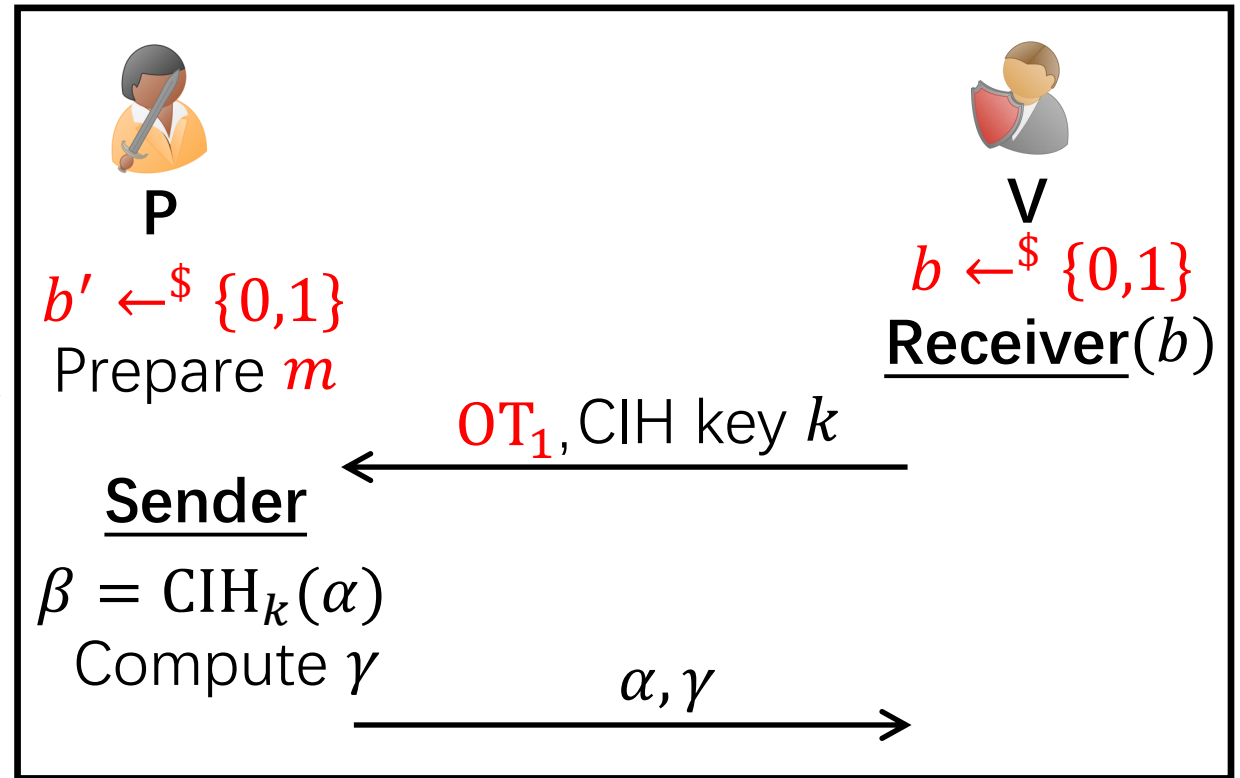
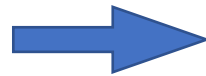
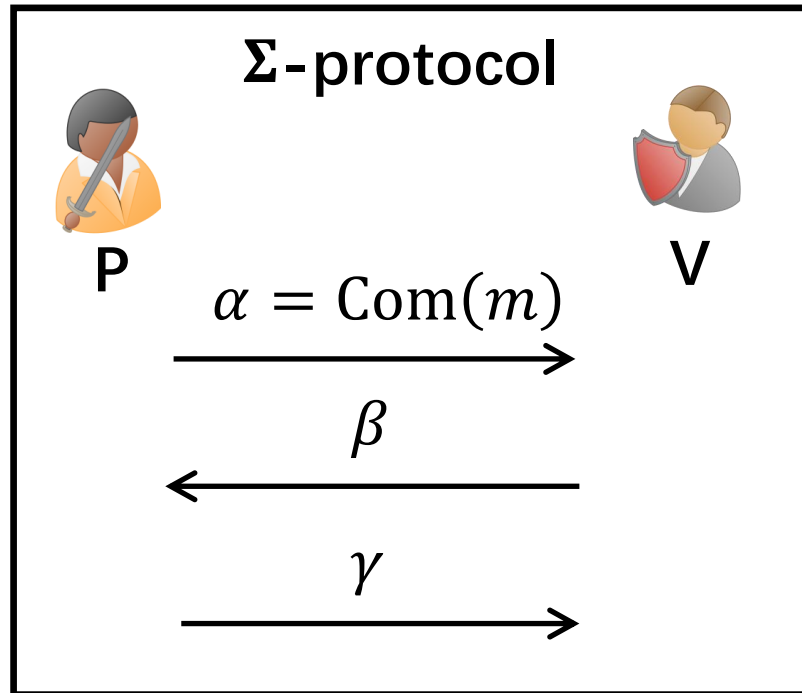
'Weakly Secure' Statistical Zaps



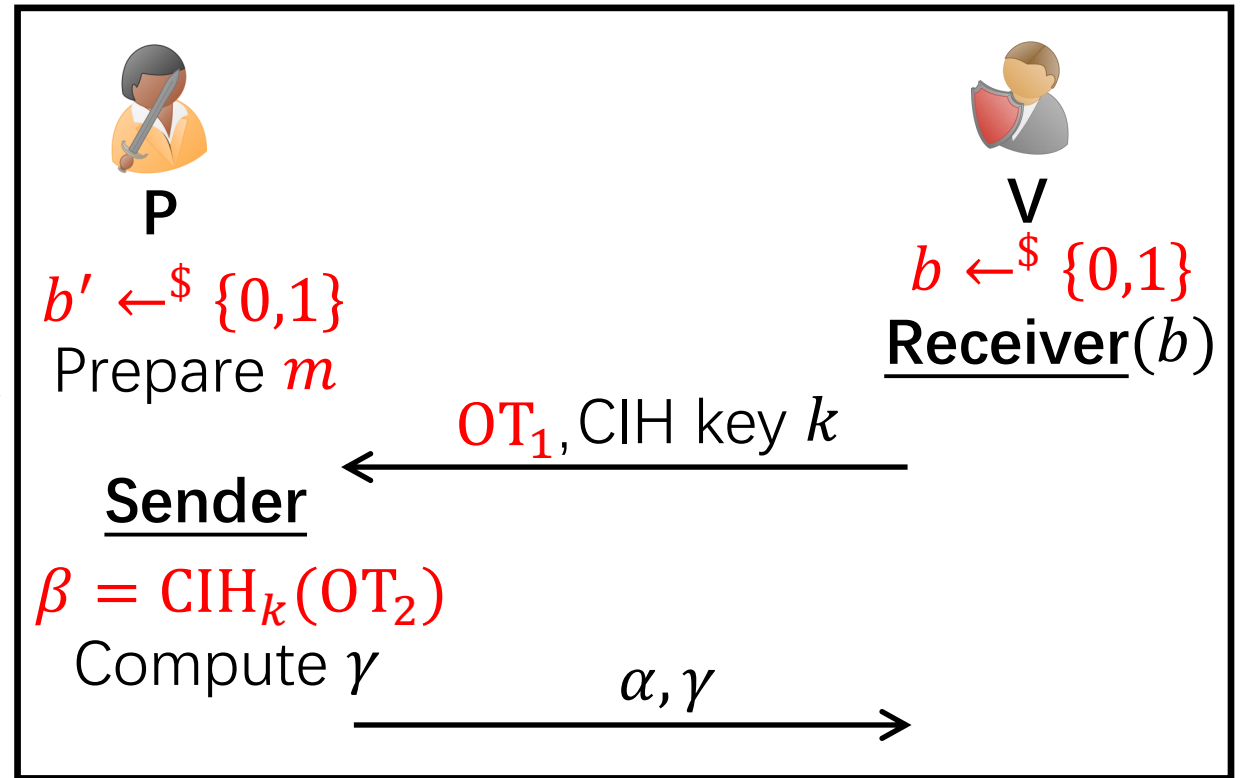
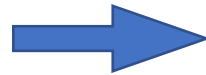
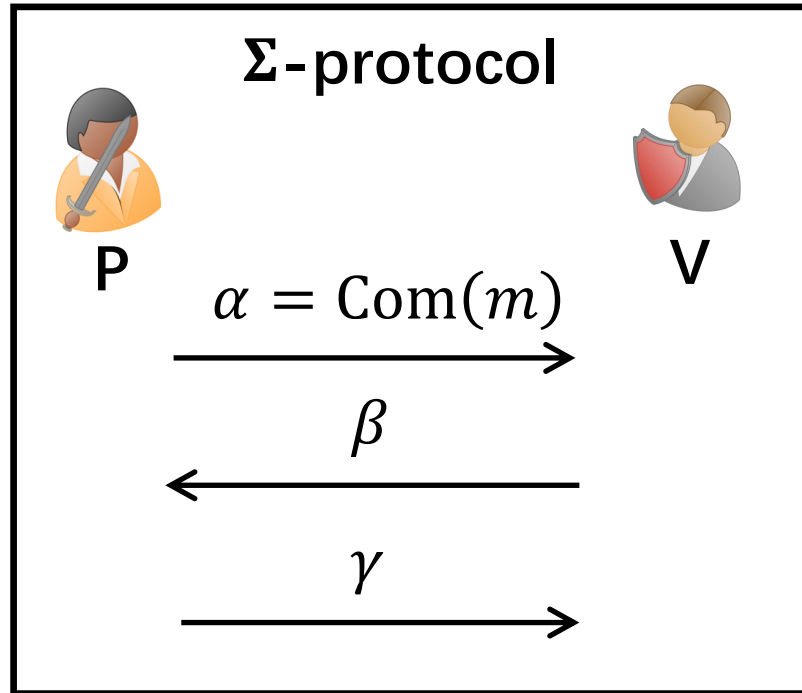
'Weakly Secure' Statistical Zaps



