Better Security for Permutation-Based Tweakable Correlation-Robust Hashing

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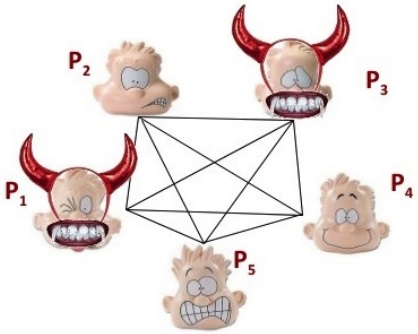
December, 2021

Symmetric-Key Primitives in Secure Multiparty Computation



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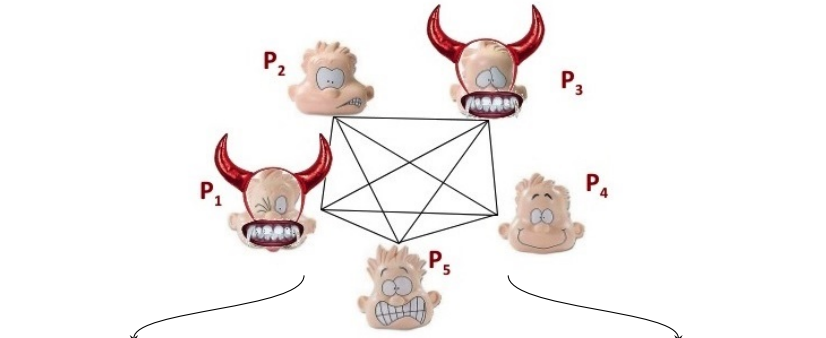

Make heavy use of symmetric-key primitives

Hash Functions as Essential Component

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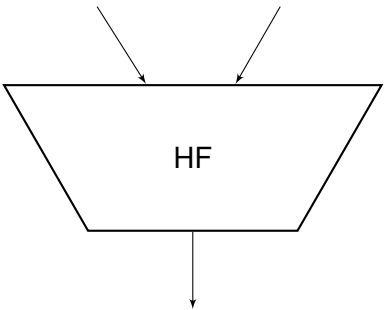
- Different constructions are needed to satisfy stronger notions

Why is the Problem Hard?



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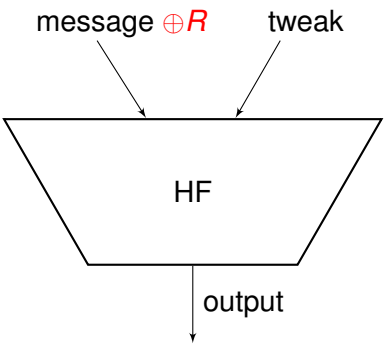

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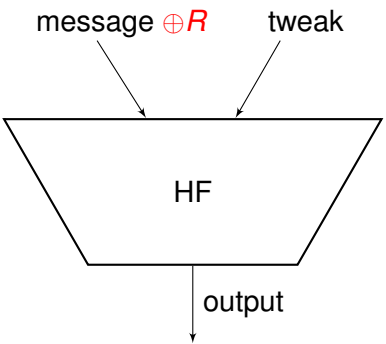

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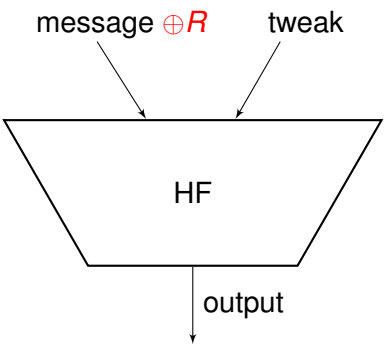

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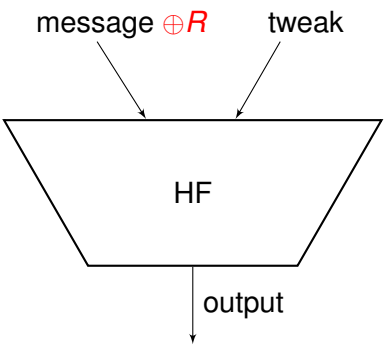

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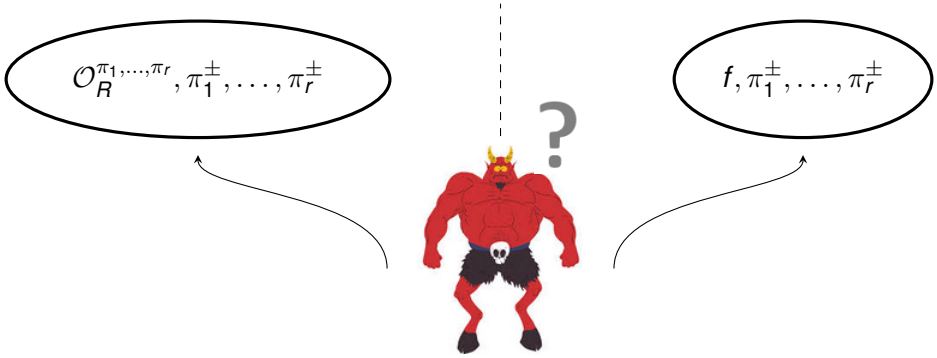
- Problem is related to related-key security for XOR

