Threshold Ring Signatures: New Security Definitions and Post-Quantum Security

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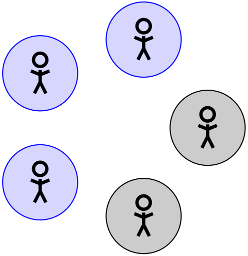
Problem Description

Threshold Ring Signature

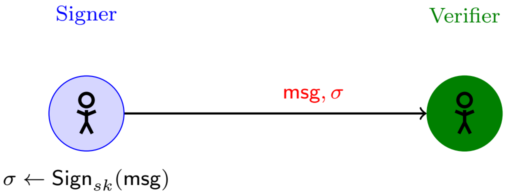
Main Definitions

Threshold ring signatures: t distinct parties anonymously sign on behalf of a ring of N public keys. The identity of the signers remains private (to any non-signers).



Threshold Ring Signature

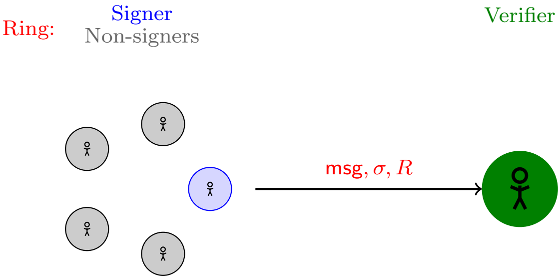
Signature



- unforgeability

Threshold Ring Signature

Ring Signature

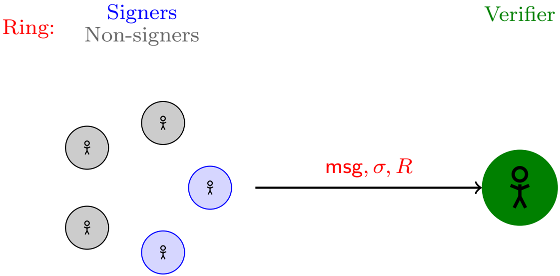


$\sigma \leftarrow \mathsf{Sign}_{sk}(\mathsf{msg}; R)$

- unforgeability
- anonymity

Threshold Ring Signature

Ring Signature

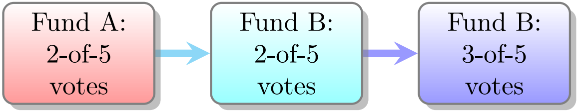


$$\sigma \leftarrow \mathsf{Sign}_{sk^i, sk^j}(\mathsf{msg}; R)$$

- unforgeability
- anonymity
- threshold

Motivation

- Increased tolerance to misbehavior of users
- Suits decentralized settings
- Settings where you need a quorum.



- an ad-hoc "voting" mechanism for community projects posted on the blockchain
- Funds: $$$

Current State of the Art

State of the Art

- Passive Security Definitions

- Post-Quantum Insecure
  1. Hardness Assumptions
  2. Techniques

Threshold Ring Signature Setting

- Ad-hoc settings where the users can generate their keys independently, and join or leave the system at any time.
- Users could join the system with dishonestly generated keys.

Passive Adversaries

- Only passive adversaries.
- Adversaries can only obtain honestly generated keys.
- Sometimes cannot even choose to add more (honest) keys (e.g., Bettaieb and Schrek (2013); Petzoldt et al. (2013)),
- Adversaries cannot corrupt parties (e.g. Okamoto et al. (2018); Petzoldt et al. (2013); Bettaieb and Schrek (2013)).
- Bender et al. (2006) observe that the above doesn't reflect the open settings of ring signatures.

State of the Art

- Passive Security Definitions
  1. passive adversaries
  2. no corruption
  3. no adding of new honest keys

- Post-Quantum Insecure
  1. Hardness Assumptions
  2. Techniques

Post-Quantum Hardness Assumptions

- Discrete log, factoring hardness assumptions are not secure against an attack from a quantum computer (Shor (1994)).
- Some constructions Melchor et al. (2011); Bettaieb and Schrek (2013); Cayrel et al. (2010); Petzoldt et al. (2013) use post-quantum secure hardness problems such as lattices or learning-with-errors.

