On the spectral features of robust probing security

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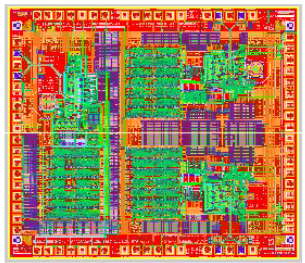
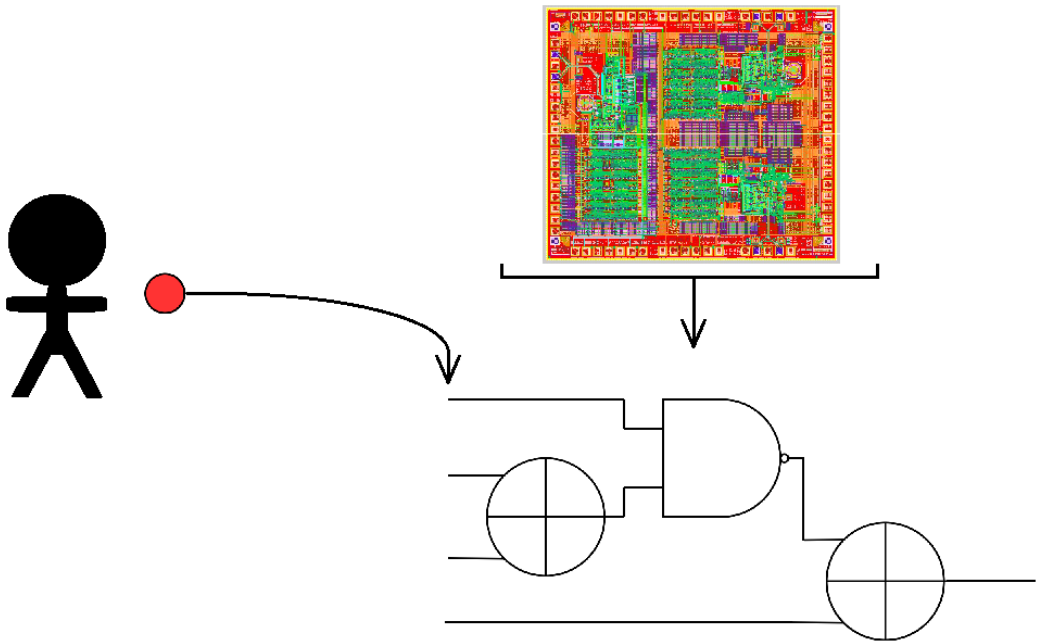
Overview

d-PROBING SECURITY



d-PROBING SECURITY



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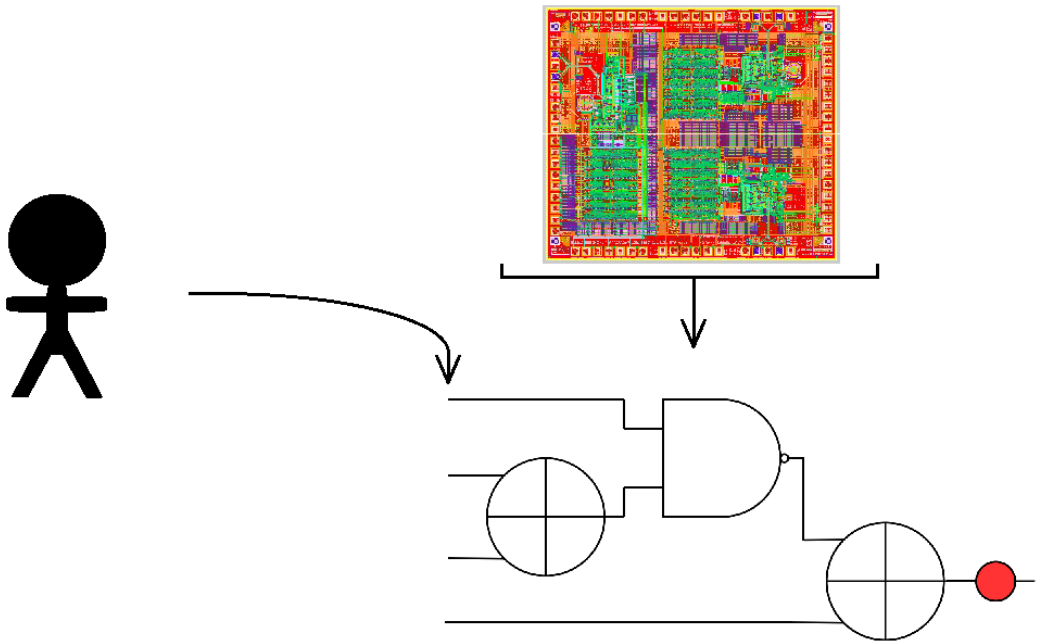
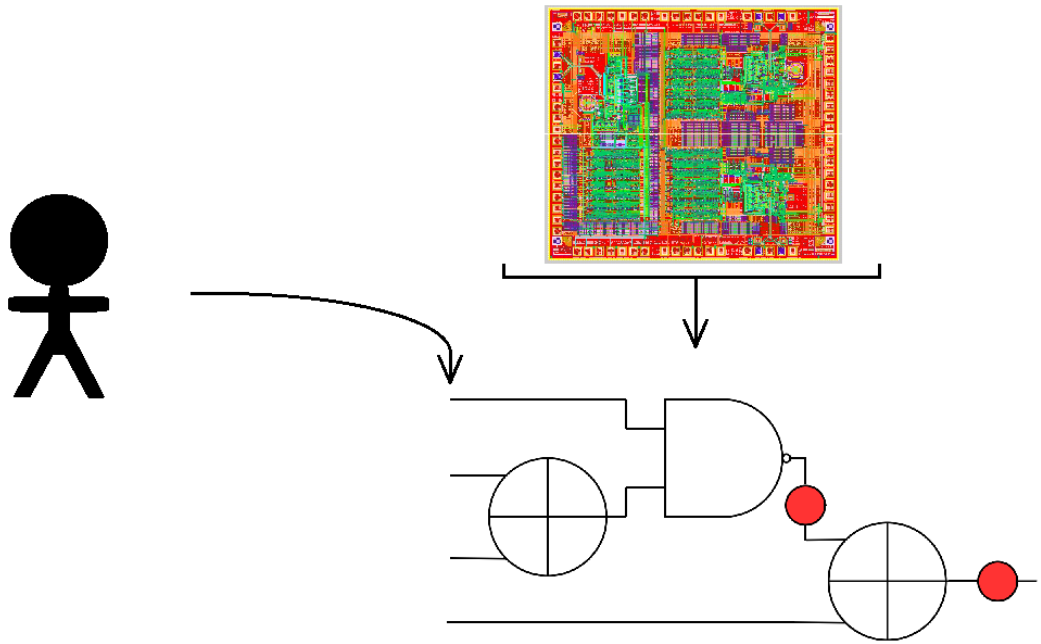


d-probing security



Probing attack

The attacker places a *probe* on a wire of interest and recover some information about the value carried along that wire during computation.