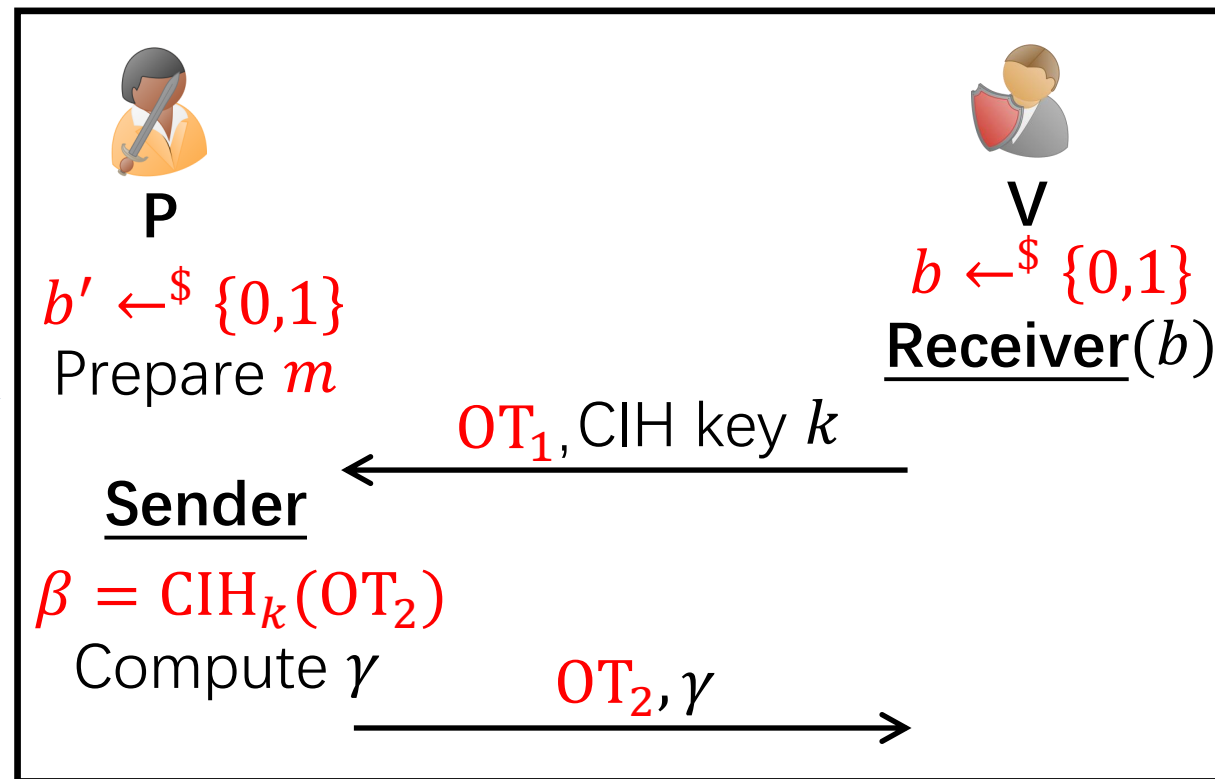
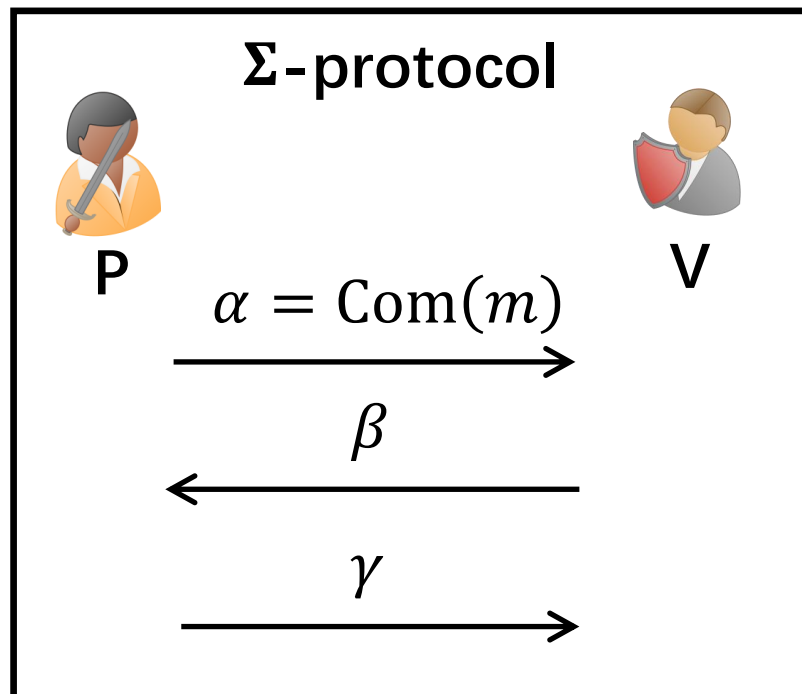
'Weakly Secure' Statistical Zaps



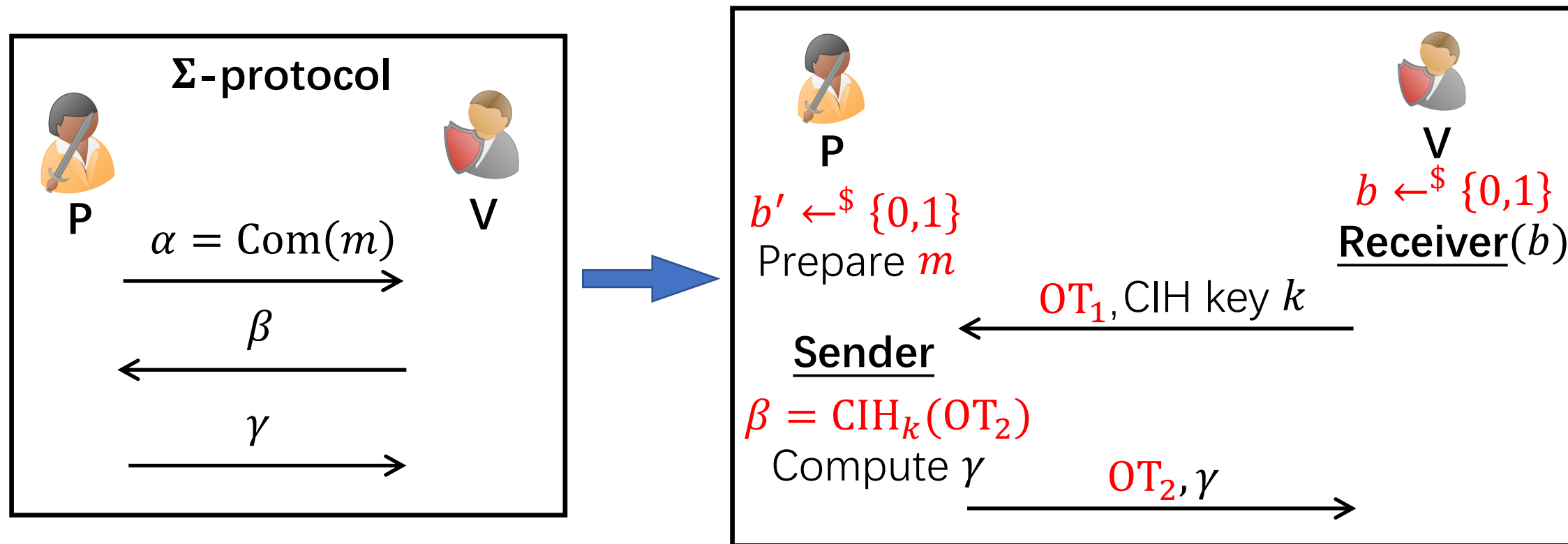
'Weakly Secure' Statistical Zaps



'Weakly Secure' Statistical Zaps

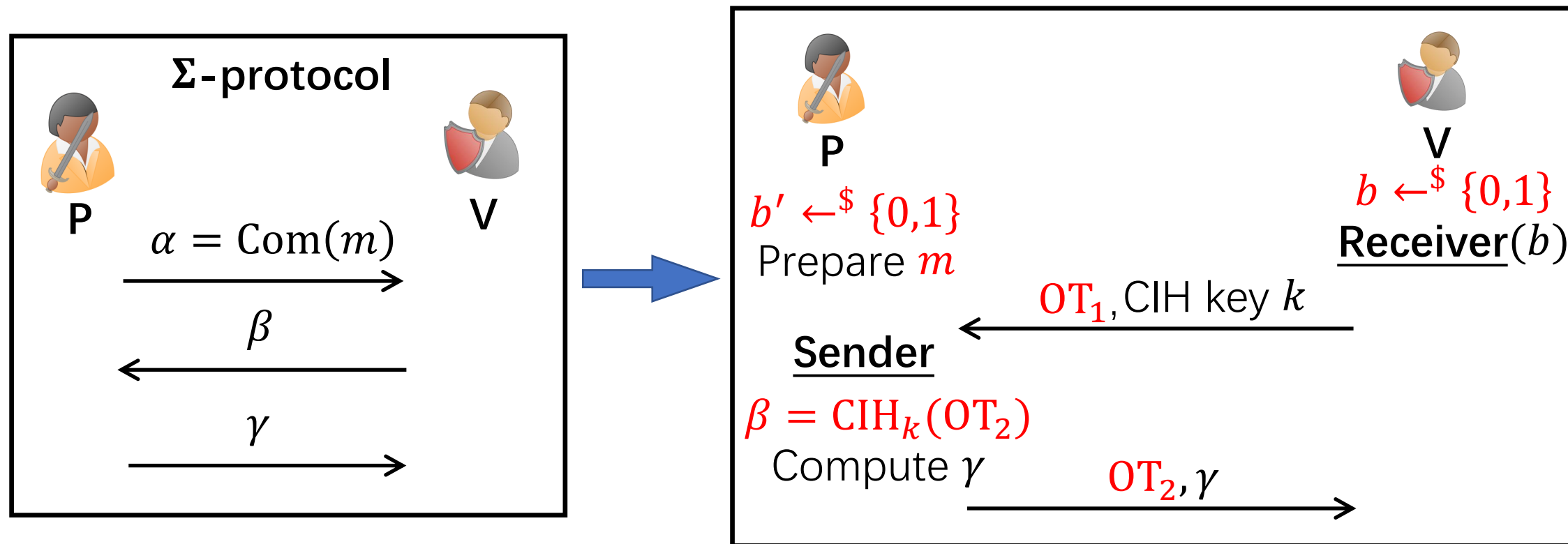


'Weakly Secure' Statistical Zaps



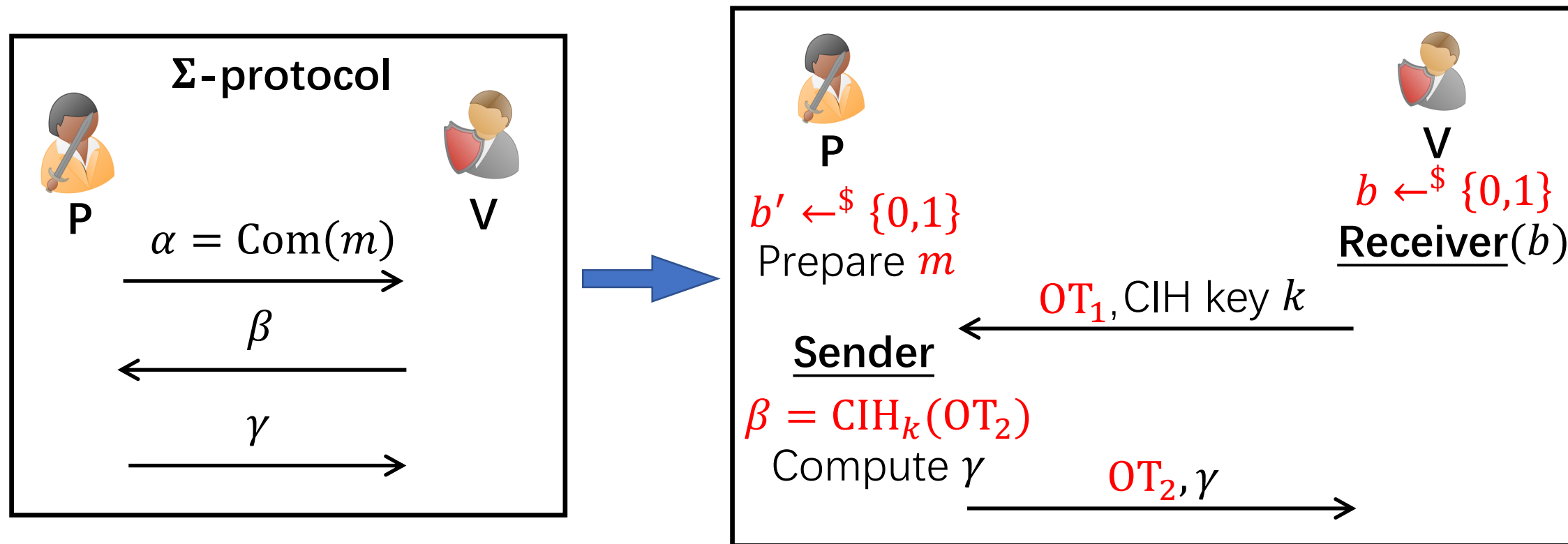
- Statistical WI with $\text{err} \approx 1/2$ (when $b \neq b'$)
- Computational Soundness