State of the Art

- Passive Security Definitions
  1. passive adversaries
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- Post-Quantum Insecure
  1. Non-PQ secure problems
  2. Techniques

Proof Techniques in Post-Quantum Setting

- Transform from Fiat and Shamir (1986) common, but security may not hold in the *quantum* setting (Boneh et al. (2011); Ambainis et al. (2014)).
- Quantum rewinding is not trivial (Watrous (2009); Ambainis et al. (2014)).
- Fiat-Shamir is post-quantum secure in certain situations (Liu and Zhandry (2019); Don et al. (2019)) but may not hold in general.

Transformation

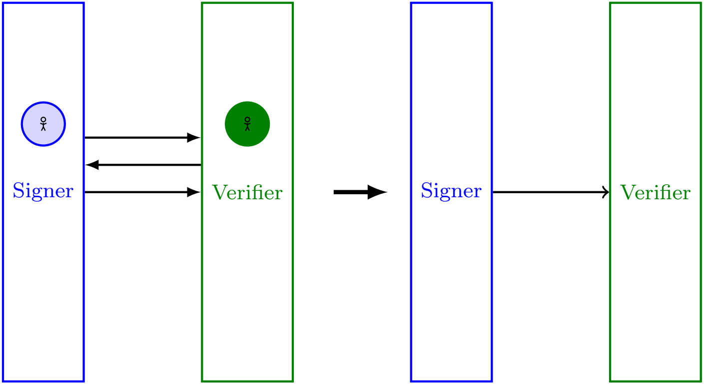


Figure: Transform an interactive protocol into a non-interactive one.

Rewinding

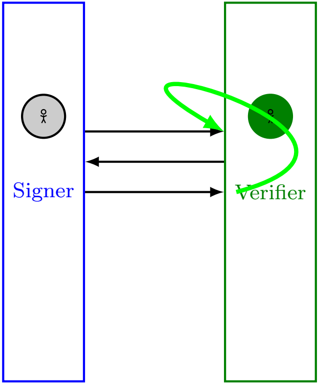
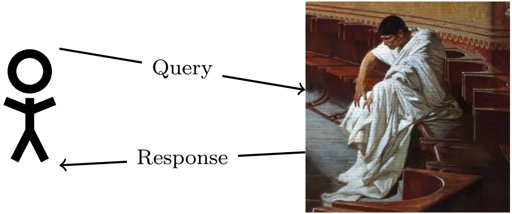


Figure: Prove scheme with rewinding. But a quantum adversary may notice!

Quantum vs Classical Access

Classical

On a single query, can only get a single response.



Quantum vs Classical Access

Quantum

Can get a *superposition* of answers.

Can define all possible outputs using only a single query. This is why we use Unruh.

State of the Art

- Passive Security Definitions
  1. passive adversaries
  2. no corruption
  3. no adding of new honest keys

- Post-Quantum Insecure
  1. Non-PQ secure problems
  2. Fiat-Shamir is not PQ-secure in general.

Our Contribution

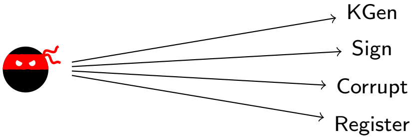
Our Contribution

1. Definitions for unforgeability and anonymity with active adversaries.
2. Post-quantum secure proof for a threshold ring signature.
   1. generalize previous approaches and provide a black-box construction from any (post-quantum) trapdoor commitment scheme.
   2. Uses Unruh Transformation to guarantee post-quantum security.

Definitions

- Make a security model by giving adversary access to oracles.
- Captures active adversaries.
- Two security notions: unforgeability and anonymity.

