Ledger Combiners for Fast Settlement

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[2]University of Edinburgh
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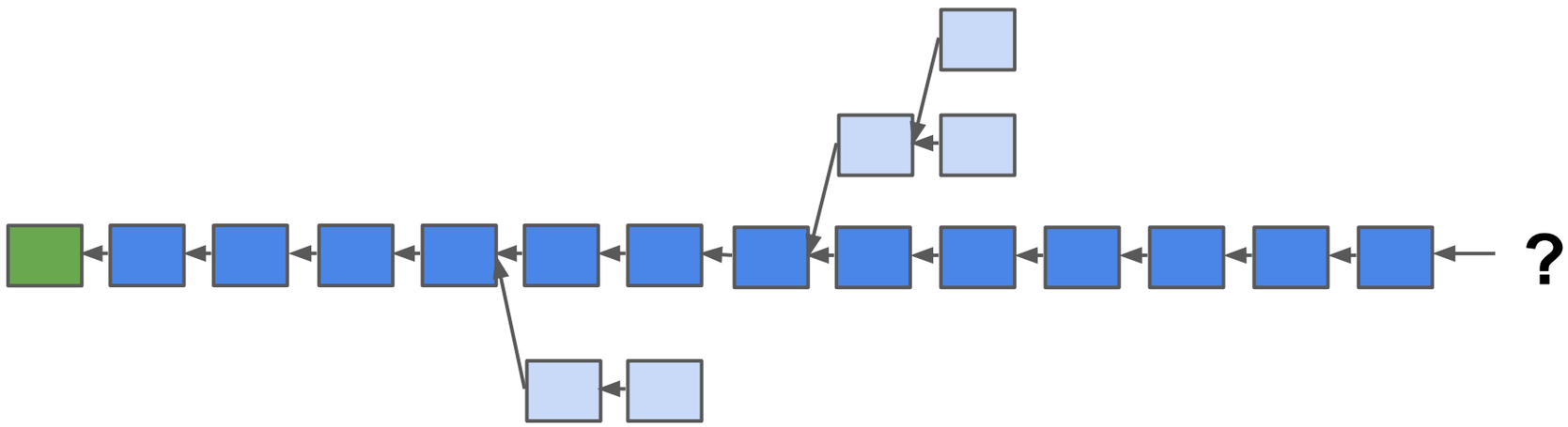
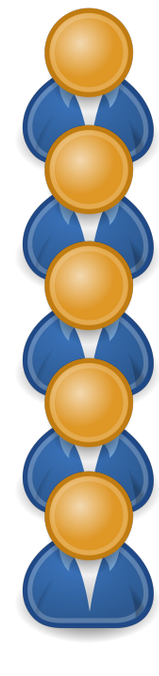
TCC 2020

Nakamoto Ledgers in a Nutshell





Cartoon description:

- transactions-carrying blocks appended in ever-growing tree
- current state: longest chain, extended by honest parties
- block-creation rights distributed “fairly” using anti-Sybil lottery

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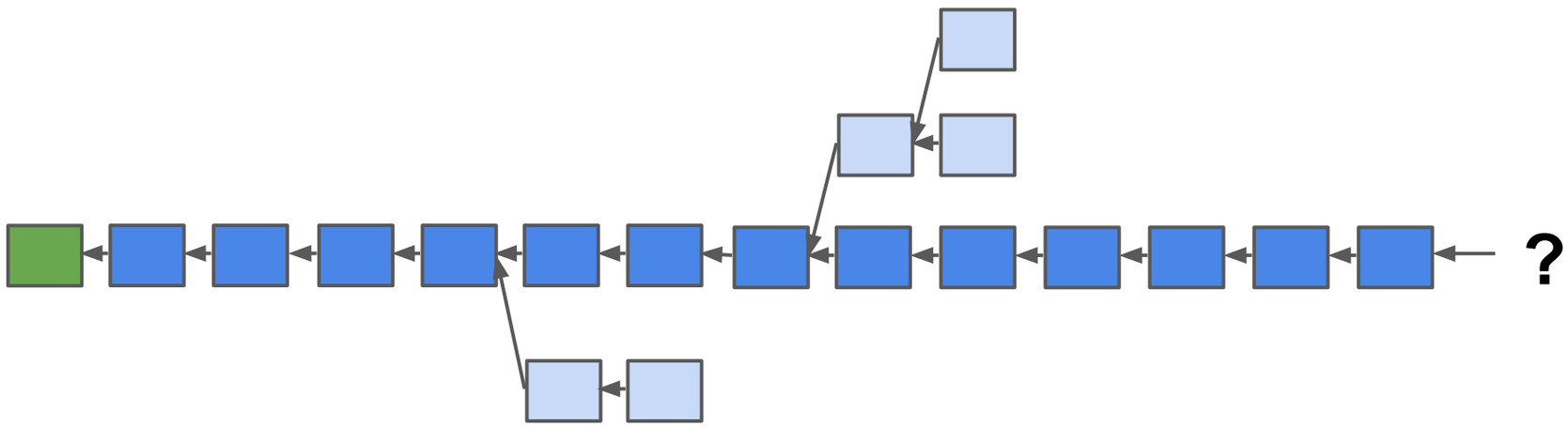

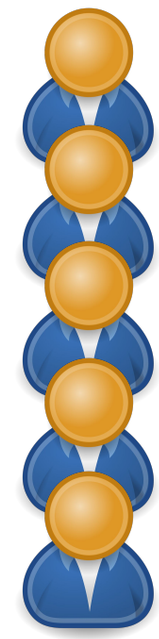

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Properties:

- sufficient fraction honest ⇒ “eventual” consensus on winning chain
- immutability measure: #blocks on top

Anti-Sybil Lotteries for Nakamoto Consensus

Proof of Work (PoW): [Bitcoin, Ethereum, ...]

- parties solve a computational puzzle
- success proportional to invested computation

Proof of Stake (PoS): [Ouroboros, Snow White, ...]

- parties own stake on the blockchain
- success proportional to owned stake

Proof of Space (PoSp): [SpaceMint, Chia, ...]

- parties dedicate storage space
- success proportional to dedicated storage

Nakamoto Consensus: Advantages

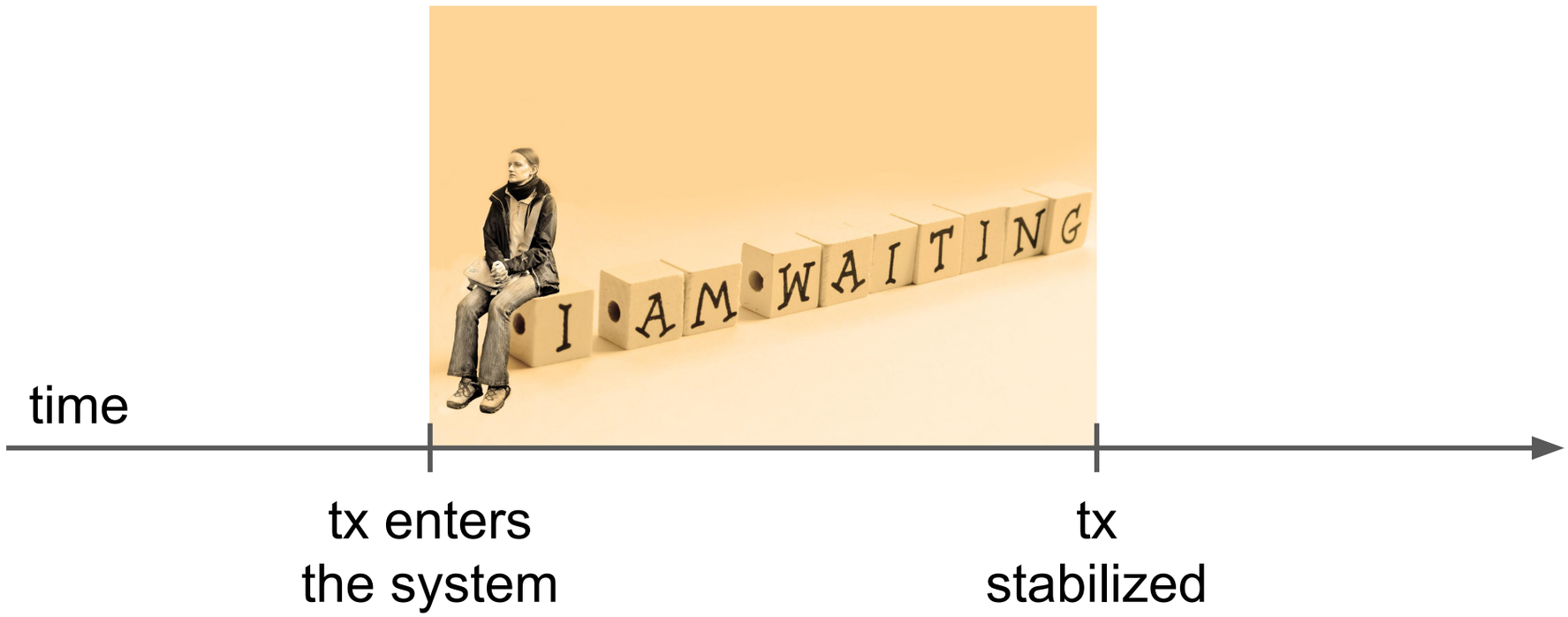
NC very resilient, can tolerate hostile environments:

- **Dishonest Minority**
  - in principle, up to 1/2 of the underlying resource can be controlled by an adversary
  - cf. classical: typically up to 1/3

- **Sleepiness/Dynamic Availability**
  - fluctuating level of participation, parties come and go without notifying others, lose access to necessary resources, ...
  - cf. classical: fix party sets, typically rely on counting

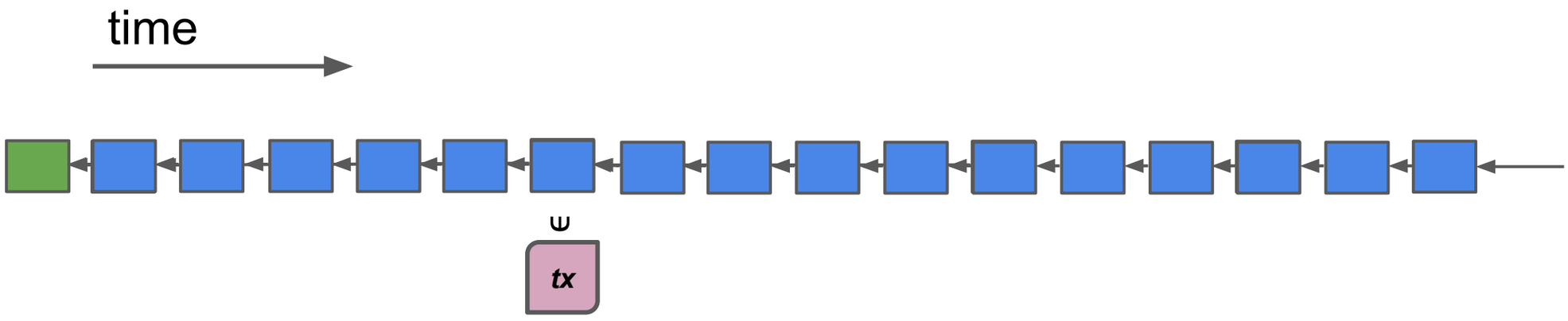
Settlement Time for Ledger Protocols

- a.k.a. latency



“How long does it take a transaction from being injected into the system until universally recognized as a stable ledger entry?”

