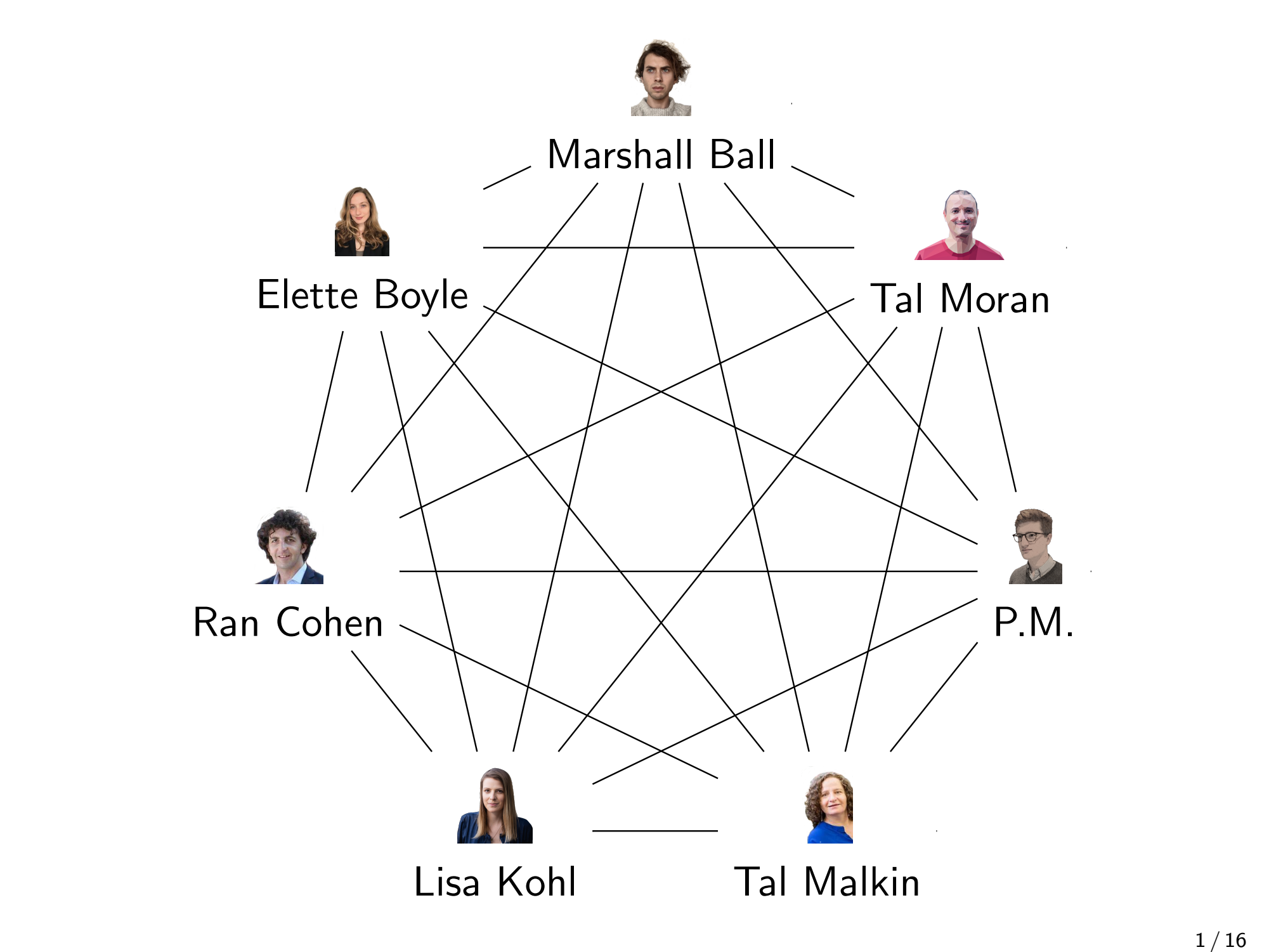
Topology-Hiding Communication from Minimal Assumptions

Marshall Ball, Elette Boyle, Ran Cohen, Lisa Kohl,
Tal Malkin, Pierre Meyer, Tal Moran

TCC 2020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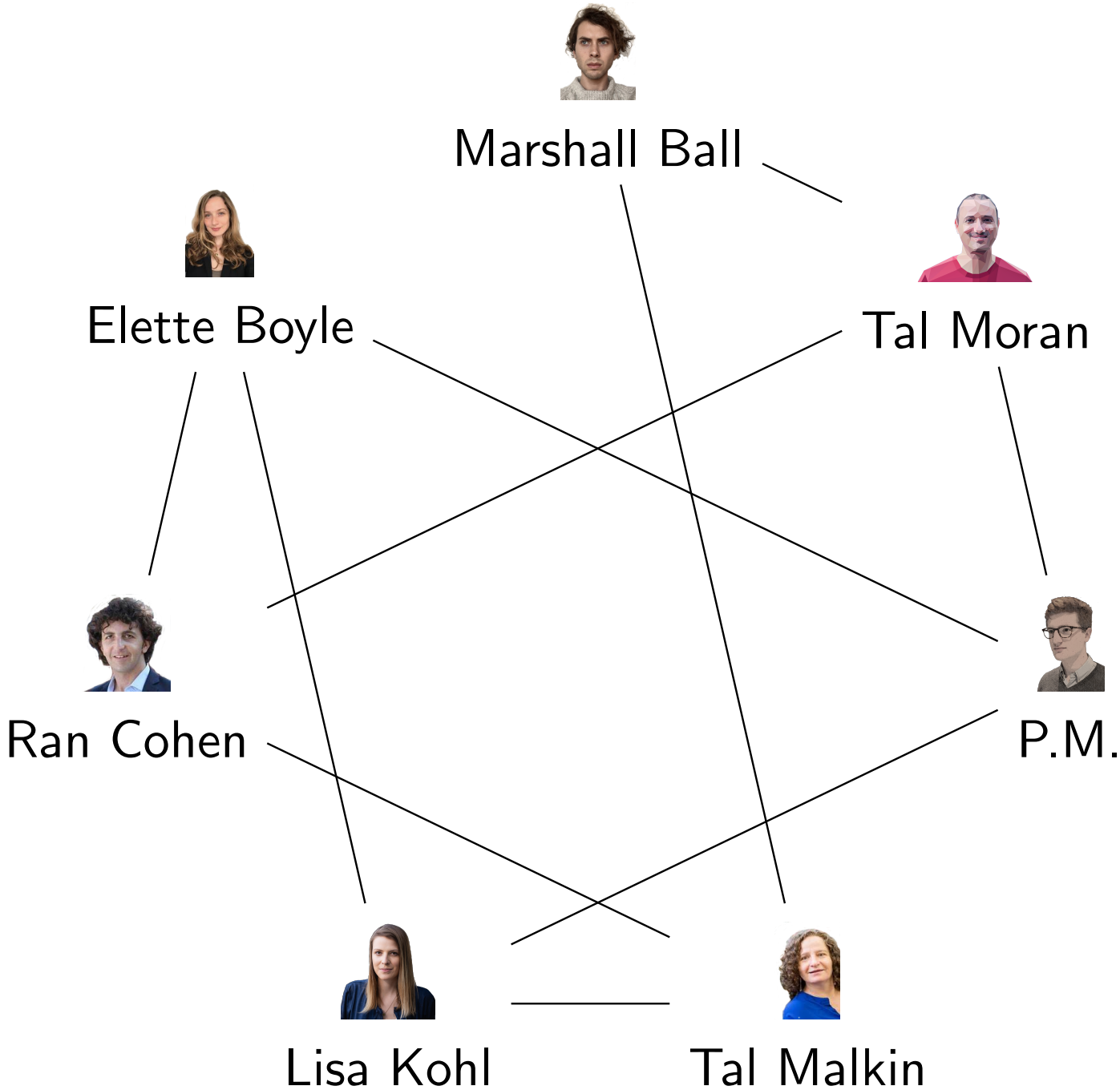
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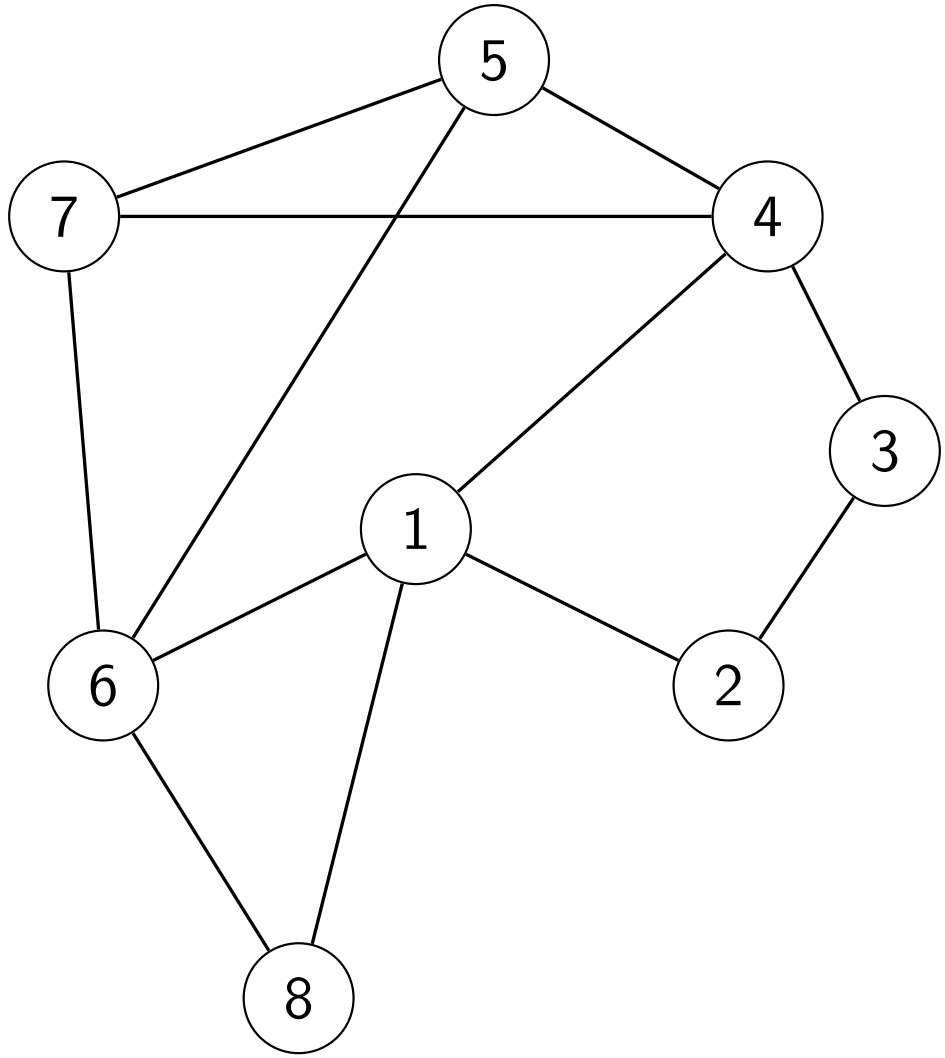
Local View of a Network



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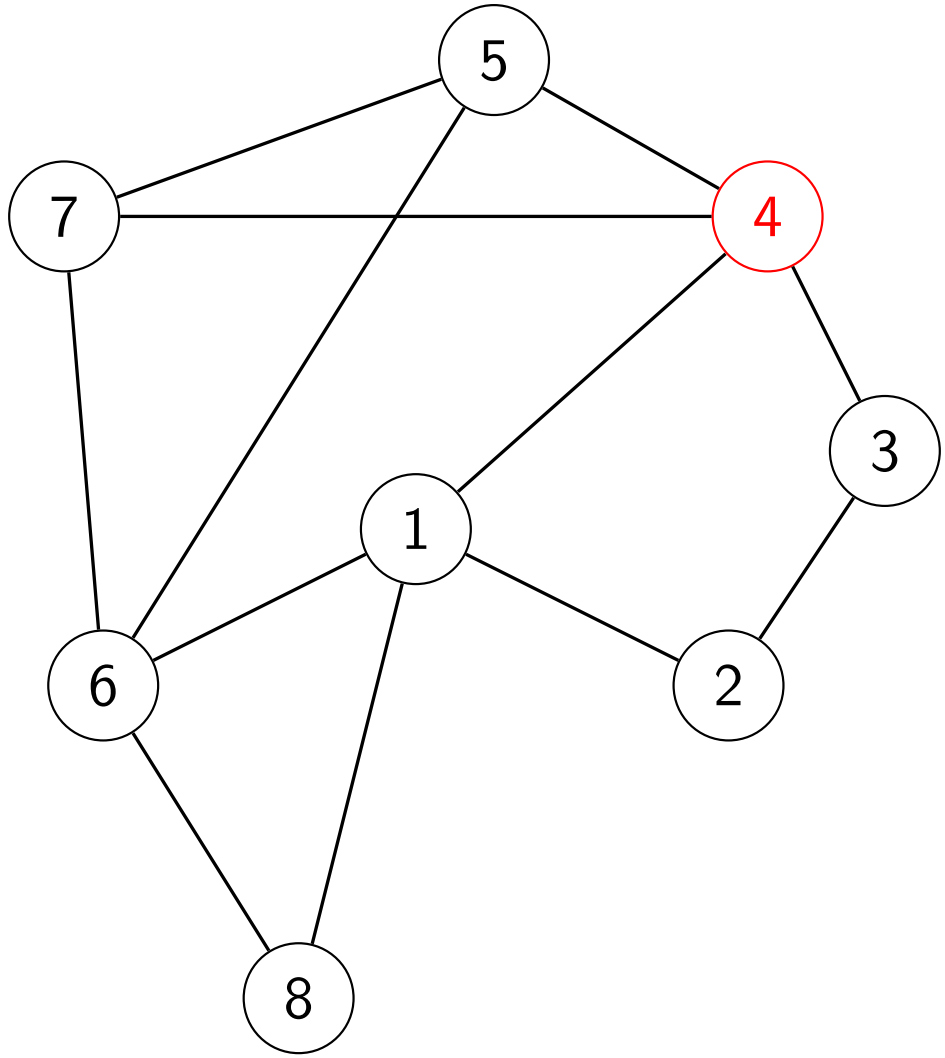