'Weakly Secure' Statistical Zaps



- Statistical WI with $\text{err} \approx 1/2$ (when $b \neq b'$)
- Computational Soundness

'Weakly Secure' Statistical Zaps



- Statistical WI with $\text{err} \approx 1/2$ (when $b \neq b'$) 
- Computational Soundness

Amplify the Security



Sender



Receiver

Amplify the Security



Sender

$$b' \leftarrow \{0,1\}^l$$



Receiver

$$b \leftarrow \{0,1\}^l$$

Amplify the Security



Sender

$$b' \leftarrow \{0,1\}^l$$



Receiver

$$b \leftarrow \{0,1\}^l$$



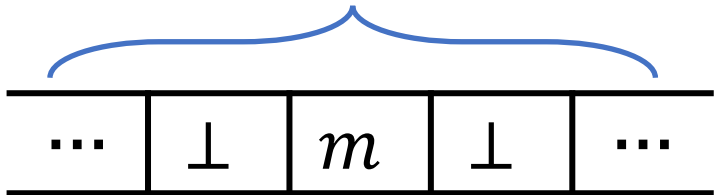
Amplify the Security



Sender

$$b' \leftarrow \{0,1\}^l$$

2^l -positions



b' -th position



Receiver

$$b \leftarrow \{0,1\}^l$$



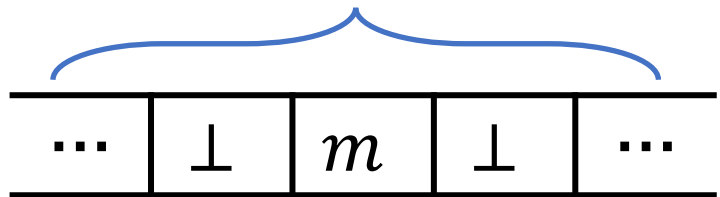
Amplify the Security



Sender

$$b' \leftarrow \{0,1\}^l$$

2^l -positions



b' -th position



Receiver

$$b \leftarrow \{0,1\}^l$$



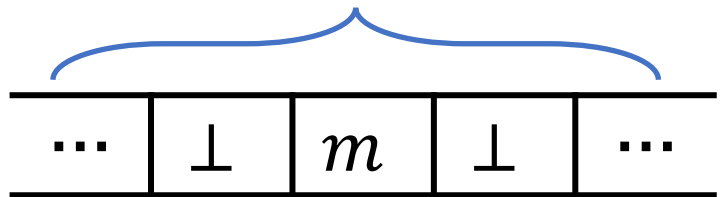
Amplify the Security



Sender

$$b' \leftarrow \{0,1\}^l$$

2^l -positions



b' -th position



Receiver

$$b \leftarrow \{0,1\}^l$$

b -th position



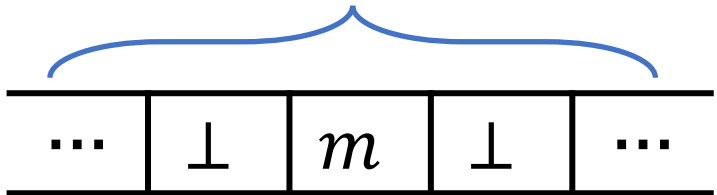