Latency Barrier of Nakamoto Consensus



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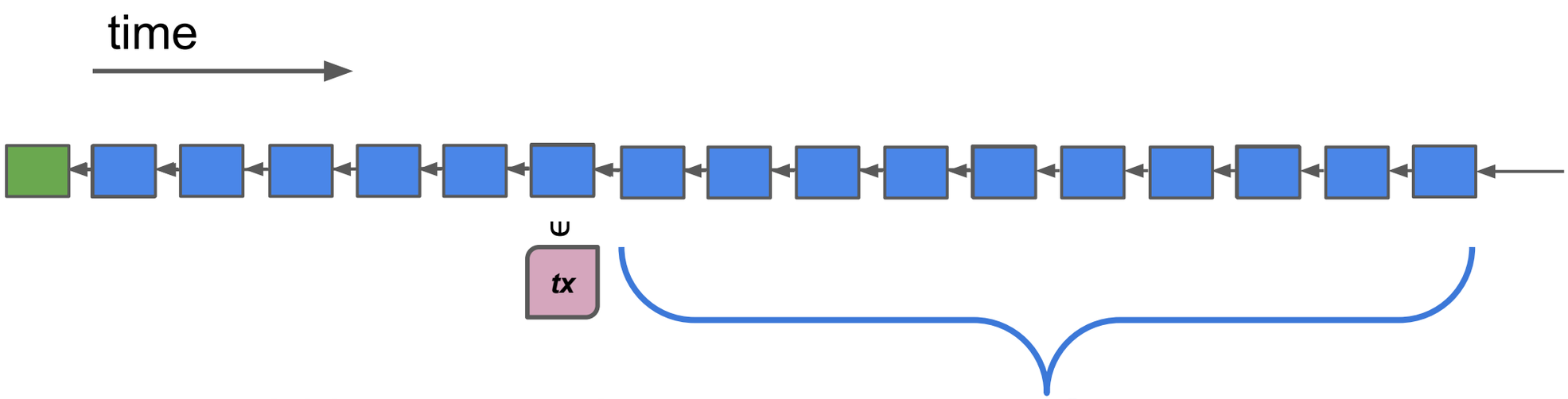

- *tx* stability proportional to number of blocks on top

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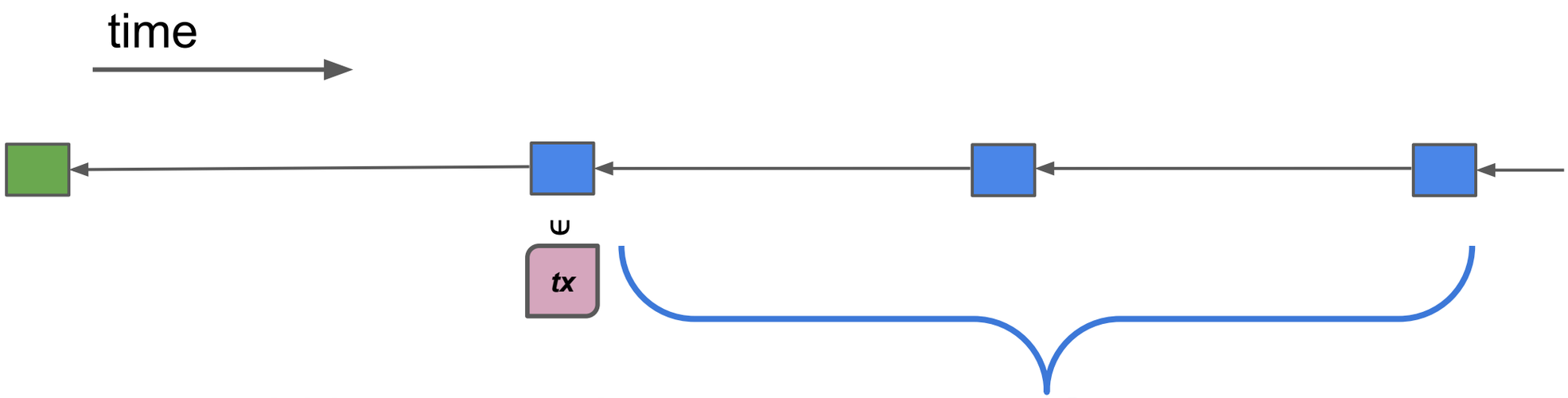

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Intrinsic Latency Barrier:

limited block creation rate
(to avoid forks) ⇒ limited settlement speed

Overcoming Nakamoto Latency Barrier

Overlay structures for improved latency:

- **stronger assumptions**
  - latency improved in optimistic settings
  - e.g. Hybrid Consensus [PS16], Thunderella [PS17], Afgjort [MMNT19]
- **layer-2 solutions**
  - no longer maintains a distributed ledger of all transactions
  - e.g. payment channels and state channels [PD16,DFH18,DEFM19,DEFHH19]

Best latency guarantees achievable by Nakamoto?

Our approach: Parallel Composition (also taken in Prism [BKTFV19])

Our Contributions

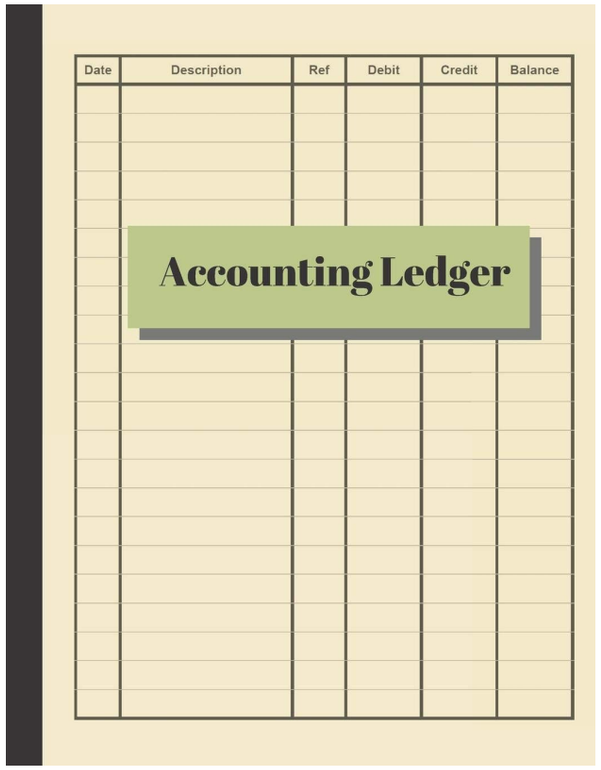
1. An Abstract Model for Ledgers

2. A Combiner for Security Amplification (= Latency Reduction)

3. A Robust Ledger Combiner

Abstract Ledgers

- **ledger**
  - a static set of transactions T
  - rank function: $T \longrightarrow \mathbb{R}^+$
    - orders transactions
    - captures their stability
    - loosely related to time
    - e.g.: block timestamp



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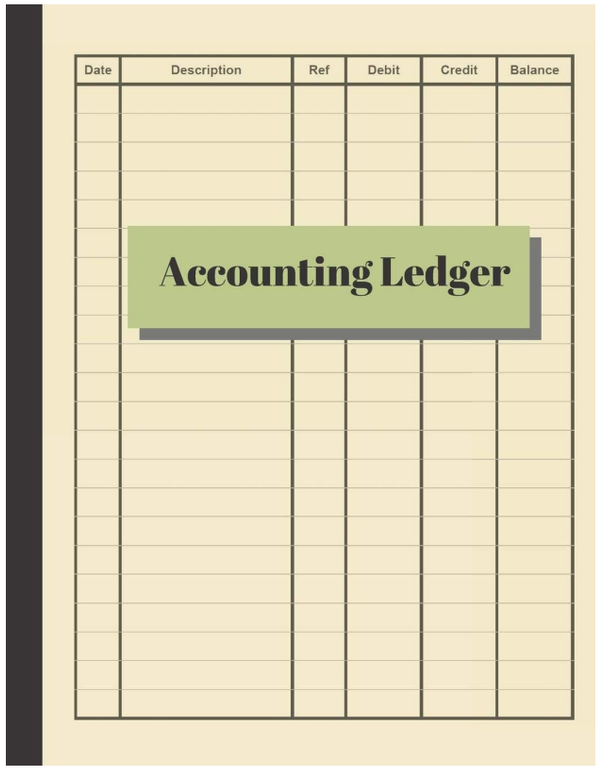