Anonymity and Unforgeability



Training: ask queries

Anonymity: $\mathcal{A}$ picks:

- message
- S_0, S_1 with respect to a ring R, where $|S_0| = |S_1| = t$.

$\mathcal{A}$ receives a signature from S_b ($b = 0$ or 1) and guesses b.

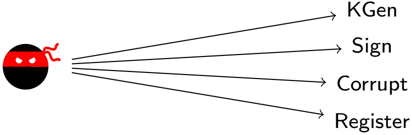
S_0, S_1 uncorrupted.

Unforgeability: $\mathcal{A}$ produces

- message
- signature
- ring

Fewer than t corrupted members in R^*.

Anonymity and Unforgeability



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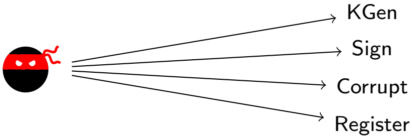

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Fewer than t corrupted members in R^*.

Oracles

- *Key Generation:* Upon query from $\mathcal{A}$, the oracle creates private-public key pair and gives the public key to $\mathcal{A}$.
- *Sign:* $\mathcal{A}$ requests a signature on message and signers w.r.t. a ring. The oracle follows the signing algorithm with the secret keys that he controls. $\mathcal{A}$ must participate in the signing procedure if there are corrupted members.
- *Corrupt:* Oracle returns requested user's secret key to $\mathcal{A}$ and updates list of corrupted users.
- *Register:* $\mathcal{A}$ provides public key to the oracle, who adds it to the ring and list of corrupted ring members.

Our Contribution

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☺

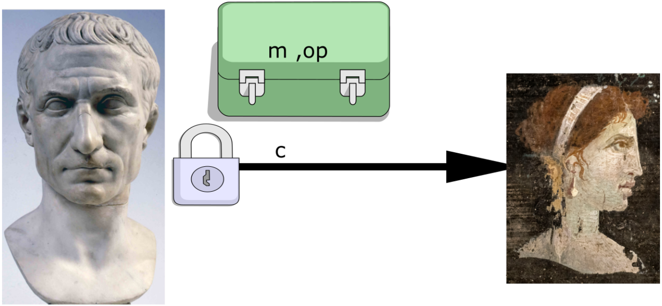
Post-Quantum Secure Problem and Technique

- Black-box use of (post-quantum) Trapdoor Commitment Scheme
- We avoid rewinding by making all outputs part of the signature (Unruh (2015)).

Our Scheme

Commitment Scheme

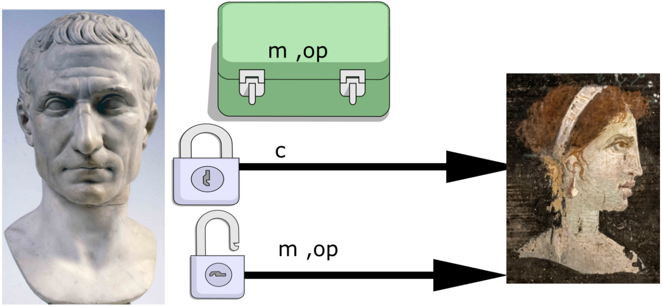
Hiding, Binding



Sender can commit to a message. Receiver cannot learn what the message is (hiding). Later sender can only open to the original message (binding).

Commitment Scheme

Hiding, Binding



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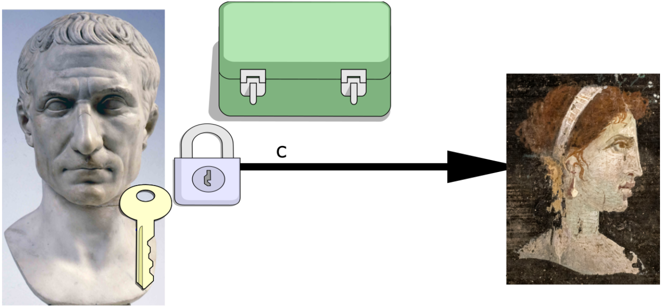
Trapdoor

Knowing a trapdoor, it's possible to 'change your mind'.

