The Usefulness of Sparsifiable Inputs: How to Avoid Subexponential iO

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Introduction

Indistinguishability obfuscation (IO) is a method to transform a program into an *unintelligible* one maintaining the original functionality.

$$
\begin{tikzcd}
\boxed{P_1} \arrow[d, "\mathtt{iO}"'] & \equiv & \boxed{P_2} \arrow[d, "\mathtt{iO}"] \\
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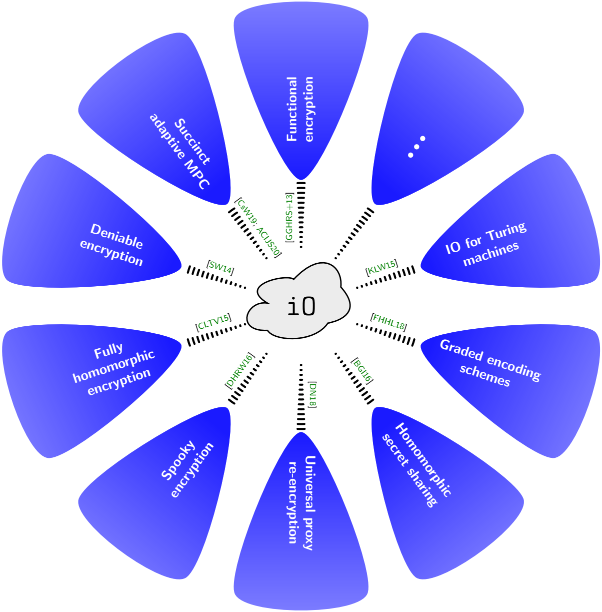
$$

Applications of iO

- We can build almost anything from iO.



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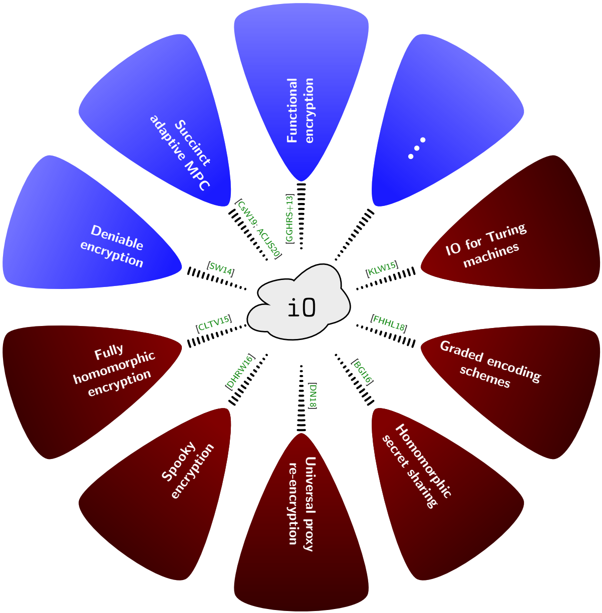

poly reduction to iO	
subexp reduction to iO	

Related work on removing subexp. IO

- Previous approaches to avoid subexponential reductions to iO: replace iO with functional encryption, [GS16; GPSZ17; LZ17; KLMR18]
  - short signatures
  - universal samplers
  - non-interactive multiparty key exchange
  - trapdoor one-way permutations
  - multi-key functional encryption
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- **But** the supported operations are relatively restricted

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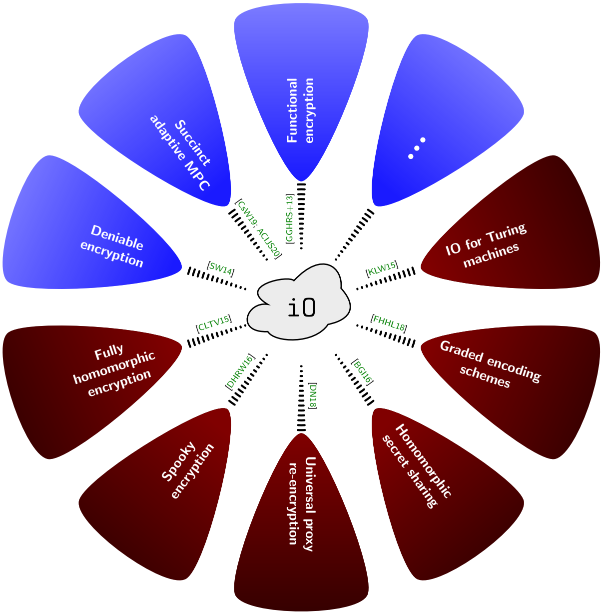