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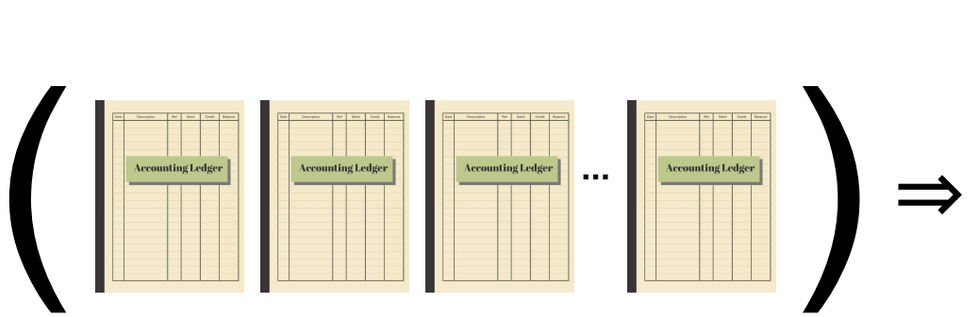
- **dynamic ledger**
  - time-indexed sequence of ledgers
  - sufficient to express persistence, liveness

Ledger Combiner for Security Amplification/Latency Reduction



⇒

***m* parallel, independent dynamic ledgers** ⇒ **1 “virtual” dynamic ledger**

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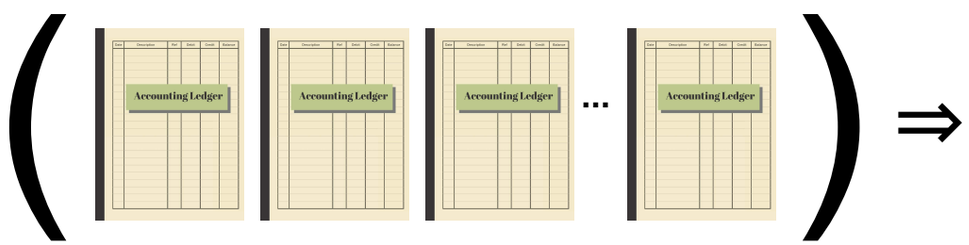

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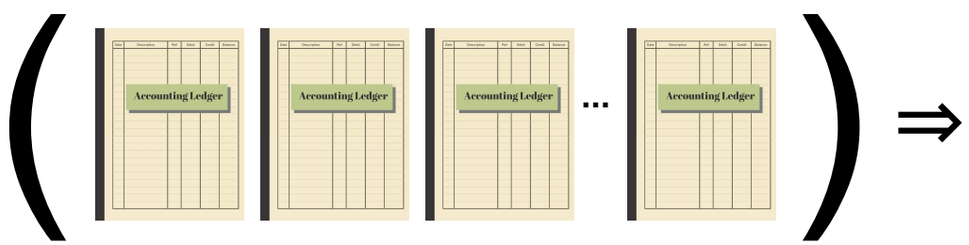
$\text{rank}_1(tx), \ldots, \text{rank}_m(tx) \quad \Rightarrow \quad \text{rank}(tx) := F(\text{rank}_1(tx), \ldots, \text{rank}_m(tx))$

Ledger Combiner for Security Amplification/Latency Reduction



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Fast (m-fold) submission:

settlement time t ⇒ t / Θ(m)

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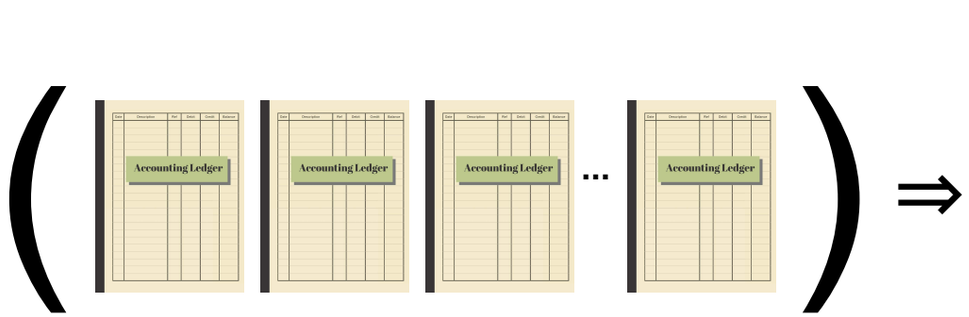

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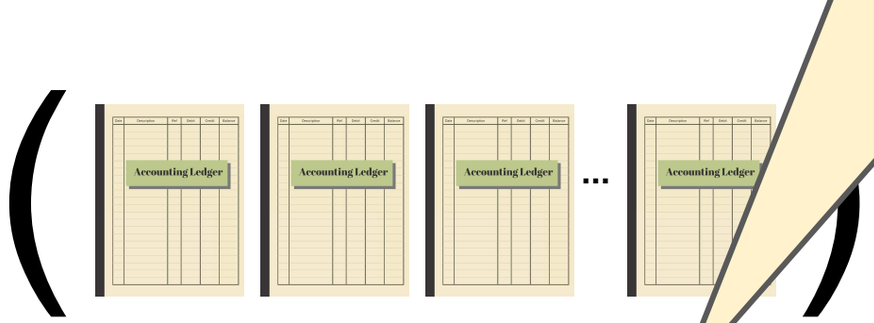
Slow (single-ledger) submission:

settlement time t ⇒ t . O(ln m)

Ledger Com… Security Amp… …y Reduction

- weaker condition suffices: **subindependence**
- achievable for PoW, PoS



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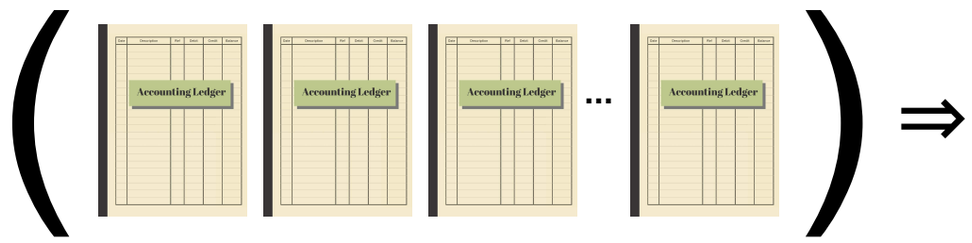

⇒

***m* parallel, independent dynamic ledgers** ⇒

1 m scales with λ: constant-time settlement with negligible error

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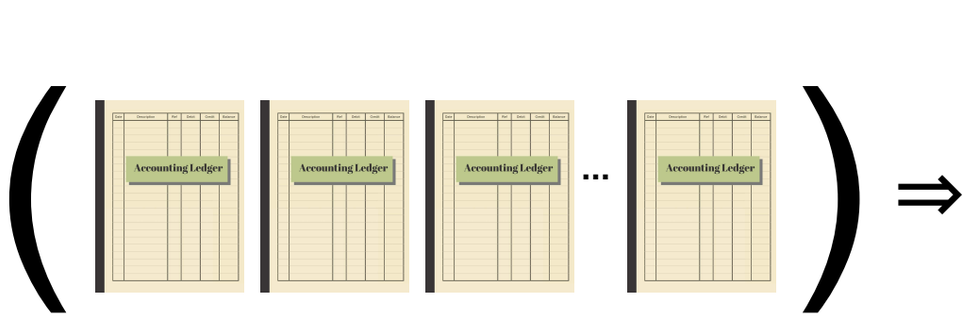

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tradeoff: tx fee ⇔ settlement time

Slow (single-ledger) submission:

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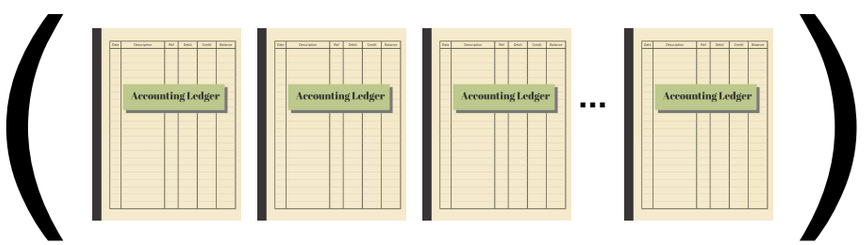
Robust Combiner for Dynamic Ledgers



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Robust Combiner for Dynamic Ledgers

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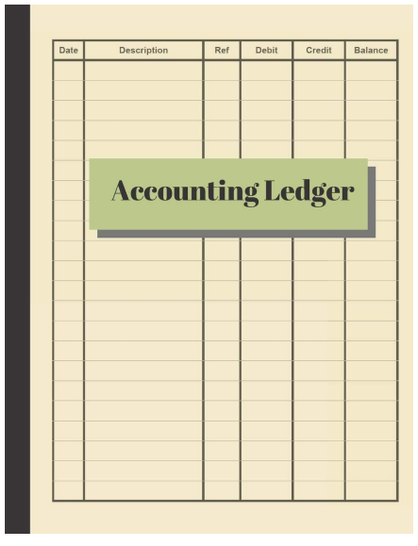
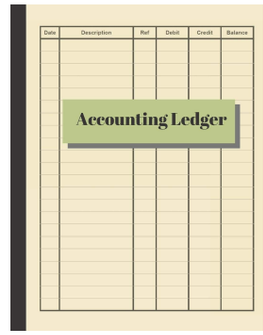
- **robust**: some persistence+liveness guarantees maintained even if minority of ledgers fully corrupted
- illustrates versatility of our dynamic ledger notion



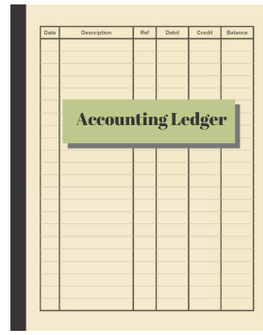
1. An Abstract Model for Ledgers

Abstract Ledgers



- **ledger:** $\mathbb{L}$ = (T, rank)
  - set of transactions T
  - rank function: T⟶ $\mathbb{R}^+$
    - transactions ordered by increasing rank

Abstract Ledgers



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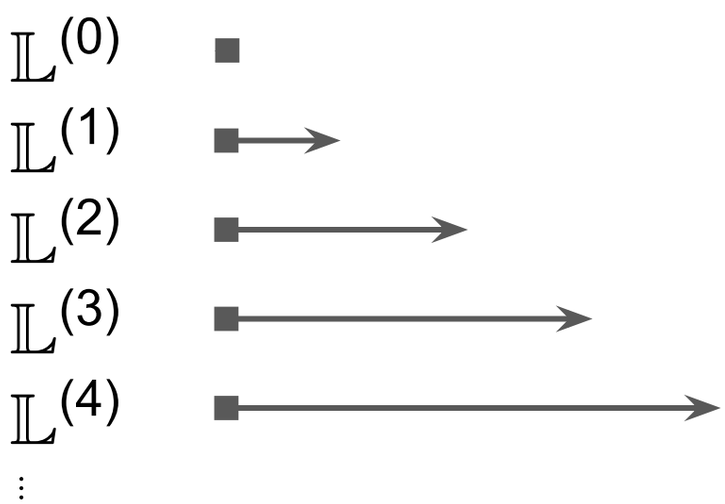
  - time-indexed sequence of ledgers