d-probing security

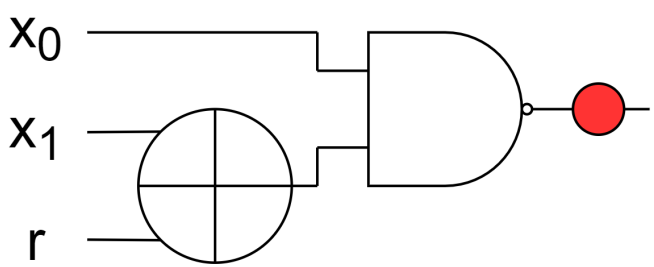
Definition

A gadget is d-**probing secure** if, given at most d probes, it is impossible to derive information about the secret values, also encoded in the masks/shares.

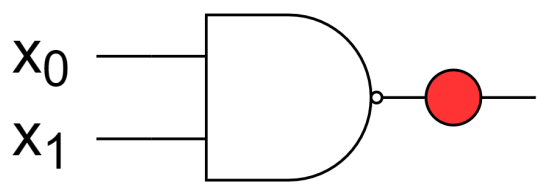
Example

x secret, x_0 and x_1 shares such that $x = x_0 + x_1$

1-probing secure



NOT 1-probing secure



d-Non Interference security

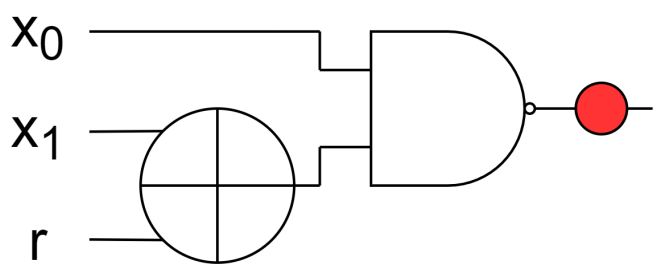
Definition

A gadget is d-**NI** if, given at most d probes, it is possible to derive information about at most d masks/shares of any secret value.

Example

x secret, x_0 and x_1 shares such that $x = x_0 + x_1$

1-NI



d-Strong Non Interference security

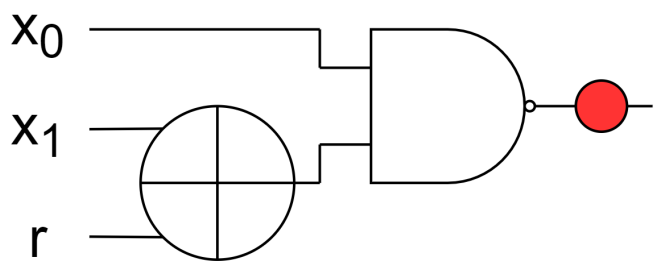
Definition

A gadget is d-**SNI** if, given at most d_1 **internal probes** and d_2 **output probes** such that $d_1 + d_2 = d$, it is possible to derive information about at most d_1 masks/shares of any secret value.

Example

x secret, x_0 and x_1 shares such that $x = x_0 + x_1$

NOT 1-SNI



d-Strong Non Interference security

Definition

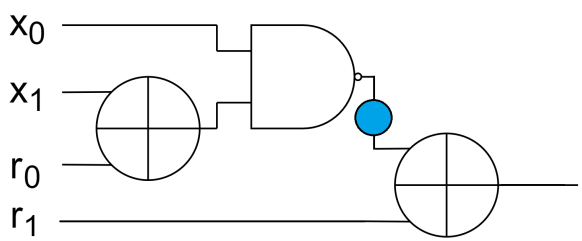
A gadget is d-**SNI** if, given at most d_1 **internal probes** and d_2 **output probes** such that $d_1 + d_2 = d$, it is possible to derive information about at most d_1 secret values, also encoded in the masks/shares.

Example

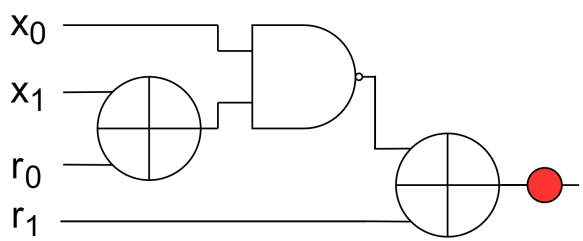
x secret, x_0 and x_1 shares such that $x = x_0 + x_1$

1-SNI

Internal probe



Output probe



Robust Probing Security

Extended Probes

Probes that model the leakage situation in presence of some physical defaults.

Types od Extended probes[1]

- Modelling glitches, i.e. combinatorial recombination
- Modelling transitions, i.e. memory recombinations
- Modelling couplings, i.e.routing recombinations

[1] S. Faust et Al., *Composable Masking Schemes in the Presence of Physical Defaults and the Robust Probing Model*

Motivation: mathematical improvement

Research standpoint

- Previous works: instance-by-instance approaches or tools (maskVerif[2])
- Our work: new conceptual tools to derive general solutions and rules

Development standpoint

- Previous works: efficient approaches might need validation
- Our work: further verification approach based on the exact theory of Boolean Functions

[2] G. Barthe et Al., *maskVerif: automated analysis of software and hardware higher-order masked implementations.*

Our contribution

Exploited tools

- Boolean Function Theory
  - Walsh Matrices
  - Tensor Product
  - String Diagrams

New contributions

- Vulnerability Profile
- Composition Rules
- Classification of Extended Probes

Our method

Walsh Matrix

- Given a Boolean function f, with m inputs and n outputs, any element of its Walsh matrix is:

$$\hat{f}_{\omega,\alpha} = \sum_{x \in \mathbb{F}_2^n} (-1)^{\omega^T f(x) \oplus \alpha^T x}$$

- Matrix that describes the results profile of a Boolean Function
- To any matrix corresponds only one function and viceversa
- Its dimension is $2^n \times 2^m$

Correlation Matrix

Matrix computed from the Walsh matrix:

$$\widetilde{W_f}(\omega, \alpha) := (\hat{f}_{\omega,\alpha} \neq 0)$$

Our method

Example

$$f(a_0, a_1, r_0, r_1) = \begin{bmatrix} o_0 \\ o_1 \\ p_0 \end{bmatrix} = \begin{bmatrix} a_0 + r_0 + r_1 \\ a_1 + r_0 + r_1 \\ a_1 + r_0 \end{bmatrix}$$

Correlation matrix $\widetilde{W_f}$:

	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	
	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	γ_{r_1}
	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	γ_{r_0}
	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	γ_{a_1}
γ_{p_0}	γ_{o_1}	γ_{o_0}															γ_{a_0}
0	0	0	1														
0	0	1												1			
0	1	0													1		
0	1	1			1												
1	0	0					1										
1	0	1										1					
1	1	0							1								
1	1	1				1											

Our method

Compact representation of $\widetilde{W_f}$

Reshaping of the Correlation matrix $\widetilde{W_f}$, by compacting the spectral coefficients, taking into account only the number of shares of each original variable.