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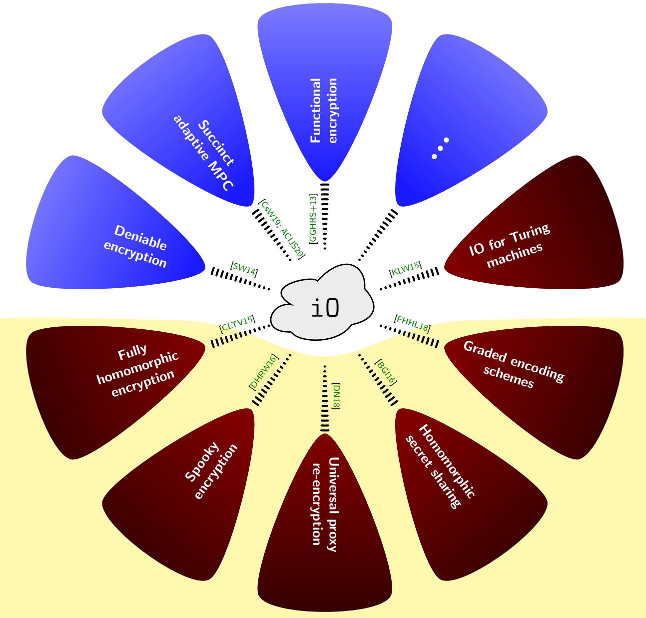
piO abstraction	



Probabilistic IO

$\mathtt{iO}$ compiles programs into
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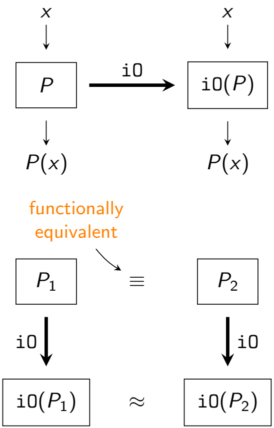

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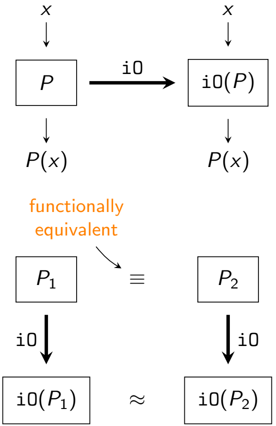