Correlation Robustness



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- Adversary $\mathcal{A}$ makes q construction queries to oracle $\mathcal{O}_R^{\pi_1,\ldots,\pi_r}(w) = H(w \oplus R)$ or $f \xleftarrow{\$} \mathrm{Func}(n, n)$

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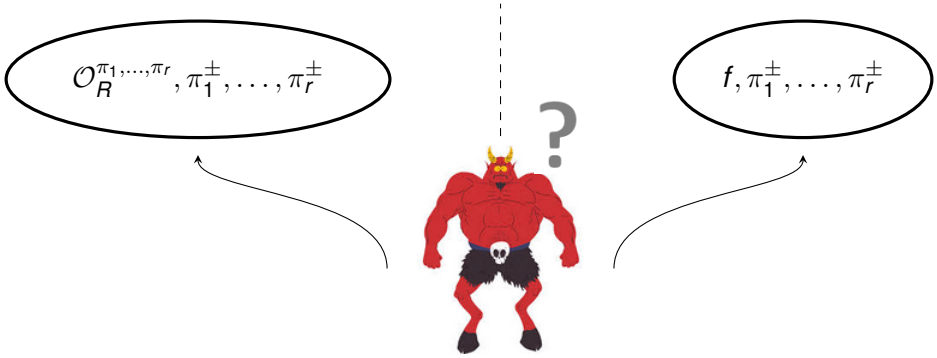

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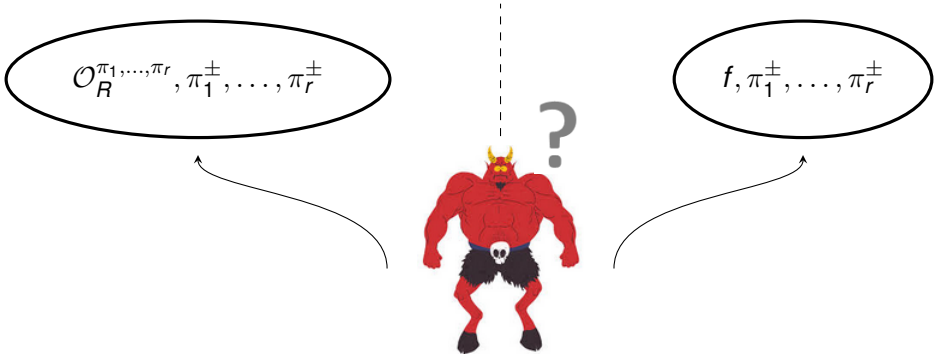

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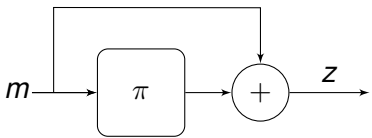
MMO Correlation Robust Hash Function



Guo, Katz, Wang, and Yu (GKWY) 2020

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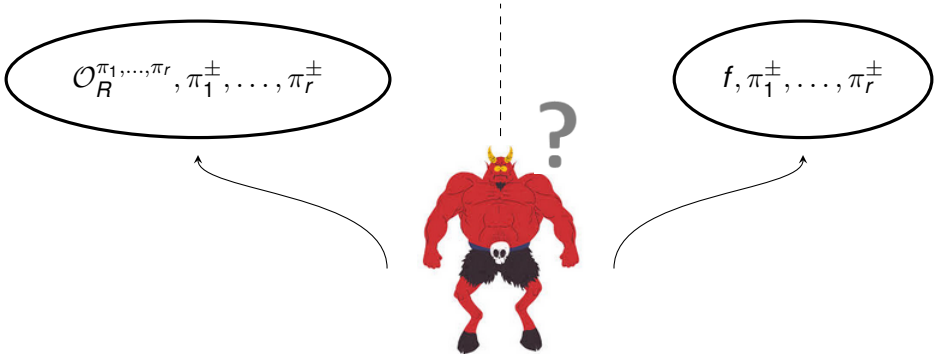
Circular Correlation Robustness



