

Shorter Lattice-Based Group Signatures via “Almost Free” Encryption and Other Optimizations

Vadim Lyubashevsky¹, Ngoc Khanh Nguyen¹², Maxime Plançon¹², and Gregor Seiler¹²

¹IBM Research Europe, Switzerland, ²ETH Zurich

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Overview of the paper

Lattice-based group signature construction from [dPLS18]

+ Lattice-based ZK proofs contributions from
[ALS20, ENS20, LNS20, BLS19, ESLL19]

→ This paper

BDLOP lattice-based commitment scheme

Notations : q prime, d power of 2, $\mathbb{Z}_q = \mathbb{Z}/q\mathbb{Z}$, $\mathcal{R}_q = \mathbb{Z}_q/(X^d + 1)$. Vectors are lower case bold letters, matrices are upper case bold letters.

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Definition of BDLOP commitment scheme

Setup : Uniformly random matrix $\mathbf{A}_0 \in \mathcal{R}_q^{n \times k}$, uniformly random vectors $\mathbf{a}_1, \dots, \mathbf{a}_\alpha \in \mathcal{R}_q^k$.

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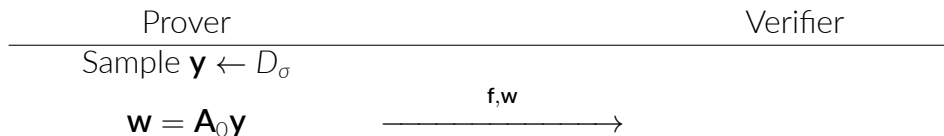
- 2 Output $\mathbf{f} = \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{a}_1^T \\ \vdots \\ \mathbf{a}_\alpha^T \end{bmatrix} \mathbf{r} + \begin{bmatrix} \mathbf{0} \\ m_1 \\ \vdots \\ m_\alpha \end{bmatrix}$

Interactive ZKPoK from BDLOP

Notations : $\mathbf{f} = (\mathbf{A}_0 \mathbf{r} \mathbf{a}_1^T \mathbf{r} + m)$, $c \in \mathcal{C}$ is such that $c \in \{-1, 0, 1\}^d$, D_σ is the discrete Gaussian of standard deviation σ over \mathbb{Z} .

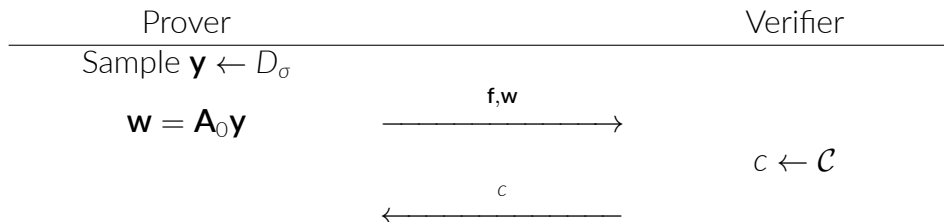
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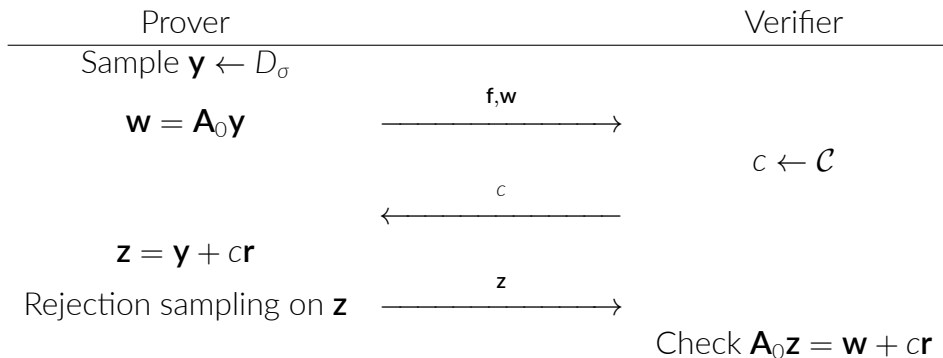
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Some statements that can be proved efficiently from BDLOP commitments :

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- 1 linear relations : $m_1 = \lambda m_2$ for some $\lambda \in \mathcal{R}_q$ (very cheap)
- 2 product relations : $m_1 = m_2 m_3$ (less cheap)
- 3 "unstructured" linear relations : $\mathbf{Q}m_1 = m_2$, where $\mathbf{Q} \in \mathbb{Z}_q^{* \times d}$, $m_1, m_2 \in \mathbb{Z}_q^d$ (less cheap)

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- The Group Manager can open σ to recover the identity i of the signer.

Security properties

- 1 **Anonymity.** The adversary who knows all the signing keys s_i cannot distinguish between signatures produced by any two users.

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- 2 **Traceability.** The adversary who possesses signing keys to all users in some set S , and the Opener's secret key, cannot create a valid signature that the Opener will decrypt to some identity $i \notin S$.

