A Practical Key-Recovery Attack on 805-Round Trivium

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Trivium

- It is a bit oriented synchronous stream cipher designed by Christophe De Cannière, and Bart Preneel.
- It was selected as one of the eSTREAM hardware-oriented finalists and it was specified as an International Standard under ISO/IEC 29192-3:2012.
- Both the key size $|K|$ and the IV size $|IV|$ are 80 bits.
- It contains a 288-bit internal state.
- The number of initialization rounds is 1152.

Trivium

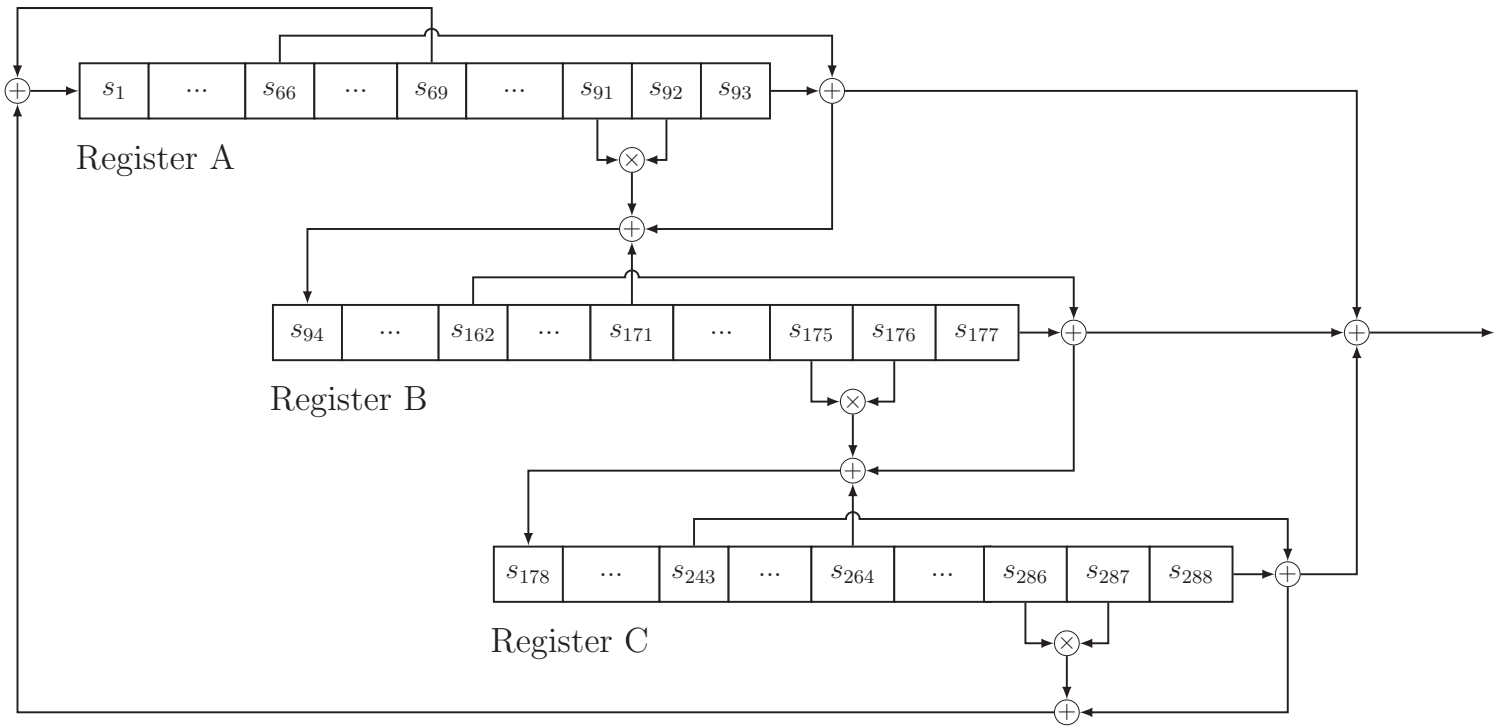

Figure: An overview of Trivium

Trivium

Algorithm 1 Key/IV initialization

$(s_1, s_2, \ldots, s_{93}) \leftarrow (k_0, k_1, \ldots, k_{79}, 0, \ldots, 0)$;
$(s_{94}, s_{95}, \ldots, s_{177}) \leftarrow (v_0, v_1, \ldots, v_{79}, 0, \ldots, 0)$;
$(s_{178}, s_{179}, \ldots, s_{288}) \leftarrow (0, \ldots, 0, 1, 1, 1)$;
for i from 1 to 1152 **do**
    $t_1 \leftarrow s_{66} \oplus s_{91} \cdot s_{92} \oplus s_{93} \oplus s_{171}$;
    $t_2 \leftarrow s_{162} \oplus s_{175} \cdot s_{176} \oplus s_{177} \oplus s_{264}$;
    $t_3 \leftarrow s_{243} \oplus s_{286} \cdot s_{287} \oplus s_{288} \oplus s_{69}$;
    $(s_1, s_2, \ldots, s_{93}) \leftarrow (t_3, s_1, \ldots, s_{92})$;
    $(s_{94}, s_{95}, \ldots, s_{177}) \leftarrow (t_1, s_{94}, \ldots, s_{176})$;
    $(s_{178}, s_{179}, \ldots, s_{288}) \leftarrow (t_2, s_{178}, \ldots, s_{287})$;
end for

Cube Attacks

- (traditional) cube attack
  - I.Dinur and A. Shamir, Cube attacks on tweakable black box polynomials, EUROCRYPT 2009. [DS09]
  - P.A. Fouque and T. Vannet, Improving key recovery to 784 and 799 rounds of Trivium using optimized cube attacks, FSE 2013. [FV13]
  - ...
- division property based cube attack
  - Y. Todo, T. Isobe, Y.L. Hao, and W. Meier. Cube attacks on non-blackbox polynomials based on division property, CRYPTO 2017. [TIHM17a]
  - Q.J. Wang, Y.L. Hao, Y. Todo, C.Y Li, T. Isobe, and E. Meier. Improved division property based cube attacks exploiting algebraic properties of superpoly, CRYPTO 2018. [WHT+18]
  - ...
- correlation cube attack