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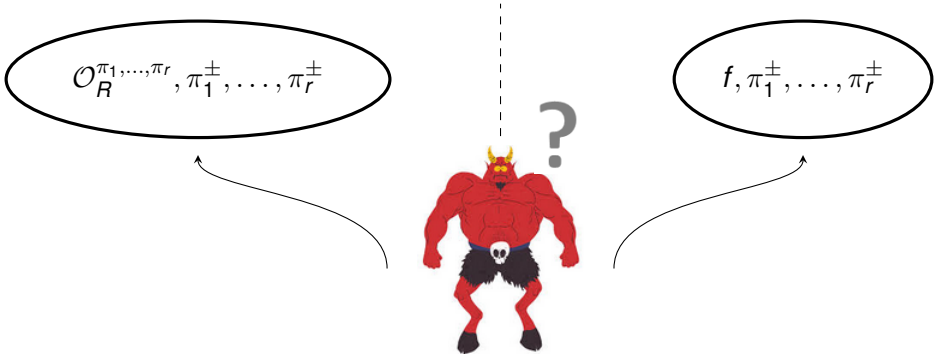

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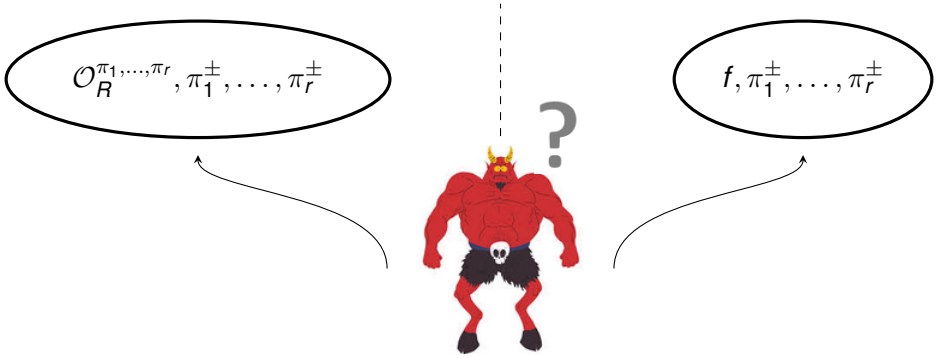

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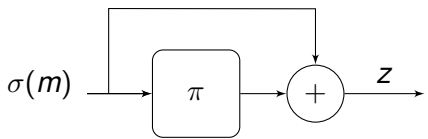
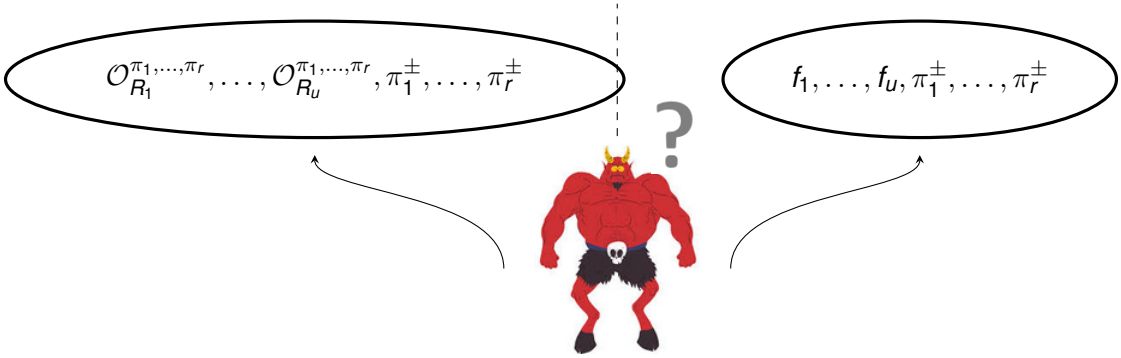


GKWY 2020

- Linear orthomorphism σ: both $\sigma(x)$ and $\sigma(x) \oplus x$ are permutations
- Security bound: $O(\frac{pq}{2^n} + \frac{q^2}{2^n})$

Tweakable (Circular) Correlation Robustness (Multi-Instance)