Amplify the Security



Sender

$$b' \leftarrow \{0,1\}^l$$

2^l -positions

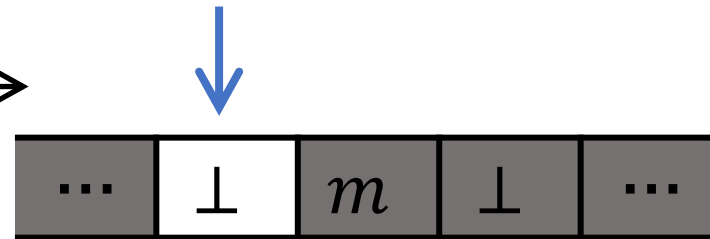


b' -th position



Receiver

$$b \leftarrow \{0,1\}^l$$



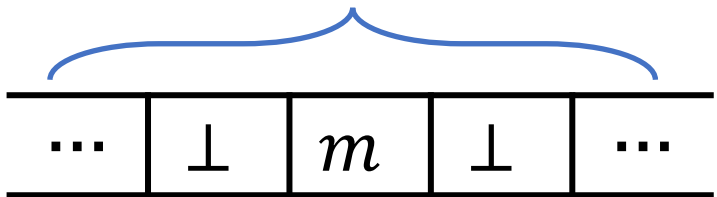
Amplify the Security



Sender

$$\mathbf{b}' \leftarrow \{0,1\}^l$$

2^l -positions



\mathbf{b}' -th position



Receiver

$$\mathbf{b} \leftarrow \{0,1\}^l$$

With $\Pr = 1 - 2^{-l}$,

$\mathbf{b} \neq \mathbf{b}'$, hide m ✓



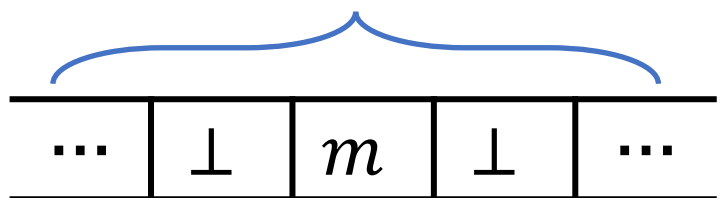
Amplify the Security



Sender

$$\mathbf{b}' \leftarrow \{0,1\}^l$$

2^l -positions



\mathbf{b}' -th position

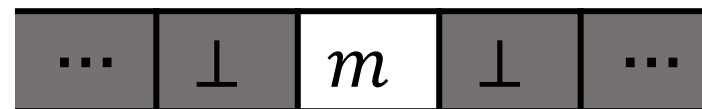


Receiver

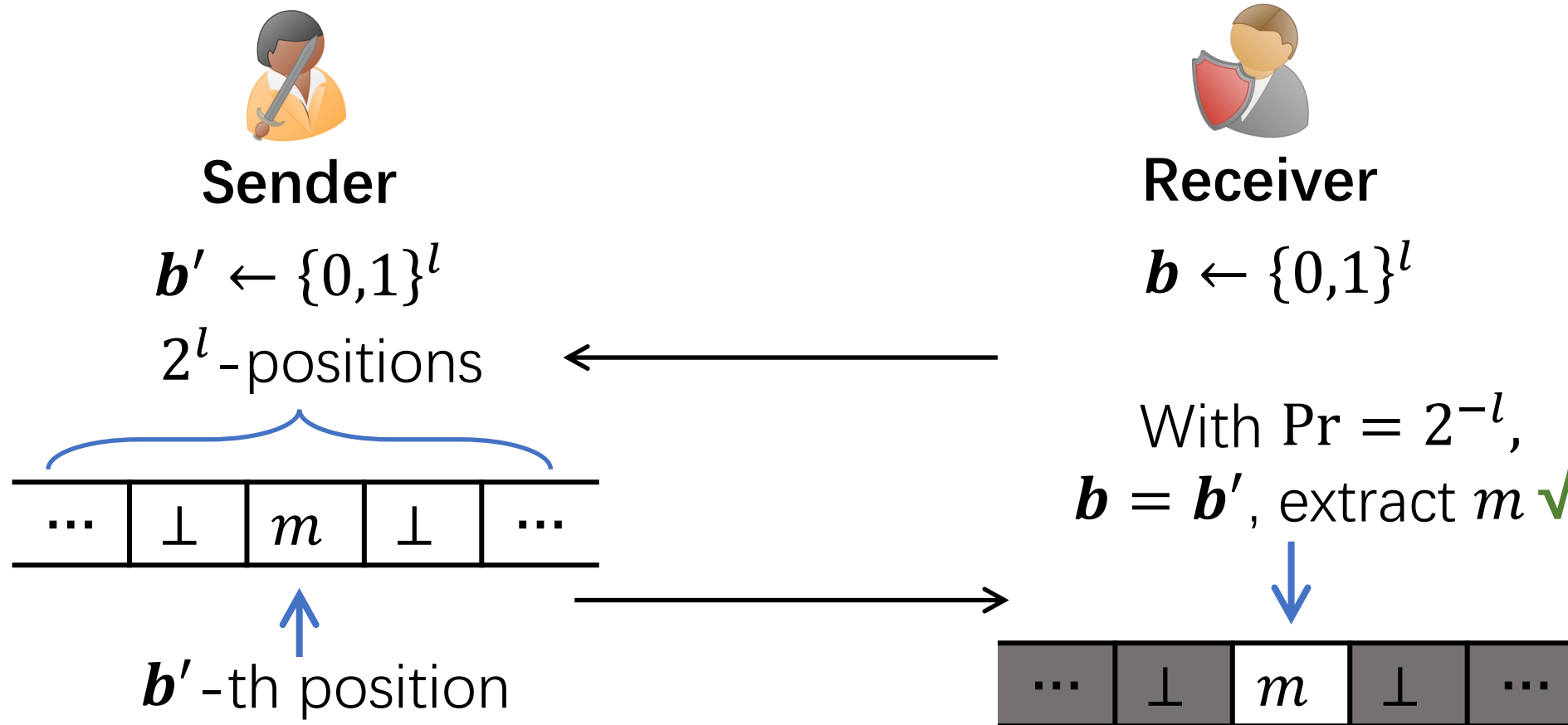
$$\mathbf{b} \leftarrow \{0,1\}^l$$

With $\Pr = 2^{-l}$,

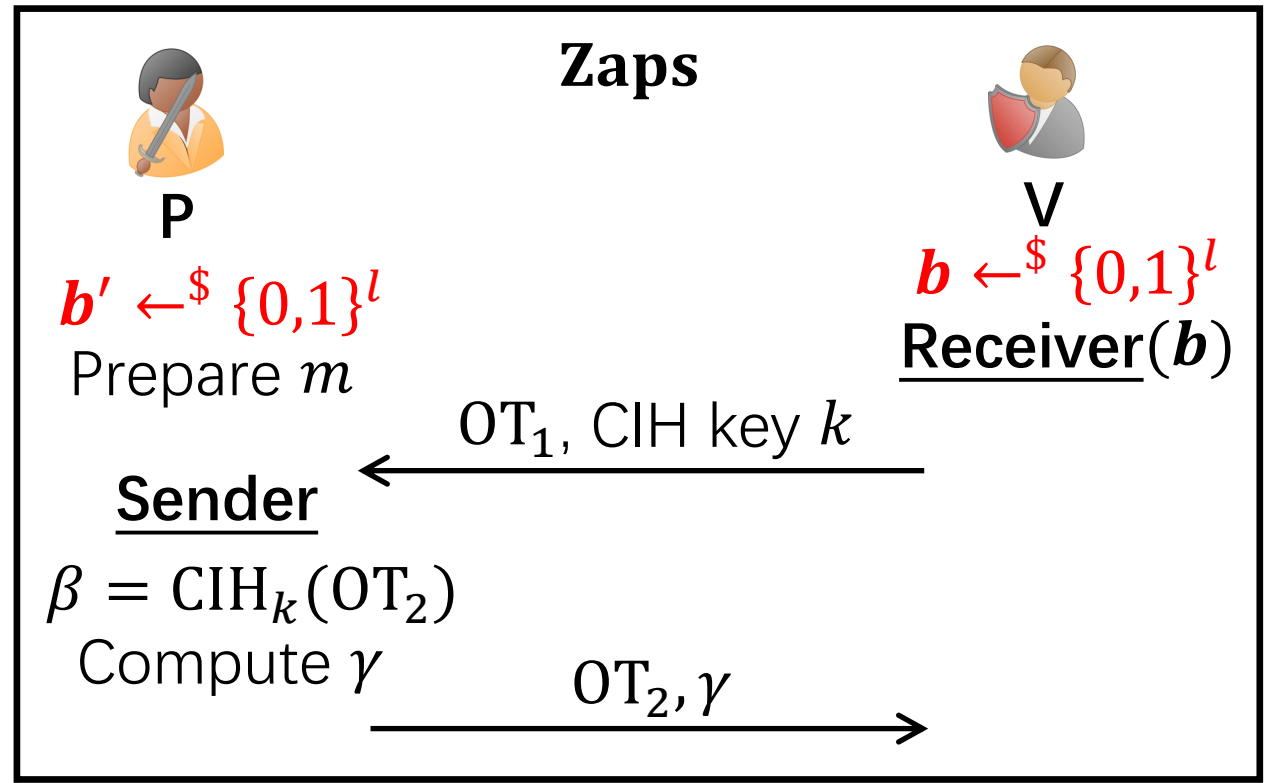
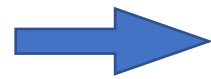
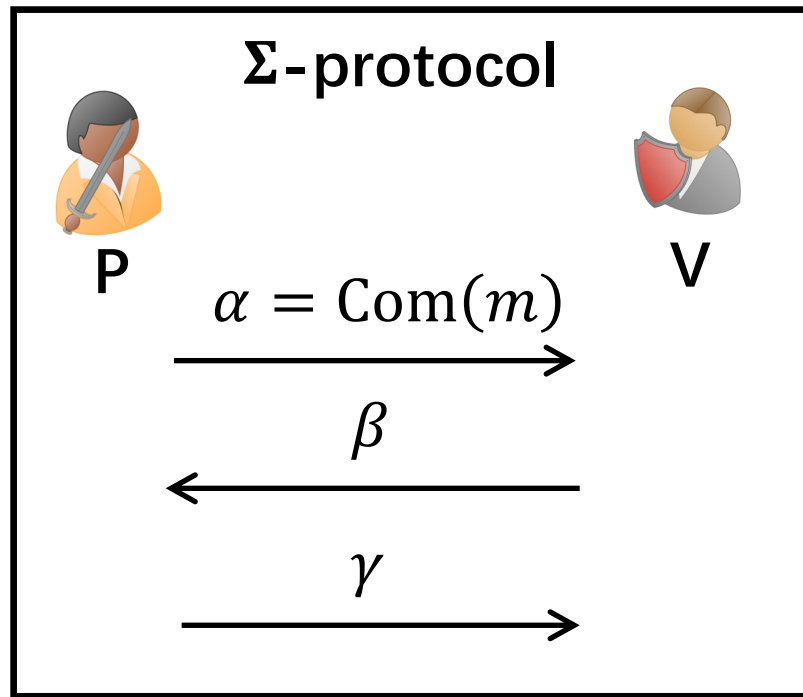
$\mathbf{b} = \mathbf{b}'$, extract m ✓

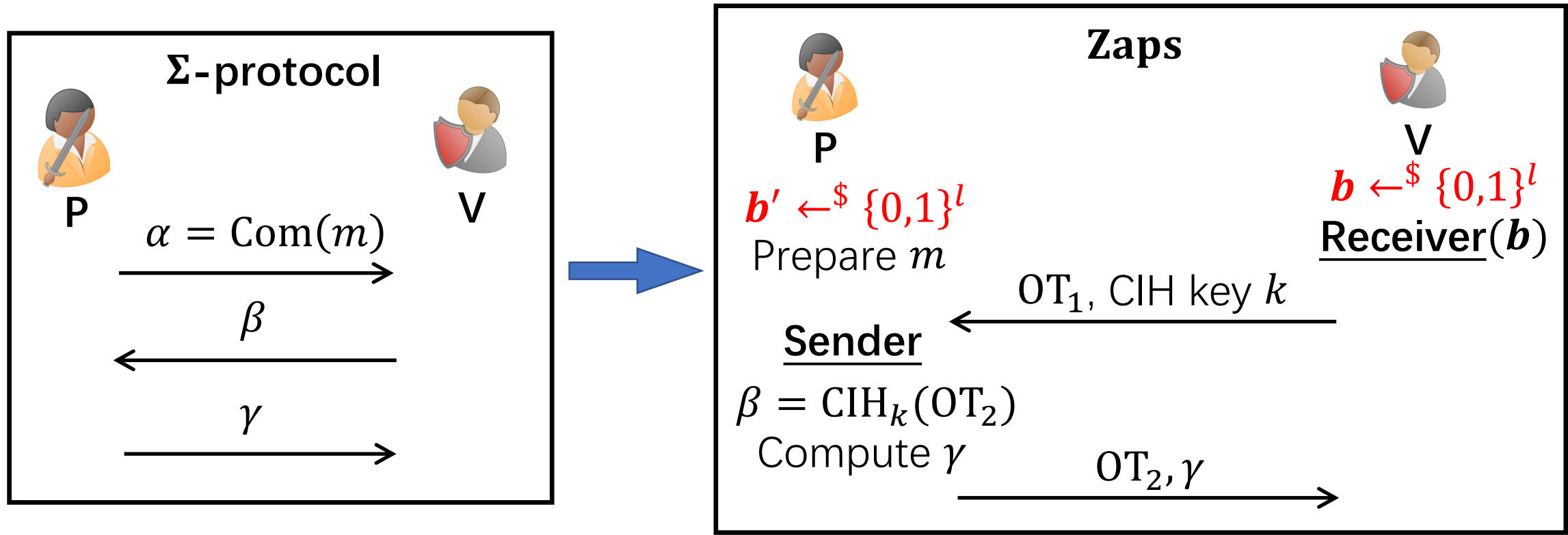


Amplify the Security

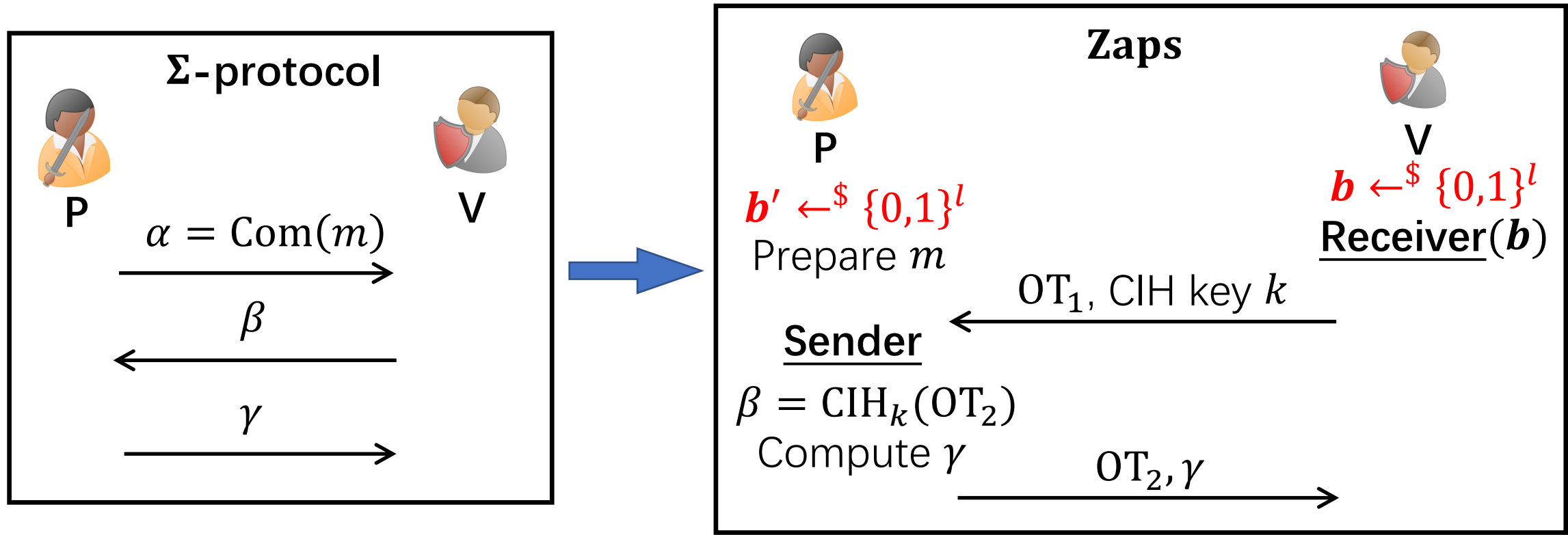


- Can be abstracted as a 2-round statistical hiding extractable commitment [KKS18]