  - M.C. Liu, J.C. Yang, W.H. Wang, D.D. Lin, Correlation cube attacks: From weak-key distinguisher to key recovery, EUROCRYPT 2018. [LYWL18]
  - ...

Basics of Cube Attacks

- Let $f(x_1, x_2, \cdots, x_n)$ be an n-variable Boolean function.
- Let $I = \{i_1, i_2, \ldots, i_d\} \subseteq \{1, 2, \ldots, n\}$ be a subset of variable indexes.
- The Boolean function f could be uniquely written as

$$f = t_I \cdot p_I \oplus q(x_1, x_2, \cdots, x_n),$$

where $t_I = \prod_{i \in I} x_i$ and q is the sum of terms not divisible by t_I.

- The polynomial p_I is called the superpoly of I in f.

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- Let $d = |I|$. A d-dimensional cube C_I is a set of assignments consisting of 2^d n-tuples in which the variables indexed by I are assigned to all the possible combinations of $0/1$ while all the other variables remain undetermined.
- Then

$$p_I(X') = \sum_{\mathbf{x} \in C_I} f(\mathbf{x}), X' = X \backslash I.$$

  - Note that to evaluate $p_I(X')$, 2^d summations are always needed.

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Cube Attacks against Stream Ciphers

- The function $f(k_1, k_2, \ldots, k_n, v_1, v_2, \ldots, v_m)$ is the polynomial representation of the first output keystream bit on key and IV variables.
- Let $I \subseteq \{1, 2, \ldots, m\}$ be a subset of IV indexes and $d = |I|$.
- Let C_I be a set of assignments for IV variables consisting of 2^d m-tuples in which the variables indexed by I are assigned to all the possible combinations of $0/1$ while all the other IV variables are assigned to 0 (or fixed constants).
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Cube Attacks against Stream Ciphers

- Off-line phase
  - independent of the secret key
  - find some useful superpolies in secret key variables
- on-line phase
  - solve a system of equations defined by superpolies under the key

$$\begin{cases} p_{I_1}(key) = a_1 \\ p_{I_2}(key) = a_2 \\ \quad \cdots \\ p_{I_u}(key) = a_u \end{cases}$$

Basics in Cube Attacks

$$f = t_I \cdot p_I \oplus q(x_1, x_2, \cdots, x_n),$$

- Using Low-Degree tests to find simple/low-degree/balanced superpolies p_I containing secrete key information.