Trapdoor Commitment Scheme

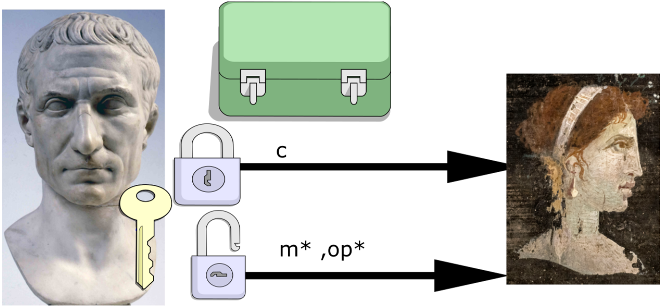
Trapdoor Indistinguishability



- With knowledge of a trapdoor t, sender can open a commitment to any message they like.
- Hiding, binding (w/o knowledge of trapdoor), and trapdoor indistinguishability.

Trapdoor Commitment Scheme

Trapdoor Indistinguishability



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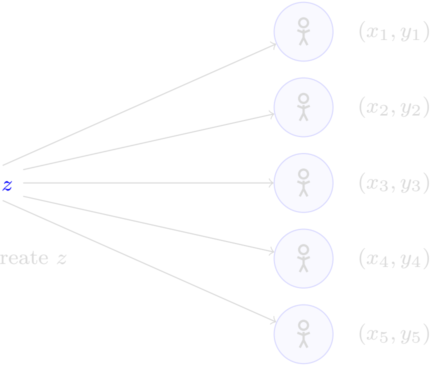
Shamir Secret Sharing

Graphic

Example: Want 3-out-of-5.

Secret: z



Combine points to uniquely create z

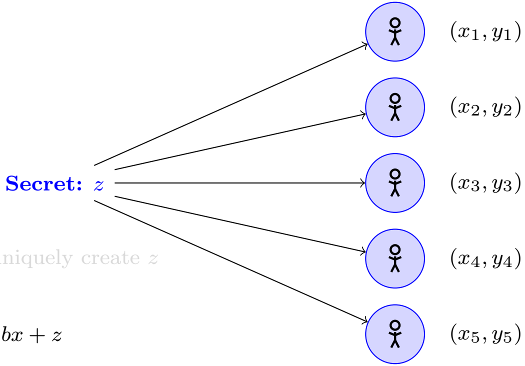
$$y = ax^2 + bx + z$$

Description

Shamir Secret Sharing

Graphic

Example: Want 3-out-of-5.



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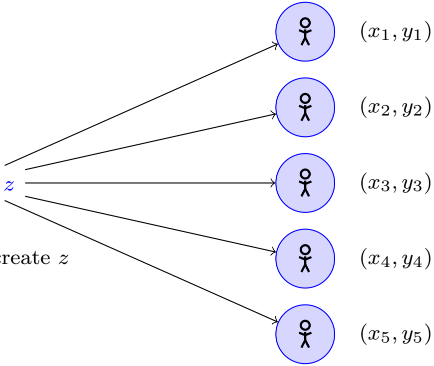