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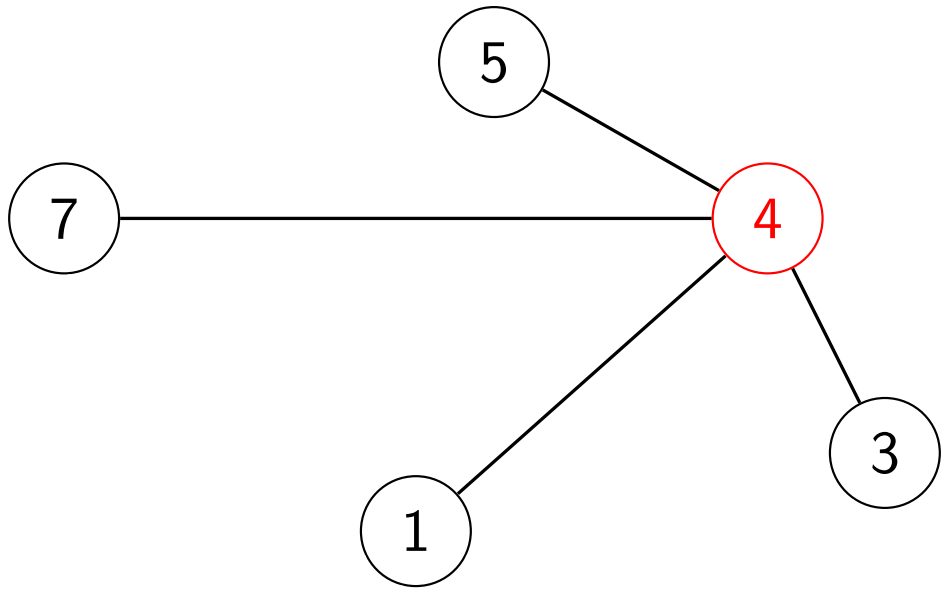

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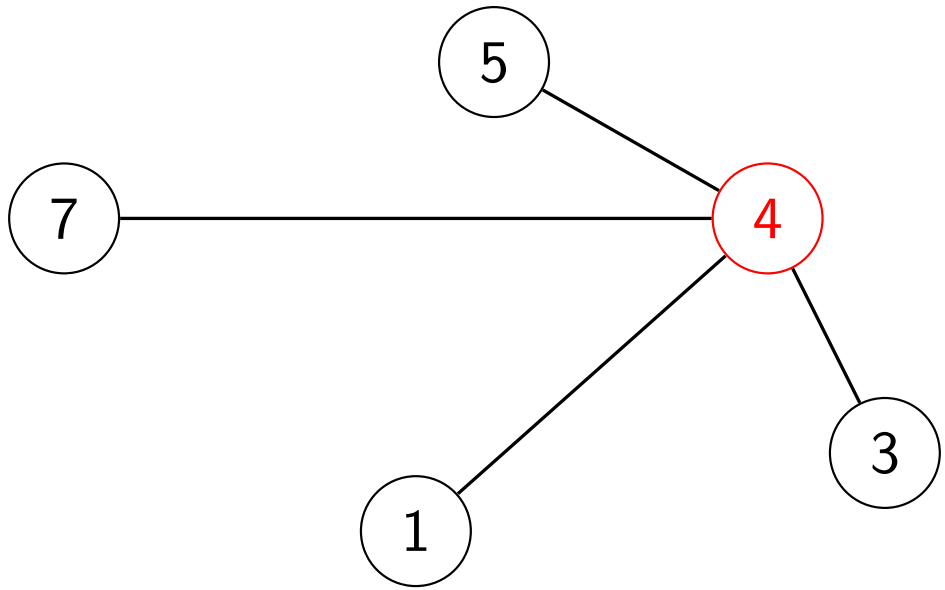


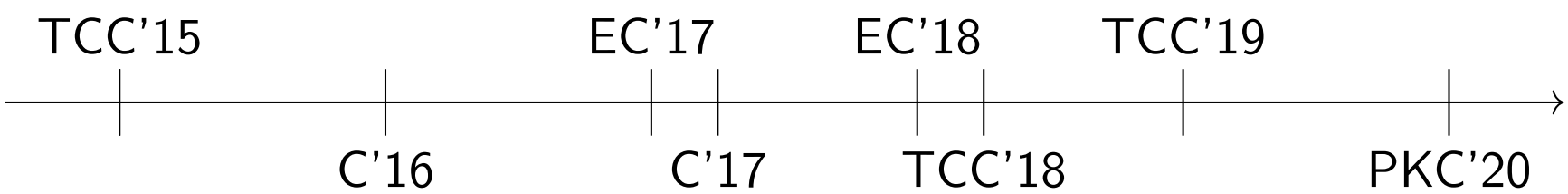
Topology-Hiding Computation:

- Parties can only see their local view
- The MPC reveals nothing else about the graph

Difficulties of Topology-Hiding Computation

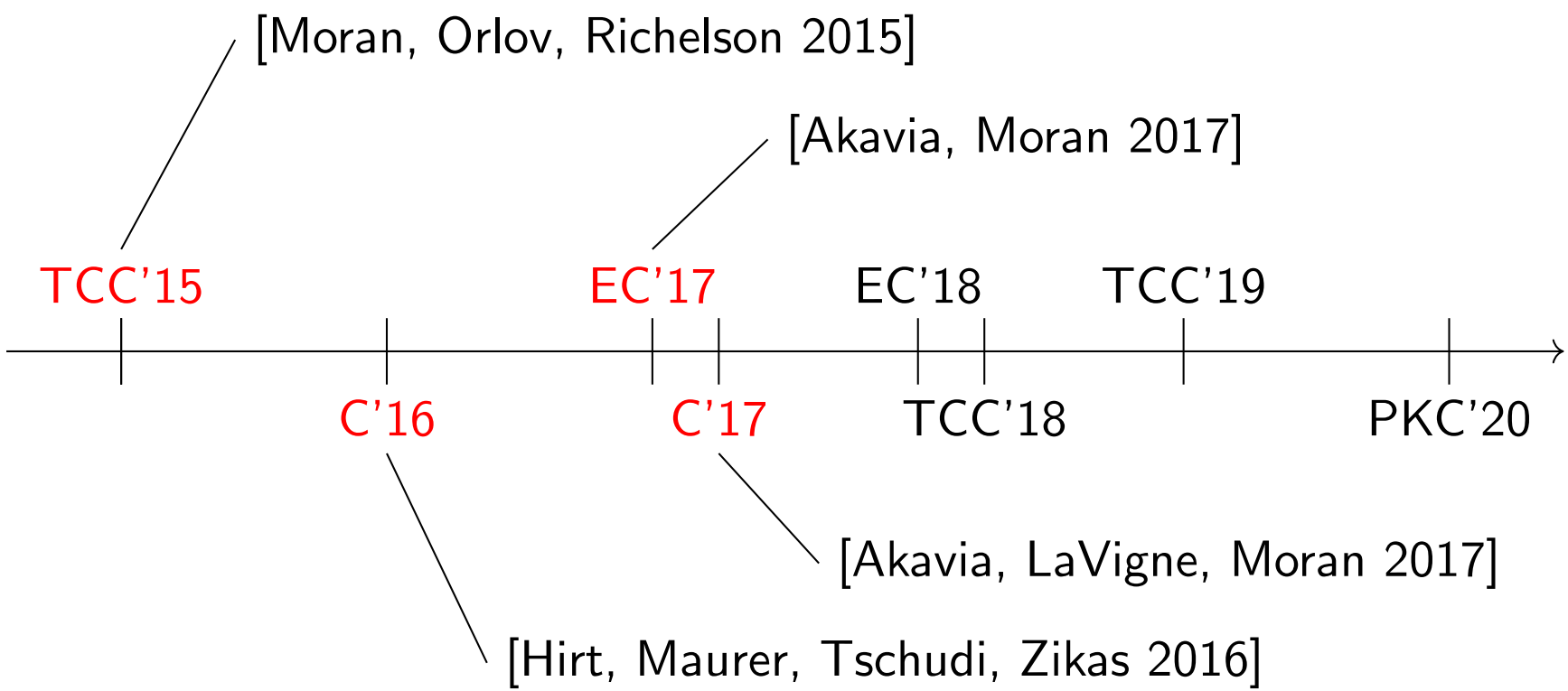
1. Fewer point-to-point secure channels
2. Only local views (and graph class) initially known
3. The topology of the network should not be leaked

Previous Works



Previous Works

- Computational (DDH, QR, or LWE), ($t = n - 1$) passive



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- Computational (DDH, QR, or LWE), ($t = n - 1$) passive
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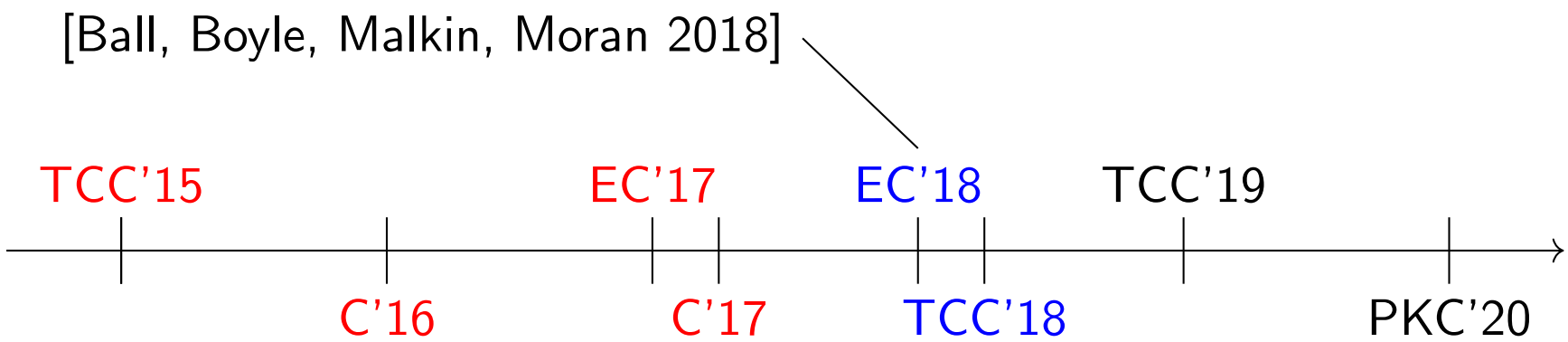

[LaVigne, Liu-Zhang, Maurer, Moran, Mularczyk, Tschudi 2018]

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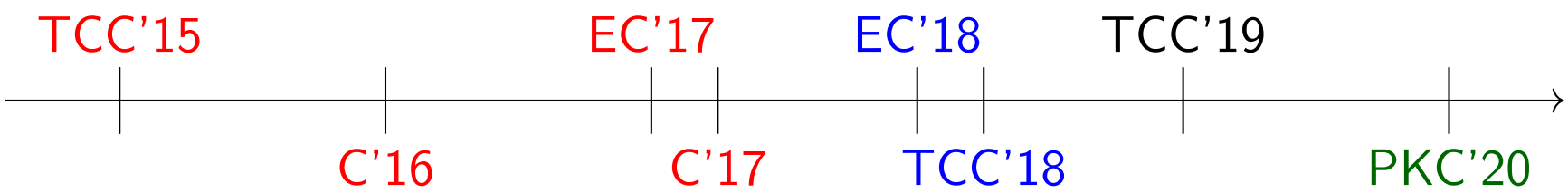
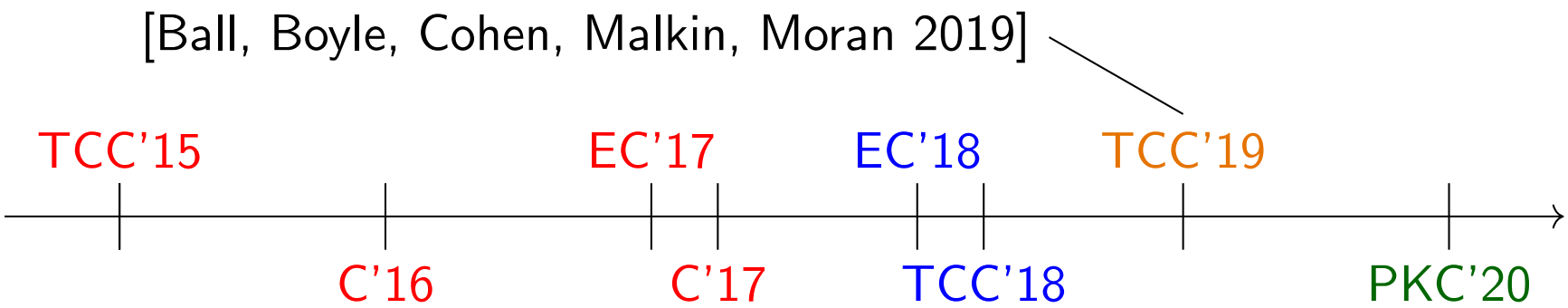
- Asynchronous model, ($t = n - 1$) passive



[LaVigne, Liu-Zhang, Maurer, Moran, Mularczyk, Tschudi 2020]

Previous Works

- Computational (DDH, QR, or LWE), $(t = n - 1)$ passive
- Computational, $(t = n - 1)$ passive + fail-stop
- Asynchronous model, $(t = n - 1)$ passive
- Revisiting information-theoretic setting, $t = 1$ passive



Our work: The Simplest Setting

- **Broadcast** Only
  (and Anonymous Broadcast)
- **One Semi-Honest** Corruption
- Synchronous Communication

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1-THB

1-THAB

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- **One Semi-Honest** Corruption
- Synchronous Communication

Trivial without Topology-Hiding

1-THB

1-THAB

Very Rich with Topology-Hiding!