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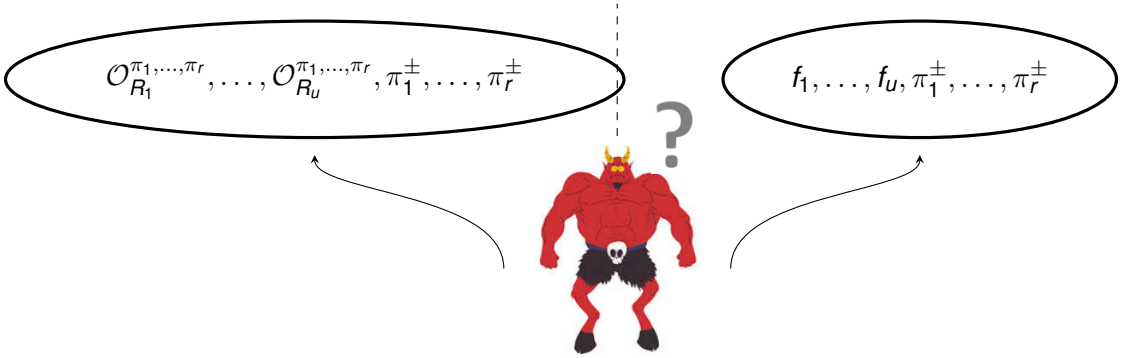

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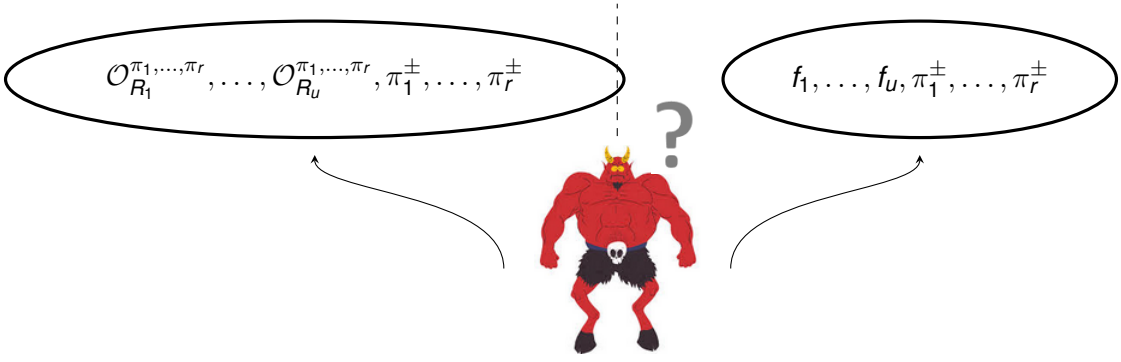

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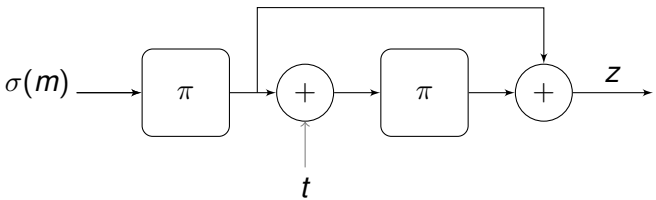

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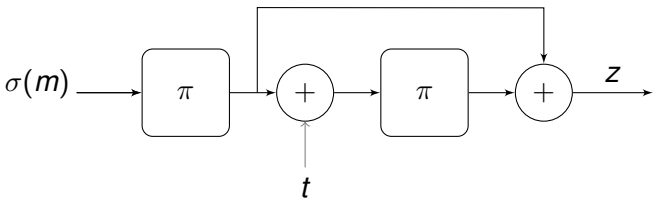

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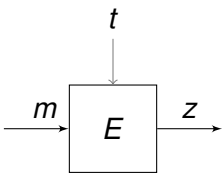
- Inefficient security when multi-user setting is considered (GKWWY 2020)

tccr Hash Function For Multi-User Setting



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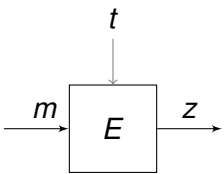

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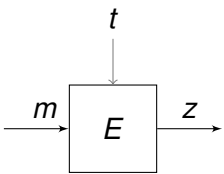

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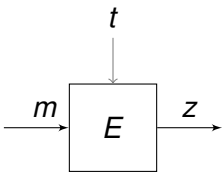

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- Ideal cipher model

Application to OT Extension Protocols and Garbled Circuits

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- Significant practical impact on efficiency of MPC

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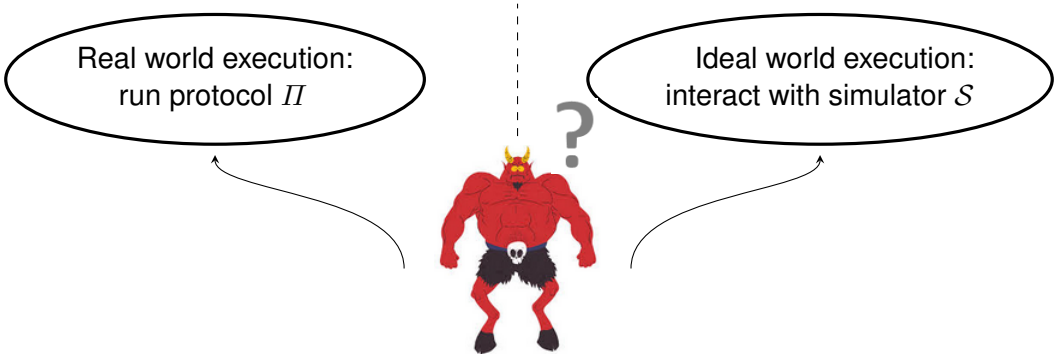