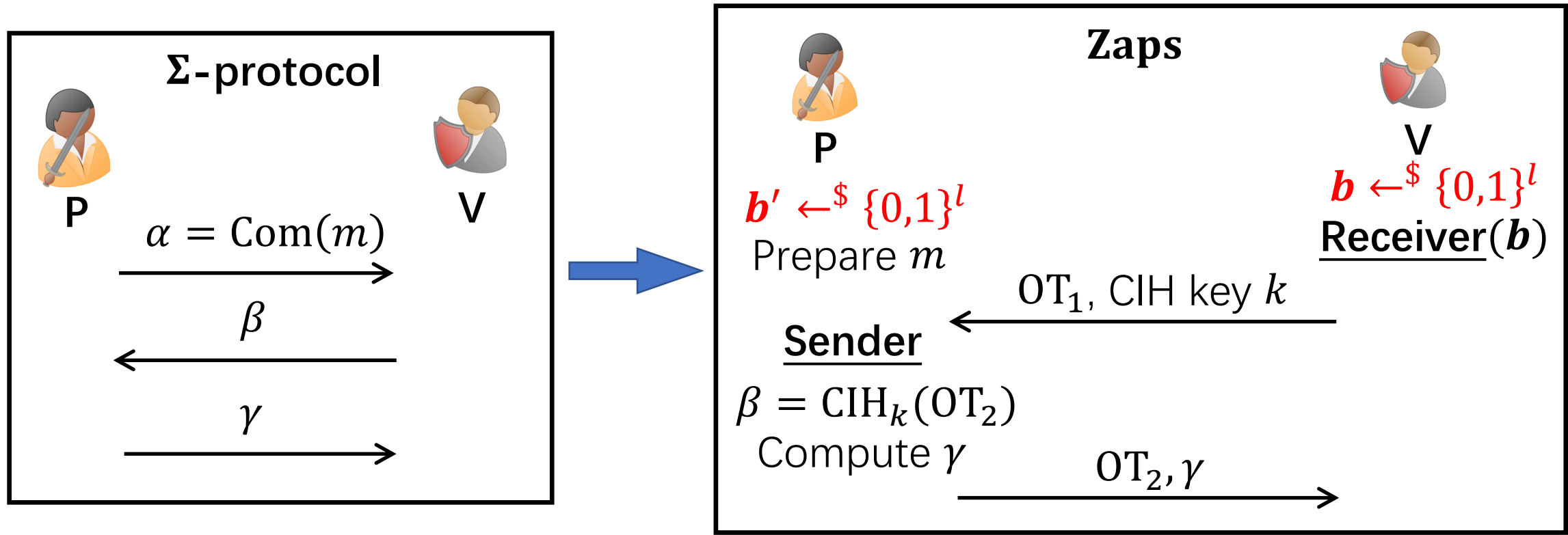




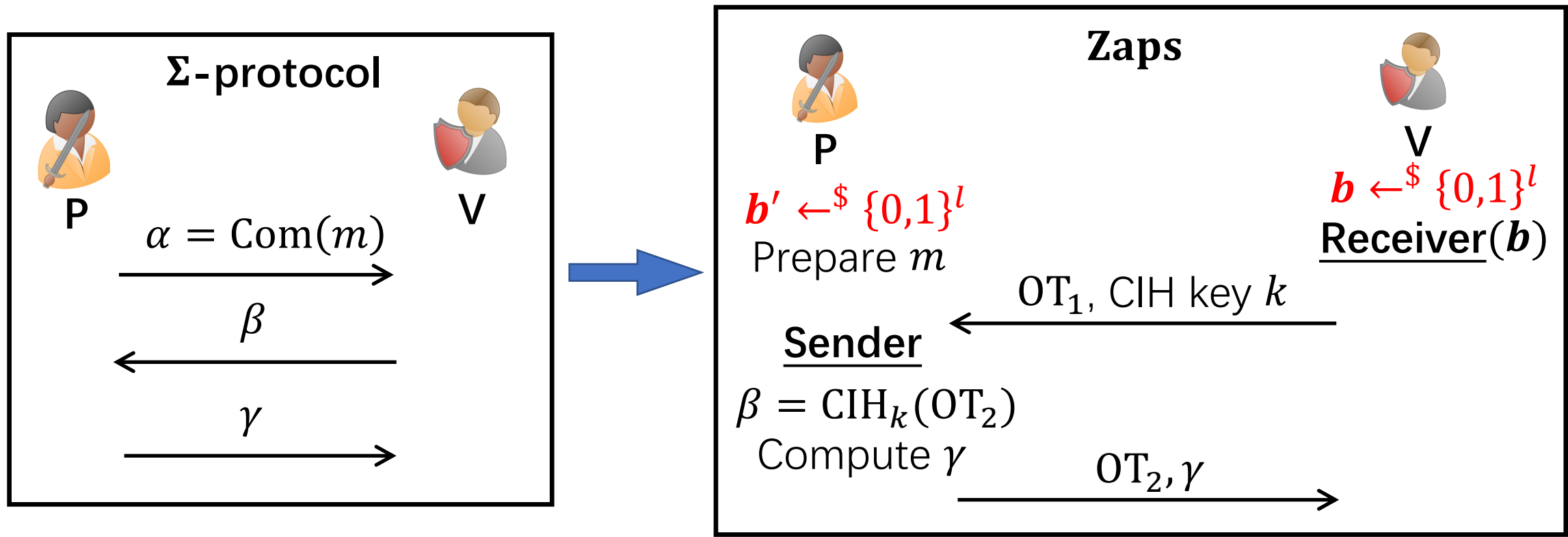
- Statistical WI with $\text{err} \approx 1/2^l$ (negligible)
- Computational Soundness via Complexity Leveraging
- Public Coin Property : OT_1 is pseudorandom



- Statistical WI with $\text{err} \approx 1/2^l$ (negligible)
- Computational Soundness via Complexity Leveraging
- Public Coin Property : OT_1 is pseudorandom



- Statistical WI with $\text{err} \approx 1/2^l$ (negligible)
- Computational Soundness via Complexity Leveraging
- Public Coin Property : OT_1 is pseudorandom



- Statistical WI with $\text{err} \approx 1/2^l$ (negligible)
- Computational Soundness via Complexity Leveraging
- Public Coin Property : OT_1 is pseudorandom

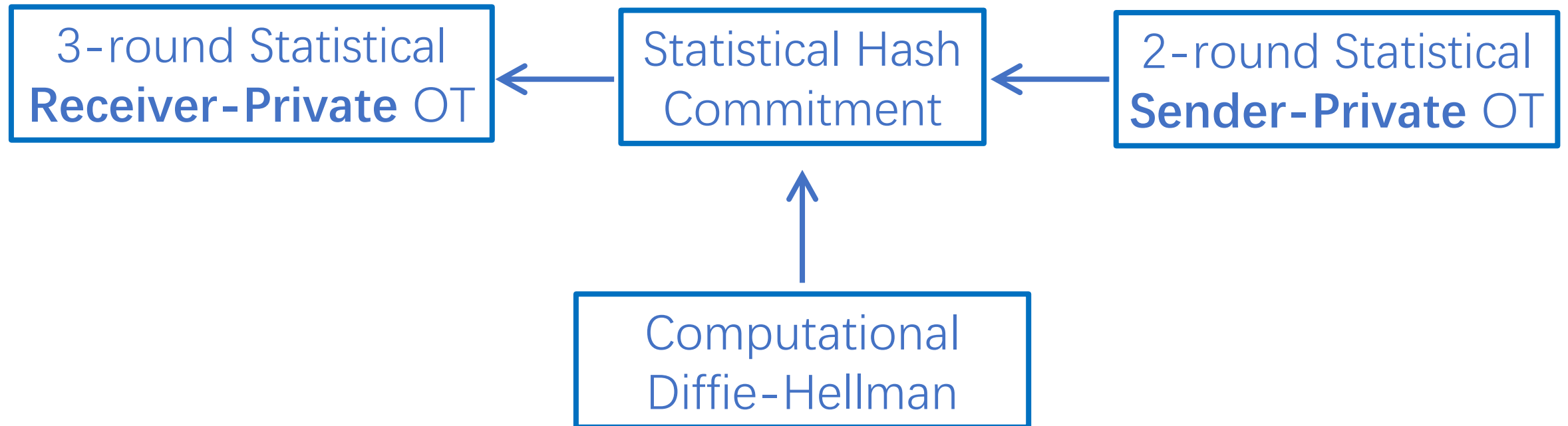
Statistical Zaps

Technical Details

Part II: Oblivious Transfer (OT)

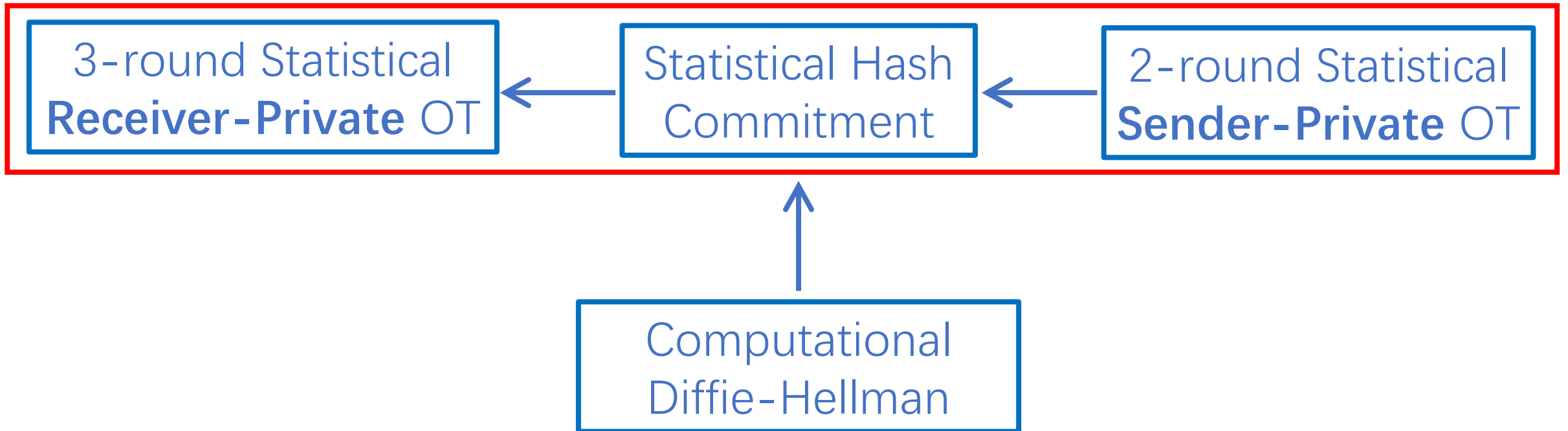
Technical Details

Part II: Oblivious Transfer (OT)