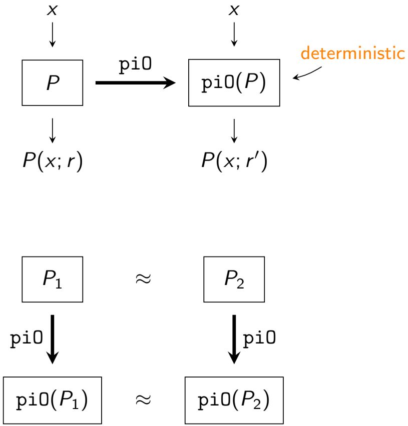

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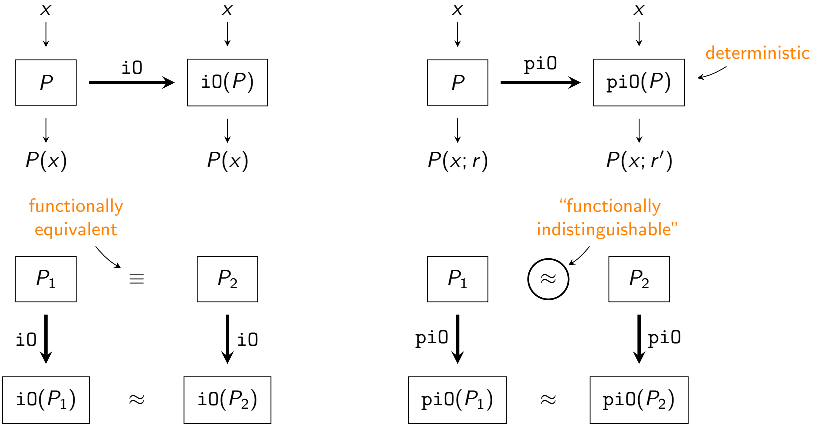

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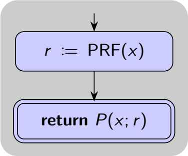

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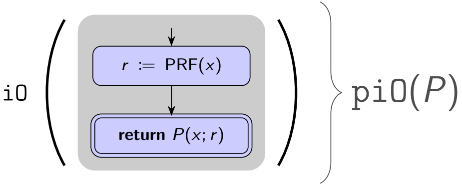

  - use iO to obfuscate this deterministic program

Construction of piO due to Canetti et al., [CLTV15]

piO construction:

$r := \mathrm{PRF}(x)$

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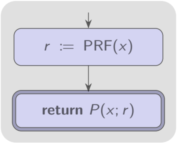

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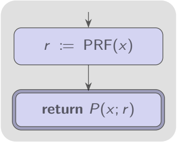

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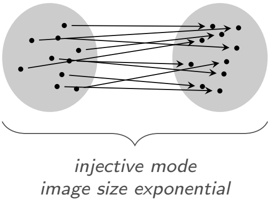
$\rightsquigarrow$ **Our goal:** reduce number of hybrids to a polynomial amount

Main tool – Extremely lossy functions

- Extremely lossy functions (ELFs) due to Zhandry, [Zha16] offer two indistinguishable modes:



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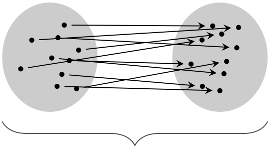
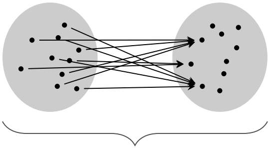


injective mode
image size exponential



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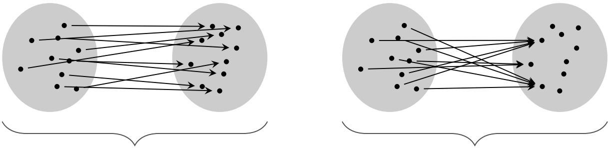

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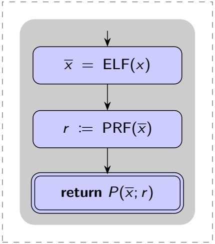
- We believe that some sort of (sub)exponential assumption is inherent for probabilistic `iO`
  - ELFs can be used to **push** this subexponentiality to a much more well-understood assumption

First try



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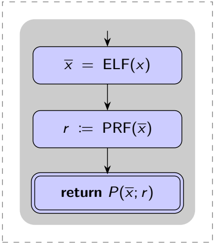

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- **First try:** reduce number of hybrids by applying the ELF on the input x
- **But** pre-processing the program input x with an ELF will not preserve the expected functionality of the circuit

Our observation

- Common ground for many applications of `piO`:

$$\begin{array}{c} \boxed{\mathcal{D}(m)} \\ \Big\downarrow x \\ \boxed{\texttt{piO}(P)} \end{array}$$

inputs x come from distributions $\mathcal{D}(m)$

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- **Approach:** reduce number of hybrids by applying the ELF *on the random tape of $\mathcal{D}$* to **sparsify** inputs