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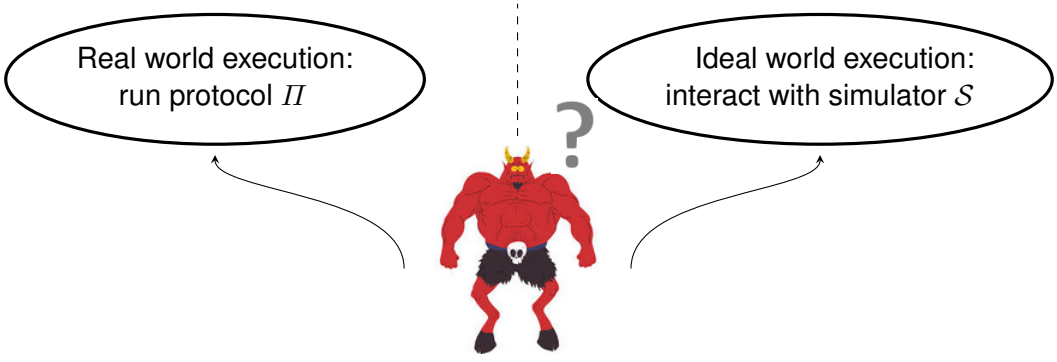

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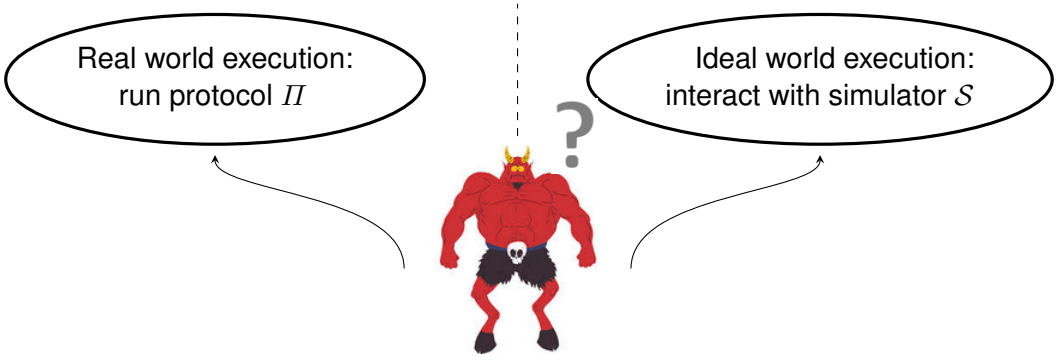

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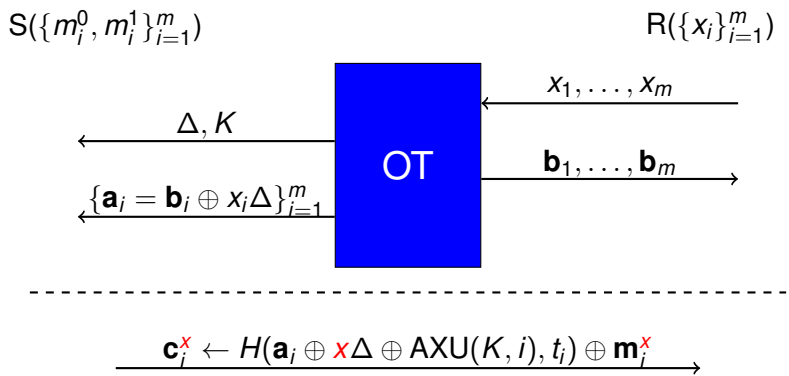
Our OT Protocol



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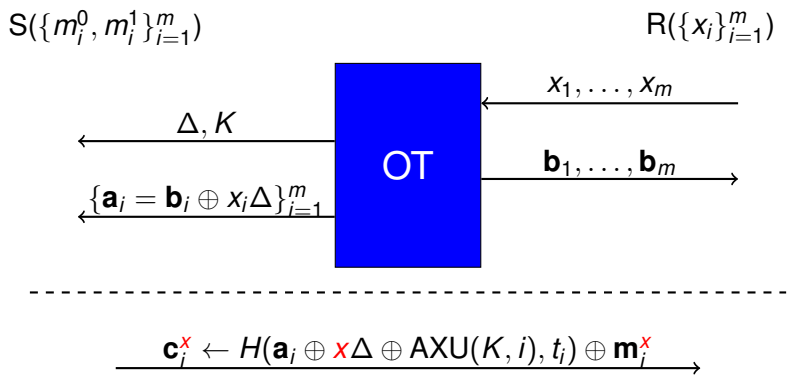