Our work: The Simplest Setting

- **Broadcast** Only
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Trivial without

For each graph class, what is the minimal (cryptographic) assumption required for 1-THB and 1-THAB?

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Trivial without

For each graph class, what is the minimal (cryptographic) assumption required for 1-THB and 1-THAB?

Information Theoretic (IT) / Key-Agreement (KA) / Oblivious Transfer (OT)

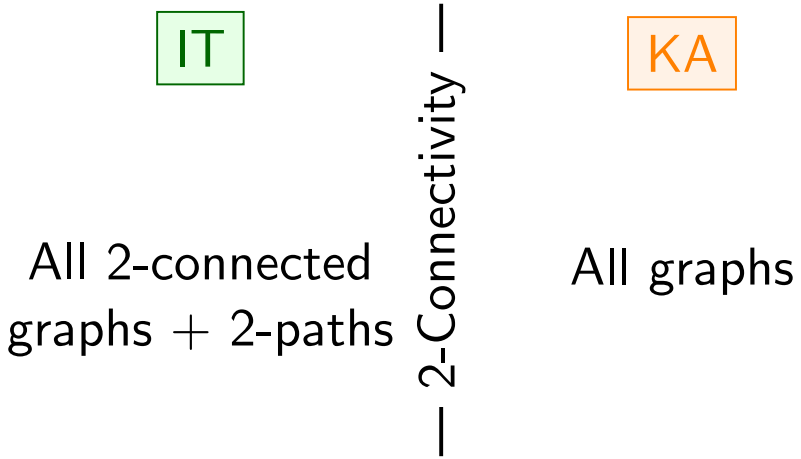
Our Results

Topology-Hiding Broadcast ($t = 1$)

IT	KA
All 2-connected graphs + 2-paths	All graphs

Topology-Hiding Anonymous Broadcast ($t = 1$)

Topology-Hiding Broadcast ($t = 1$)



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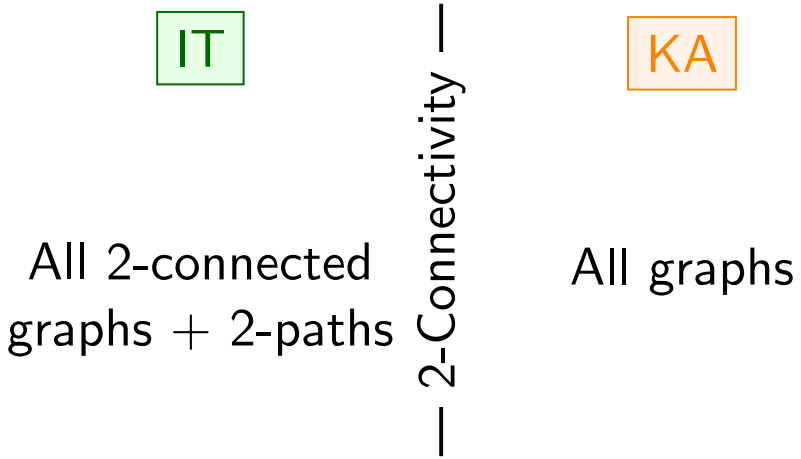

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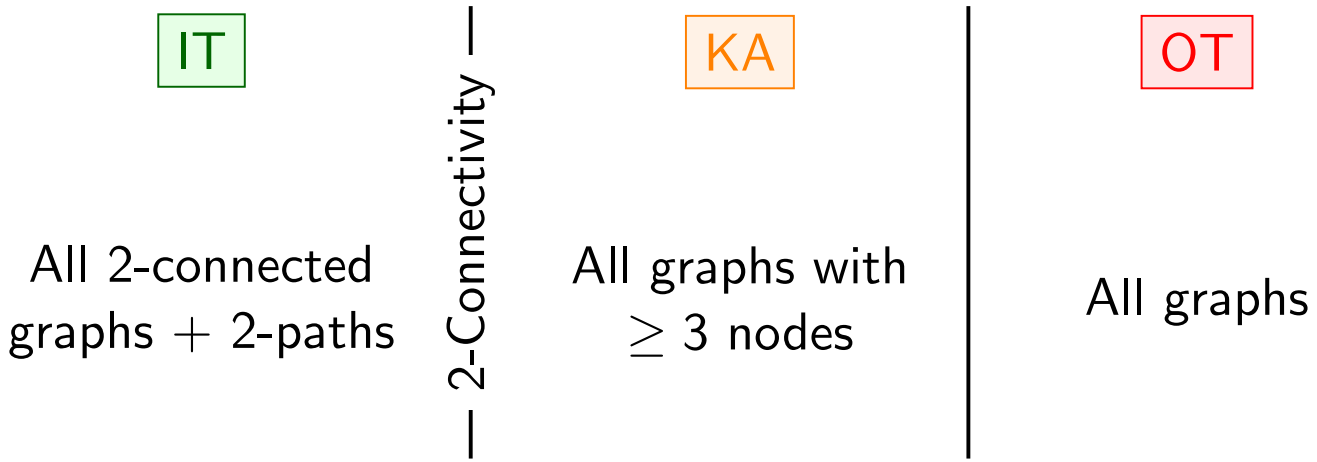

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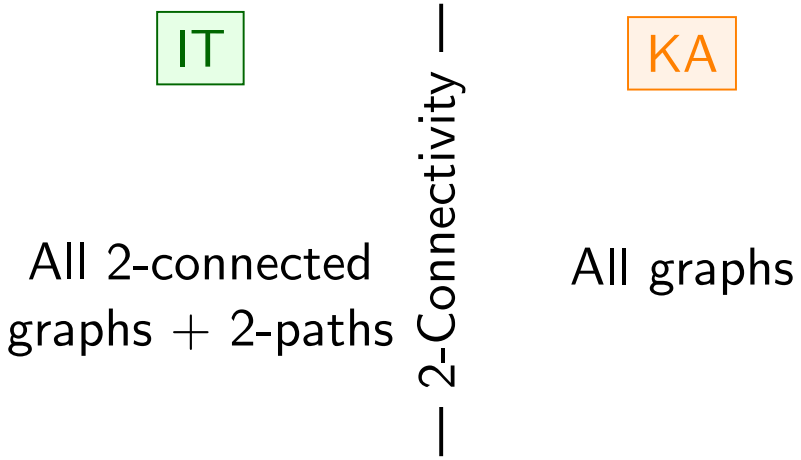

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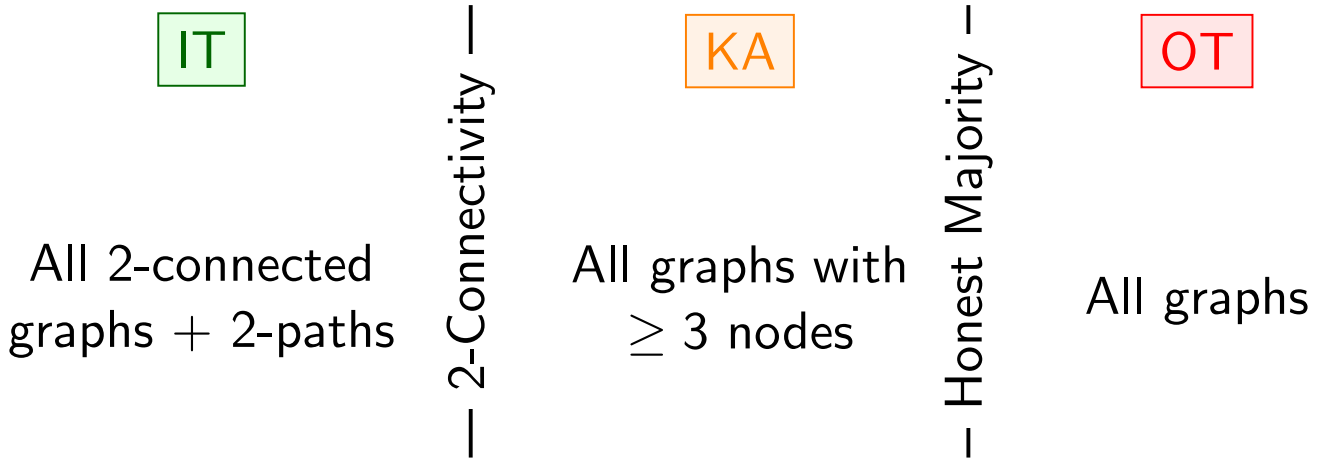
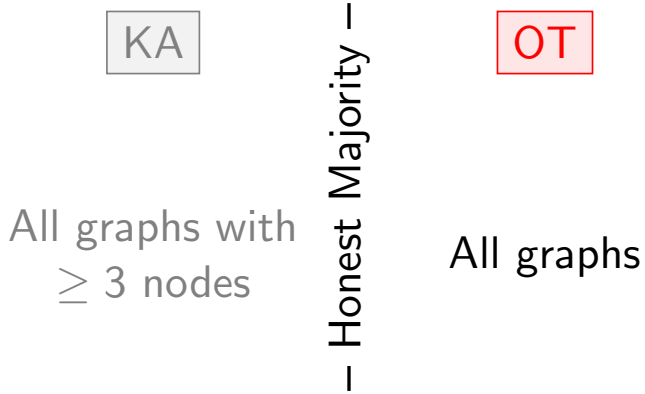
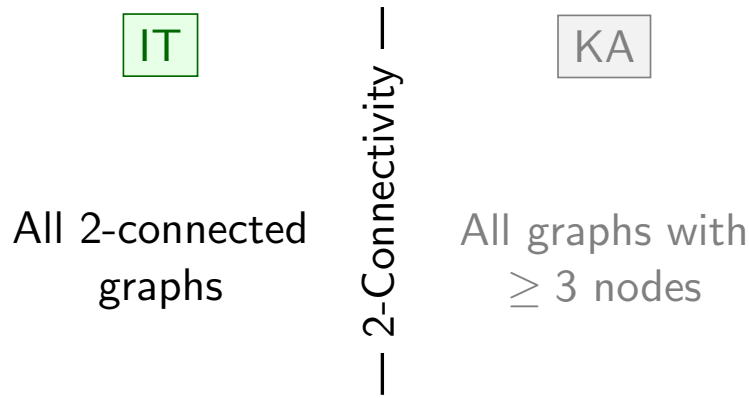


This Talk

1. 'Paths of Length Two and Three': 1-THAB requires OT



2. 'All 2-connected Graphs': 1-THAB is possible Information-Theoretically



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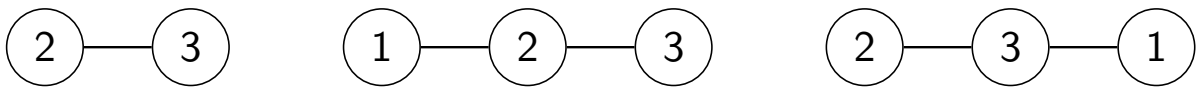
- *Functionality:* Anonymous Broadcast
- *Player Pool:* $\{①, ②, ③\}$
- *Graph Class:* $\mathcal{G}_{P_2\text{-}vs\text{-}P_3}$



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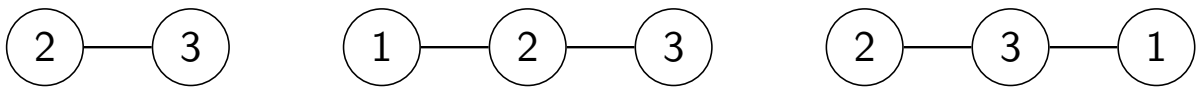
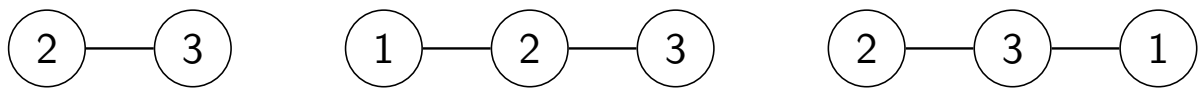


$$\text{1-THAB}(\mathcal{G}_{P_2\text{-}vs\text{-}P_3}) \Rightarrow \text{OT}$$

‘Paths of Length Two and Three’: 1-THAB requires OT

- *Functionality:* Anonymous Broadcast in 2 Rounds
- *Player Pool:* $\{①, ②, ③\}$
- *Graph Class:*