Doubly-Probabilistic IO

- Our framework: **doubly-probabilistic IO (dpiO)**

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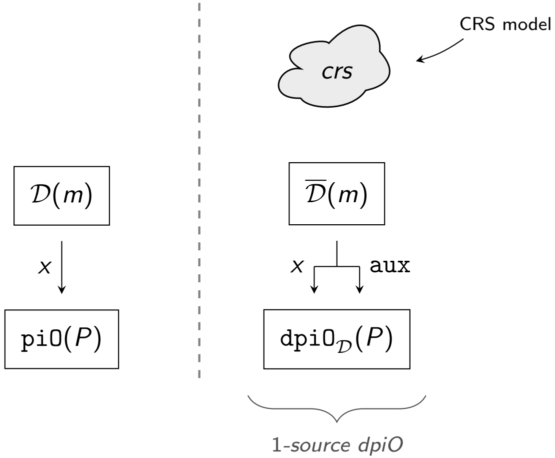

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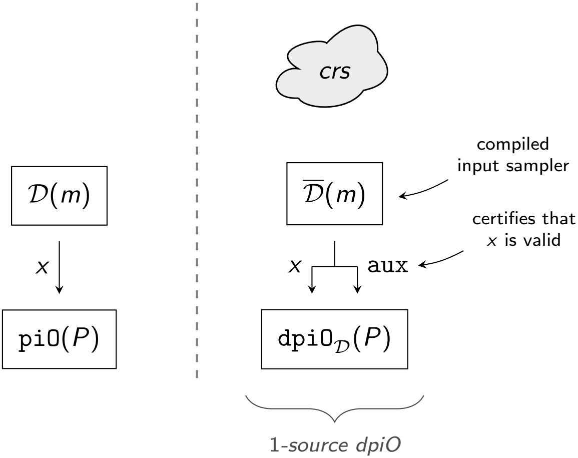

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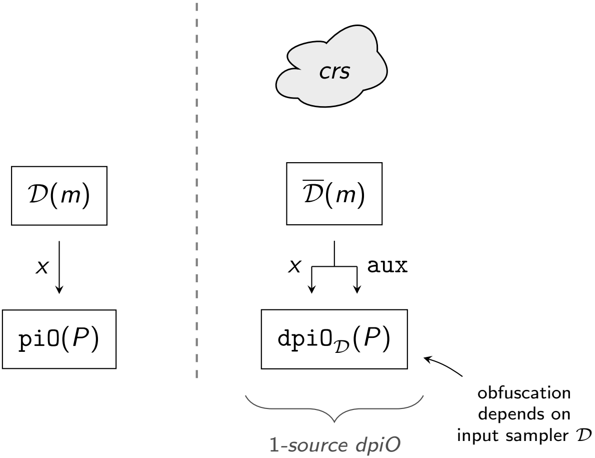

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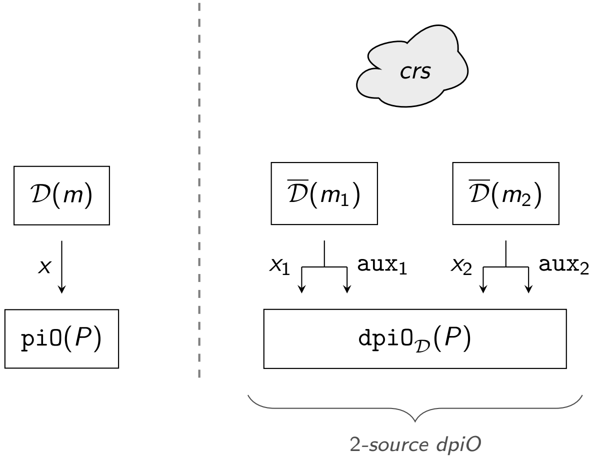

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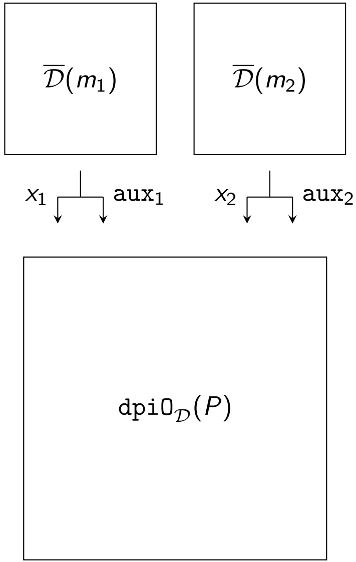