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- Solution: universal hash function AXU

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- Security dominated by $O(\sqrt{m}p/2^n)$

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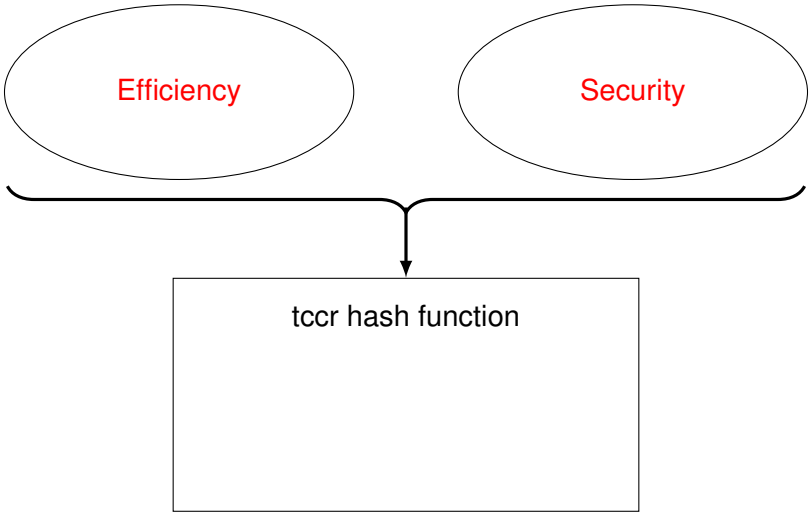

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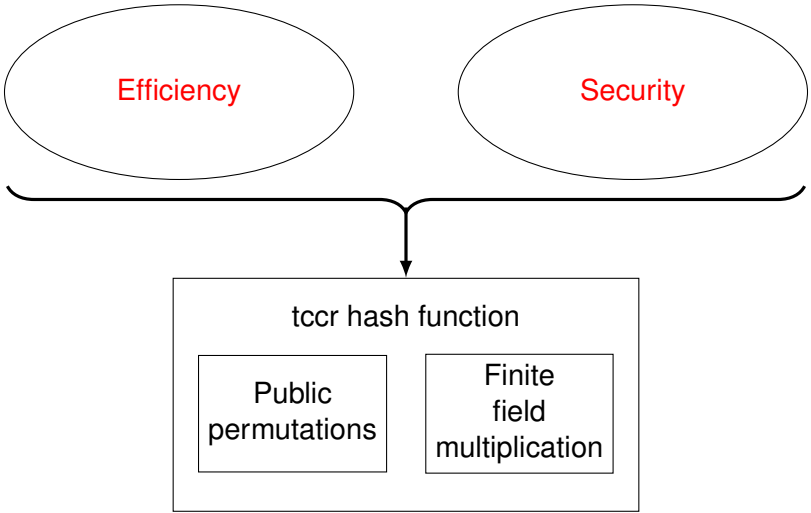
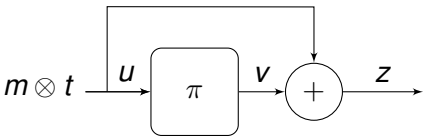
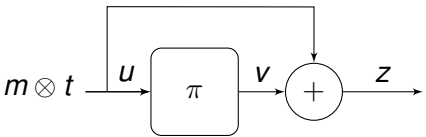


tccr Hash Function Based on One Permutation Call



Security bound: $O(\frac{pq}{2^n} + \frac{q^2}{2^n})$

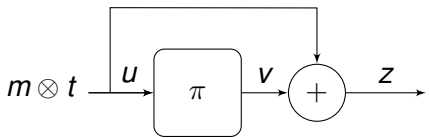
tccr Hash Function Based on One Permutation Call