Example

		0	0	0	1	1	1	2	2	2	ρ
		0	1	2	0	1	2	0	1	2	α
π	ω										
0	0	1									
0	1								1		
0	2			❶							
1	0					1					
1	1				1		1				
1	2					1					

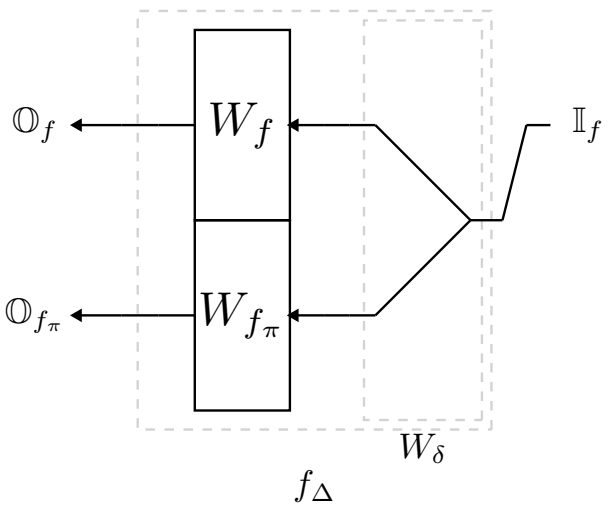
α,ρ, ω and ϕ are called the *compact spectral indexes* of the input, randoms, output and probe respectively

Vulnerability Profile

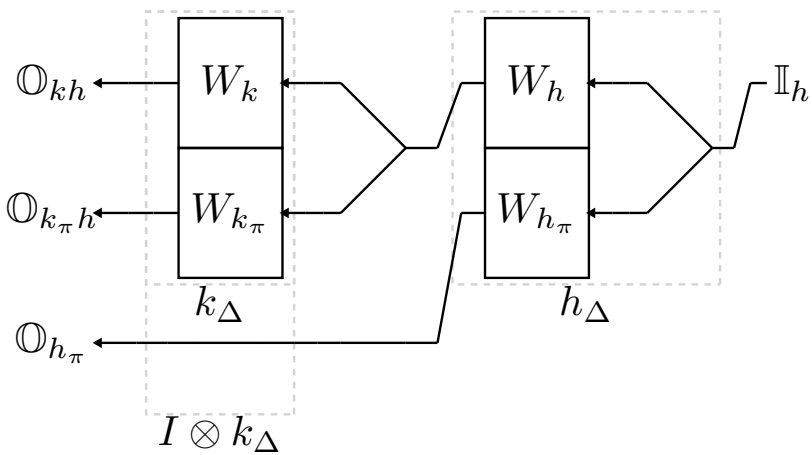
Vulnerability Profile of a function

Tensor product of the regular Walsh transform of a function f and of its probes f_π, multiplied by W_δ

Function f



Composition $h \bullet k$

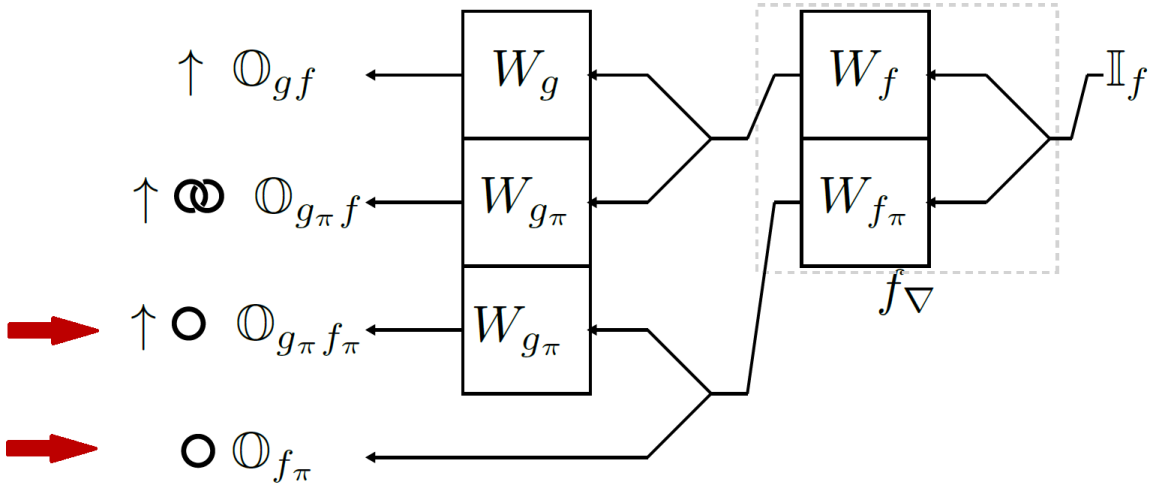


Classification of the Extended Probes

Classification

1. *Pure Probe* ($\circ$): placed on a wire computing $w(x)$, it gives information about all the inputs of the function:

$$w_\pi(x) = \bigwedge_{x_i \in \text{support}(w)} x_i$$



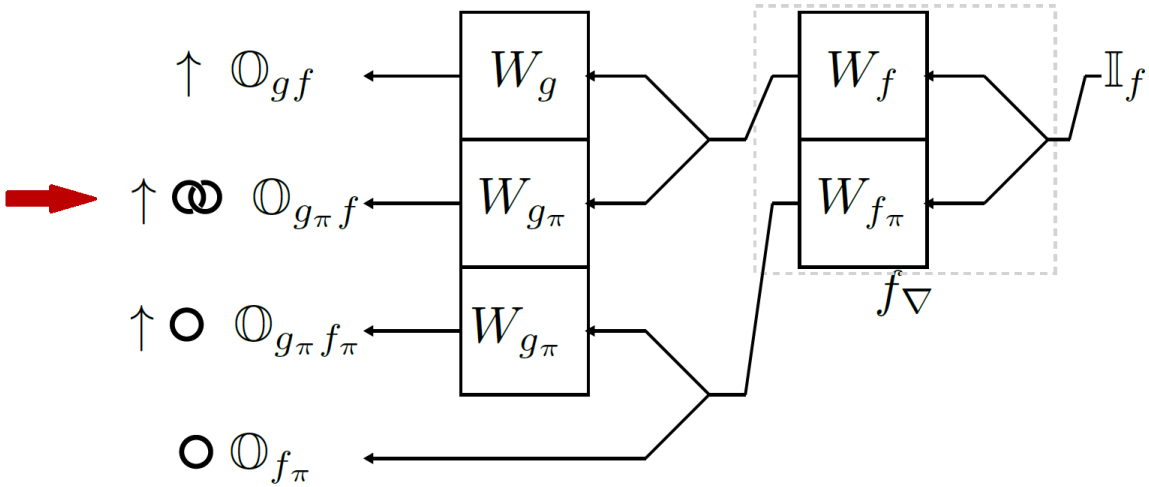
Classification of the Extended Probes

Classification

2. *Composed Probe* (⊚): placed on a wire computing $w(x) = w^a \bullet w^b$, it gives information about the values:

$$w_k(x) = (w^a_\pi \bullet w^b)(x)$$

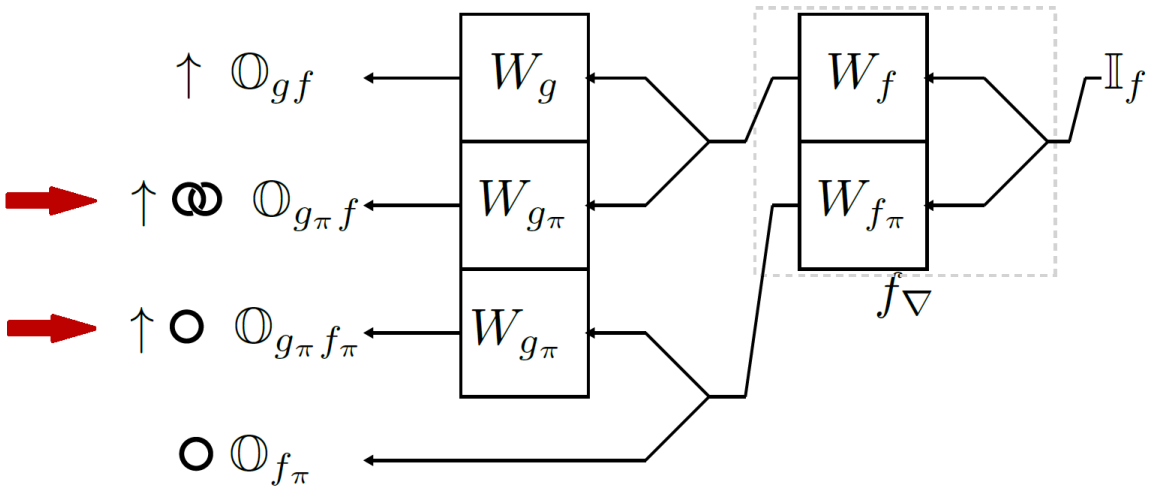
where $w^b(x)$ is different from the identity.



Classification of the Extended Probes

Classification

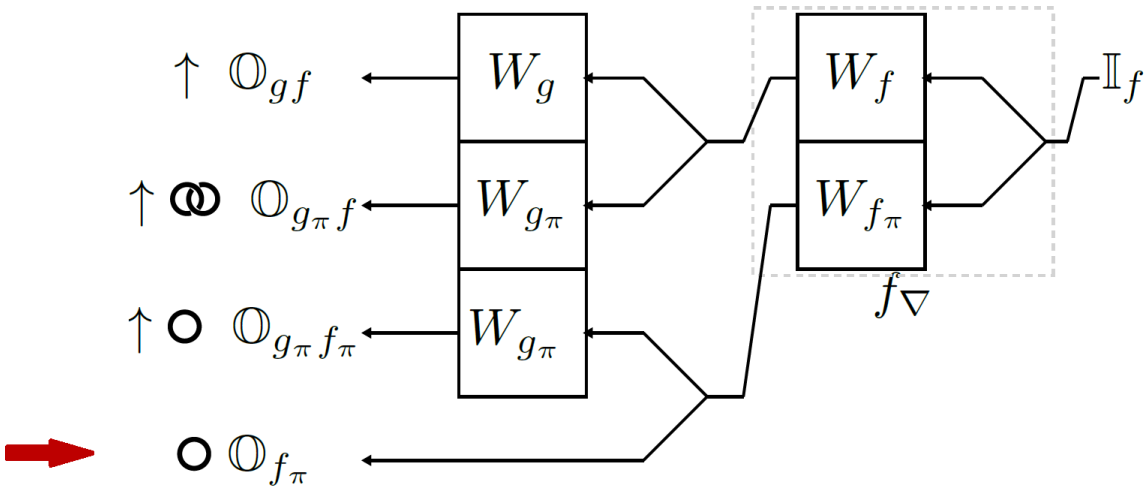
3. *Output Probe* (↑): placed on an actual output of the function; during composition of functions, it could produce new probes



Classification of the Extended Probes

Classification

4. *Internal Probe*: placed on an internal wire; it couldn't produce new probes when composing functions