[2-round 1-THAB($\mathcal{G}_{P_2\text{-}vs\text{-}P_3}$)] $\Rightarrow$ Semi-honest AND $\Rightarrow$ OT

[2-round 1-THAB($\mathcal{G}_{P_2\text{-}vs\text{-}P_3}$)] $\Rightarrow$ OT (Correctness)

Alice | Bob

If $x = 0$

If $x = 1$

If $y = 0$

If $y = 1$

$[\text{2-round 1-THAB}(\mathcal{G}_{P_2\text{-vs-}P_3})] \Rightarrow \text{OT}$ (Correctness)

Alice | Bob

$r \xleftarrow{\$} \{0,1\}^\lambda \longrightarrow$

If $x = 0$

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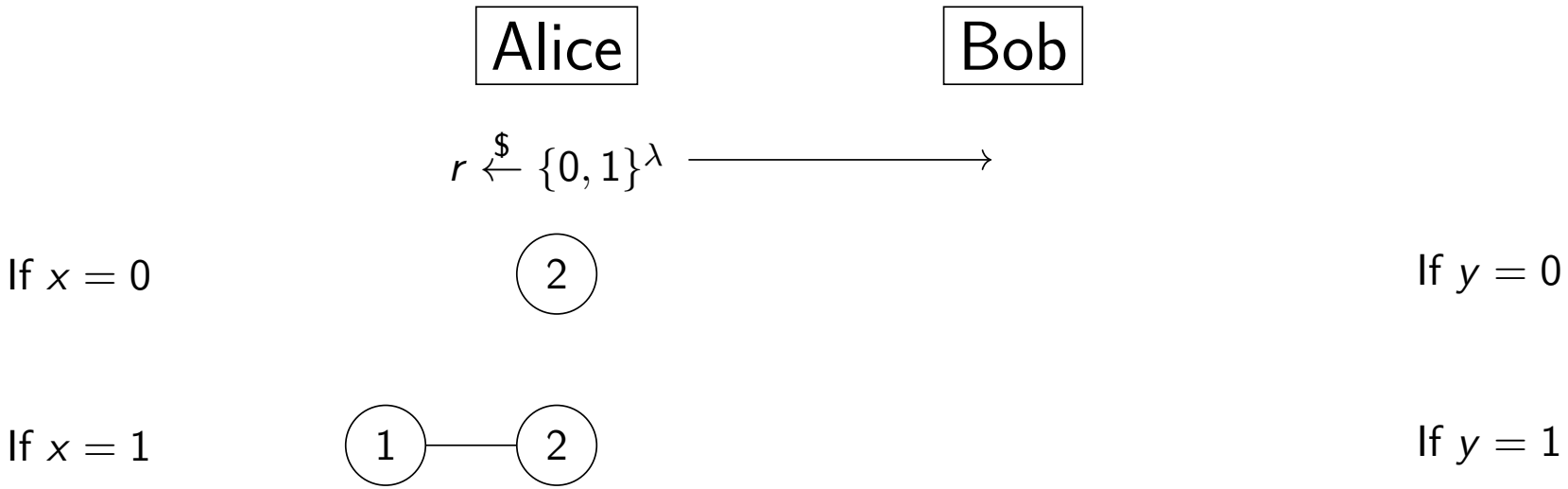

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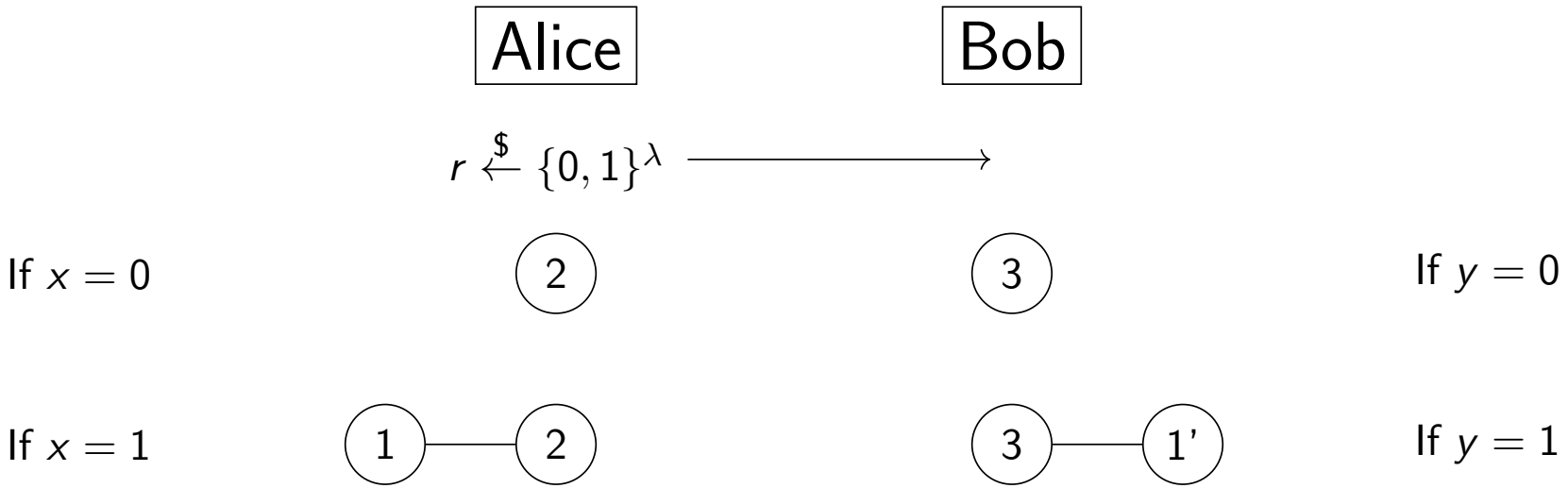

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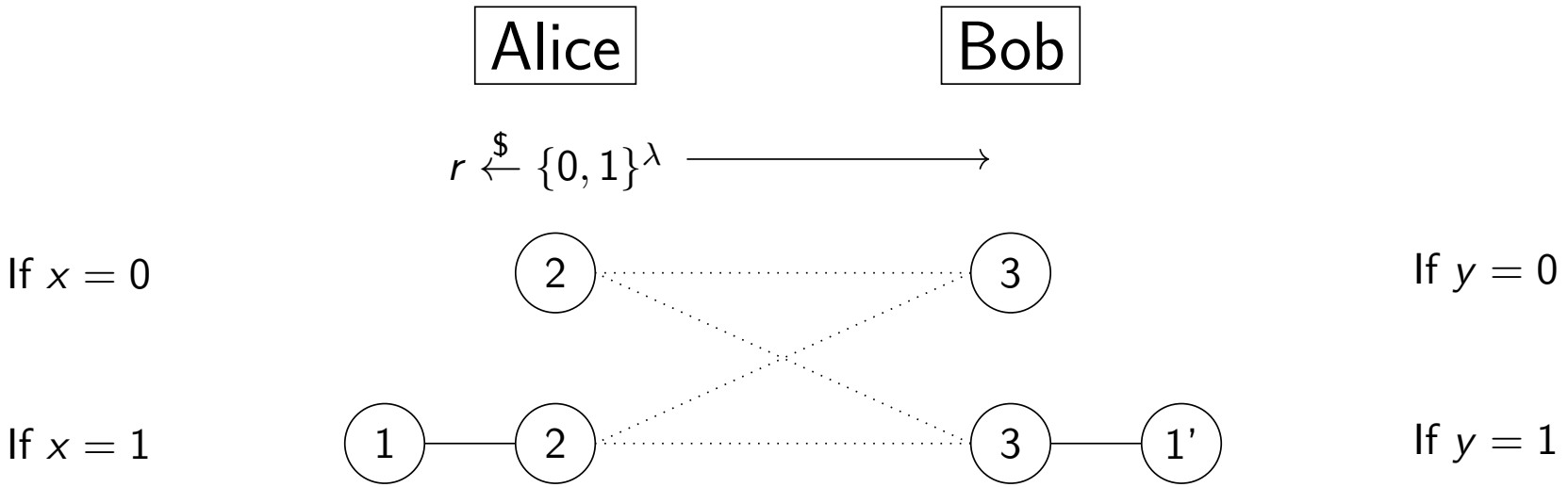

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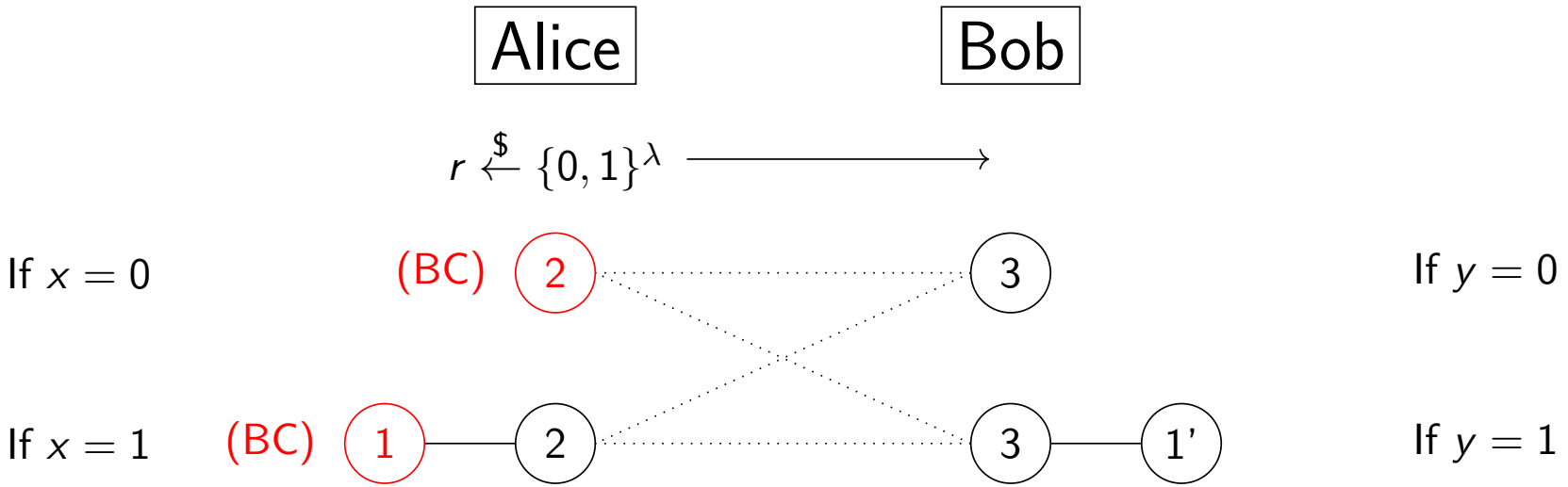

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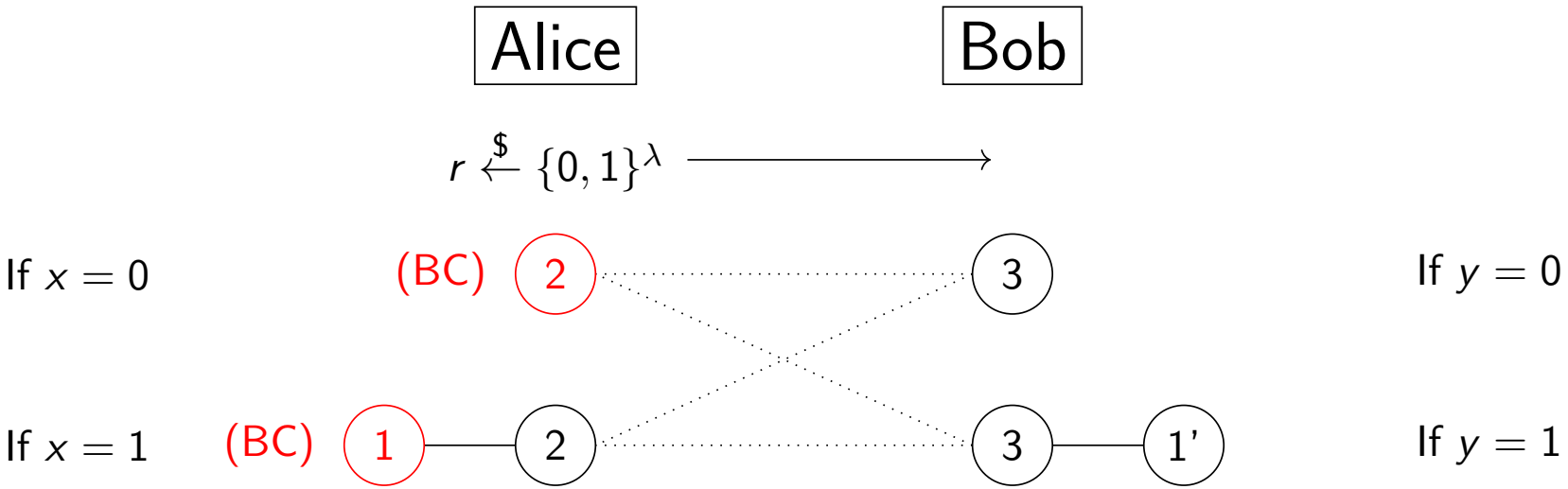