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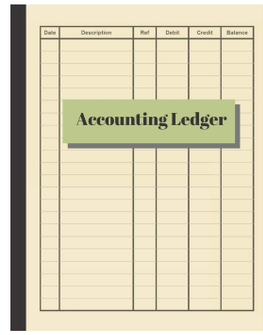

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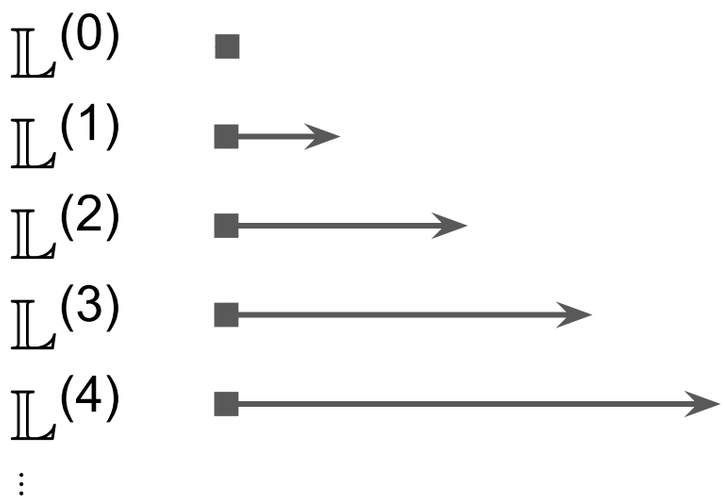
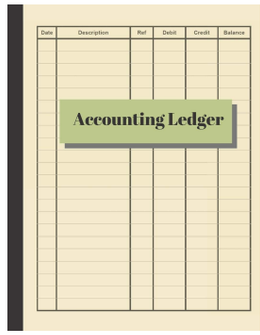
  - time-indexed sequence of ledgers
  - sufficient to express liveness, persistence

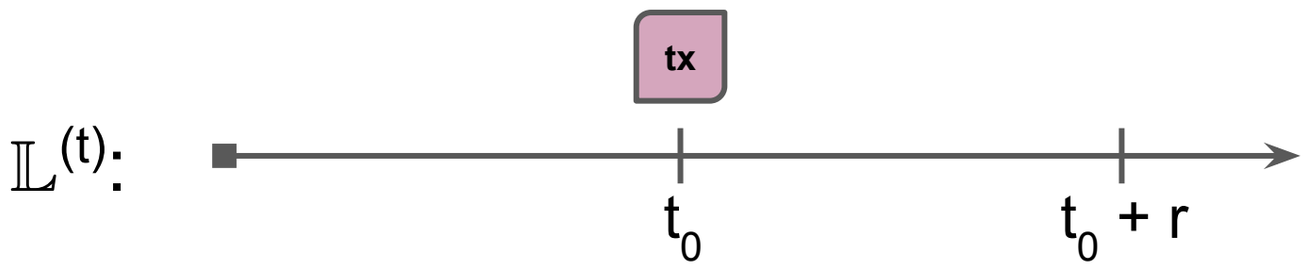


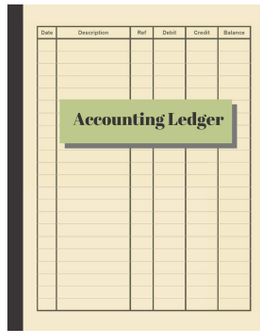


Dynamic Ledger Properties

Dynamic ledger $\mathbb{D} = \mathbb{L}^{(0)}, \mathbb{L}^{(1)}, \mathbb{L}^{(2)}, \ldots$

➢ Liveness. For any r, t_0, $t \geq t_0$+r:





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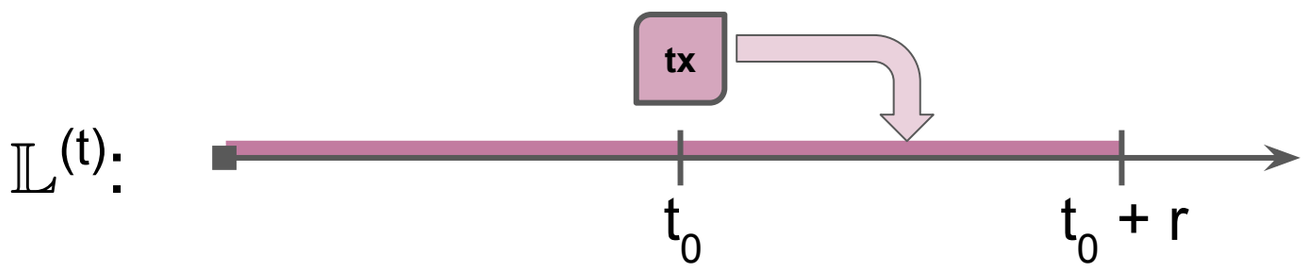

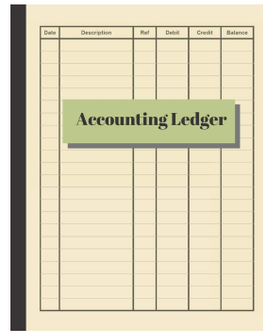

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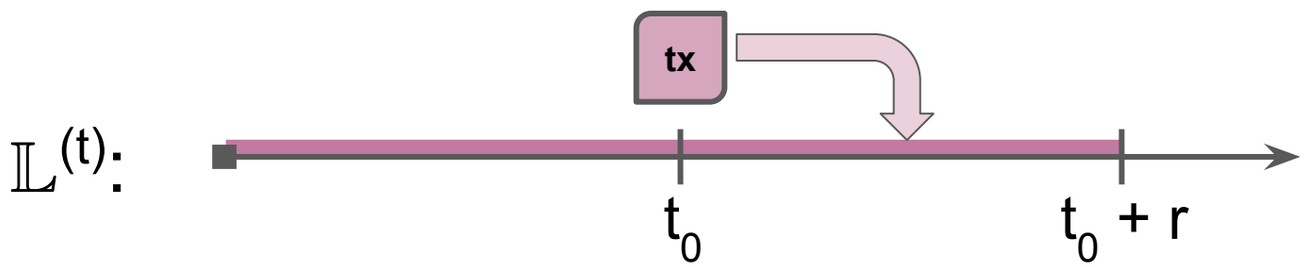

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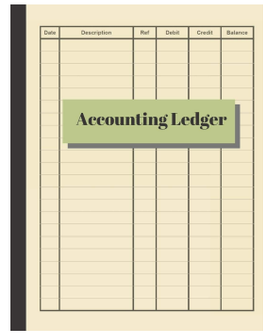

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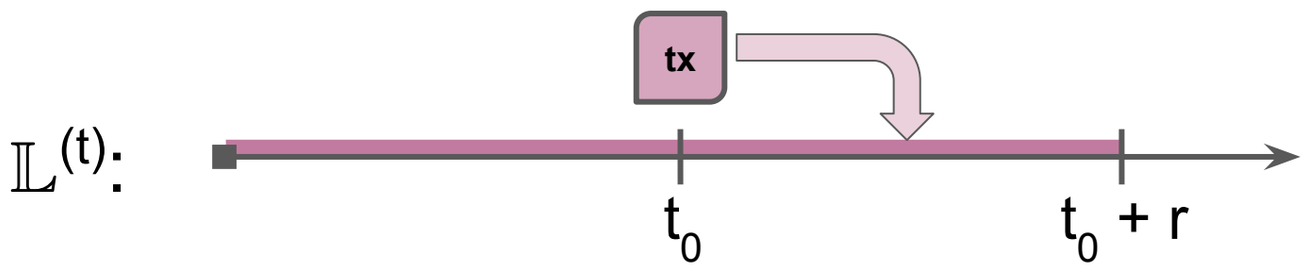

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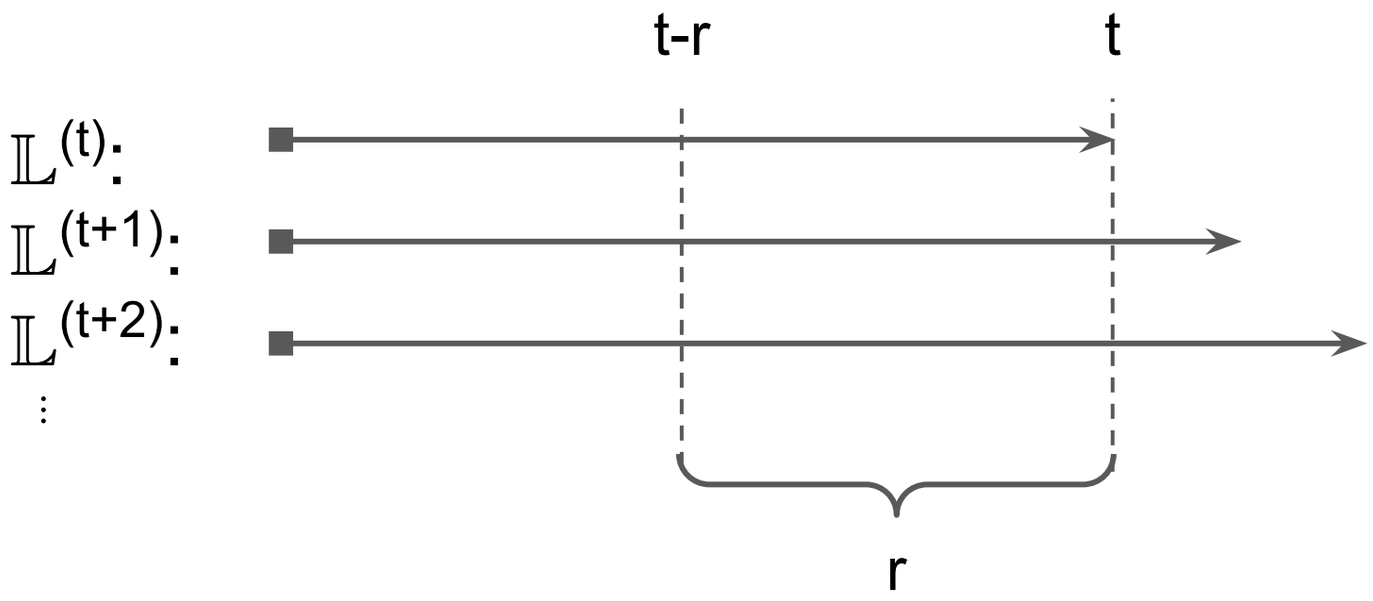
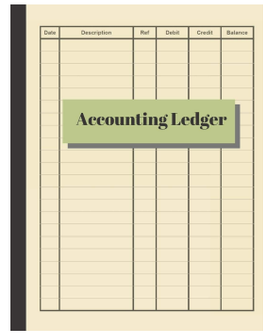
➢ Persistence. For any r, t:





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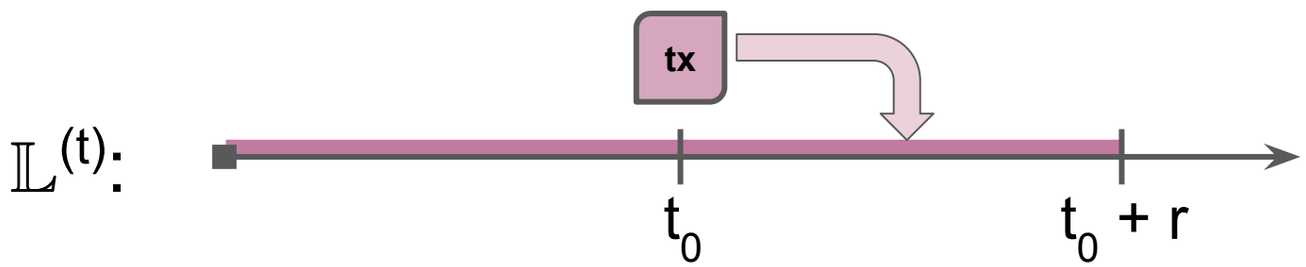

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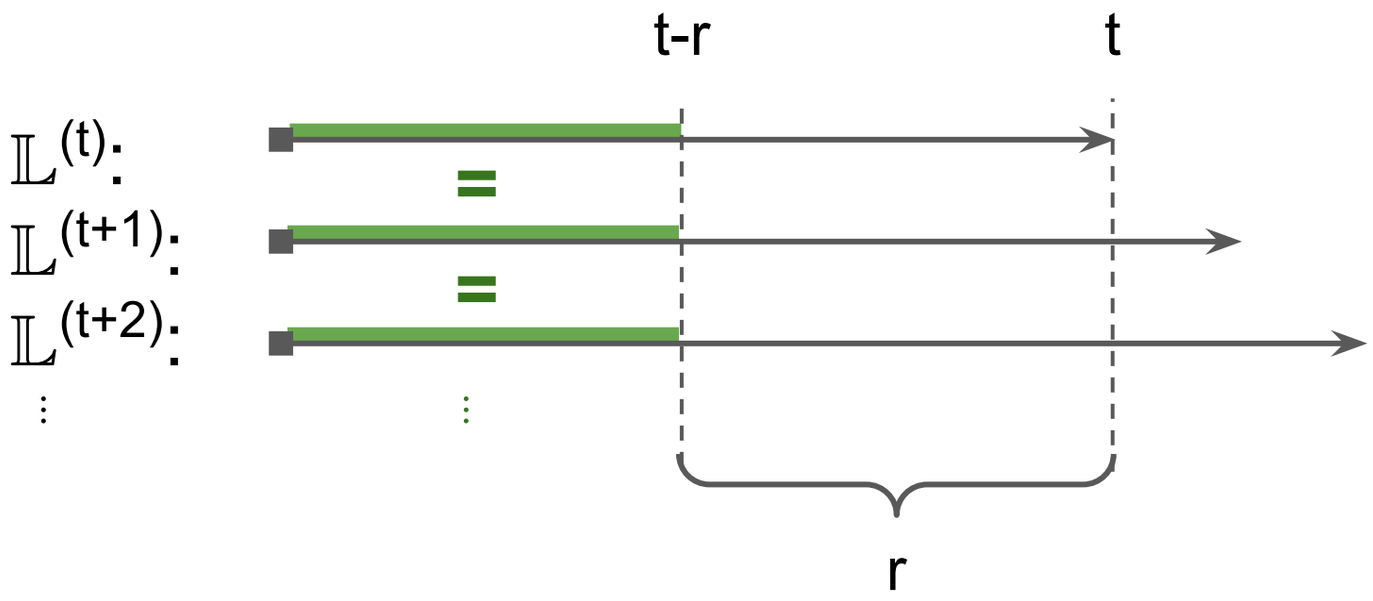
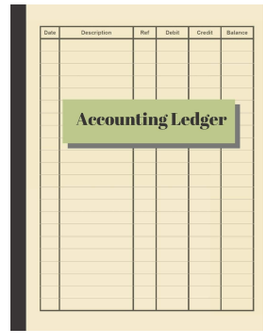


except with error p(r).



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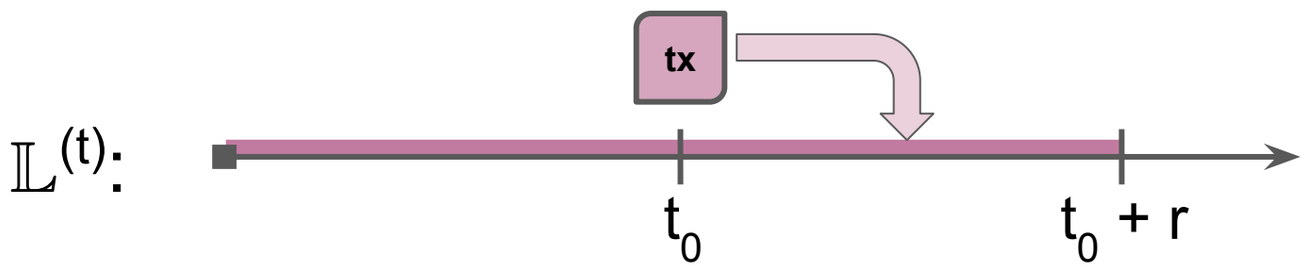
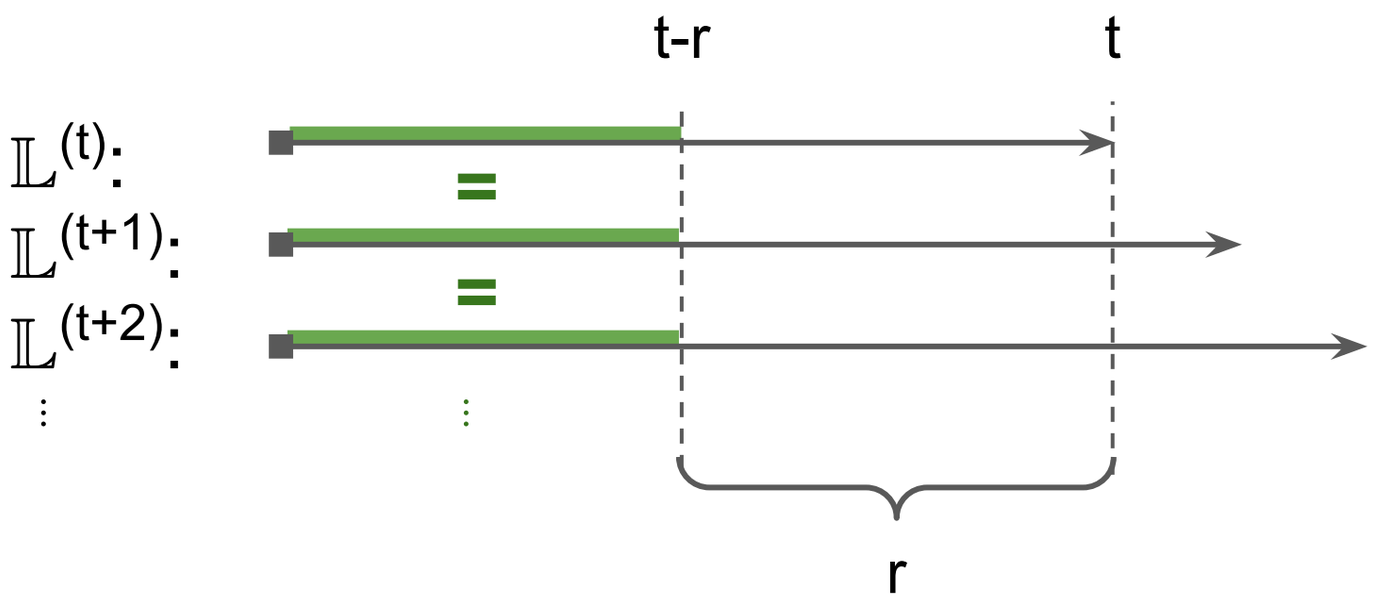


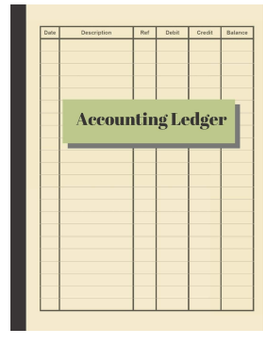
except with error l(r).

➢ Absolute persistence. For any r, t:



except with error $p_A(r)$.