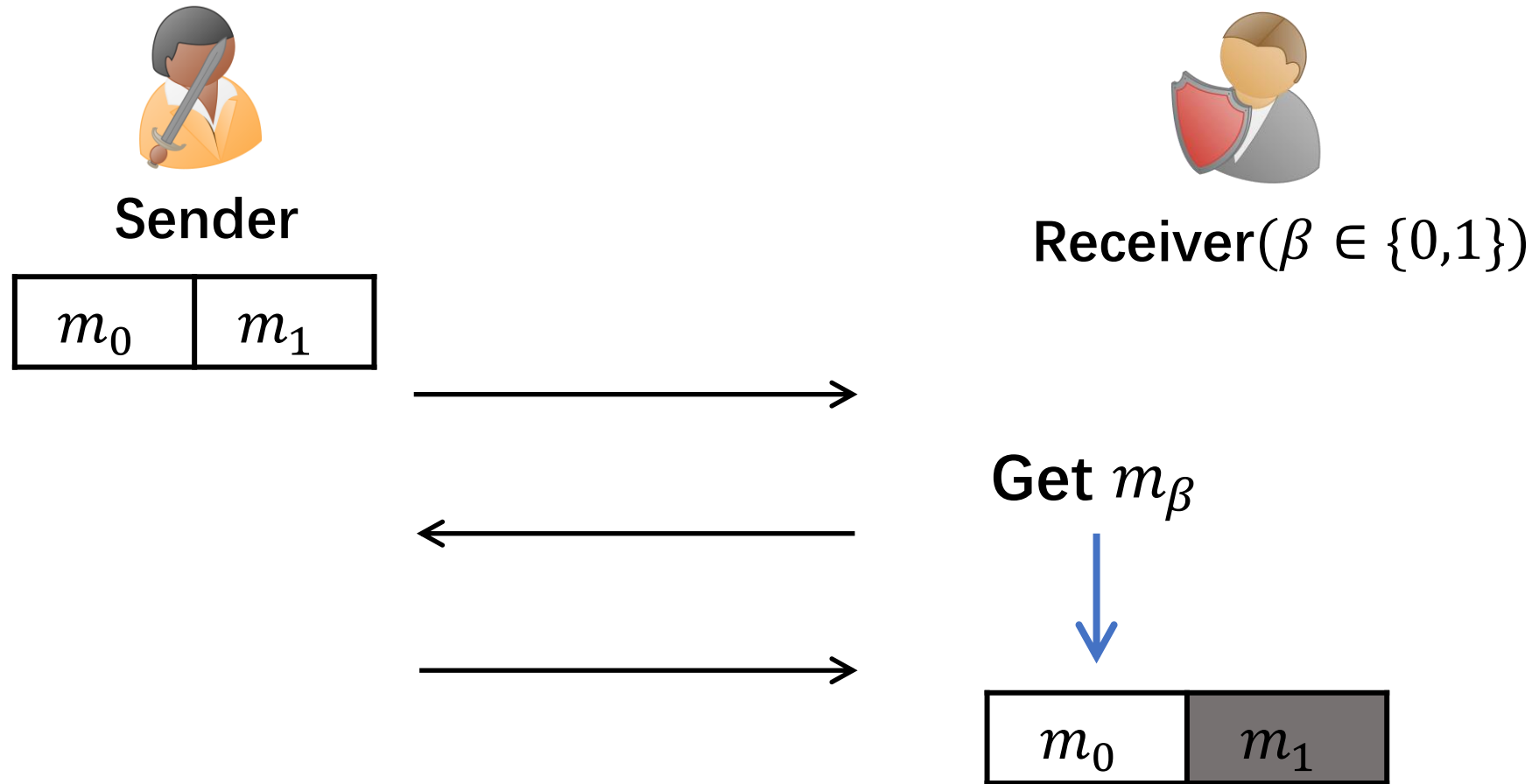


Technical Details

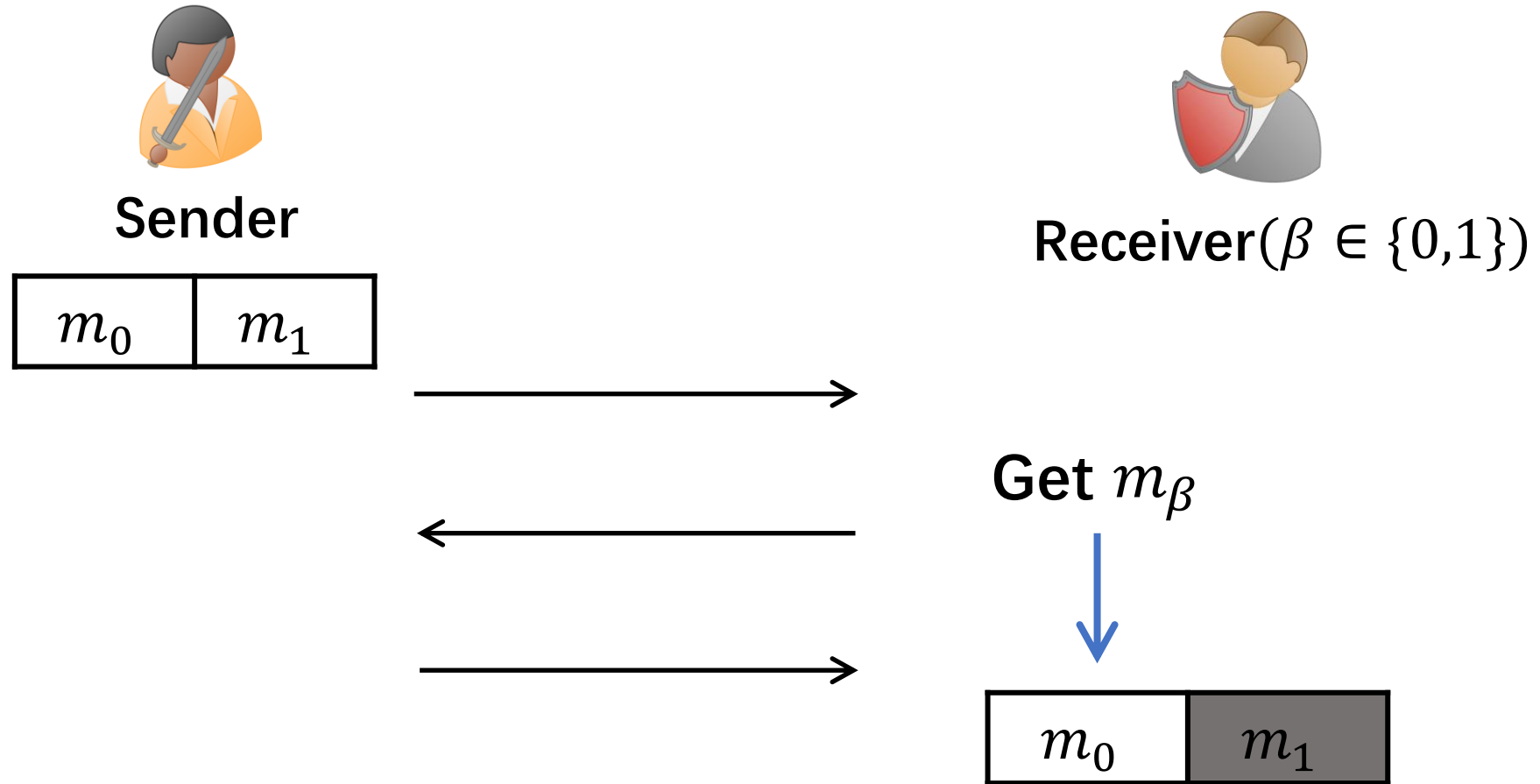
Part II: Oblivious Transfer (OT)



Statistical Receiver-Private OT



Statistical Receiver-Private OT



Statistical Receiver-Privacy: β is statistical hidden

Main Tool: **S**tatistical **H**ash **C**ommitments (SHC)

Main Tool: Statistical Hash Commitments (SHC)



Receiver



Committer($\beta \in \{0,1\}$)

Main Tool: Statistical Hash Commitments (SHC)



Receiver

Committing Phase:



Committer($\beta \in \{0,1\}$)

Main Tool: Statistical Hash Commitments (SHC)



Receiver

Committing Phase:



Committer($\beta \in \{0,1\}$)



Main Tool: Statistical Hash Commitments (SHC)



Receiver

Committing Phase:



Committer($\beta \in \{0,1\}$)

Opening Phase:

Main Tool: Statistical Hash Commitments (SHC)



Receiver

Committing Phase:



Committer($\beta \in \{0,1\}$)

Opening Phase:

Hash value for $\beta = 0$: 

Hash value for $\beta = 1$: 

Main Tool: Statistical Hash Commitments (SHC)



Receiver

Committing Phase:



Committer ($\beta \in \{0,1\}$)



Opening Phase:

Hash value for $\beta = 0$: 

Hash value for $\beta = 1$: 

β , 



Main Tool: Statistical Hash Commitments (SHC)



Receiver

Committing Phase:



Committer ($\beta \in \{0,1\}$)

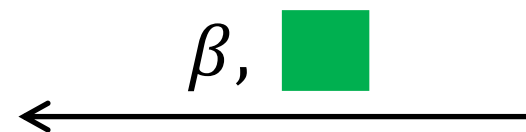


Opening Phase:

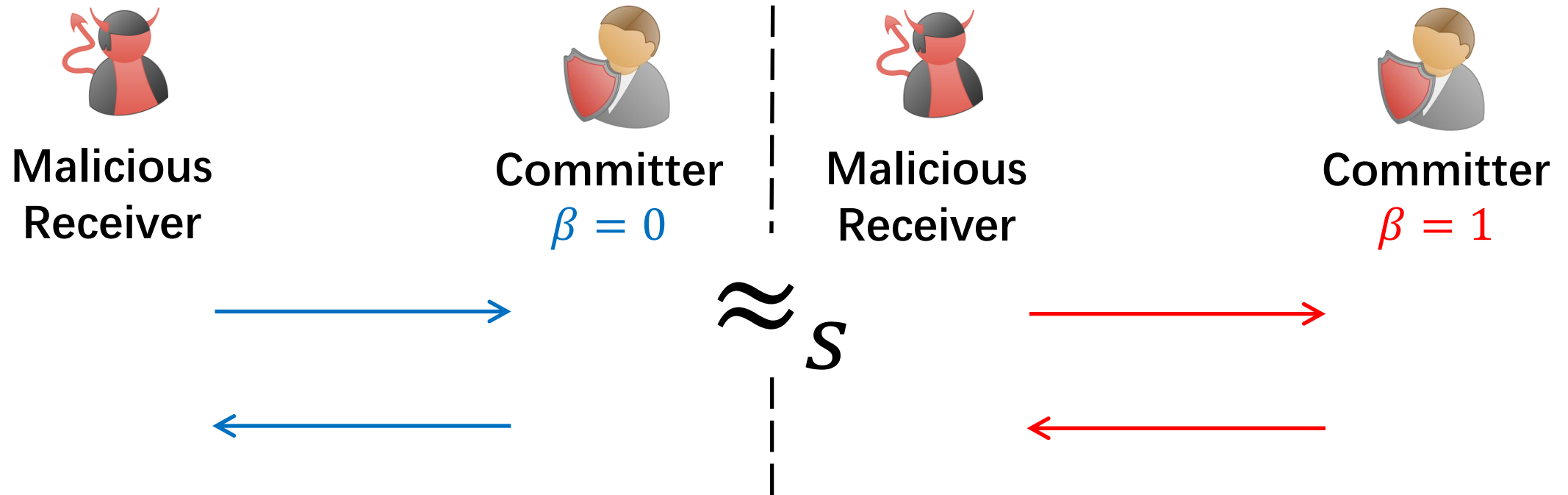
Hash value for $\beta = 0$: 

Hash value for $\beta = 1$: 

Check  =? 