If $(x, y) = (0, 0)$:

If $(x, y) = (0, 1)$:

If $(x, y) = (1, 0)$:

If $(x, y) = (1, 1)$:

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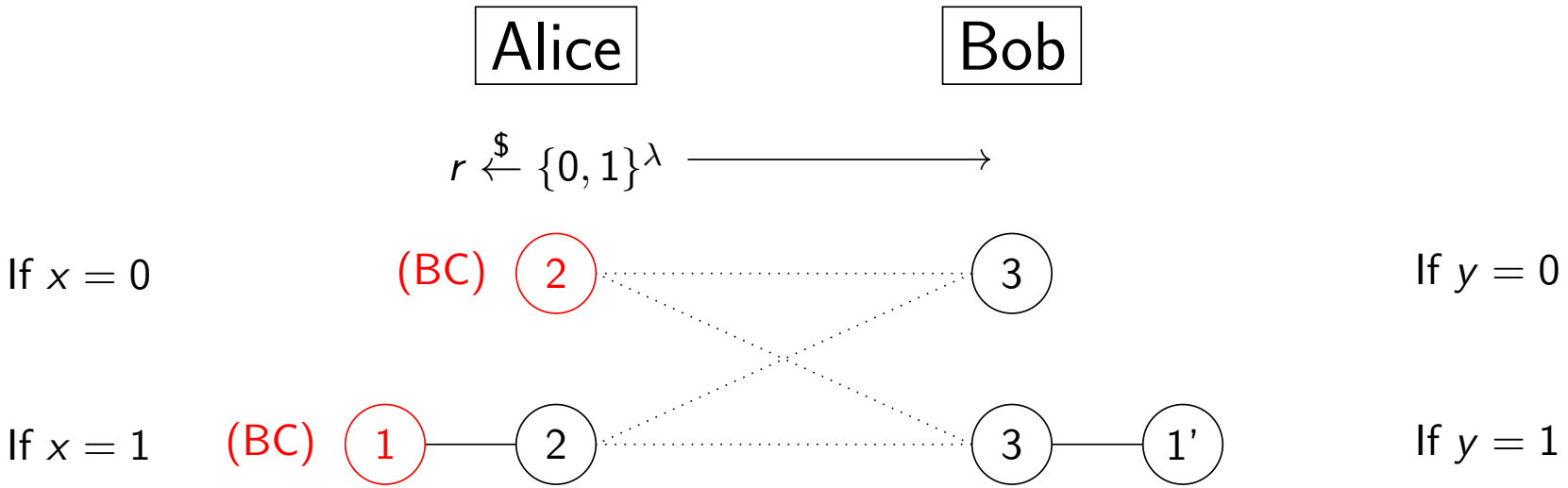

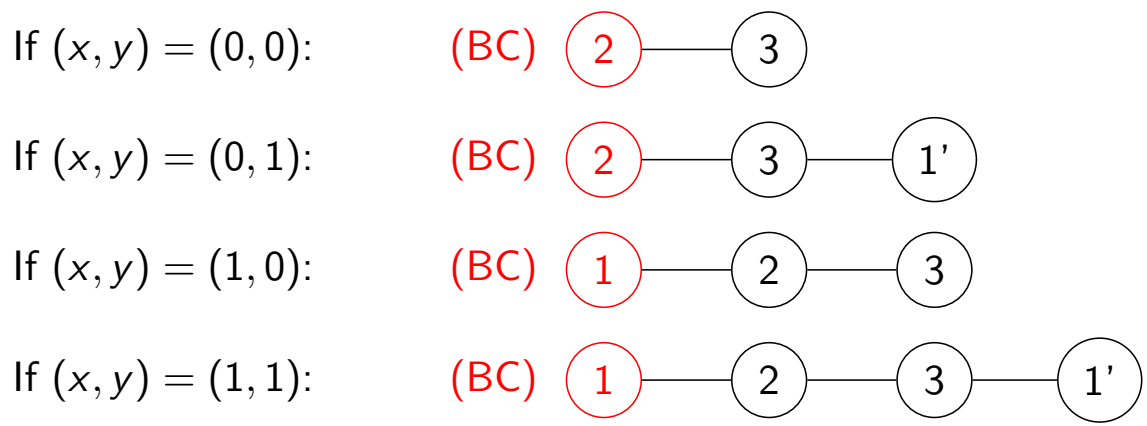

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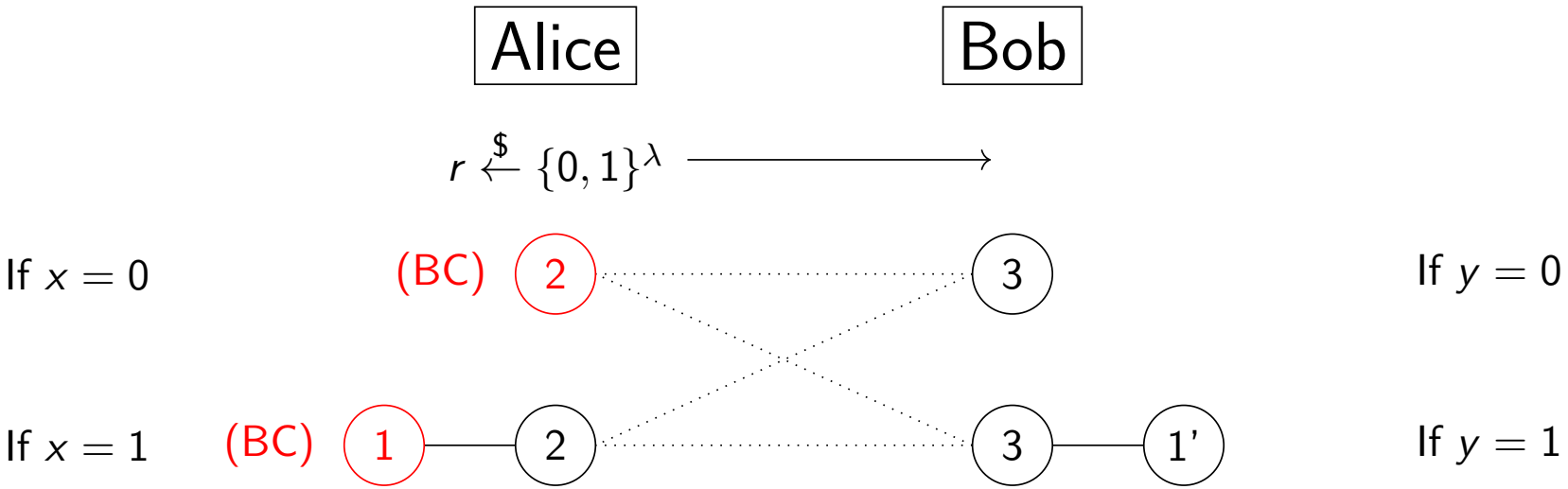

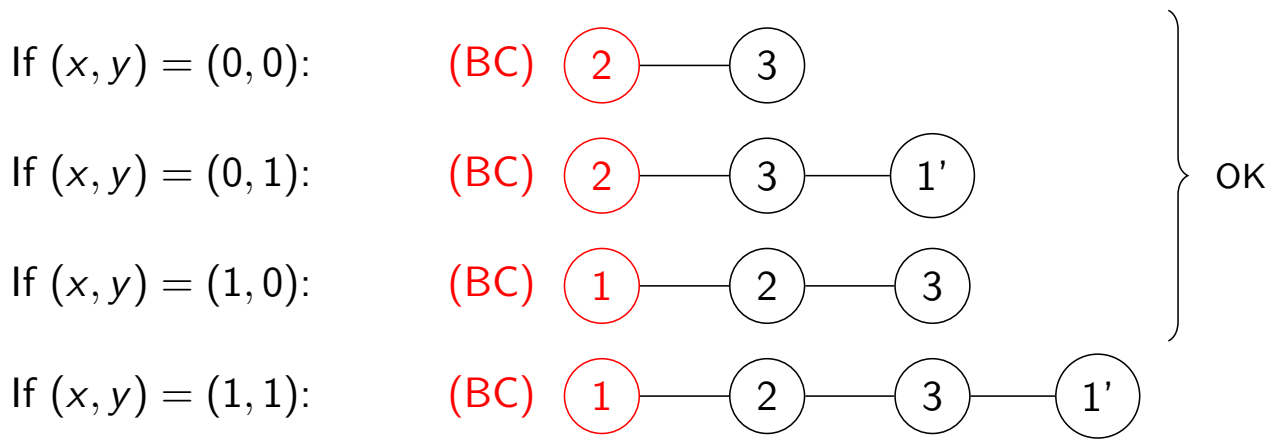

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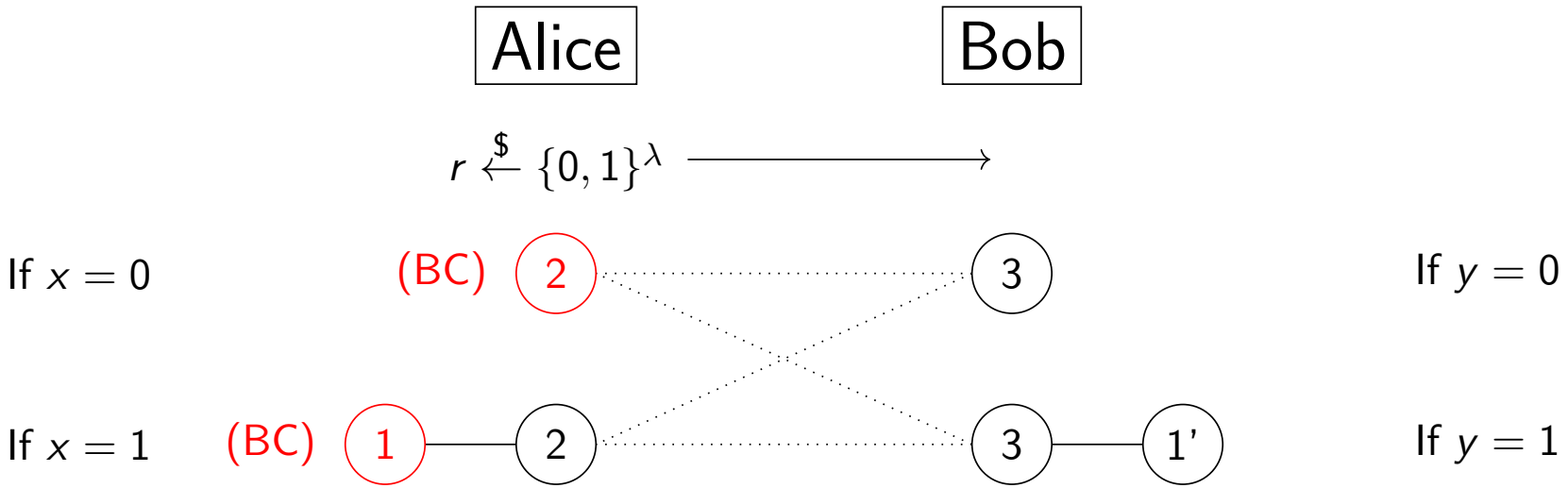

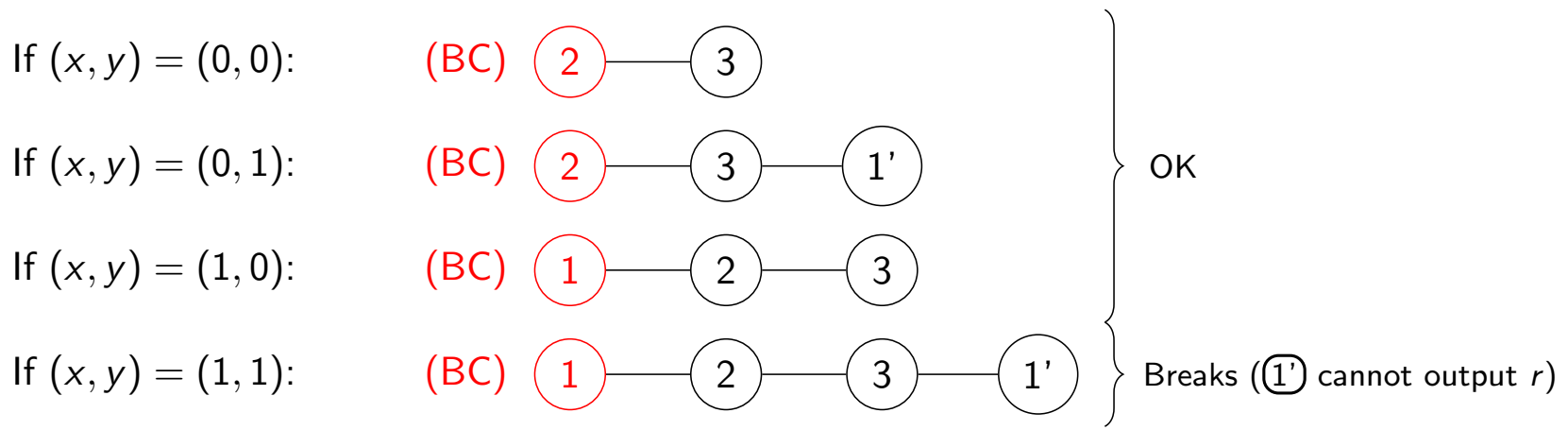

[2-round 1-THAB($\mathcal{G}_{P_2\text{-vs-}P_3}$)] $\Rightarrow$ OT (Security)