Dynamic Ledger Properties



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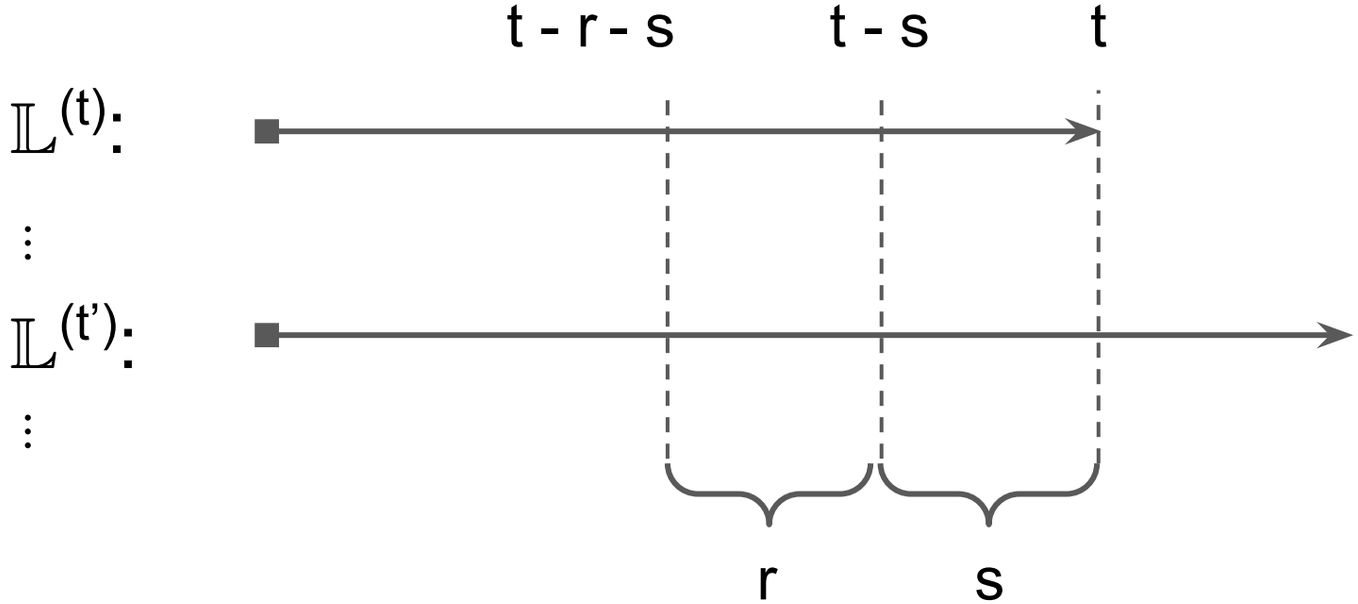

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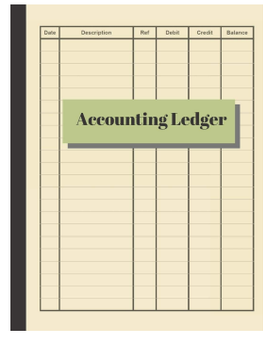

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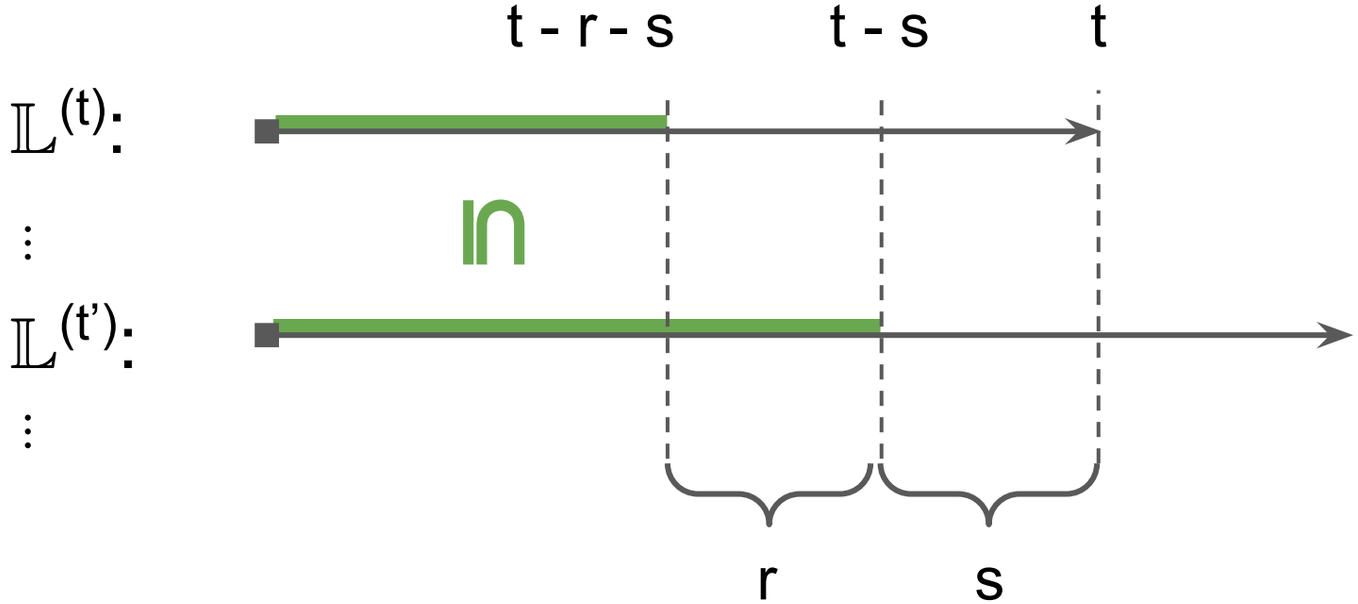

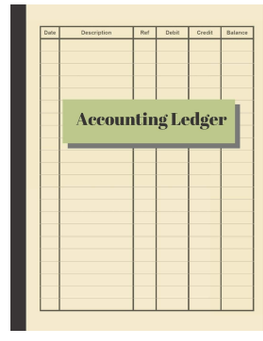

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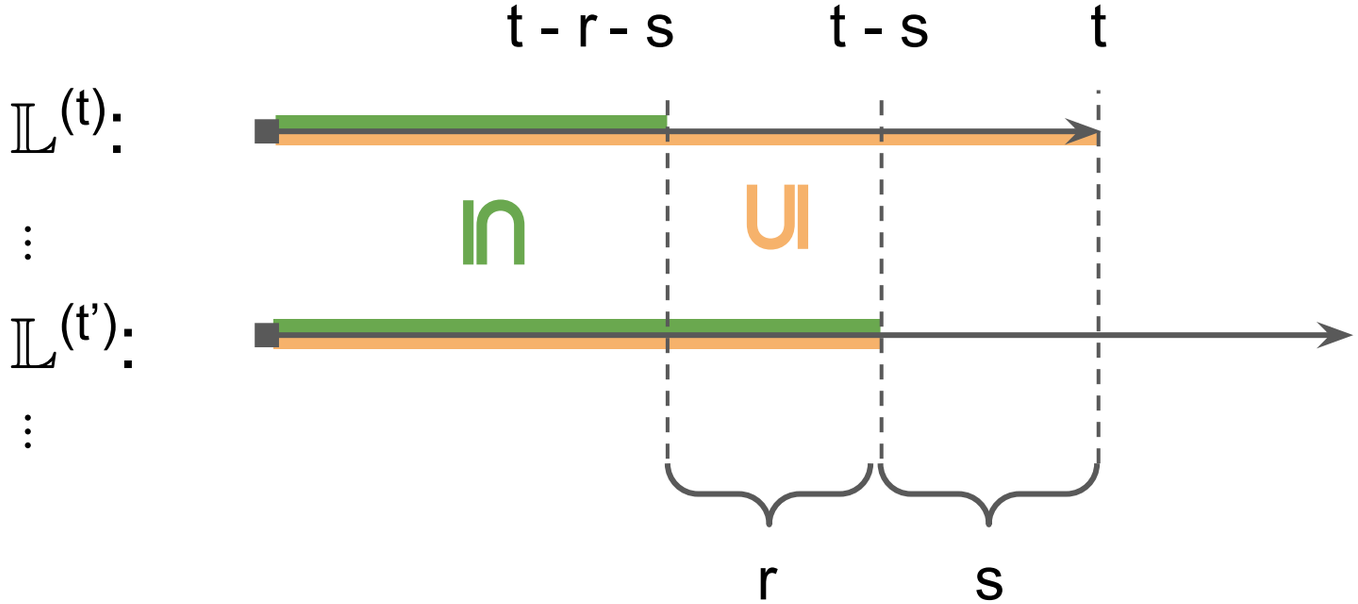

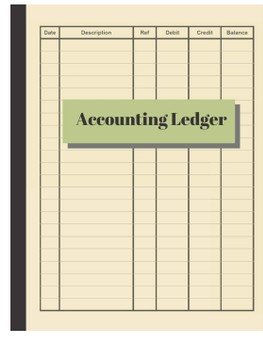

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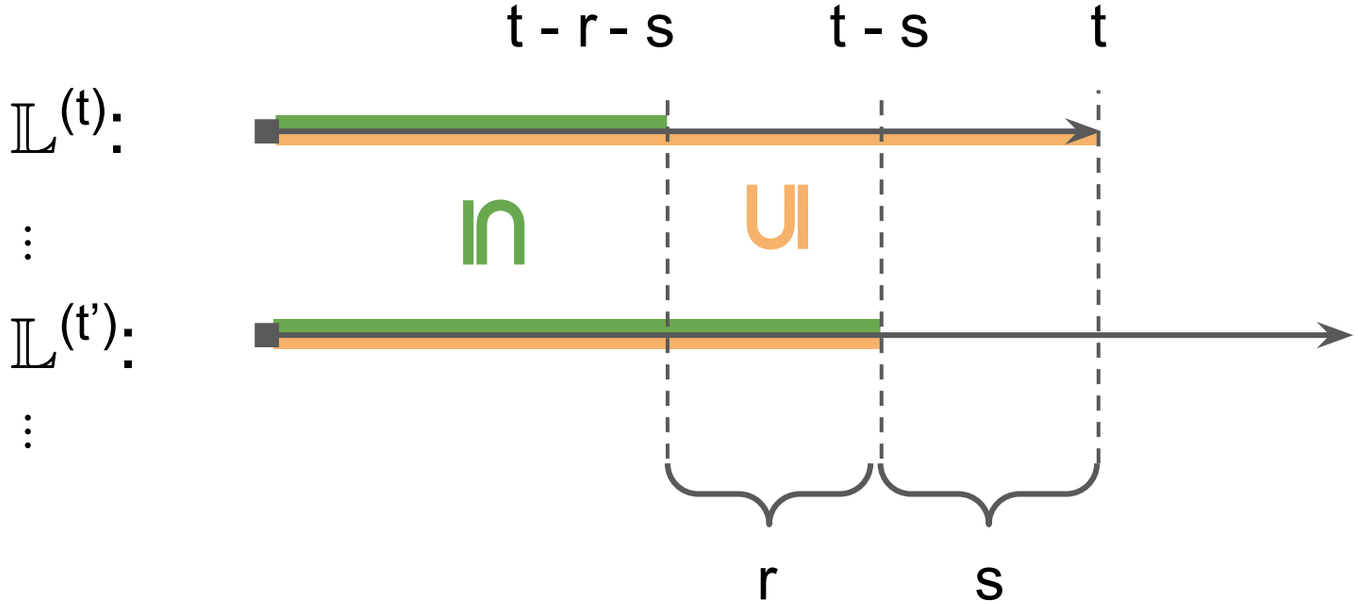
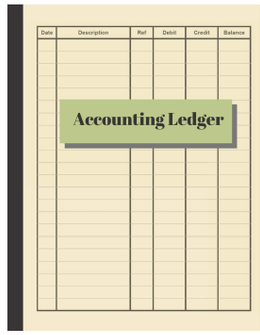


except with error $p_R(r,s)$.