Low-Degree Tests

- BLR Linearity Tests: Finding linear superpolies.
- Quadraticity Tests: Finding quadratic superpolies.

Using Möbius Transformation [FV13]

- In FSE 2013, Fouque and Vannet used **Möbius Transformation** to simultaneously compute a large number of subcubes from a large cube.
- Möbius Transformation: Given the truth table of a Boolean function, compute its algebraic normal form.
  - store $\{f(key, IV), IV \in C_I\}$
  - compute $\sum_{J \subseteq I} p_J(key) \prod_{j \in J} x_j$

Remark: Much memory is needed to store the truth table for a large cube.

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Division property based cube attacks

- The division property was proposed by Todo at EUROCRYPT 2015 [Tod15] as a generalization of integral property used in integral cryptanalysis against block ciphers.
- The bit-based division property as well as the three-subset division property was introduced by Todo and Morii at FSE 2016 [TM16] to search new integral distinguishers for SIMON family.

Division property based cube attacks

Division property based cube attacks were first proposed by Yosuke Todo et al. at CRYPTO 2017 [TIHM17a, TIHM17b].

- Limitation of traditional cube attacks: the cube dimension could hardly be larger than 40.
- Advantage of division property based cube attacks: large cubes could be exploited (for example, 78-dimensional cubes).

Division property based cube attacks

Definition (Bit-Based Division Property)

Let $\mathbb{X}$ be a multiset whose elements take a value of $\mathbb{F}_2^n$. Let $\mathbb{K}$ be a subset of $\mathbb{F}_2^n$. When the multiset $\mathbb{X}$ has the division property $\mathcal{D}_{\mathbb{K}}^{1^n}$, it fulfills the following conditions:

$$\bigoplus_{\mathbf{x}\in\mathbb{X}} \mathbf{x}^{\mathbf{u}} = \begin{cases} \text{unknown} & \text{if there exists } \mathbf{k} \text{ in } \mathbb{K} \text{ s.t. } \mathbf{u} \succeq \mathbf{k}, \\ 0 & \text{otherwise.} \end{cases}$$

where $\mathbf{u} \succeq \mathbf{k}$ if $u_i \geq k_i$ for all i and $\mathbf{x}^{\mathbf{u}} = \prod_{i=1}^{n} x_i^{u_i}$.

Division property based cube attacks

- Propagation rules of division property for basic operations (AND, XOR and COPY) are studied and proved in [Todo15, TM16].
- The propagation of division property could be evaluated as

$$\{\boldsymbol{k}\} = \mathbb{K}_0 \to \mathbb{K}_1 \to \cdots \to \mathbb{K}_r,$$

  where $\mathbb{K}_i$ is the division property of the internal state after i rounds.
- For $(\boldsymbol{k}_0, \boldsymbol{k}_1, \ldots, \boldsymbol{k}_r) \in \mathbb{K}_0 \times \mathbb{K}_1 \times \cdots \mathbb{K}_r$, if $\mathbf{k}_i$ could propagate to $\mathbf{k}_{i+1}$ for all $i \in \{0, 1, \ldots, r-1\}$, then

$$\boldsymbol{k}_0 \to \boldsymbol{k}_1 \to \cdots \to \boldsymbol{k}_r$$

  is called an r-round division trail.

Division property based cube attacks

- At ASIACRYPT 2016, Xiang et al. showed that the propagation could be efficiently evaluated by using MILP (Mixed Integer Linear Program) models [XZBL16].
- An MILP model covers all division trails, and the solver evaluates the feasibility whether there are division trails from the input division property to the output one or not.