Applications

Applications to multiplication gadgets

- CMS: analysis and improvement
- DOM-indep: analysis

Consolidating Masking Scheme

CMS[3] multiplication scheme

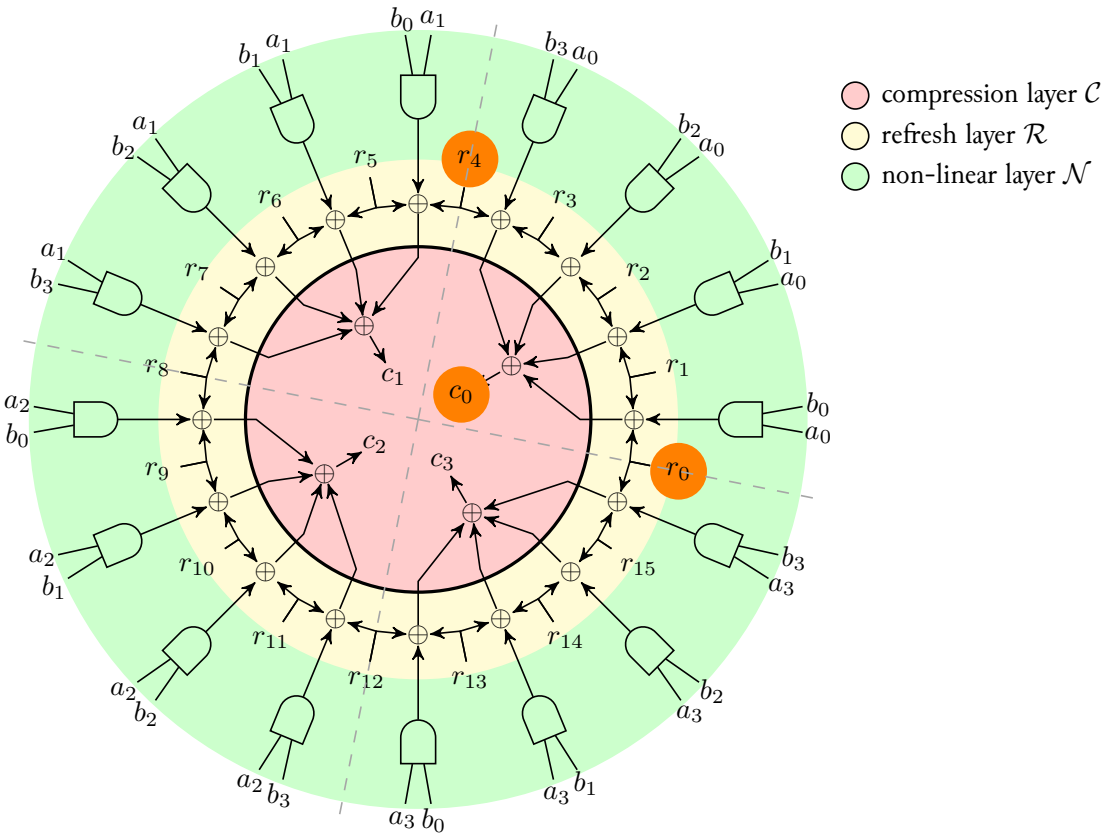
- Evolution of the ISW scheme, meant to provide d-probing security and protection against glitches
- $s = d + 1$ is the number of shares, a_i and b_i are the inputs' shares and c_i are the output's shares
- Every c_i is computed in a *logic cone*, which involves s pairs (a_i, b_h)
- *Adjacent cones* share only a random bit
- Internal bits within a cone preserve uniformity
- Three layers: non-linear ($\mathcal{N}$), refresh ($\mathcal{R}$) and compression ($\mathcal{C}$), the latter two separated by a register

[3] O. Reparaz et Al., *Consolidating Masking Schemes*

CMS and probing security

Problem

This scheme is not robust-d-probing secure for $d \geq 3$ [4]



[4] T. Moos et Al., *Glitch-Resistant Masking Revisited*

Analysis of the CMS probing security

through our classification of extended probes

Types of probes

- **Pure internal probes** at the output of $\mathcal{R}$: information about $\{a_i, b_j, r_{h_1}, r_{h_2}\}$
- **Composed output probes** at the output of $\mathcal{C}$: information about d values computed as $a_i \cdot b_j + r_{h_1} + r_{h_2}$

Fail of CMS, for $d \geq 3$

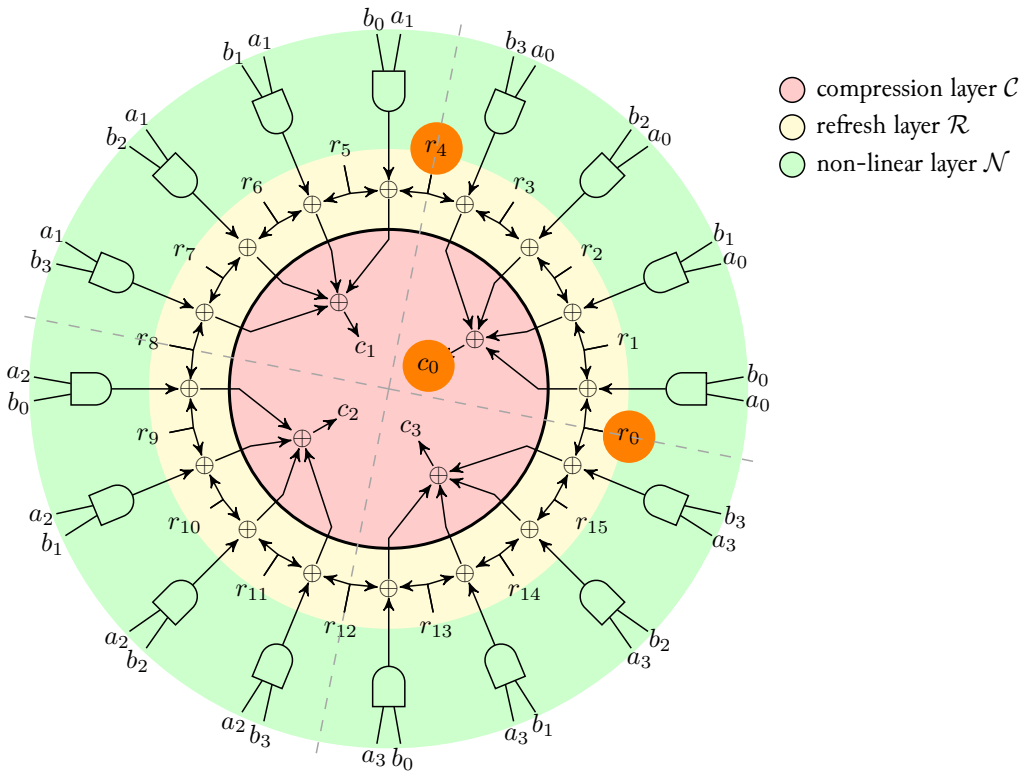
	b_0	b_1	b_2	...	b_d
a_0	c_0	c_0	c_0	...	c_0
a_1	c_1	c_1	c_1	...	c_1
a_2	c_2	c_2	c_2	...	c_2
$\vdots$					
a_d	c_d	c_d	c_d	...	c_d