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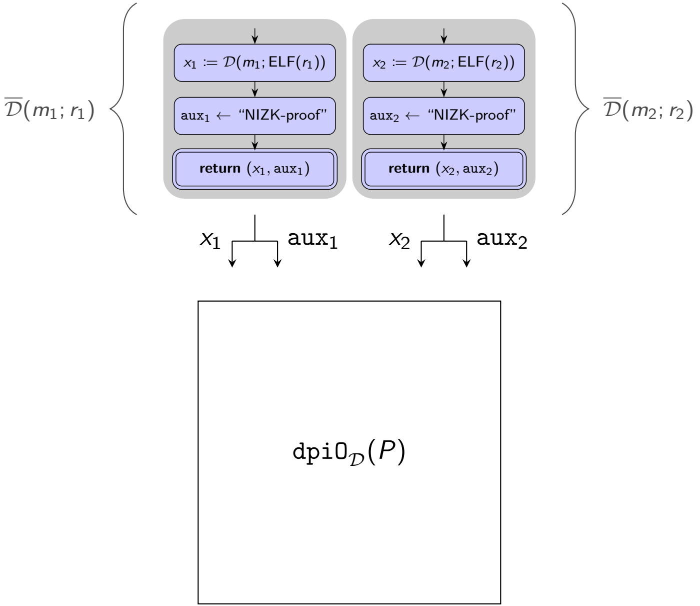

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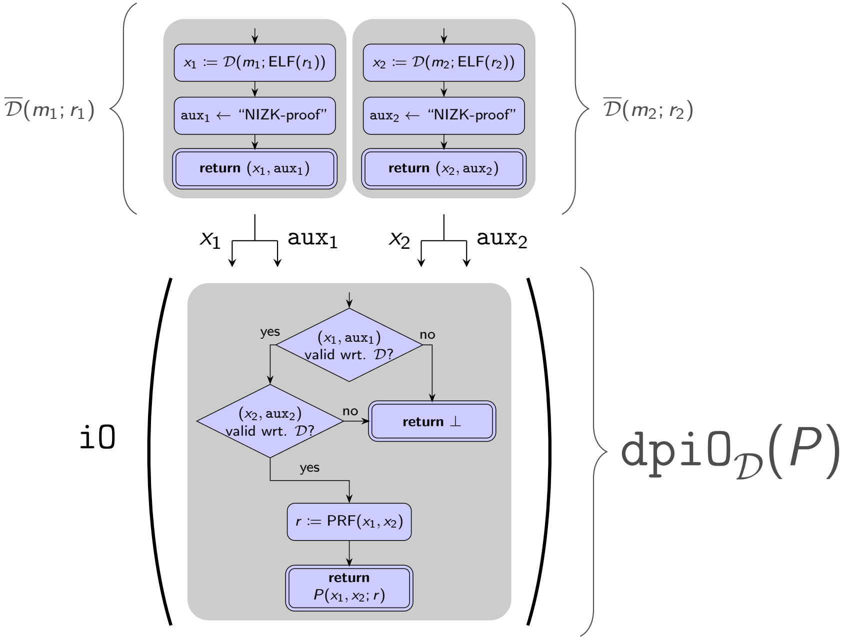