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Degree evaluation method

Proposition ([WHT+18])

Let $f(\boldsymbol{x}, \boldsymbol{v})$ be a polynomial, where $\boldsymbol{x}$ and $\boldsymbol{v}$ denote the secret and public variables, respectively. For a set of indices $I = \{i_1, i_2, \ldots, i_{|I|}\} \subset \{1, 2, \ldots, m\}$, let C_I be a set of $2^{|I|}$ values where the variables in $\{v_{i_1}, v_{i_2}, \ldots, v_{i_{|I|}}\}$ are taking all possible combinations of values. Let $\boldsymbol{k}_I$ be an m-dimensional bit vector such that $\boldsymbol{v}^{\boldsymbol{I}} = t_I = v_{i_1} v_{i_2} \cdots v_{i_{|I|}}$. Let $\boldsymbol{k}_\Lambda$ be an n-dimensional bit vector. If there is no division trail such that $(\boldsymbol{k}_\Lambda || \boldsymbol{k}_I) \xrightarrow{f} 1$, then the monomial $x^{\boldsymbol{k}_\Lambda}$ is not involved in the superpoly of the cube C_I.

- If there is $d \geq 0$ such that for all $\boldsymbol{k}_\Lambda$ of Hamming Weight $hw(\boldsymbol{k}_\Lambda) > d$, the division trail $x^{\boldsymbol{k}_\Lambda}$ does not exist, then it can be seen that d is an upper bound of the algebraic degree of the superpoly.

Previous result on contructing good cubes

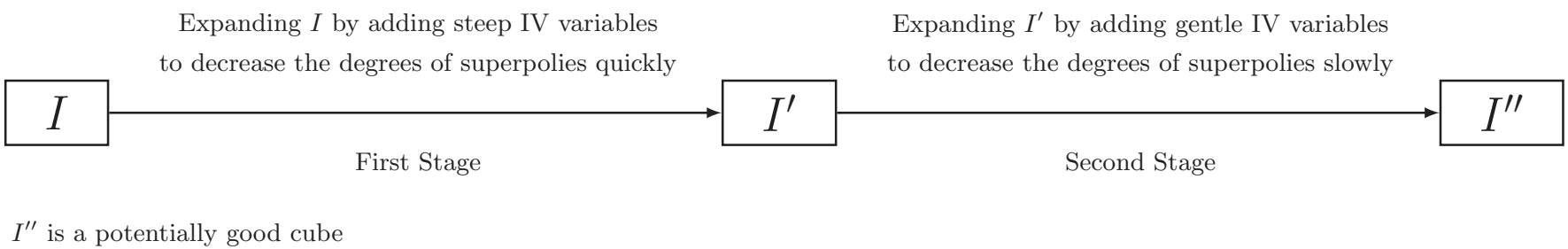
- a random walk algorithm [DS09] linear superpolies
- the union of two disjoint small cubes [FV13] linear superpolies
- cubes with no adjacent indexes [Liu17] zero-sum distinguishers
- a GreedyBitSet algorithm [SMB17, KRSM20] zero-sum distinguishers

Our aim and idea

- Our Aim: construct good cubes for recovering linear superpolies.
- Our Idea: Start from a set of cube variables indexed by I. We extend I to be a large candidate cube indexes.
  - How to determine a proper set of cube variables indexed by I?
  - How to extend I to be a large candidate cube indexes?

An Overview of How to Extend I

We propose a two-stage method to extend I.



- Steep IV Variable: an IV variable to I which makes the degree of the superpoly decrease as fast as possible.
- Gentle IV Variable: an IV variable of I which decreases the degree of the superpoly as slowly as possible.

A Heuristic Algorithm of Constructing Cubes

- Stage I: Every time, add a steep IV variable to decrease the degree of superpoly as fast as possible.