Secret b placing only one composed probe and two pure probes

Analysis of the CMS probing security

through our classification of extended probes

Example



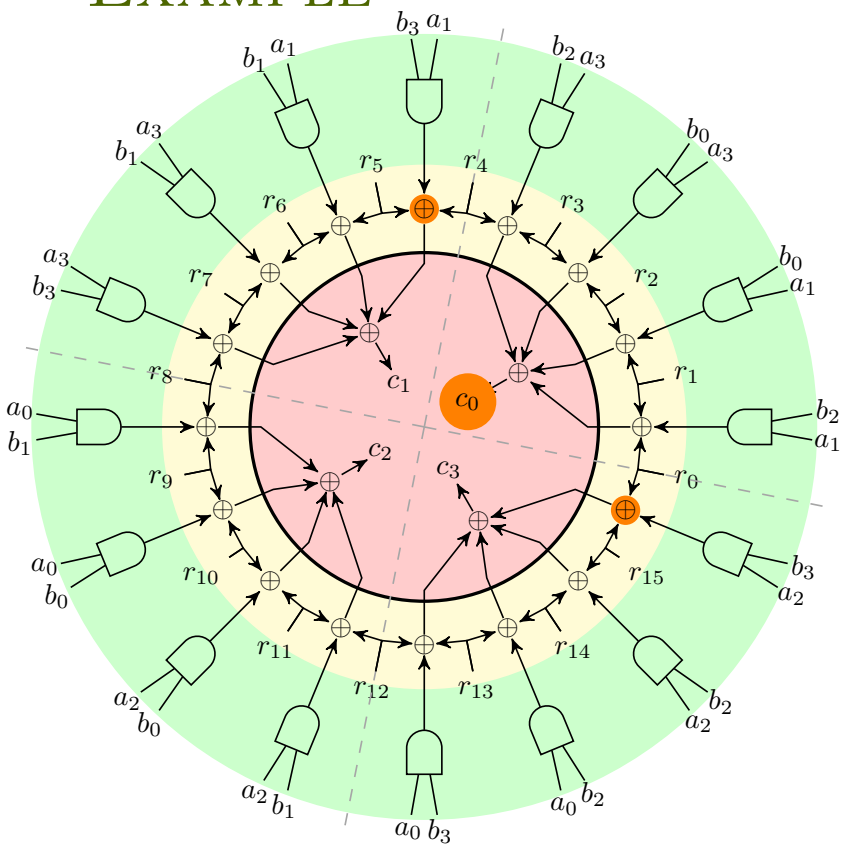
	b_0	b_1	b_2	b_3
a_0	c_0	c_0	c_0	c_0
a_1	c_1	c_1	c_1	c_1
a_2	c_2	c_2	c_2	c_2
a_3	c_3	c_3	c_3	c_3

- Output composed probe c_0
- Internal pure probes to recover r_0 and r_4

1st solution: CMS robust-d-probing secure

Non-completeness

Example



	b_0	b_1	b_2	b_3
a_0	c_2	c_2	c_3	c_3
a_1	c_0	c_1	c_0	c_1
a_2	c_2	c_2	c_3	c_3
a_3	c_0	c_1	c_0	c_1

- No information from any combination of 3 probes

1st solution: CMS robust-d-probing secure

Non-completeness

Example

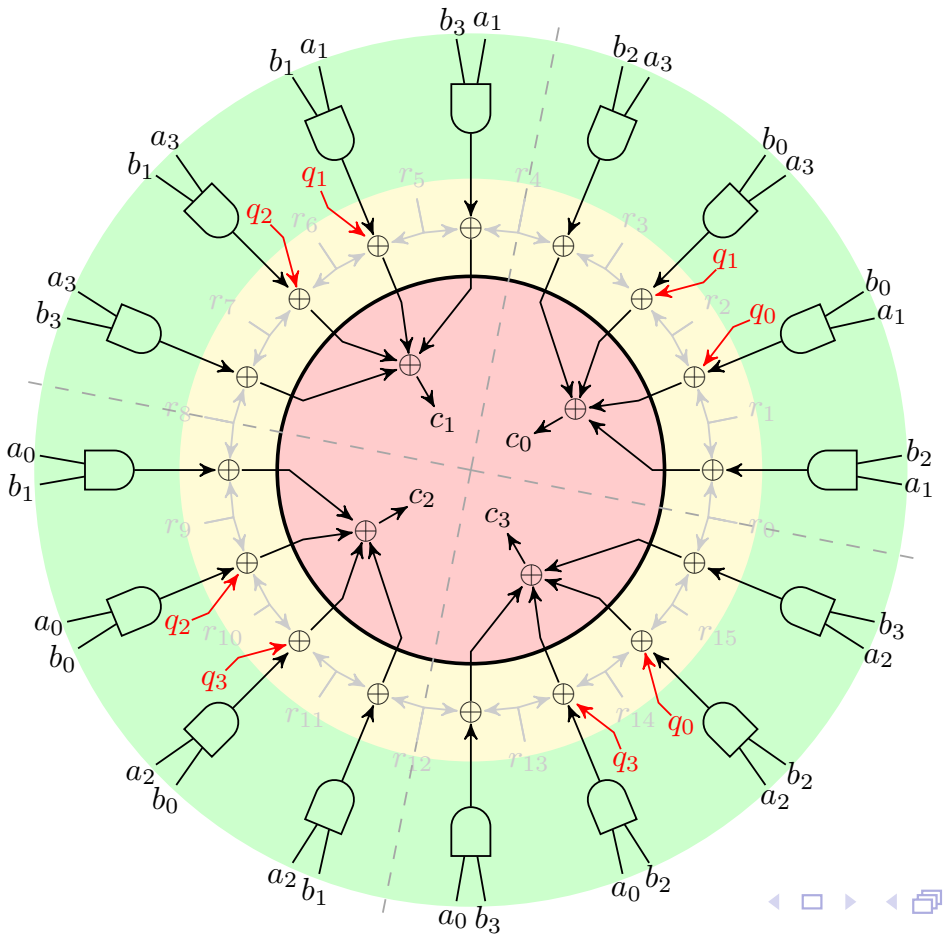
The Compact correlation matrix highlights that, in our first solution, the scheme with $s = 4$ is robust-3-probing secure but not robust-3-SNI

		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\dots \rho$
		0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	$\dots \beta$
		0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	$\dots \alpha$
ω_{f_π}	$\omega_{g_\pi f}$																									
$\dots$	$\dots$																									
0	3																									
$\dots$	$\dots$																									
1	2																									
$\dots$	$\dots$																									
2	1	1	1	1	❶		1	1	1	❶		1	1	1	❶		1	1	1	❶						
$\dots$	$\dots$																									
3	0	1	1	1	1		1	1	1	1		1	1	1	1		1	1	1	1						
$\dots$	$\dots$																									

2nd solution: CMS robust-d-SNI

Non-completeness + more randoms

Example



2nd solution: CMS robust-d-SNI

Non-completeness + more randoms

Example

The Compact correlation matrix highlights that, in our second solution, the scheme with $s = 4$ is robust-3-SNI