Security bound: $O(\frac{pq}{2^n} + \frac{q^2}{2^n})$

- Construction query (m, t, z) and primitive query $(u, v) \Rightarrow m \otimes t = u$

tccr Hash Function Based on One Permutation Call

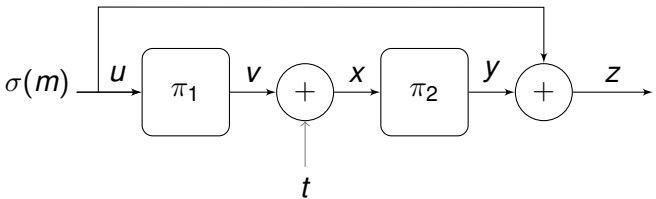


Security bound: $O(\frac{pq}{2^n} + \frac{q^2}{2^n})$

- Construction query (m, t, z) and primitive query $(u, v) \Rightarrow m \otimes t = u$
- Construction query inputs (m, t, z) and $(m', t', z') \Rightarrow m \otimes t = m' \otimes t'$

tccr Hash Function Based on Two Permutation Calls

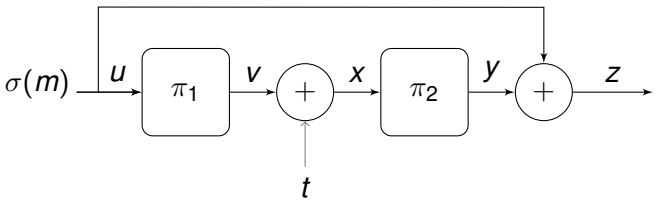
FPTP (Feed-forward Permutation-Tweak-Permutation)



Security bound: $O(\frac{B\sqrt{qp}}{2^n} + \frac{q^2}{2^n})$

tccr Hash Function Based on Two Permutation Calls

FPTP (Feed-forward Permutation-Tweak-Permutation)

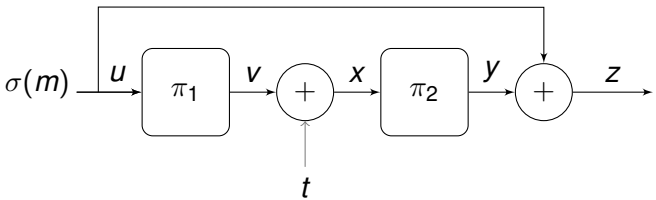


Security bound: $O(\frac{B\sqrt{qp}}{2^n} + \frac{q^2}{2^n})$

- B queries per tweak

tccr Hash Function Based on Two Permutation Calls

FPTP (Feed-forward Permutation-Tweak-Permutation)



Security bound: $O(\frac{B\sqrt{qp}}{2^n} + \frac{q^2}{2^n})$

- B queries per tweak
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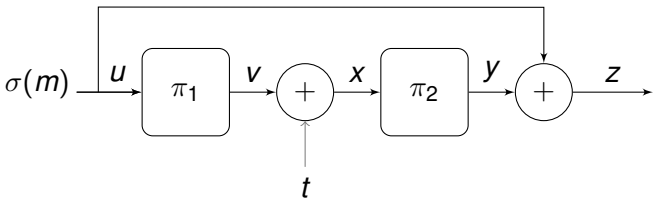
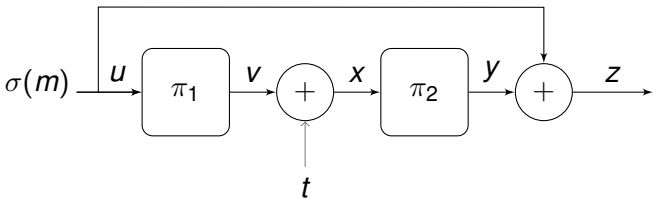


Security bound: $O(\frac{B\sqrt{qp}}{2^n} + \frac{q^2}{2^n})$

- B queries per tweak
- Tweaks chosen from nice combinatorial subset $T \subseteq \{0,1\}^n$ (e.g. random subset)
- Distinct and uniform input messages

tccr Hash Function Based on Two Permutation Calls

FPTP (Feed-forward Permutation-Tweak-Permutation)



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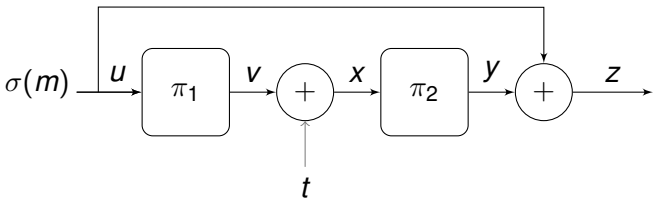
- B queries per tweak
- Tweaks chosen from nice combinatorial subset $T \subseteq \{0,1\}^n$ (e.g. random subset)
- Distinct and uniform input messages
- Arbitrary input messages using $m \otimes t$ instead of m

tccr Hash Function Based on Two Permutation Calls

FPTP (Feed-forward Permutation-Tweak-Permutation)



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tccr Hash Function Based on Two Permutation Calls