/* The first stage */
while $ds > 1$ and $|I|$ is less than a given bound **do**
  for $v \in B$ **do**
    Estimate the upper bound of $ds(I \cup \{v\})$ using the division property based method;
  end for
  $I \leftarrow I \cup \{v\}$, where v is the first steep IV variable of I;
  $B \leftarrow B \setminus v$;
  $ds \leftarrow DS(I \cup \{v\})$, where $DS(I \cup \{v\})$ is the upper bound of $ds(I \cup \{v\})$
end while
if $ds(I) == 1$ **then**
  return I
end if

A Heuristic Algorithm of Constructing Cubes

- Stage II: add a gentle IV variable to decrease the degree of the superpoly gentlely.

/* The second stage */
if $ds(I) == 0$ **then**
  $I \leftarrow I \setminus \{v\}$, where v is the steep IV variable added in the last iteration of the first stage.
  $I \leftarrow I \cup \{v'\}$, where $DS(I \cup \{v'\})$ attains minimum except 0 in the last iteration of the first stage.
  $B \leftarrow \{v_0, v_1, \ldots, v_{m-1}\} \setminus I$;
  while $ds > 1$ and $|I|$ is less than a given bound **do**
    for $v \in B$ **do**
      Estimate the upper bound of $ds(I \cup \{v\})$ using the division property based method;
    end for
    $I \leftarrow I \cup \{v\}$, where v is the first gentle IV variable
    $B \leftarrow B \setminus v$;
    $ds \leftarrow DS(I \cup \{v\})$
  end while
end if

The start cube set

- $z = s_{66} \oplus s_{93} \oplus s_{162} \oplus s_{177} \oplus s_{243} \oplus s_{288}$
- If a cube C has linear superpoly, then it is likely only one of the six terms contributes the linear superpoly and the other five terms contribute constant 0.
- Take s_{66}^t for example:

$$s_{66}^t = s_{243}^{t-66} \oplus s_{286}^{t-66} \cdot s_{287}^{t-66} \oplus s_{288}^{t-66} \oplus s_{69}^{t-66}.$$

- Choose a set of cube variables indexed by I and search some of its subcubes to find a cube with linear superpolies either in s_{286}^{t-66} or in s_{287}^{t-66}. Such cube is a starting cube set.

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- Choose a set of cube variables indexed by I and search some of its subcubes to find a cube with linear superpolies either in s_{286}^{t-66} or in s_{287}^{t-66}. Such cube is a starting cube set.

Predict the Preference Bit

$$z^r = s_{66} \oplus s_{93} \oplus s_{162} \oplus s_{177} \oplus s_{243} \oplus s_{288}$$
$$\Rightarrow \quad p_I = p_I^{66} \oplus p_I^{93} \oplus p_I^{162} \oplus p_I^{177} \oplus p_I^{243} \oplus p_I^{288}$$

- Note that a linear superpoly usually only has one term, say $p_I = k_j$, one of the six superpolies $p_I^{66}, p_I^{93}, p_I^{162}, p_I^{177}, p_I^{243}, p_I^{288}$ is equal to k_j.
- For a given r, we would like to predict

$$k \in \{66, 93, 162, 177, 243, 288\}$$

with a large success probability such that a linear superpoly in z^r implies a linear superpoly in s_k^r(Preference Bit).

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Predict the Preference Bit

Our Idea

Predicting the numbers of terms in form of $t_I \cdot k_j$ (called VK-terms) in $s_{66}, s_{93}, s_{162}, s_{177}, s_{243}, s_{288}$.

- It is difficult to predict the number of called VK-terms in $s_{66}, s_{93}, s_{162}, s_{177}, s_{243}, s_{288}$ directly.

Our Solution

Exploiting the feedback function of Trivium to predict the number of called VK-terms iteratively.