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Bob

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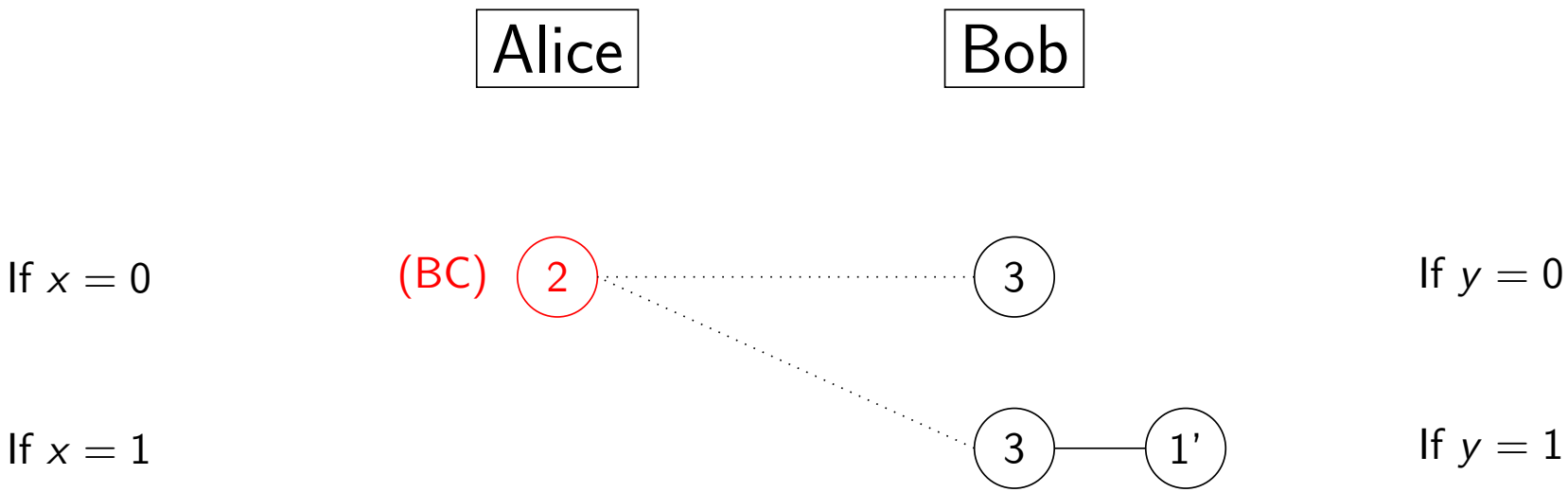

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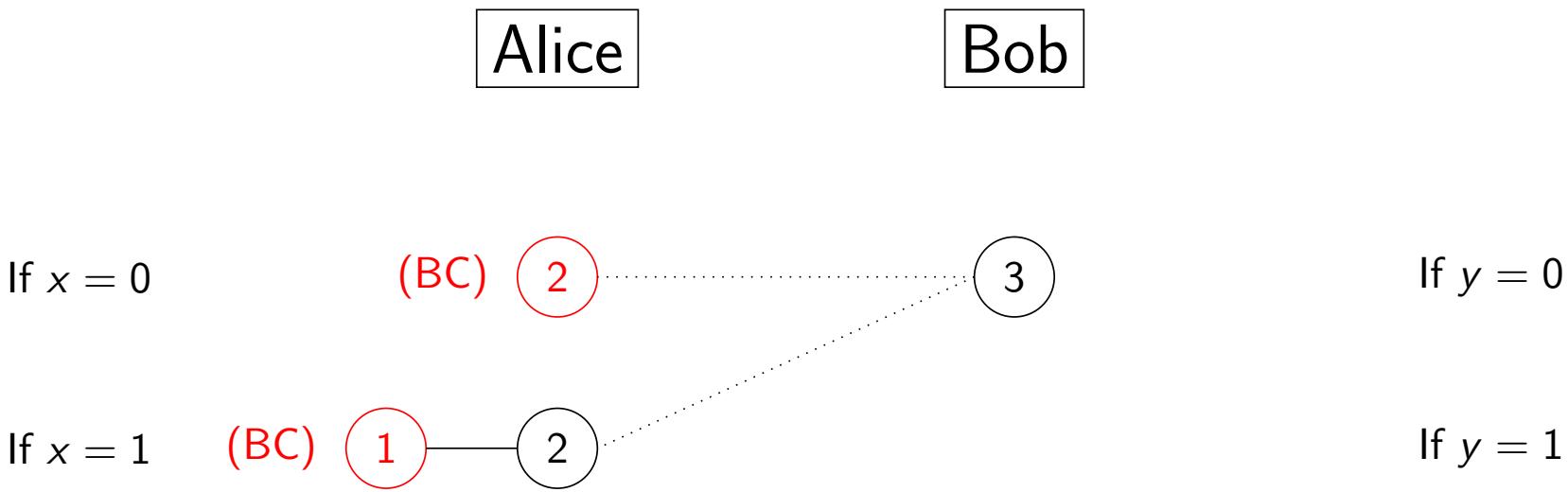

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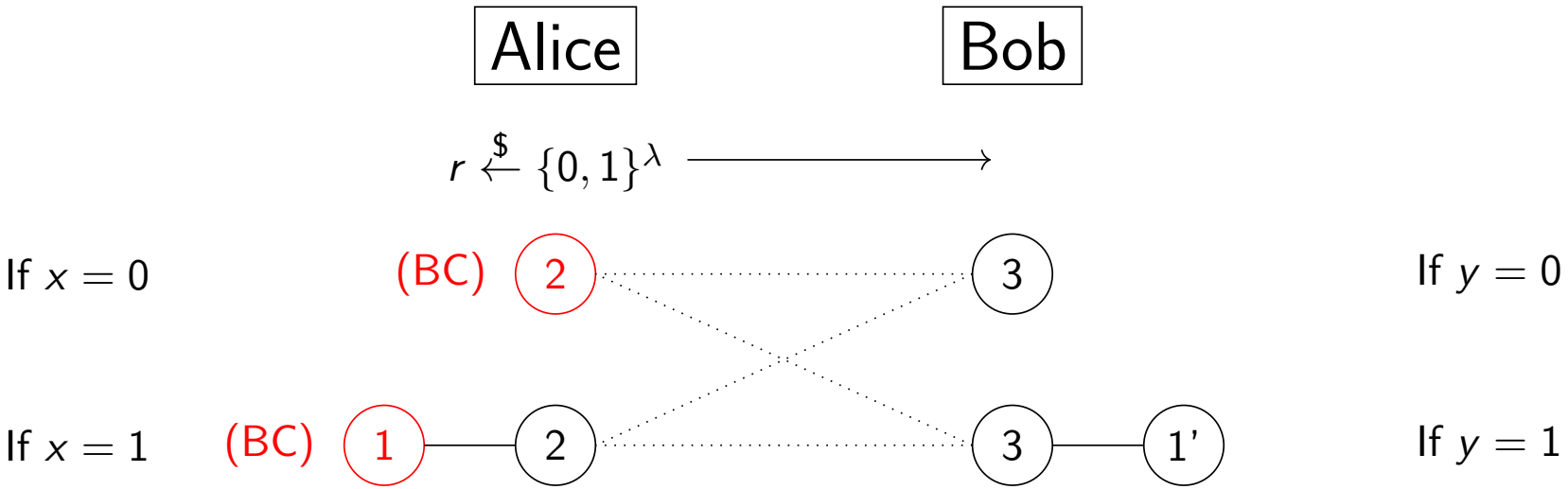
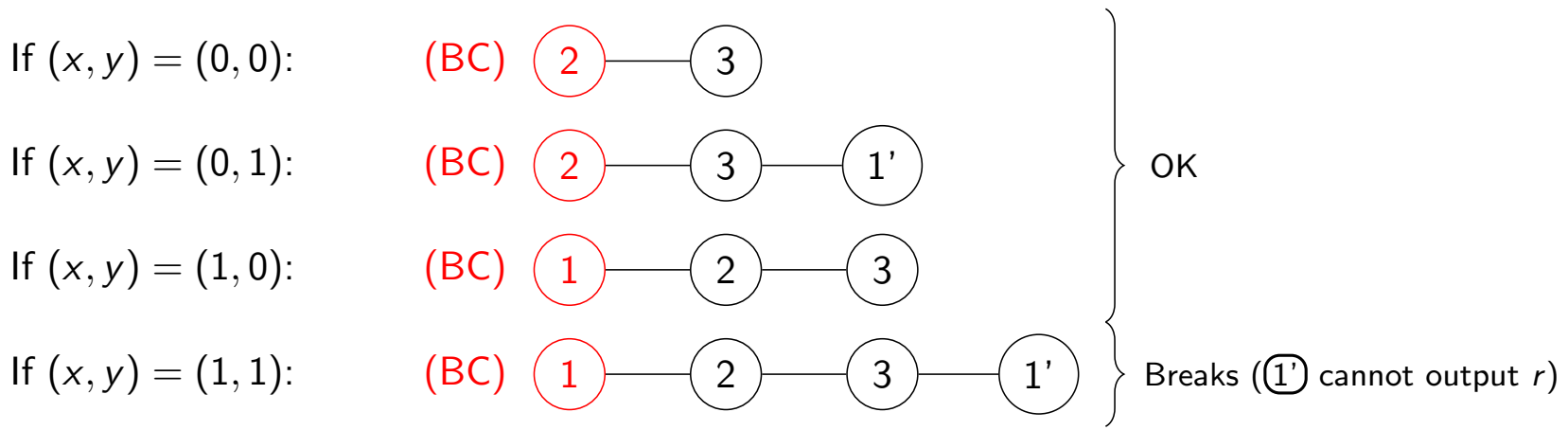




‘All 2-connected Graphs’: 1-THAB is possible Information-Theoretically

‘All 2-connected Graphs’: 1-THAB is possible IT

- *Functionality:* Anonymous Broadcast
- *Player Pool:* $\left\{\boxed{P_1}, \ldots, \boxed{P_N}\right\}$
- *Graph Class:* All two-connected graphs

1-THAB($\mathcal{G}_{2\text{-}conn}$) is possible

- *Functionality:* Anonymous Broadcast
- *Player Pool:* $\{\boxed{P_1}, \ldots, \boxed{P_N}\}$
- *Graph Class:* All two-connected graphs with all the players

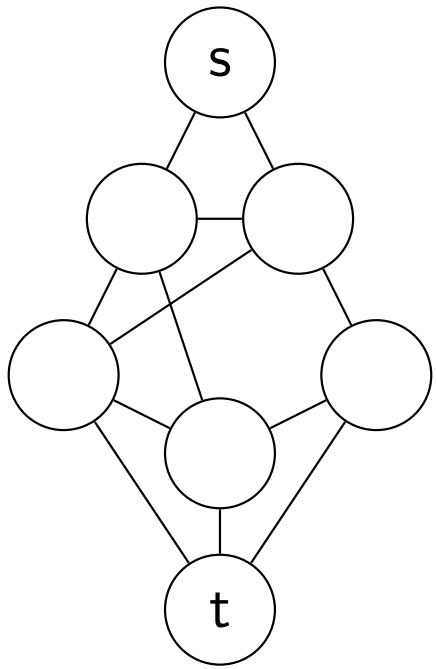
1-THAB($\mathcal{G}_{2\text{-}conn}$) is possible Unconditionally

1-THAB($\mathcal{G}_{2\text{-}conn}$) is possible

- *Functionality:* Secure Message Transmission from Ⓢ to Ⓣ
- *Player Pool:* $\left\{\boxed{P_1}, \ldots, \boxed{P_N}\right\}$
- *Graph Class:* All two-connected graphs with all the players

1-SMT$_{s \to t}(\mathcal{G}_{P_2\text{-}vs\text{-}P_3})$ is possible Unconditionally