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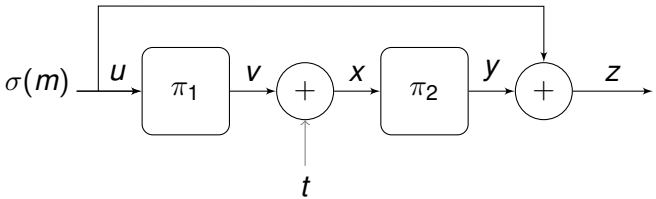

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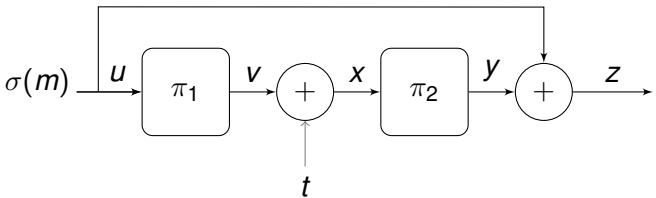
- Both for two independent and same permutation calls
- Multi-user security

FPTP (technical overview)



Security bound: $O(\frac{B\sqrt{\bar{q}p}}{2^n} + \frac{q^2}{2^n})$

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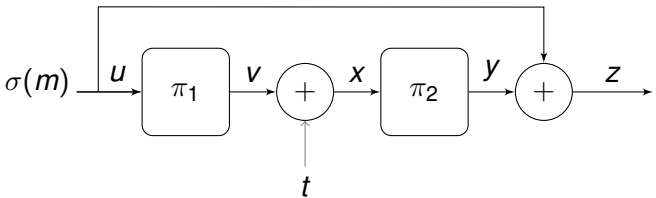

Security bound: $O(\frac{B\sqrt{qp}}{2^n} + \frac{q^2}{2^n})$

- Chains: construction query (m, t, z), and primitive queries (u, v) and (x, y)
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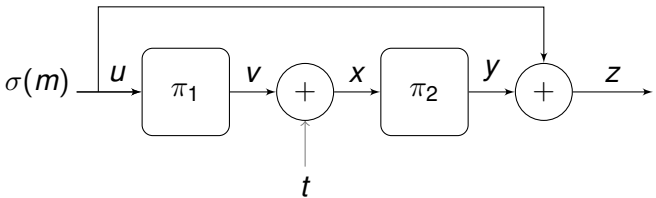

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- Chains: construction query (m, t, z), and primitive queries (u, v) and (x, y) $\Rightarrow \sigma(m) = u$ and $v \oplus t = x$
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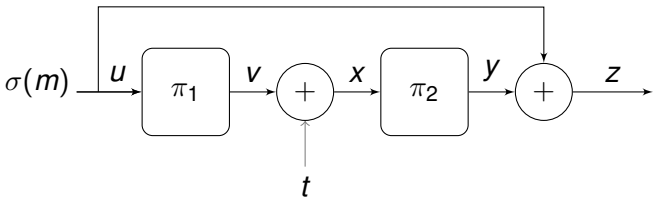

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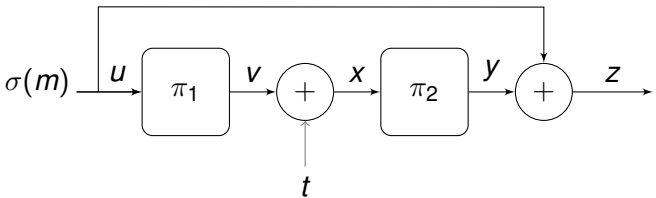

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- Sum-capture theorems (Babai 02, Steinberger 12, Cogliati and Seurin 18)

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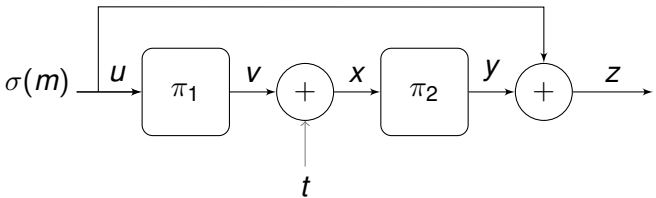

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- Sum-capture theorems (Babai 02, Steinberger 12, Cogliati and Seurin 18)
- New balls-into-bins combinatorial lemmas

Conclusion

New results

- Concrete security treatment of oblivious transfer extension
- $O(qp/2^n + q^2/2^n)$ secure tccr HF using one permutation call
- $O(\sqrt{q}p/2^n + q^2/2^n)$ secure tccr HF (FPTP) using two permutation calls

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Future research

- Improve the security of FPTP by improving $q^2/2^n$ to $q^3/2^{2n}$
- Formalize the limitation of the tweak set used in FPTP
- Extend FPTP to multiple rounds

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Thank you for your attention!