Predict the Preference Bit

$$s_{94} \leftarrow s_{66} \oplus s_{91} \cdot s_{92} \oplus s_{93} \oplus s_{171}$$

- The number of terms of the form $t_I \cdot k_j$ in s is denoted by $VK(s)$.
- The number of pure IV terms in s is denoted by $V(s)$.
- Then

$$VK(s_{94}) \leftarrow VK(s_{66}) + VK(s_{91}s_{92}) + VK(s_{93}) + VK(s_{171})$$

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- The number of terms of the form $t_I \cdot k_j$ in s is denoted by $VK(s)$.
- The number of pure IV terms in s is denoted by $V(s)$.
- Then $VK(s_{94})$ is set as

$$\begin{aligned} VK(s_{66}) \quad &+ \quad VK(s_{93}) + VK(s_{171}) \\ &+ \quad V(s_{91}) \cdot VK(s_{92}) + V(s_{92}) \cdot VK(s_{91}) \end{aligned}$$

- Base on the above method, we propose Algorithm 4 which could iteratively predict the number of VK-terms $s_{66}^{(r)}, s_{93}^{(r)}, s_{162}^{(r)}, s_{177}^{(r)}, s_{243}^{(r)}, s_{288}^{(r)}$. Thus, the preference bit could be predicted successfully.

Algorithm 4 The algorithm of predicting the preference bit

Calculate the ANFs of $s_i^{(280)}$ to initialise $NVK^{(280)}$ and $NV^{(280)}$;
for $280 \leq t \leq r-1$ **do**
  $NVK_{t_1} \leftarrow NV_{91}^{(t)} \cdot NVK_{92}^{(t)} + NV_{92}^{(t)} \cdot NVK_{91}^{(t)} + NVK_{93}^{(t)} + NVK_{66}^{(t)} + NVK_{171}^{(t)}$;
  $NV_{t_1} \leftarrow NV_{91}^{(t)} \cdot NV_{92}^{(t)} + NV_{93}^{(t)} + NV_{66}^{(t)} + NV_{171}^{(t)}$;
  $NVK_{t_2} \leftarrow NV_{175}^{(t)} \cdot NVK_{176}^{(t)} + NV_{176}^{(t)} \cdot NVK_{175}^{(t)} + NVK_{177}^{(t)} + NVK_{162}^{(t)} + NVK_{264}^{(t)}$;
  $NV_{t_2} \leftarrow NV_{175}^{(t)} \cdot NV_{176}^{(t)} + NV_{177}^{(t)} + NV_{162}^{(t)} + NV_{264}^{(t)}$;
  $NVK_{t_3} \leftarrow NV_{286}^{(t)} \cdot NVK_{287}^{(t)} + NV_{287}^{(t)} \cdot NVK_{286}^{(t)} + NVK_{288}^{(t)} + NVK_{243}^{(t)} + NVK_{69}^{(t)}$;
  $NV_{t_3} \leftarrow NV_{286}^{(t)} \cdot NV_{287}^{(t)} + NV_{288}^{(t)} + NV_{243}^{(t)} + NV_{69}^{(t)}$;
  for $288 \geq j \geq 2$ **do**
    $NVK_j^{(t)} \leftarrow NVK_{j-1}^{(t)}$;
    $NV_j^{(t)} \leftarrow NV_{j-1}^{(t)}$;
  end for
  $NV_{94}^{(t)} \leftarrow NV_{t_1}$; $NV_{178}^{(t)} \leftarrow NV_{t_2}$; $NV_1^{(t)} \leftarrow NV_{t_3}$;
  $NVK_{94}^{(t)} \leftarrow NVK_{t_1}$; $NVK_{178}^{(t)} \leftarrow NVK_{t_2}$; $NVK_1^{(t)} \leftarrow NVK_{t_3}$;
end for
Choose the bit $s_b^{(t)}$ such that

$$NVK_b^{(t)} = \max\{NVK_\lambda^{(t)} | \lambda \in \{66, 93, 162, 171, 243, 288\}\}$$

as the preference bit, where $b \in \{66, 93, 162, 171, 243, 288\}$;

Improved Möbius Transformation

- Advantage: compute a large number of subcubes from a large cube simultaneously.
- Weakness: Much memory is needed to store the truth table for a large cube.

Our Aim

Testing a large number of subcubes from a large cube simultaneously with reasonable memory.

Improved Möbius Transformation

- Let $f(x_0, \ldots, x_{n-1})$ be a Boolean function on $x_0, \ldots, x_{n-1}$.

Our Idea

Breaking the original Möbius transformation into a two-stage one to recover a part of ANF of f.