1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible



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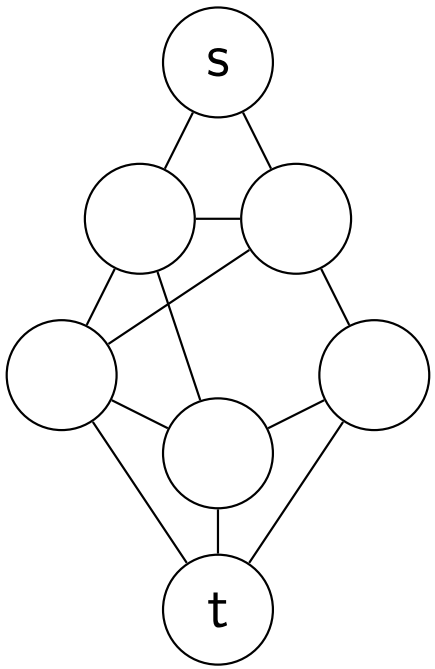
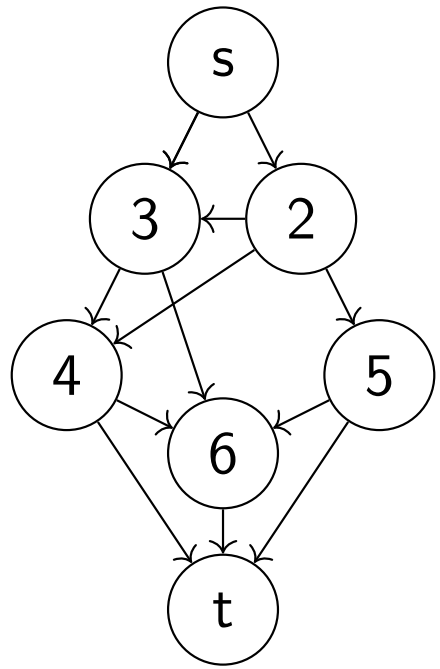
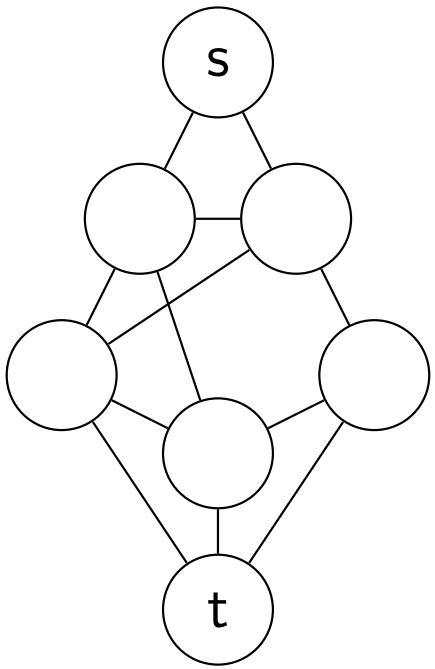
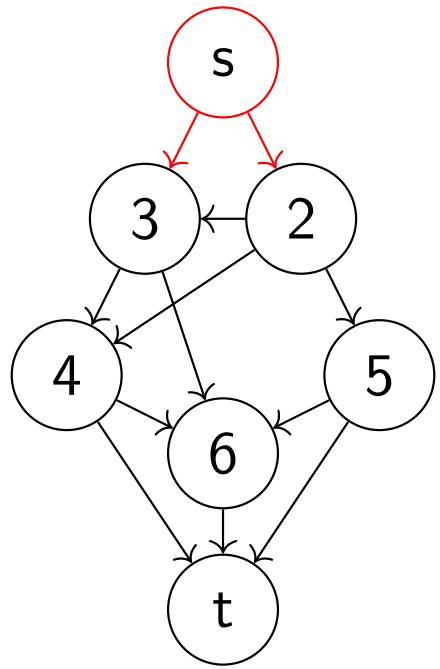




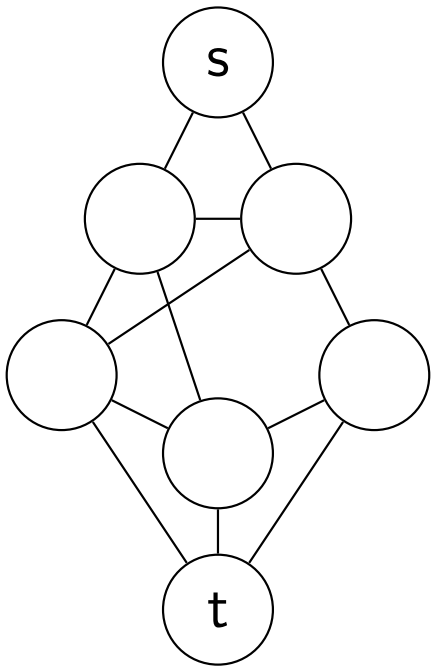
bipolar orientation from s to t:
orientation as D.A.G. with single source s and single sink t

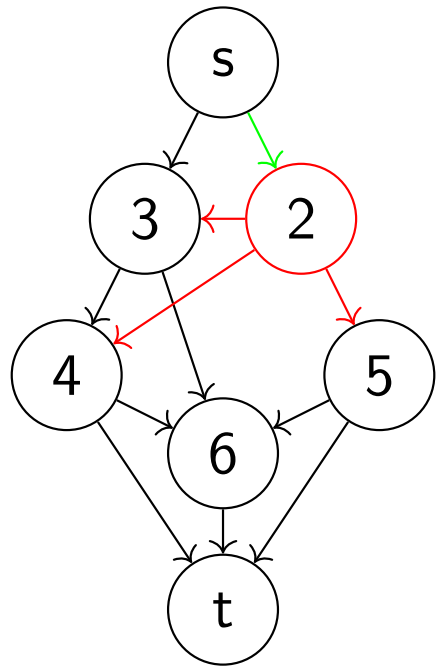
1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible



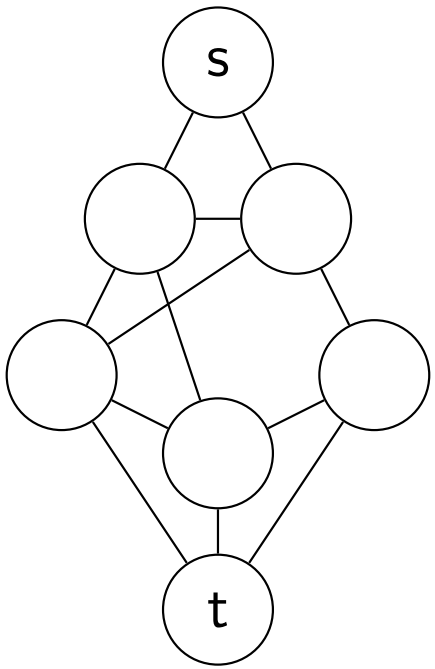


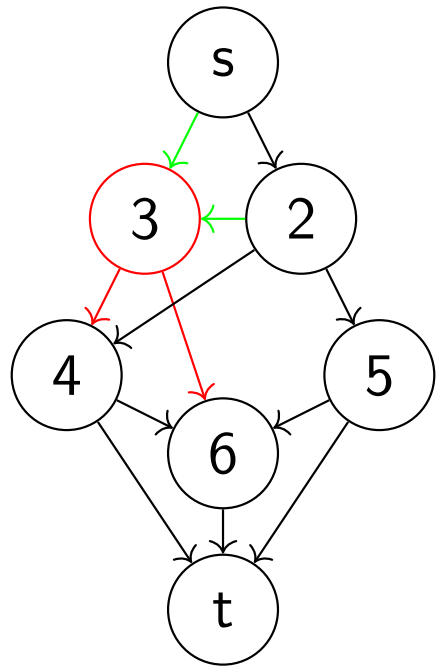
1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible



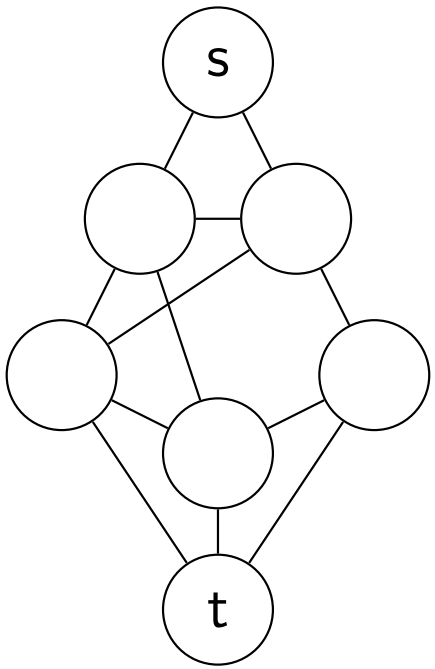


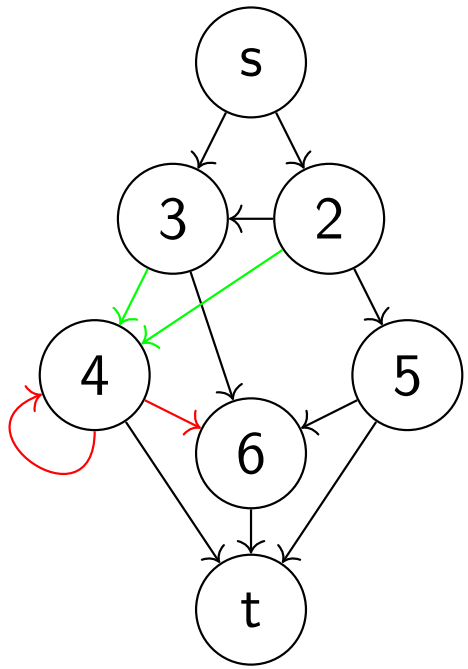
1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible



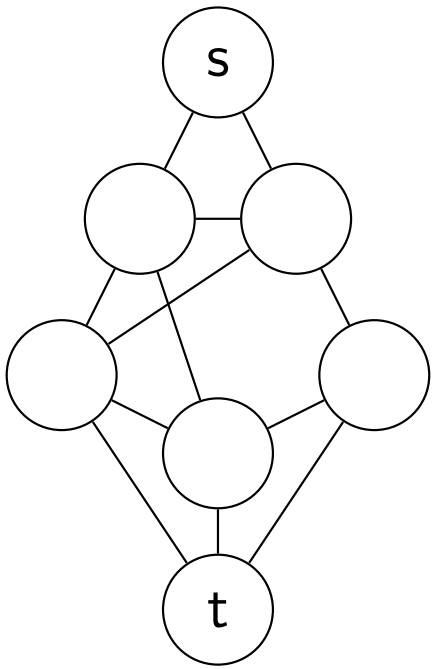


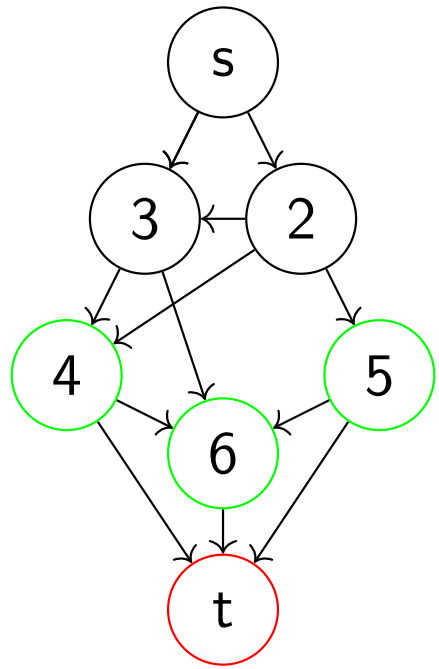
1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible





1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible





1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible

Public:

- $\mathcal{G} = \{G_1, G_2, \ldots, G_k\}$ on $V = [N]$
- st-orientations $s \to t$: $H_1, H_2, \ldots H_k$

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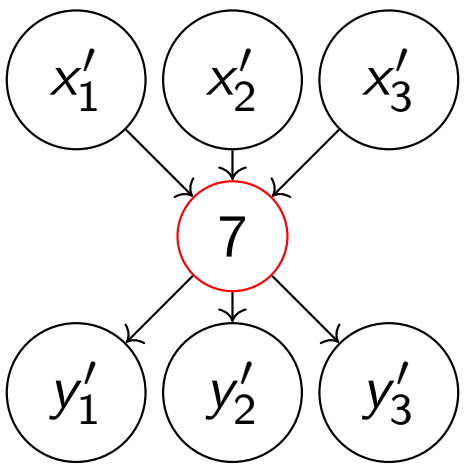
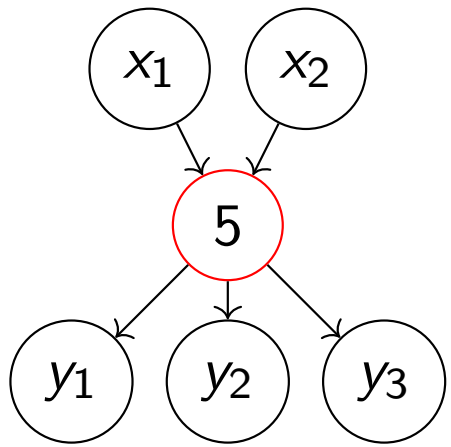
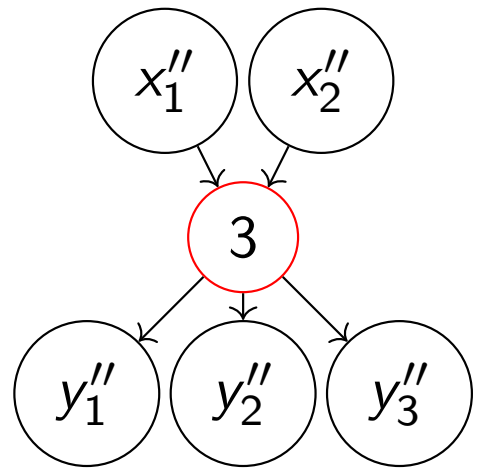
$(H_{i'})$



(H_i)