Intuition : If $S = \emptyset$, then the adversary shall not be able to produce a valid signature.

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- 3 User i 's signing key : Sample $(\mathbf{s}_1^i, \mathbf{s}_2^i) \leftarrow D_s$ such that (conditioned on) $[\mathbf{A}|\mathbf{AR} - i\mathbf{G}] \begin{bmatrix} \mathbf{s}_1^i \\ \mathbf{s}_2^i \end{bmatrix}$, where \mathbf{G} is a simple matrix that allows such sampling using the trapdoor \mathbf{R} .

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- 4 Generate a key pair of a verifiable encryption scheme.

Signing

User i 's signature of a message M :

- (f_0, f_1) BDLOP commitment to i with randomness \mathbf{r} and compute a NIZK π_1 that $i \in \mathcal{I}$ (that is $\sigma(i) = i$ for both automorphisms defining \mathcal{I})

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- Compute a NIZKP π_2 that $[\mathbf{A}|\mathbf{A}\mathbf{R} - i\mathbf{G}] \begin{bmatrix} \mathbf{s}_1 \\ \mathbf{s}_2 \end{bmatrix}^*$.
- Encrypt \mathbf{r} with the group manager's verifiable encryption public key, and compute a NIZK π_3 that the ciphertext \mathbf{c} is well-formed.

The signature is $(\mathbf{f}, \mathbf{c}, \pi_1, \pi_2, \pi_3)$. Verification is simply verifying π_1, π_2, π_3 . Opening is simply decrypting \mathbf{r} from \mathbf{c} .

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- 3 More efficient membership proof for $\pi_1 : i \in \mathcal{I}$.
- 4 Encryption almost comes for free, so does the well-formedness ZKP

Overall, we shrink the size of the signature from 580 KB to 203 KB.

Proving knowledge of $(\mathbf{s}_1^i, \mathbf{s}_2^i)$

We need to prove knowledge of $(\mathbf{s}_1^i, \mathbf{s}_2^i)$ such that

$$[\mathbf{A} | \mathbf{A}\mathbf{R} - i\mathbf{G}] \begin{bmatrix} \mathbf{s}_1^i \\ \mathbf{s}_2^i \end{bmatrix} = \mathbf{u},$$

where $\mathbf{A}, \mathbf{B}, \mathbf{G}, \mathbf{u}$ are public, and i is committed to.

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Can we use the product proof from BDLOP commitments ?

The equation to be proven can be written as $\mathbf{A}\mathbf{s}_1^i + \mathbf{B}\mathbf{s}_2^i - \mathbf{G}\mathbf{s}_2^i = \mathbf{u}$. Can we do a product proof ? Yes, but committing to \mathbf{s}_1^i and \mathbf{s}_2^i would be a disaster for the length of the signature.

Proving knowledge of (s_1^i, s_2^i)

$$\mathbf{A}' = [\mathbf{A} | \mathbf{B} - i\mathbf{G}].$$

Our solution

If \mathbf{A}' was public, the verification equation would be $\mathbf{A}'\mathbf{z} = \mathbf{w} + c\mathbf{u}$, where \mathbf{w} would be $\mathbf{A}'\mathbf{y}$ for some Gaussian \mathbf{y} , c a challenge and $\mathbf{z} = \mathbf{y} + c[\mathbf{s}_1^i \mathbf{s}_2^i]^T$.

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From the homomorphic properties of the BDLOP commitment scheme, the verifier can infer a commitment to $\mathbf{A}' = [\mathbf{A}|\mathbf{B} - i\mathbf{G}]$ from the commitment to i .

Our solution is to commit to \mathbf{w} instead of sending it. The equation $\mathbf{A}'\mathbf{z} = \mathbf{w} + c\mathbf{u}$ is then linear in the committed messages, so we can give a (relatively cheap) BDLOP linear proof of the verification equation.

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ZK proof that $i \in \mathcal{I}$

We add an extra commitment to i_b , the inverse NTT of the binary representation of i . Only remains to prove two things :

- 1 i_b 's NTT is binary $\rightarrow \text{NTT}(i_b) \odot (\text{NTT}(i_b) - 1) = 0 \rightarrow \text{NTT}(i_b(i_b - 1)) = 0 \rightarrow i_b(i_b - 1) = 0$ is a product proof.

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- 2 $\mathbf{Q} \cdot \text{NTT}(i_b) = \text{NTT}(i)$, where $\mathbf{Q} = \begin{bmatrix} 1 & 2 & 4 & \dots & 2^{d-1} \\ 1 & 2 & 4 & \dots & 2^{d-1} \\ \vdots & \vdots & & & \vdots \\ 1 & 2 & 4 & \dots & 2^{d-1} \end{bmatrix}$. This means

that all of i 's NTT coefficients are equal, hence i is an integer.

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All in all, i is an integer whose binary representation has length d , i.e $i \in \mathcal{I}$.