		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\ldots \rho$
		0	0	0	0	0	1	1	1	1	1	2	2	2	2	2	3	3	3	3	3	4	4	4	4	4	$\ldots \beta$
		0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	0	1	2	3	4	$\ldots \alpha$
ω_{f_π}	$\omega_{g_\pi f}$																									
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0	3																									
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3	0	1	1	1	1		1	1	1	1		1	1	1	1		1	1	1	1						
$\ldots$	$\ldots$																									

2nd solution: CMS robust-d-SNI

Non-completeness + more randoms

Generalization for any d

Let s be the number of shares ($s \geq 4$); any generalized CMS scheme can become robust-$(s-1)$-SNI by adding $s \cdot (\lfloor \frac{s}{2} \rfloor - 1)$ randoms to the refresh layer such that each pair of adjacent cones shares $\lfloor \frac{s}{2} \rfloor - 1$ of them

Domain Oriented Masking

DOM[5] multiplication scheme

- d-probing security by using $\frac{d(d+1)}{2}$ random bits
- $s = d + 1$ is the number of shares, a_i and b_i are the inputs' shares and c_i are the output's shares
- DOM with independent shares is called DOM-indep
- Terms in the DOM-indep equations are inner-domain terms $(a_i b_i)$ and cross-domain $(a_i b_j)$; cross-domain are masked by random bits
- Before the compression phase, partial solutions are saved in registers

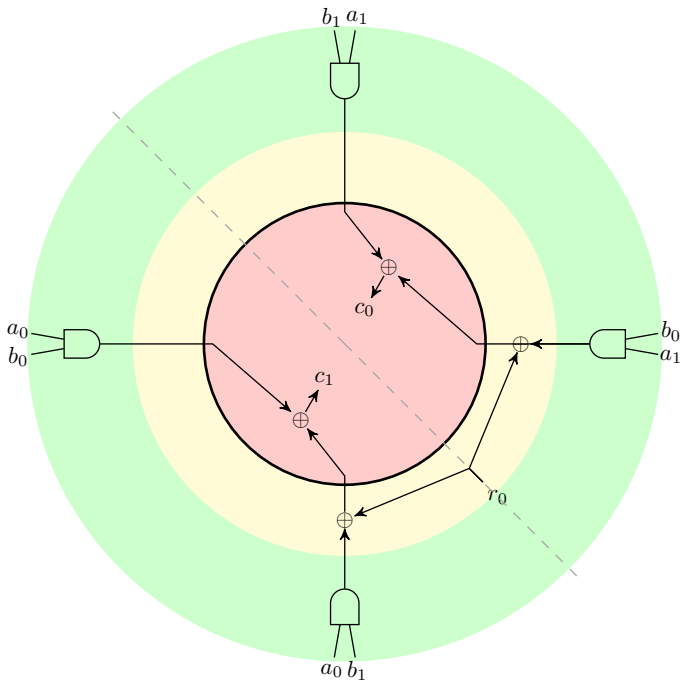
[5] H. Gross et Al., *Domain-Oriented Masking: Compact Masked Hardware Implementations with Arbitrary Protection Order.*

DOM-indep and probing security

Problem

This scheme is not robust-d-SNI, for any d [6]

Example



[6] T. Moos et Al., *Glitch-Resistant Masking Revisited*

DOM-indep and probing security

Example

The Compact correlation matrix highlights that the scheme with $s = 2$ is robust-1-probing secure but not robust-1-SNI

		0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	ρ
		0	0	0	1	1	1	2	2	2	0	0	0	1	1	1	2	2	2	β
		0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	α
ω_i	ω_o																			
0	0	1																		
0	1	1	❶		❶	❶					1	1		1	1		1	1		
...	...																			
4	4	1	1					1	1						1					

DOM-indep robust-d-SNI

Output registers[7]

Example

The Compact correlation matrix highlights that, with an output register, the scheme with $s = 2$ is robust-1-SNI

		0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	ρ
		0	0	0	1	1	1	2	2	2	0	0	0	1	1	1	2	2	2	β
		0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	0	1	2	α
ω_i	ω_o																			
0	0	1																		
0	1										1	1					1	1		
0	2	1		1				1		1										
1	0	1	1		1	1					1	1		1	1		1	1		
...	...																			
6	2	1	1					1	1						1					

[7] S. Faust et Al., *Composable Masking Schemes in the Presence of Physical Defaults and the Robust Probing Model*

Trade off randomness / registers

To ensure the robust-d-SNI:

- CMS: addition of random bits
- DOM-indep: addition of output registers

Trade off randomness / registers

To ensure the robust-d-SNI:

- CMS: addition of random bits
- DOM-indep: addition of output registers

Example

With $d = 3$:

	random	register (per bit)
CMS	+4	+0
DOM	+0	+4

Trade off randomness / registers

To ensure the robust-d-SNI:

- CMS: addition of random bits
- DOM-indep: addition of output registers

Example

With $d = 3$:

	random	register (per bit)
CMS	$+4$	$+0$
DOM	$+0$	$+4$

Ratio of random usage

$$\frac{2\left(\frac{s^2}{2} + \left(\frac{s}{2} + 1\right) s\right)}{(s-1)\, s}$$

Complexity of the proposed approach

Complexity problem

With the increasing of the variables, the number of elements in the Walsh matrices becomes too large $\rightarrow$ its complete computation becomes impracticable

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Solution

- Store only the rows that refer to single outputs and probes

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- Compute on-demand the remaining rows by using convolution