Why Relative Persistence?



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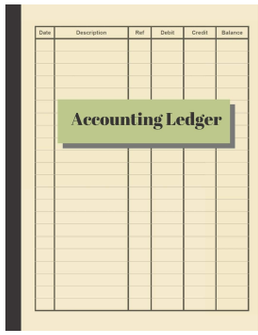

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$p_A(s)$-absolute persistence
} ⇒
(absolute) settlement in *r*+*s* steps with error $l(r) + p_A(s)$

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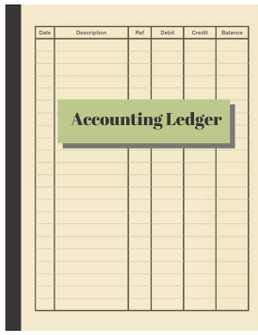

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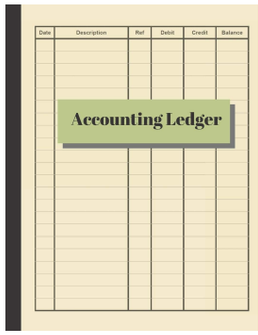

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⇒

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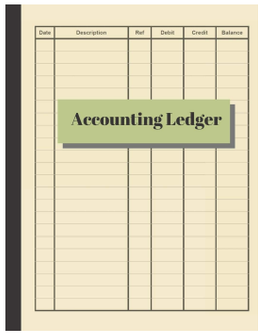

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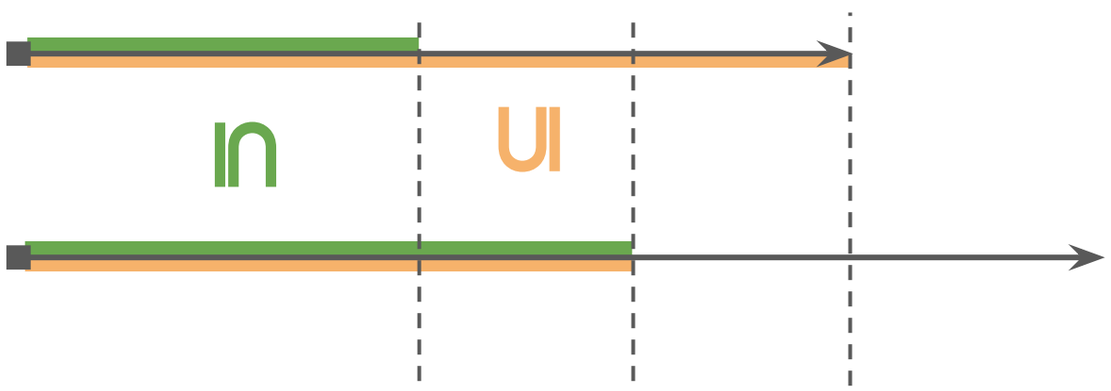

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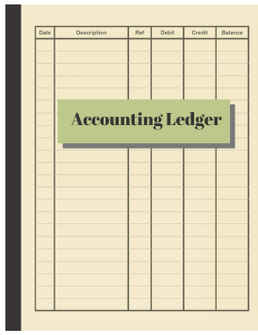

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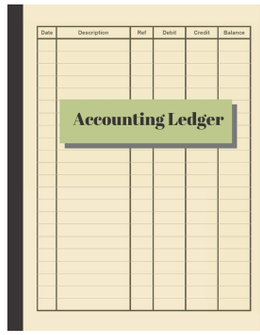
⇒

If:
- all inputs of *tx* absolutely settled
- *tx* relatively settled
- no conflicting *tx'* currently in ledger

then tx will not be invalidated

- tx stays in ledger
- any conflicting *tx'* that could overtake *tx* in the future is already in the ledger



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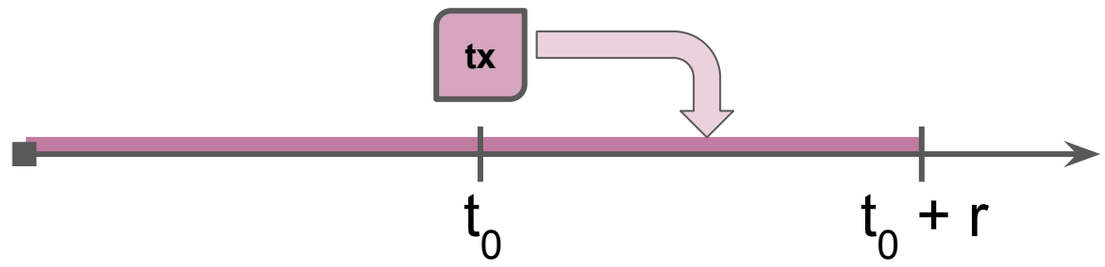
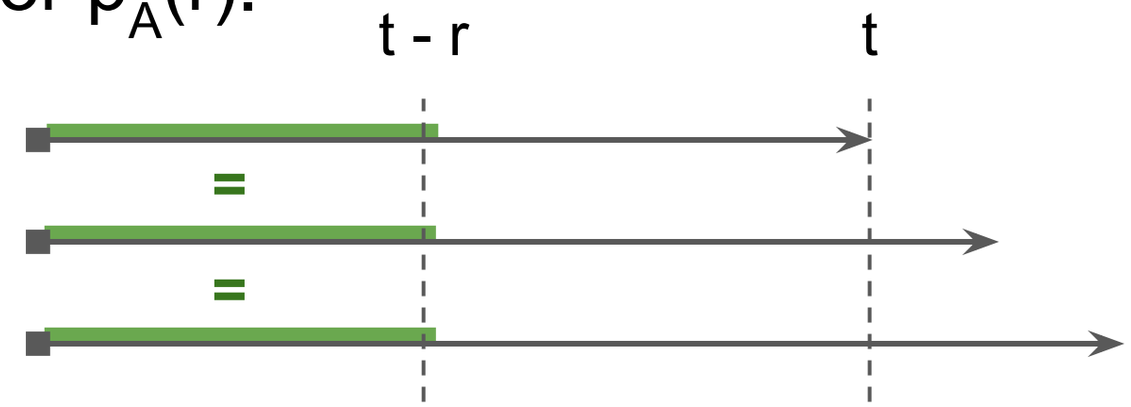
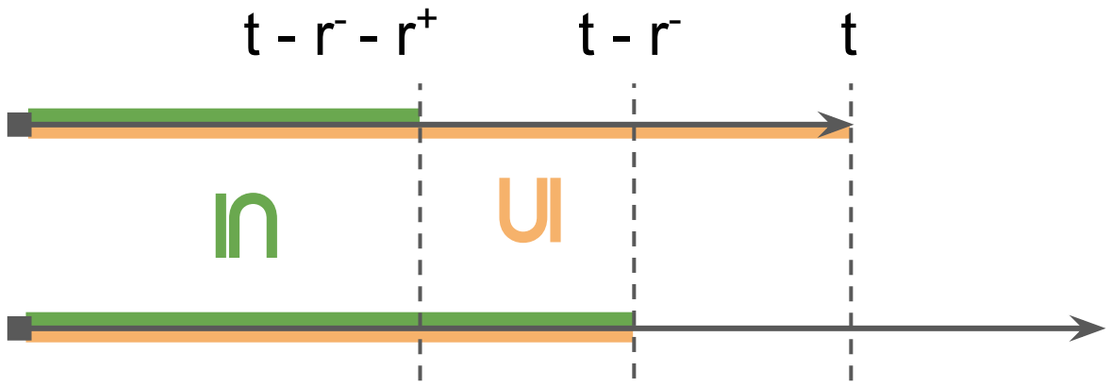
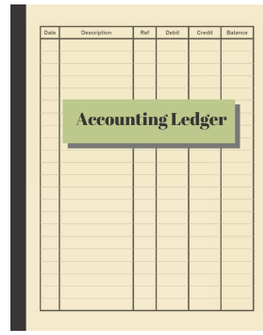


- Absolute persistence with error $p_A(r)$.



- Relative persistence with error $p_R(r^+, r^-)$.