Verifiable arbitrary-message encryption almost for free

We add an extra commitment to \sqrt{qi} and use the commitment to i, \sqrt{qi} as the ciphertext.

$$\begin{bmatrix} \mathbf{f}_0 \\ f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{a}_1 \\ \mathbf{a}_2 \end{bmatrix} \mathbf{r} + \begin{bmatrix} \mathbf{0} \\ i \\ \sqrt{qi} \end{bmatrix} .$$

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The setup authority plants a decryption key as follows

$$\mathbf{a}_1^T = \mathbf{h}_1^T \mathbf{A}_0 + \mathbf{e}_1^T,$$

$$\mathbf{a}_2^T = \mathbf{h}_2^T \mathbf{A}_0 + \mathbf{e}_2^T, \text{ for small } \mathbf{h}_1, \mathbf{h}_2, \mathbf{e}_1, \mathbf{e}_2.$$

The new $\mathbf{a}_1, \mathbf{a}_2$ are indistinguishable from uniform under the Module-LWE assumption, therefore this change is secure.

Decryption

Given the ciphertext

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decryption is as follows :

- Compute $x_1 = f_1 - \mathbf{h}_1^T \mathbf{f}_0$
- Compute $x_2 = f_2 - \mathbf{h}_2^T \mathbf{f}_0$
- Compute $k = (\sqrt{q}x_1 - x_2) \bmod \sqrt{q} \leftarrow k = (\sqrt{q}\mathbf{e}_1^T - \mathbf{e}_2^T)\mathbf{r}$
- return $(x_2 + k)/\sqrt{q} \leftarrow$ if $\mathbf{e}_1, \mathbf{e}_2$ small enough then $(x_2 + k)/\sqrt{q} = i$.

Well-formedness ZK proof for almost free

To prove that the ciphertext \mathbf{f}_0, f_1, f_2 is well formed, the user needs to prove that

- 1 he knows a short \mathbf{r} such that

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1) is proven already and 2) is cheap (about 2KB extra to the signature).

Unreliable decryption

The soundness of the well-formedness proof only ensures that there exists $\bar{\mathbf{r}}$ slightly bigger than \mathbf{r} and $\bar{c} \in \mathcal{C}$ such that

$$\bar{c} \begin{bmatrix} \mathbf{f}_0 \\ f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{a}_1^T \\ \mathbf{a}_2^T \end{bmatrix} \bar{\mathbf{r}} + \bar{c} \begin{bmatrix} \mathbf{0} \\ i \\ i' \end{bmatrix} .$$

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The verifier is not sure that the decryption is correct !

He only knows that there exists c' such that with $\bar{c} = c - c'$, $\bar{c}\mathbf{f}$ is a valid ciphertext for the message $\bar{c}i$.

Verifiable decryption



Verifiable decryption

Actual decryption algorithm

- Sample $c' \leftarrow \mathcal{C}$
- $\bar{c} = c - c'$
- Compute $x_1 = f_1 - \mathbf{h}_1^T \mathbf{f}_0$ (Try and decrypt $\bar{c}(\mathbf{f}_0, f_1, f_2)$)
- Compute $x_2 = f_2 - \mathbf{h}_2^T \mathbf{f}_0$
- Compute $k = \bar{c}(\sqrt{q}x_1 - x_2) \bmod \sqrt{q}$
- If $\|\bar{c}(\sqrt{q}x_1 - x_2)\|_\infty \leq \frac{q}{4\|\bar{c}\|}$, then
- return $(x_2 + k)/(\sqrt{q}\bar{c})$ (Divide the plaintext by \bar{c}).

Verifiable decryption

Actual decryption algorithm

- Sample $c' \leftarrow \mathcal{C}$
- $\bar{c} = c - c'$
- Compute $x_1 = f_1 - \mathbf{h}_1^T \mathbf{f}_0$ (Try and decrypt $\bar{c}(\mathbf{f}_0, f_1, f_2)$)
- Compute $x_2 = f_2 - \mathbf{h}_2^T \mathbf{f}_0$
- Compute $k = \bar{c}(\sqrt{q}x_1 - x_2) \pmod{\sqrt{q}}$
- If $\|\bar{c}(\sqrt{q}x_1 - x_2)\|_\infty \leq \frac{q}{4\|\bar{c}\|}$, then
- return $(x_2 + k)/(\sqrt{q}\bar{c})$ (Divide the plaintext by \bar{c}).

We prove that correctness follows from the condition and the correctness of the original decryption algorithm, and that the condition is met with non-negligible probability.

Open question

For the traceability reduction, we need to add a uniformly random copy of **B** which we call **B'**. The public matrix becomes $[\mathbf{A}|\mathbf{B}|\mathbf{B}']$, which costs about 31KB in the signature.

Open question

For the traceability reduction, we need to add a uniformly random copy of **B** which we call **B'**. The public matrix becomes $[A|B|B']$, which costs about 31KB in the signature.

Adding **B'** seems to be an artefact of the proof, and removing it does not seem to affect security. It is a very good open question to find a proof that does not rely on a copy of **B** (without affecting other parameters) to reduce the signature (and public key) size.

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