- Calculate the Möbius transformations of $f(x_0, \ldots, x_{n-q-1}, 0, \ldots, 0)$, $f(x_0, \ldots, x_{n-q-1}, 1, \ldots, 0)$, $\ldots, f(x_0, \ldots, x_{n-q-1}, 1, \ldots, 1)$, which are denoted by $g_0, g_1, \ldots, g_{2^q-1}$. Keep a part information of $g_0, g_1, \ldots, g_{2^q-1}$.
- Recover a part of ANF of f according to the kept information of $g_0, g_1, \ldots, g_{2^q-1}$.

Improved Möbius Transformation

Algorithm 5 An Improved Möbius Transformation

Require: A Boolean function f, the parameter q, the bound ω

/* the first stage */

for $(c_0, c_1, \ldots, c_{q-1})$ from $(0, 0, \ldots, 0)$ to $(1, 1, \ldots, 1)$ **do**
  $S \leftarrow$ the truth table of $f(x_0, x_1, \ldots, x_{n-q-1}, c_0, c_1, \ldots, c_{q-1})$;
  Call Algorithm 2 to do Möbius transformation on S;
  $t \leftarrow 0$, $j \leftarrow \sum_{l=0}^{q-1} 2^l c_l$;
  for i from 0 to $2^{n-q} - 1$ **do**
    $tmp \leftarrow (b_0, b_1, \ldots, b_{n-q-1})$, where $i = \sum_{l=0}^{n-q-1} b_l \cdot 2^l$;
    if $wt(tmp) \geq \omega$ **then**
      $FS[j][t] \leftarrow S[i]$;
      $t \leftarrow t + 1$;
    end if
  end for
end for

/* the second stage */

for i from 1 to q **do**
  $Sz \leftarrow 2^i$, $Pos \leftarrow 1$;
  while $Pos < 2^q$ **do**
    for b from 0 to $Sz - 1$ **do**
      for a from 0 to $t - 1$ **do**
        $FS[Pos + Sz + b][a] \leftarrow FS[Pos + Sz + b][a] \oplus FS[Pos + b][a]$;
      end for
    end for
    $Pos \leftarrow Pos + 2 \times Sz$;
  end while
end for

Memory Complexity of Improved Möbius Transformation

- Memory Complexity of Algorithm 5.
  - The size of S is 2^{n-q}, and so it costs 2^{n-q} bits memory.
  - For each j, the size of $FS[j]$ is t, and so it requires $2^q \times t$ bits memory totally.

- Memory Complexity Comparison
  - Improved Möbius transformatio: $2^q \times t + 2^{n-q}$.
  - Original Möbius transformatio: 2^n.

A Practical Key-Recovery Attack on 805-Round Trivium

- The preference bit to find a linear superpoly is $s_{66}^{(805)}$.
- $s_{66}^{(805)} = s_{286}^{(739)} \cdot_{287}^{(739)} \oplus s_{243}^{(739)} \oplus s_{288}^{(739)} \oplus s_{69}^{(739)}$
- Choose cubes of sizes 22 and use the Moebius transformation to find a proper cube whose superpoly in $s_{286}^{(739)}$ is linear.
- Start from

$$I_1 = \{v_4, v_6, v_{10}, v_{11}, v_{15}, v_{17}, v_{19}, v_{21}, v_{25}, v_{29}, v_{32},$$

$$v_{34}, v_{36}, v_{39}, v_{41}, v_{43}, v_{50}\}$$

A Practical Key-Recovery Attack on 805-Round Trivium

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$$I_1 = \{v_4, v_6, v_{10}, v_{11}, v_{15}, v_{17}, v_{19}, v_{21}, v_{25}, v_{29}, v_{32},$$

$$v_{34}, v_{36}, v_{39}, v_{41}, v_{43}, v_{50}\}$$

A Practical Key-Recovery Attack on 805-Round Trivium

- First stage: We extend I_1 of size 17 to I_2 of size 34 as follows:

$$I_2 = \{v_4, v_6, v_{10}, v_{11}, v_{15}, v_{17}, v_{19}, v_{21}, v_{25}, v_{29}, v_{32}, v_{34}, v_{36}, v_{39}, v_{41}, v_{43}, v_{50},$$
$$v_2, v_{69}, v_{79}, v_8, v_{27}, v_0, v_1, v_{28}, v_{71}, v_{13}, v_{45}, v_{23}, v_{26}, v_{38}, v_{76}, v_{47}, v_{56}\}.$$