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- under “safe conditions” (e.g. honest majority, bounded delays, ...)
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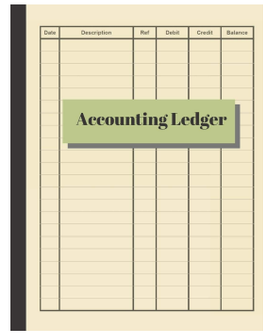

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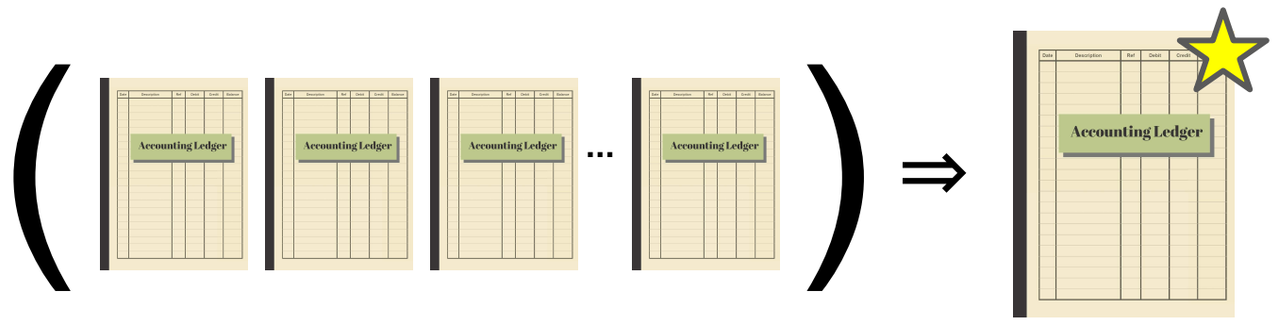
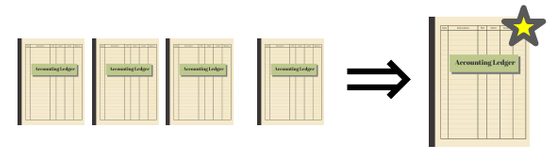
- error functions already cover the presence of the adversary
  - no formal adversary needed in the model



2. Ledger Combiner for Security Amplification and Latency Reduction



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- full independence seems unlikely

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- any collection of failure events $\{ F_i \}_{i \in I}$ where F_i concerns $\mathbb{D}_i$

we have

$$\Pr[\bigwedge_{i \in I} F_i \mid E] \leq \prod_{i \in I} \Pr[F_i]$$

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Achieving ε-subindependence:

- **PoS:** independent lottery for each chain
- **PoW:** m-for-1 PoW

Our Combiner: Fast and Slow Tx Submission

For $\mathbb{D}_1$, …, $\mathbb{D}_m$ we define C($\mathbb{D}_1$, …, $\mathbb{D}_m$) that allows for:

- **“Fast” tx submission**
  - tx submitted to all *m* underlying ledgers simultaneously
- **“Slow” tx submission**
  - tx submitted to a single ledger only

- also all intermediate possibilities
- can be chosen per-tx by user

- tradeoff: fees ⇔ settlement time

Our Combiner: The Rank Function

For $\mathbb{D}_1, \ldots, \mathbb{D}_m$ we define $C(\mathbb{D}_1, \ldots, \mathbb{D}_m)$ via

$$\mathrm{rank}_L(\mathrm{tx}) \stackrel{\mathrm{def}}{=} -L \ln\left(\frac{1}{m} \sum_{r_i \le \theta(\mathrm{tx})} \exp\left(-\frac{r_i}{L}\right)\right)$$

where

$L \in \mathbb{N}$ (parameter)

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Some intuition:

- average under exponential functional exp(-rank(.)/*L*):

$$\exp(-\mathrm{rank}_L(\mathrm{tx})/L) = \frac{1}{m} \sum_i \exp(-r_i/L)$$

- inspired by theory of regret minimization

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Simple lower and upper bound:

$$\min_{i \in [m]} r_i \le \mathrm{rank}_L(\mathrm{tx}) \le \left(\min_{i \in [m]} r_i\right) + L \ln m$$

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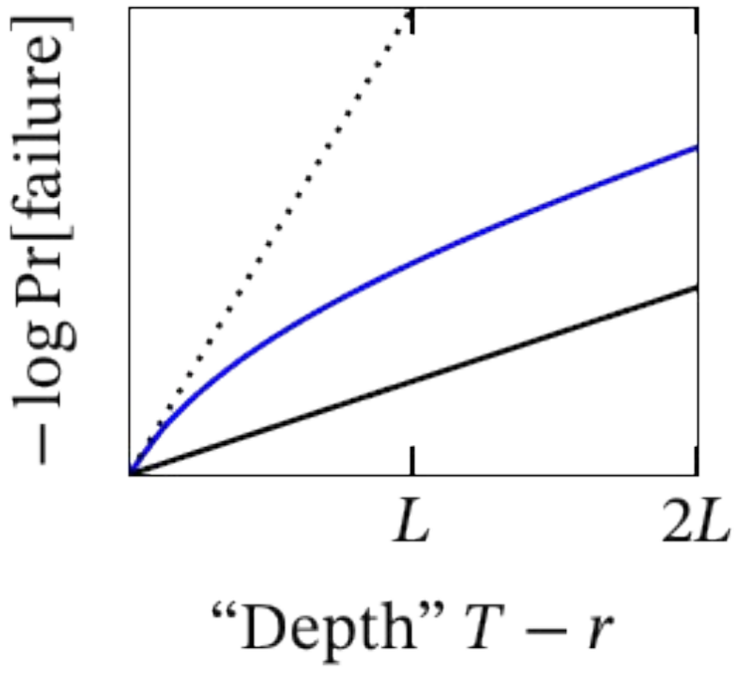
Role of the L parameter:

- seemingly contradicting requirements for “stabilization speed”:
  - *m*-fold speedup for fast submissions
  - in the long term, slow submissions only as secure as a single ledger
- L parametrizes this transition
- choose proportional to sec. parameter
  - $\exp(-\Theta(L))$ is acceptable error



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Role of the θ threshold:

- absolute persistence is sensitive to small changes in rank
- the θ(*tx*) cut-off point guarantees eventual absolute stability

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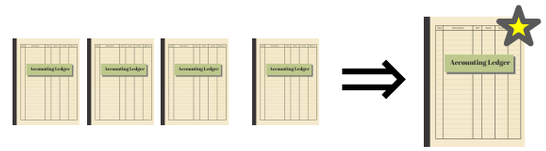
Preemptive version:

- contribution from each $\mathbb{D}_i$ only counts if *tx* not preceded by a conflicting transaction *tx'* there



Security Amplification and Latency Reduction

Theorem:

For *m* dynamic ledgers $\mathbb{D}_1, \ldots, \mathbb{D}_m$ with

- exponential liveness $l(r) = \exp(-\Omega(r))$
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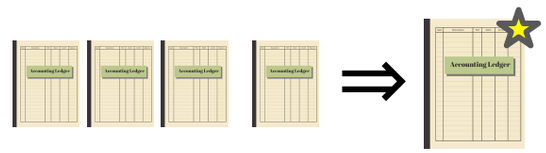
- subindependence
- a conflict relation



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the construction $C(\mathbb{D}_1, \ldots, \mathbb{D}_m)$ achieves

- fast submission:

$$\Pr[\exists\, tx \text{ not rel. settled after } 2r] \leq$$

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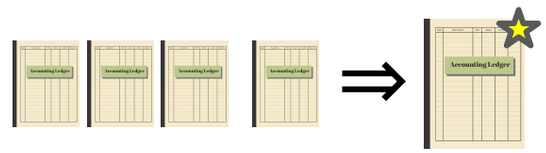

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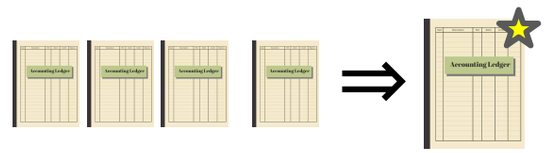
- dominant while $rm < L$



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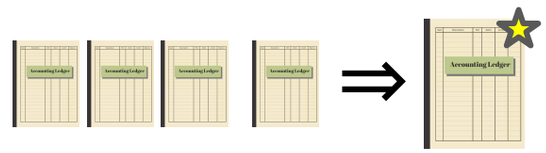
- standard settlement speed
- dominant after $rm > L$



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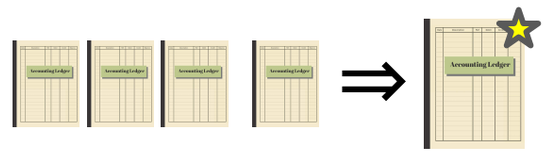


Security Amplification and Latency Reduction

Theorem:

For *m* dynamic ledgers $\mathbb{D}_1, \ldots, \mathbb{D}_m$ with

- exponential liveness $l(r) = \exp(-\Omega(r))$
- exponential absolute persistence $p_A(r) = \exp(-\Omega(r))$
- subindependence
- a conflict relation

the construction $C(\mathbb{D}_1, \ldots, \mathbb{D}_m)$ achieves

- fast submission:

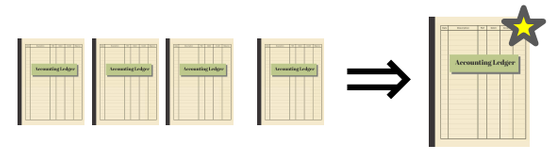
$$\Pr[\exists\, tx \text{ not rel. settled after } 2r] \leq$$

$$\exp(- r\Omega(m) + O(m)) + \exp(- \Omega(r) - \Omega(L.\ln(m)))$$

- slow submission:

$$\Pr[\exists\, tx \text{ not abs. settled after } 2r] \leq \exp(- \Omega(r) + O(L.\ln(m)))$$

standard settlement speed



Constant-Time Settlement

Theorem:

For *m* dynamic ledgers $\mathbb{D}_1, \ldots, \mathbb{D}_m$ with

- exponential liveness $l(r) = \exp(-\Omega(r))$
- exponential absolute persistence $p_A(r) = \exp(-\Omega(r))$
- subindependence
- a conflict relation

the construction $C(\mathbb{D}_1, \ldots, \mathbb{D}_m)$ achieves

- fast submission:

$$\Pr[\exists\, tx \text{ not rel. settled after } 2r] \leq$$

$$\exp(- r\Omega(m) + O(m)) + \exp(- \Omega(r) - \Omega(L.\ln(m)))$$

Corollary:

If the number of chains *m* scales with the security parameter, then $C(\mathbb{D}_1, \ldots, \mathbb{D}_m)$ with fast submission achieves constant-time settlement except with a negligible error.

Thank you for your attention.

M. Fitzi, P. Gaži, A. Kiayias, A. Russell:
Parallel Chains: Improving Throughput and Latency of Blockchain Protocols via Parallel Composition
[eprint 2018/1119]

M. Fitzi, P. Gaži, A. Kiayias, A. Russell:
Ledger Combiners for Fast Settlement
[eprint 2020/675]