$(H_{i''})$



1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible

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- $\mathcal{G} = \{G_1, G_2, \ldots, G_k\}$ on $V = [N]$
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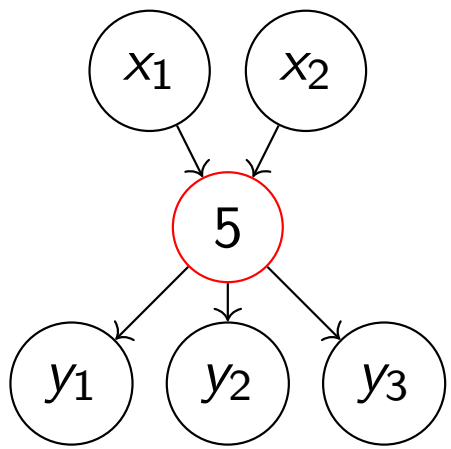
(H_i)



1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible

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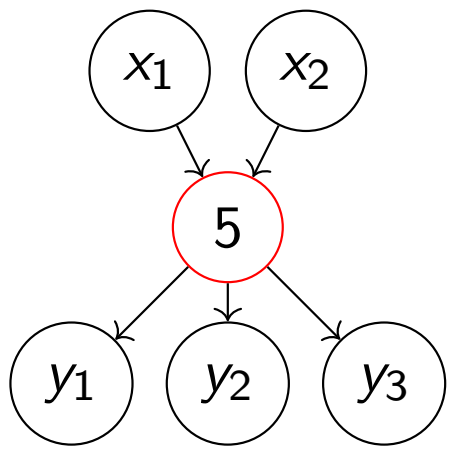
If $N_G = N_{G_i}$

(H_i)



1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible

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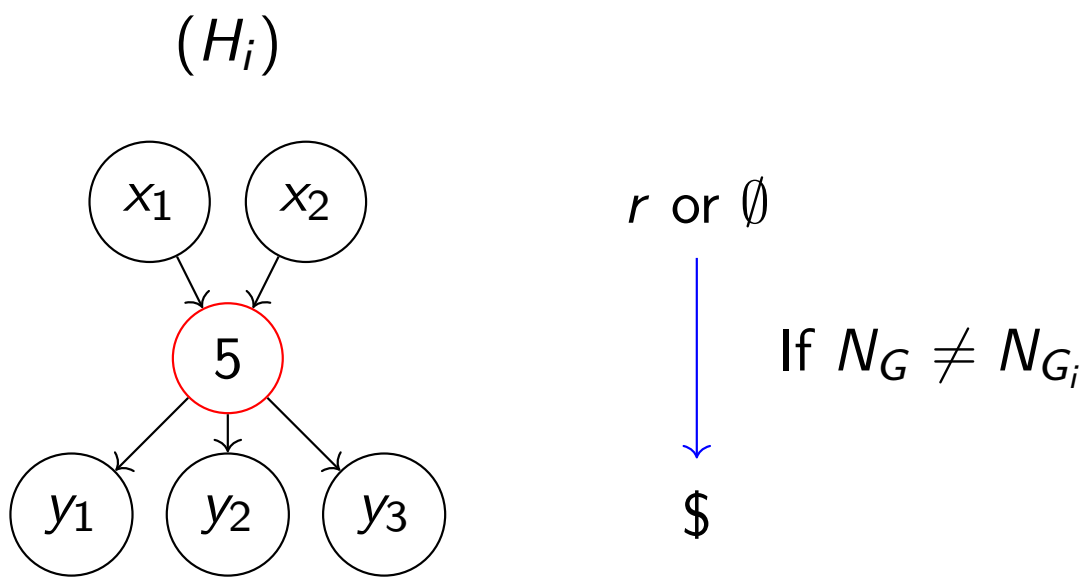
- $\mathcal{G} = \{G_1, G_2, \ldots, G_k\}$ on $V = [N]$
- st-orientations $s \to t$: $H_1, H_2, \ldots H_k$



1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible

Public:

- $\mathcal{G} = \{G_1, G_2, \ldots, G_k\}$
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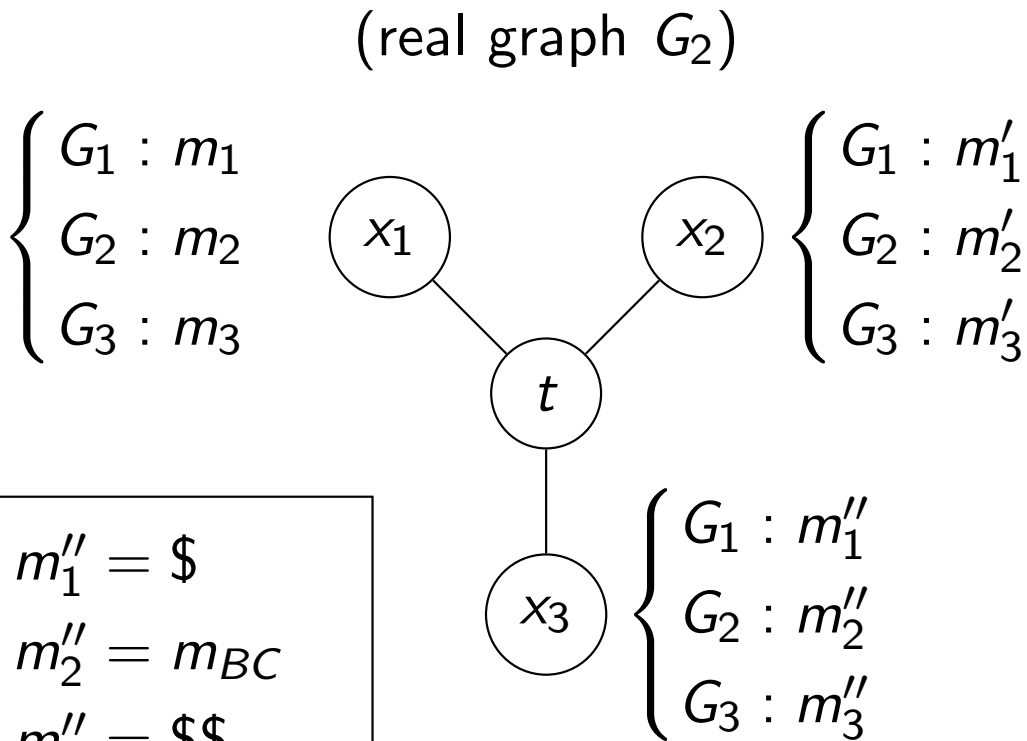
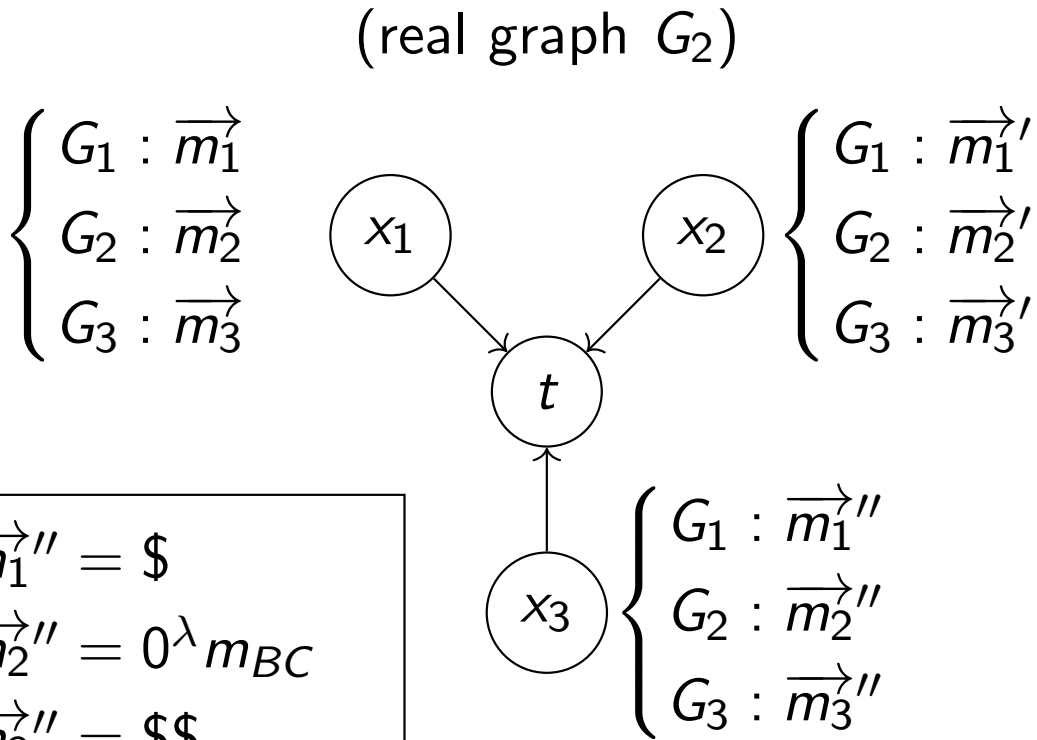


$$\begin{cases} m_1 + m'_1 + m''_1 = \$ \\ m_2 + m'_2 + m''_2 = m_{BC} \\ m_3 + m'_3 + m''_3 = \$\$ \end{cases}$$

1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible

Public:

- $\mathcal{G} = \{G_1, G_2, \ldots, G_k\}$
- st-orientations $s \to t$: $H_1, H_2, \ldots H_k$



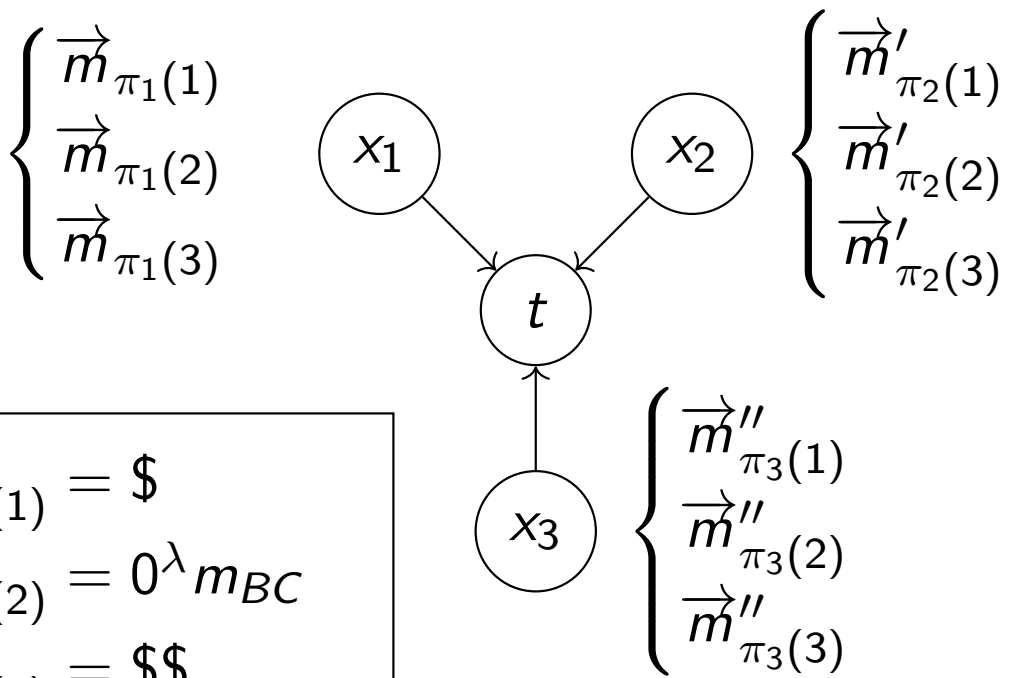
$$\begin{cases} \overrightarrow{m_1} + \overrightarrow{m_1}' + \overrightarrow{m_1}'' = \$ \\ \overrightarrow{m_2} + \overrightarrow{m_2}' + \overrightarrow{m_2}'' = 0^{\lambda} m_{BC} \\ \overrightarrow{m_3} + \overrightarrow{m_3}' + \overrightarrow{m_3}'' = \$\$ \end{cases}$$

1-SMT$_{s\to t}(\mathcal{G}_{2\text{-}conn})$ is possible

Public:

- $\mathcal{G} = \{G_1, G_2, \ldots, G_k\}$
- st-orientations $s \to t$: $H_1, H_2, \ldots H_k$

(real graph G_2)



$$\begin{cases} \overrightarrow{m}_{\pi_1(1)} + \overrightarrow{m}'_{\pi_2(1)} + \overrightarrow{m}''_{\pi_3(1)} = \$ \\ \overrightarrow{m}_{\pi_1(2)} + \overrightarrow{m}'_{\pi_2(2)} + \overrightarrow{m}''_{\pi_3(2)} = 0^{\lambda} m_{BC} \\ \overrightarrow{m}_{\pi_1(3)} + \overrightarrow{m}'_{\pi_2(3)} + \overrightarrow{m}''_{\pi_3(3)} = \$\$ \end{cases}$$

Our Results (Extended)

Topology-Hiding Anonymous Broadcast ($t = 1$)

IT	KA	OT
All 2-connected graphs + 2-paths	All graphs with ≥ 3 nodes	All graphs

Topology-Hiding Anonymous Broadcast ($t = 1$)

IT	KA	OT
Only 2-connected graphs + 2-paths	Only graphs with ≥ 3 nodes	Contains paths of length 2 and 3

Topology-Hiding Anonymous Broadcast ($t = 1$)

IT	KA	???	OT
Only 2-connected graphs + 2-paths	Only graphs with ≥ 3 nodes	Contains a path of length 2 and a graph with ≥ 3 nodes not 2-connected	Contains paths of length 2 and 3

Thank You!