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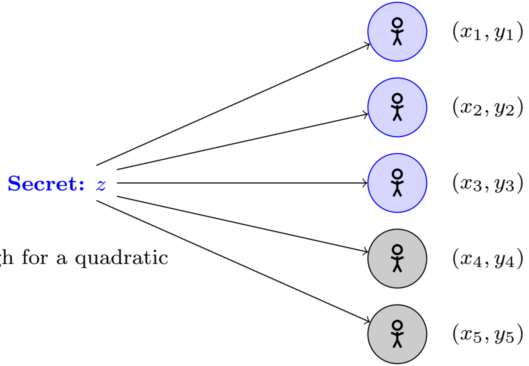
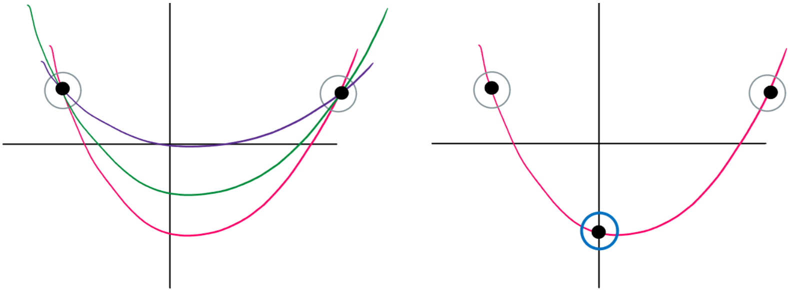


2 points is not enough for a quadratic

▸ Description

Shamir Secret Sharing

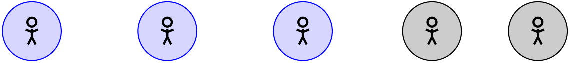


- With 2 points there are lots of solutions to the quadratic polynomial.
- By adding the third point we uniquely define the polynomial.

Protocol

Ring Members

For a 3-out-of-5 threshold ring signature:



$(vk^1, sk^1), (vk^2, sk^2), (vk^3, sk^3)$ vk^4 vk^5

Where $vk^s = (pk^s, x^s)$.

Template

Signer

$c^s \leftarrow \mathsf{TCom}(sk^s)$

$com = \boxed{c^1}\boxed{c^2}\boxed{c^3}\boxed{c^4}\boxed{c^5}$

$\mathsf{msg}, com \longrightarrow \boxed{\mathcal{H}} \longrightarrow z$

Verifier

$com, \boxed{\{(y^i, op^i)\}_{i=1}^5} \longrightarrow$

Non-signer

$(pk^q, ?)$

$c^q, op^q \leftarrow \mathsf{Com}_{pk^q}(y^q)$

Unruh

Template

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msg, $com \longrightarrow \boxed{\mathcal{H}} \longrightarrow z$

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Non-signer

$(pk^q, ?)$

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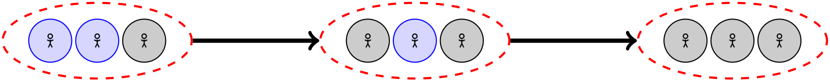
Unruh

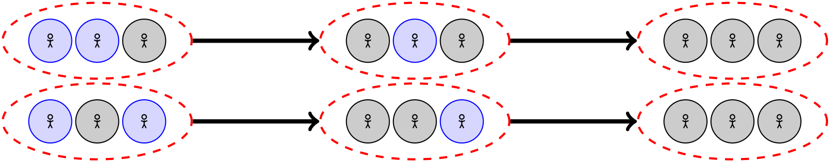
Core Technique

- Swap every trapdoor commitment out with an honest commitment step-by-step.
- At the end signers and non-signers look perfectly alike!



Anonymity

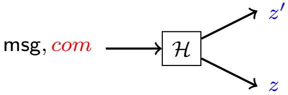
With all honest trapdoors two signatures look exactly alike.

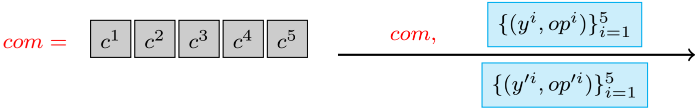


Replacing a trapdoor commitment with an honest commitment is indistinguishable.