Statistical Hash Commitments (SHC): Statistical Hiding Property



Statistical Hash Commitments (SHC): Computational Binding



Receiver
Committing Phase:



Malicious
Committer



Hash value for $\beta = 0$: 

Hash value for $\beta = 1$: 

Statistical Hash Commitments (SHC): Computational Binding



Receiver
Committing Phase:



Malicious
Committer



Hash value for $\beta = 0$: 

Hash value for $\beta = 1$: 

Computational Binding:

it's hard for committer to find both  

3-round Statistical Receiver-Private OT from SHC

3-round Statistical Receiver-Private OT from SHC

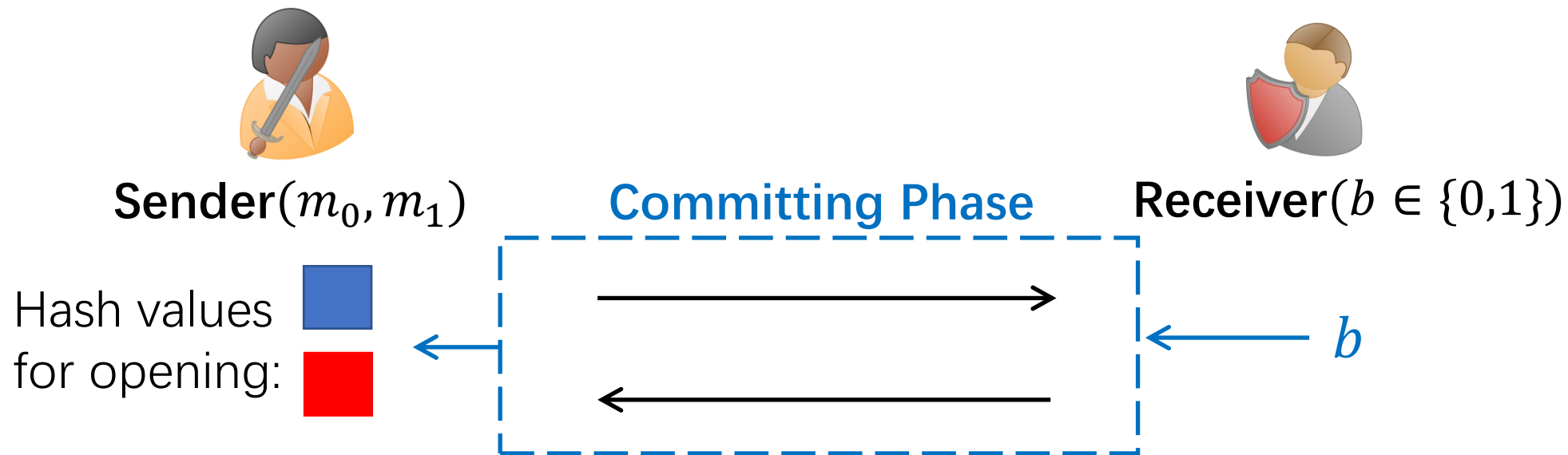


Sender(m_0, m_1)



Receiver($b \in \{0,1\}$)

3-round Statistical Receiver-Private OT from SHC



3-round Statistical Receiver-Private OT from SHC



Sender(m_0, m_1)

Hash values
for opening:



$hc(\cdot)$: Goldreich-Levin
hardcore predicate

Committing Phase

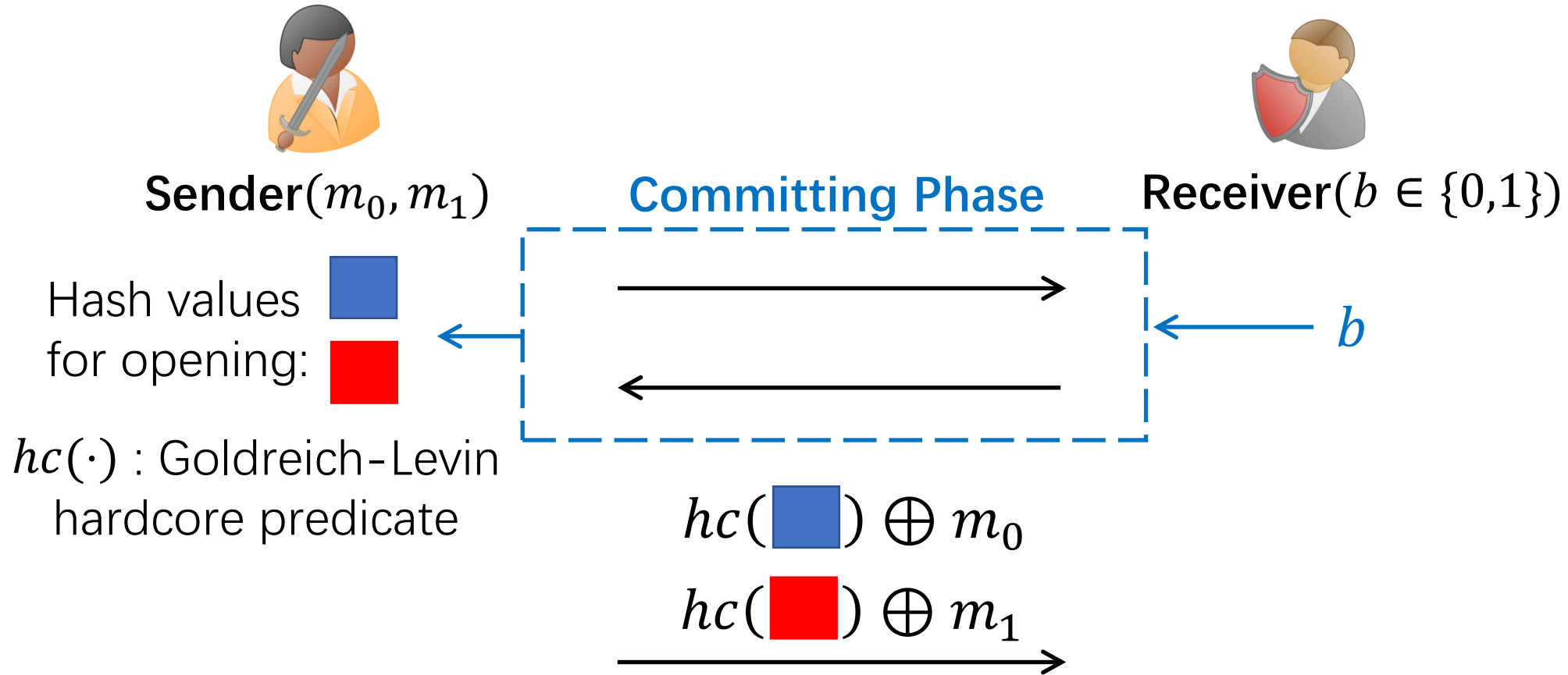


Receiver($b \in \{0,1\}$)

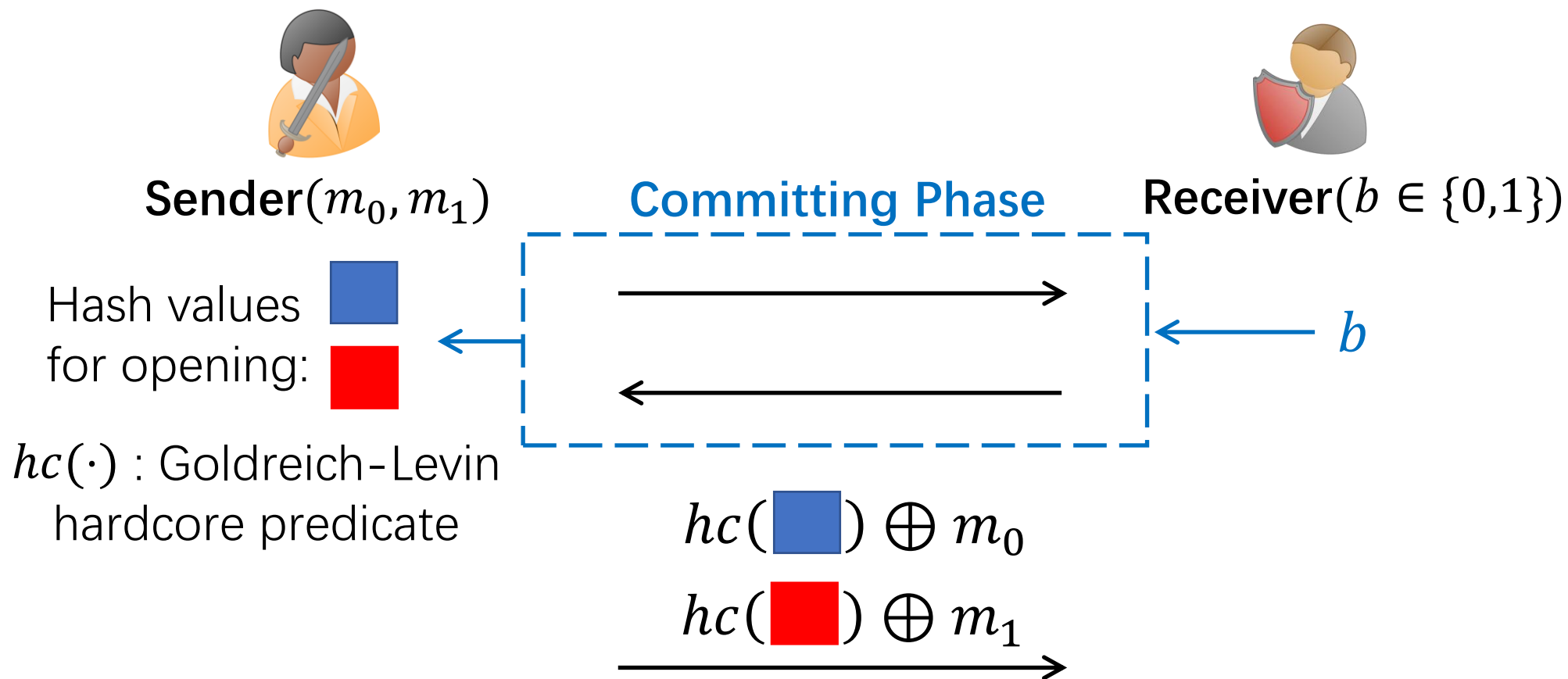
b



3-round Statistical Receiver-Private OT from SHC



3-round Statistical Receiver-Private OT from SHC



- Statistical Hiding \Rightarrow Statistical Receiver-Private
- Computational Binding \Rightarrow Computational Sender-Private