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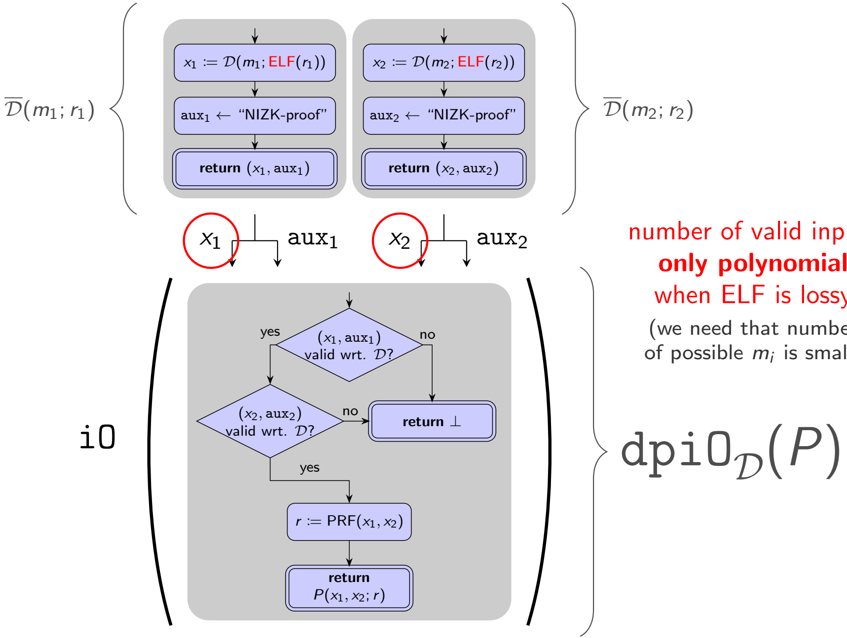

number of valid inputs
only polynomial
when ELF is lossy
(we need that number of possible m_i is small)

Leveled Homomorphic Encryption

- $\mathsf{LHE}_L = (\textsc{Gen}, \textsc{Enc}, \textsc{Dec}, \textsc{Eval})$ is a LHE scheme for depth-L circuits C, if
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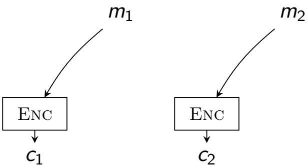

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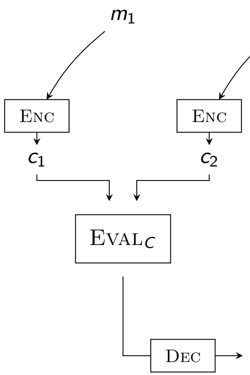

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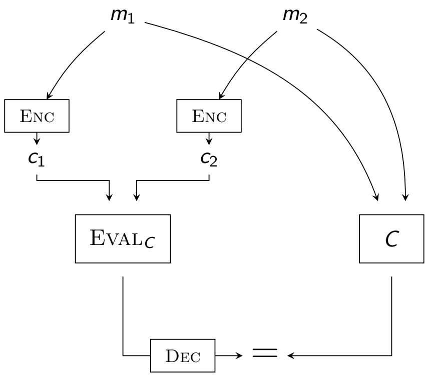

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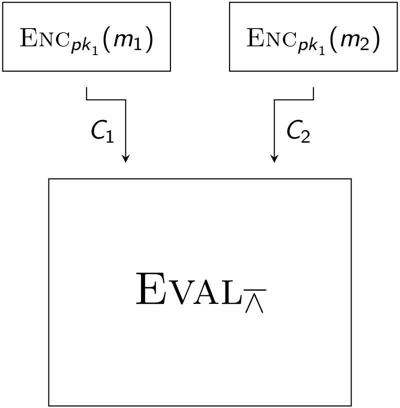

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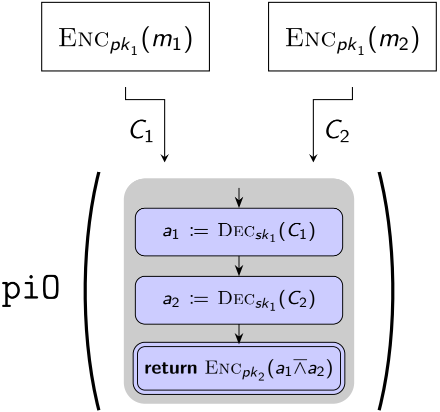