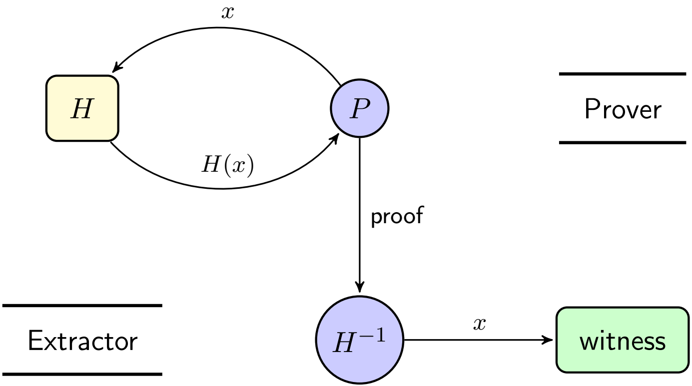
Unforgeability

With all honest commitments use a forgery to break binding.



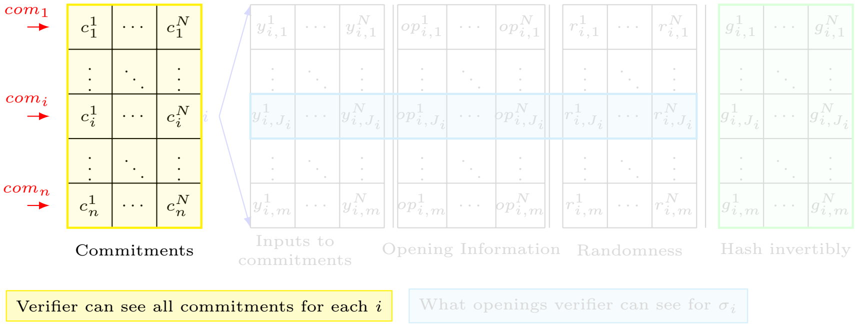


Unruh Transformation



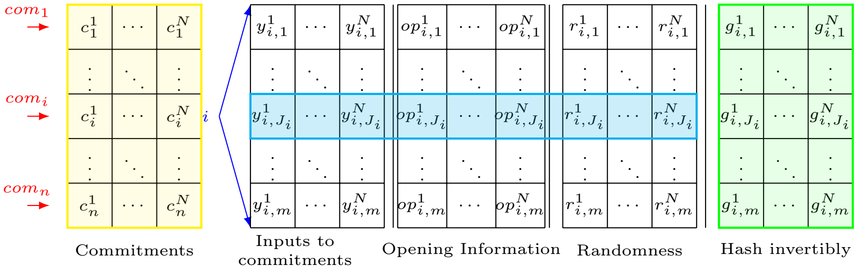
- make the RO invertible
- include all outputs in the proof

Unruh Transformation



Instead of making a single commitment, make n commitments and answer m challenges.

Unruh Transformation



Verifier can see all commitments for each i

What openings verifier can see for σ_i

Instead of making a single commitment, make n commitments and answer m challenges. 1

Unforgeability

$g_{i,1}^{1}$	$\cdots$	$g_{i,1}^{N}$
$\vdots$	$\ddots$	$\vdots$
g_{i,J_i}^{1}	$\cdots$	g_{i,J_i}^{N}
$\vdots$	$\ddots$	$\vdots$
$g_{i,m}^{1}$	$\cdots$	$g_{i,m}^{N}$

W.h.p. 2 commitments have 2 valid responses.

Summary

Summary

1. First formal definitions for a t-out-of-N threshold ring signature scheme in the presence of active adversaries that leverage malicious keys in their attacks. Generalized the definitions of Bender et al. (2006) from 1-out-of-N ring signatures to threshold t-out-of-N ring signatures.
2. Created a scheme which uses black-box trapdoor commitments, meaning that the parties can use any (post-quantum) trapdoor commitment scheme.
3. First construction that is provably secure against quantum adversaries that have quantum access to the random oracle.

Questions for Future Research

- Can we use Fiat-Shamir for thring signatures in a way that's provably post-quantum secure?
- Can we make a post-quantum secure thring signature which has anonymity amongst signers?

The End

https://eprint.iacr.org/2020/135

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