Statistical Hash Commitment from 2-round OT

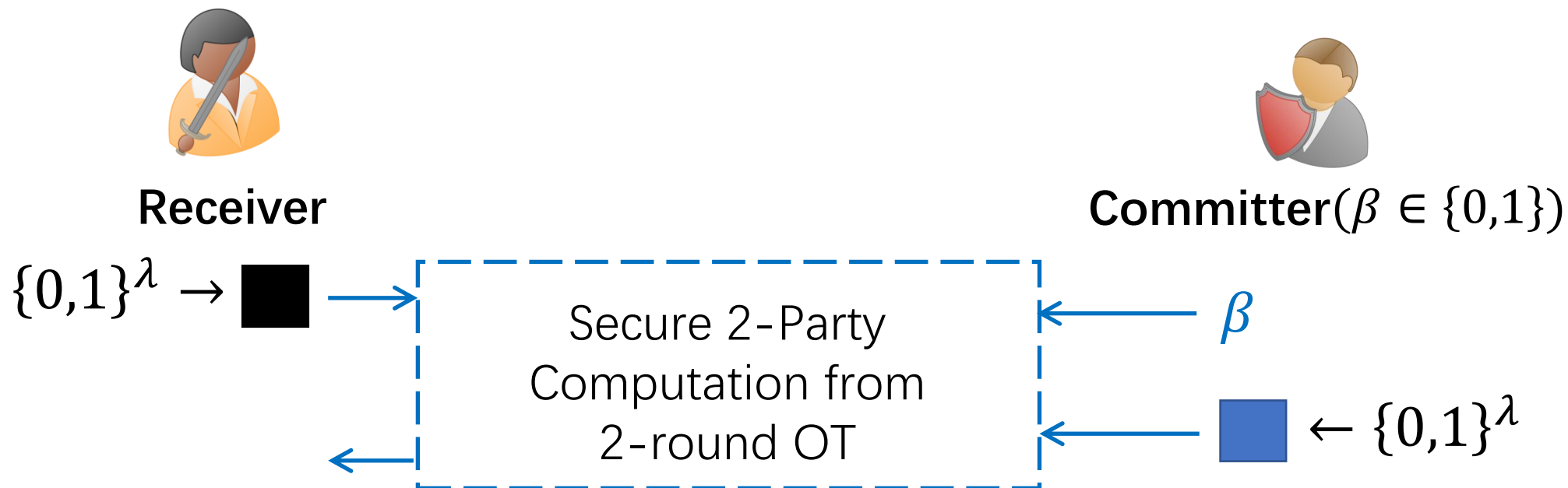


Receiver

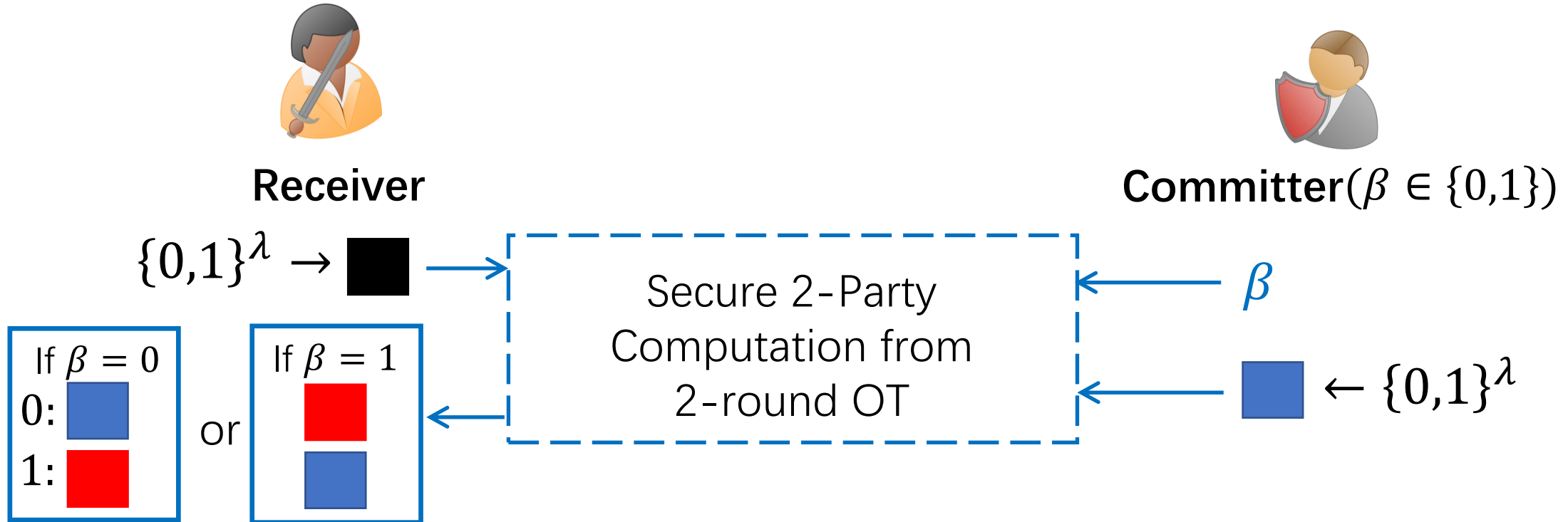


Committer($\beta \in \{0,1\}$)

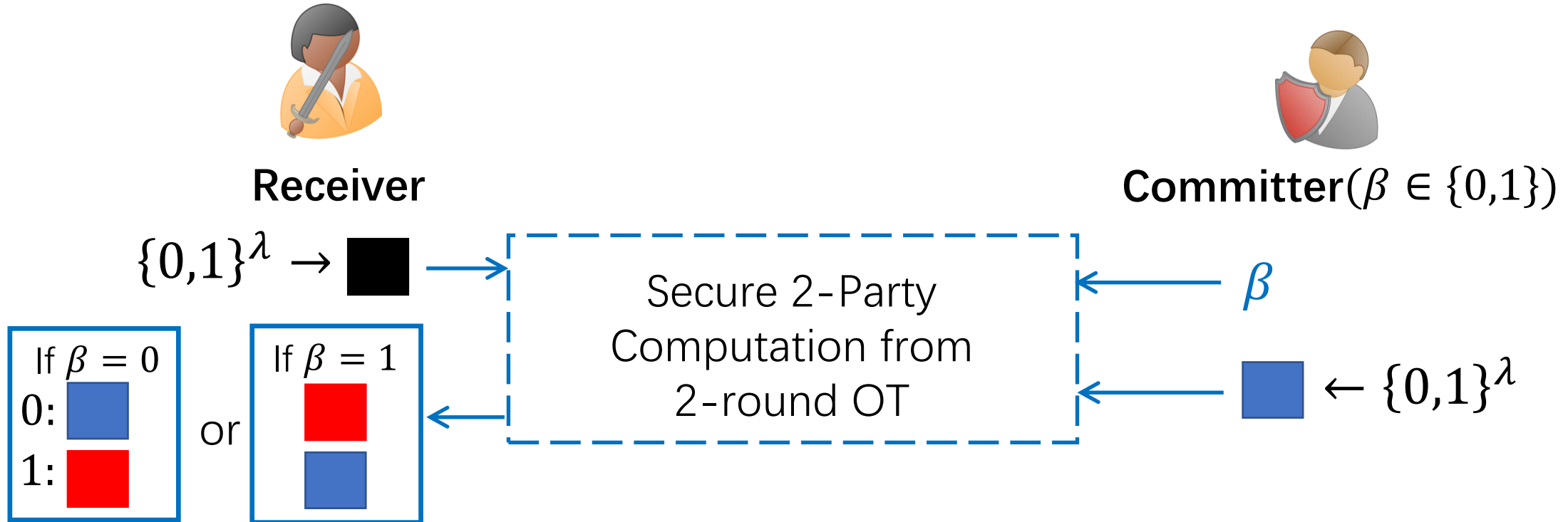
Statistical Hash Commitment from 2-round OT



Statistical Hash Commitment from 2-round OT



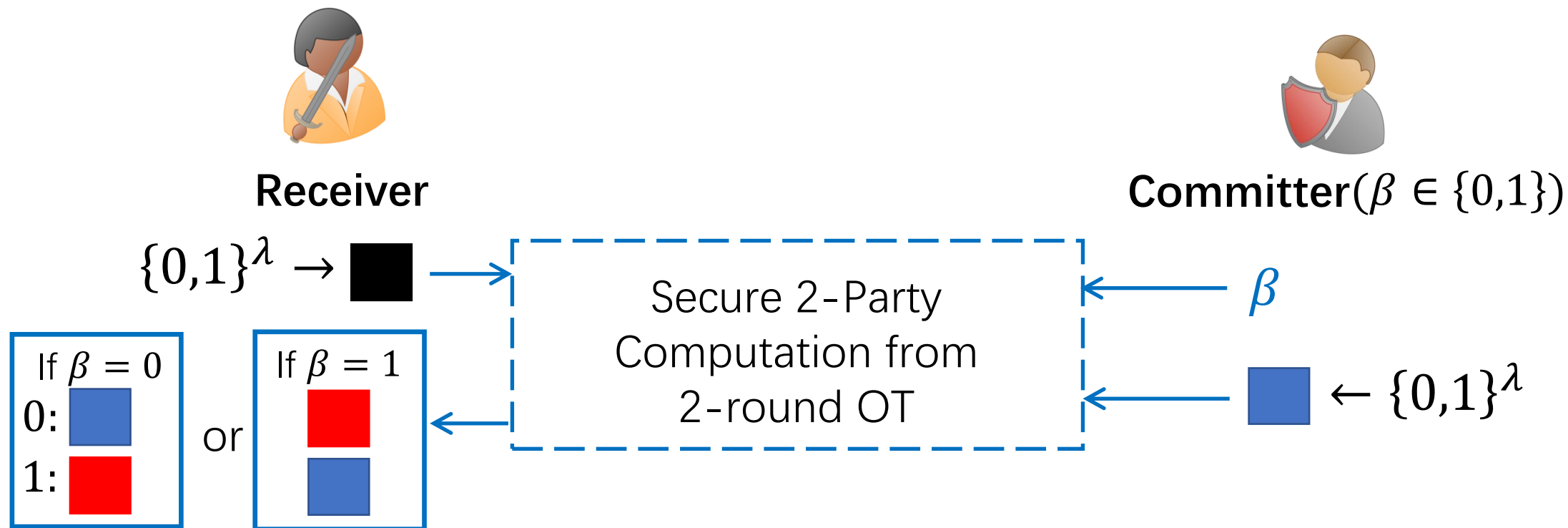
Statistical Hash Commitment from 2-round OT



Where

$$\text{red} = \text{blue} \oplus \text{black}$$

Statistical Hash Commitment from 2-round OT



Where

$$\blacksquare = \blacksquare \oplus \blacksquare$$

- Statistical Sender-Privacy of OT \Rightarrow Statistical Hiding
- Computational Hiding of $\blacksquare \Rightarrow$ Computational Binding

Summary of Results

- Statistical Zaps from quasi-poly hardness Learning with Errors
- 3-round statistical receiver-private oblivious transfer from poly hardness
 - 2-round statistical sender-private oblivious transfer
 - Computational Diffie-Hellman Assumption

Full version : ia.cr/2020/235

Thank you!