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Complexity problem

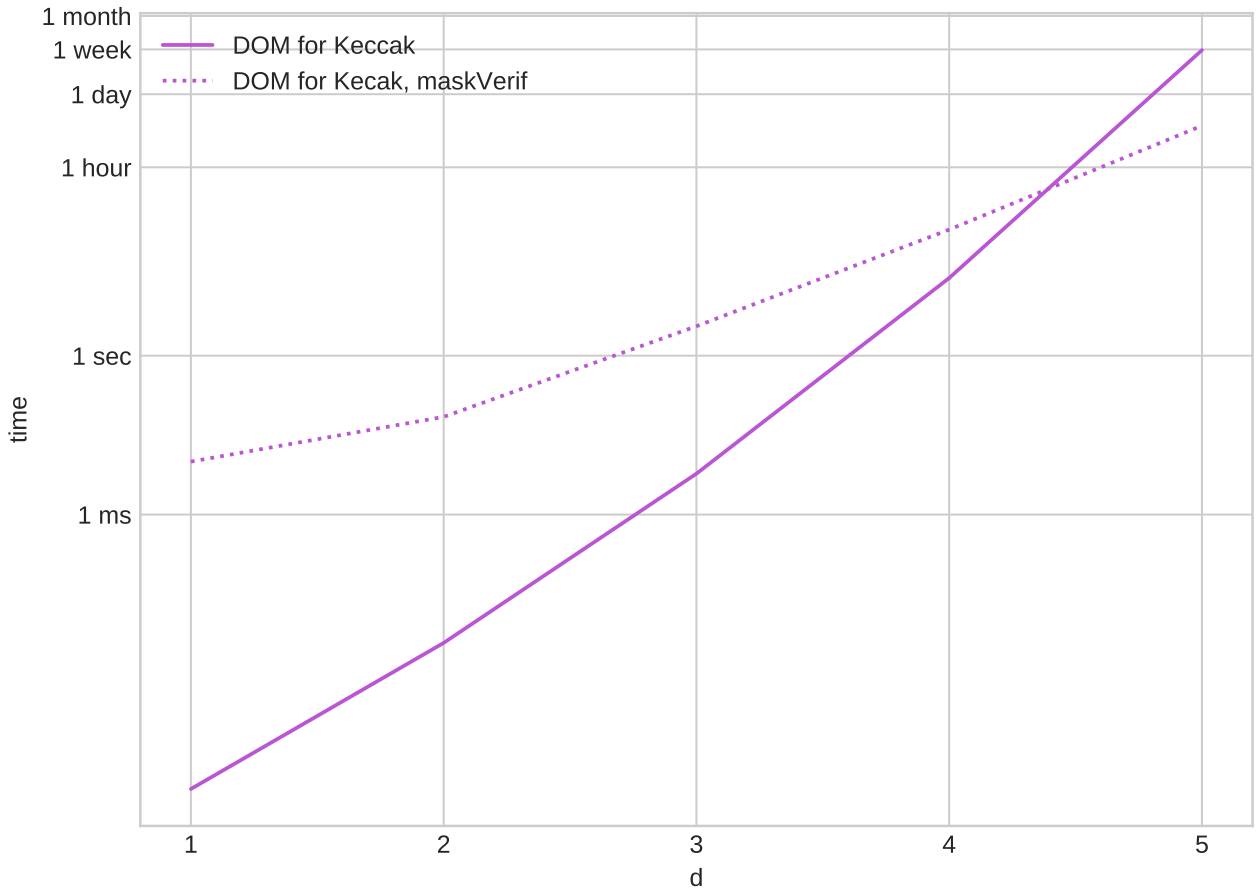
With the increasing of the variables, the number of elements in the Walsh matrices becomes too large $\rightarrow$ its complete computation becomes impracticable

Solution

- Store only the rows that refer to single outputs and probes
- Compute on-demand the remaining rows by using convolution
- Exploit the sparsity of the correlation matrices

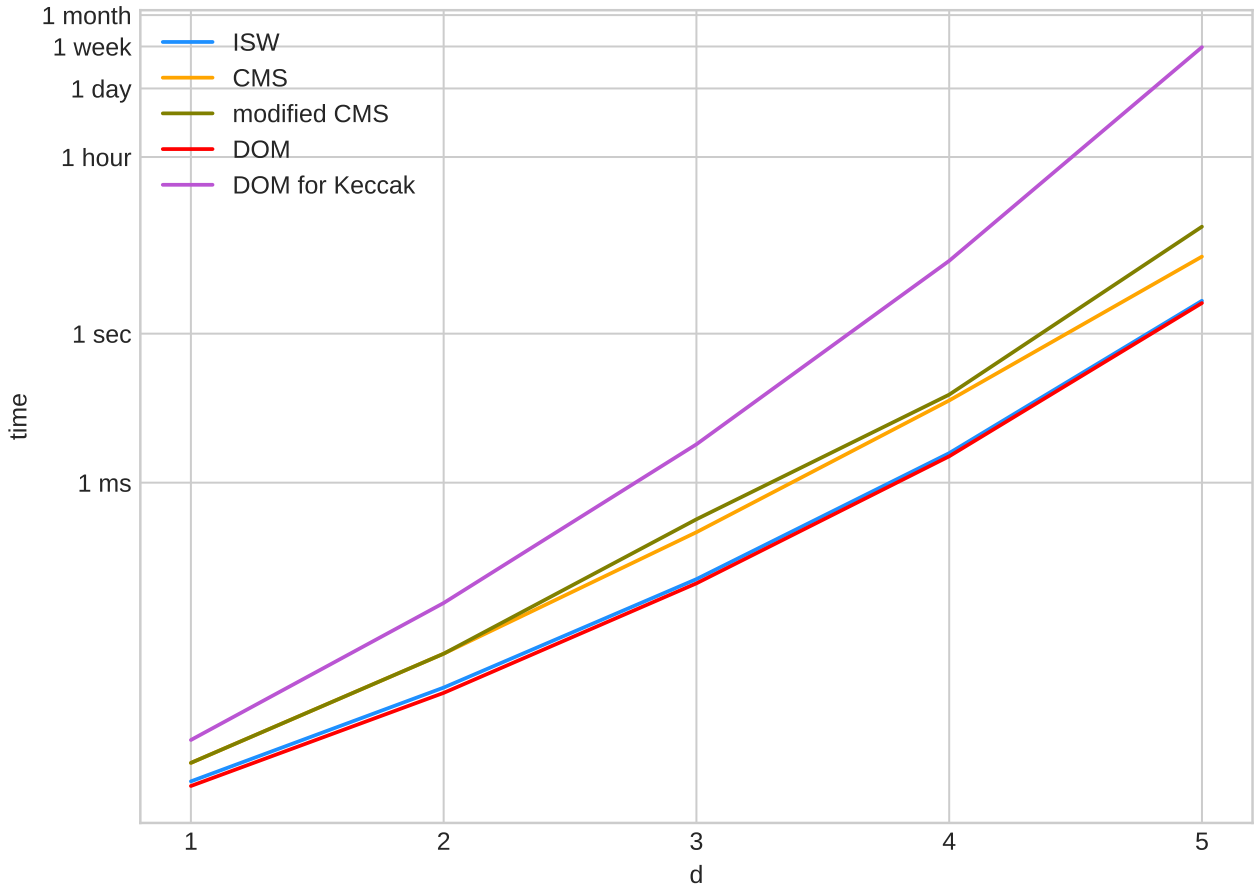
Scalability of the proposed approach

Security verification of χ of Keccak with DOM-indep



Scalability of the proposed approach

Estimated time to compute the compact correlation matrix for gadgets



Conclusion

- Alternative view of robust probing security

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- New mathematical framework and approach, based on the Walsh matrices

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- Applications to multiplication gadgets:
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- Investigate the ring structure of multiplication gadgets: more efficient refresh layers?

THANK YOU FOR THE ATTENTION

Any question?

You can also write to me at the address
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