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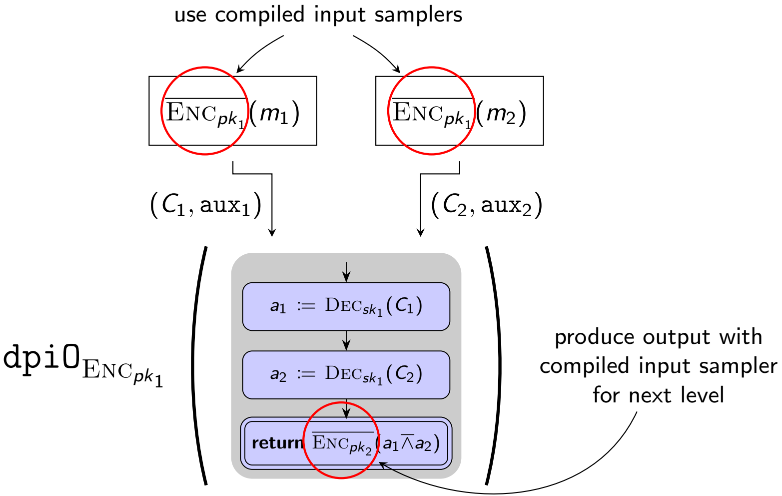

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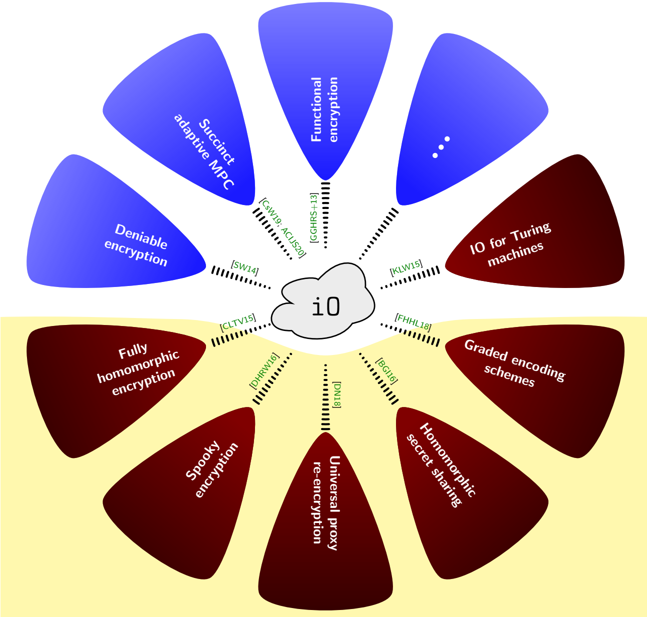

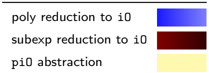

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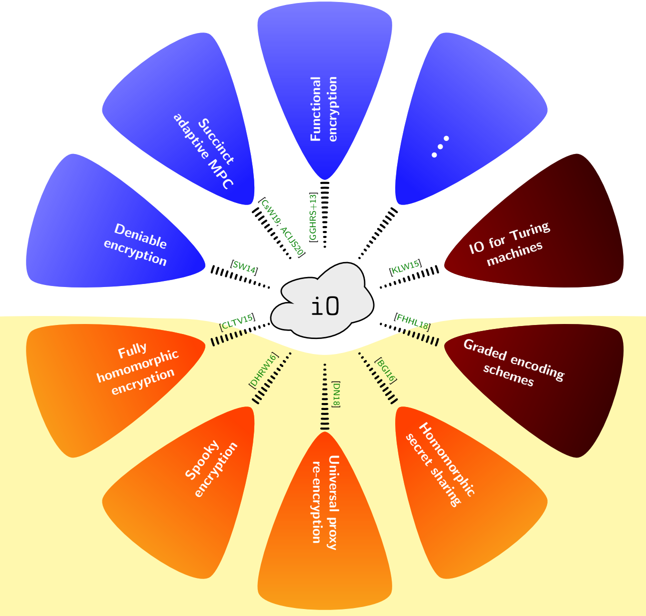

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