- Second stage: We start with

$$I_3 = \{v_4, v_6, v_{10}, v_{11}, v_{15}, v_{17}, v_{19}, v_{21}, v_{25}, v_{29}, v_{32}, v_{34}, v_{36}, v_{39}, v_{41}, v_{43}, v_{50},$$
$$v_2, v_{69}, v_{79}, v_8, v_{27}, v_0, v_1, v_{28}, v_{71}, v_{13}, v_{45}, v_{23}, v_{26}, v_{38}, v_{76}, v_{47}, v_{52}\},$$

Table: The chosen cube variables in the last iteration

Chosen cube	$I_3 \cup \{v_{48}\}$	$I_3 \cup \{v_{59}\}$	$I_3 \cup \{v_{58}\}$	$I_3 \cup \{v_{63}\}$
degree of superpolies	1	1	2	3

A Practical Key-Recovery Attack on 805-Round Trivium

- Finally, I_4 of size 40 is a candidate large cube, which is given by

$$I_4 = \{v_4, v_6, v_{10}, v_{11}, v_{15}, v_{17}, v_{19}, v_{21}, v_{25}, v_{29}, v_{32}, v_{34}, v_{36}, v_{39},$$
$$v_{41}, v_{43}, v_{50}, v_2, v_{69}, v_{79}, v_8, v_{27}, v_0, v_1, v_{28}, v_{71}, v_{13},$$
$$v_{45}, v_{23}, v_{26}, v_{38}, v_{76}, v_{47}, v_{52}, v_{57}, v_{42}, v_{48}, v_{58}, v_{59}, v_{63}\}.$$

A Practical Key-Recovery Attack on 805-Round Trivium

- Together with some other candidate cubes, we find more than 1000 cubes with linear superpolies in the output of 805-round Trivium.
- There are 38 independent linear superpolies.
- We also find 16 linear superpolies for 806-round Trivium.

A Practical Key-Recovery Attack on 805-Round Trivium

Table: Linear superpolies of 805-round Trivium (incomplete)

cube indies	superpolies
0,1,2,4,6,8,11,13,15,17,19,21,23,26,27,28,29,32,34, 36,38,39,41,42,45,47,48,50,52,53,57,69,71,75,76,79	$1 \oplus k_2 \oplus k_{65}$
0,1,2,4,6,8,10,11,12,13,15,16,19,21,23,25,26,27,29, 31,34,36,38,39,40,43,45,47,49,62,64,70,74,77,79	$1 \oplus k_3$
0,1,2,4,6,8,10,11,13,15,17,19,21,23,26,27,29,31, 34,36,38,39,40,41,43,45,47,49,58,62,64,77,79	$k_4 \oplus k_{19} \oplus k_{34}$
0,1,2,4,6,8,10,13,15,17,19,21,23,25,26,27,28,29,32,34, 36,38,39,41,42,43,47,48,50,52,57,59,69,71,75,76,79	k_{14}
0,1,2,4,6,8,10,11,13,15,17,19,21,23,25,26,27,29,32,34, 36,38,39,41,42,43,47,48,50,52,53,57,59,69,71,76,79	k_{15}
0,1,2,4,6,8,10,13,15,17,19,21,23,25,26,27,28,29,32, 34,36,38,39,41,42,43,47,48,50,52,59,69,71,75,76,79	$1 \oplus k_{16}$
0,1,2,4,6,8,10,11,13,15,17,19,21,23,25,26,27,28,29,32, 34,36,38,39,41,42,43,45,47,48,50,53,57,69,71,75,76,79	$1 \oplus k_{17}$
0,1,2,4,6,8,10,11,12,13,15,16,19,21,23,25,27,28, 29,34,36,38,40,41,43,45,47,49,50,64,70,74,77,79	k_{18}
0,1,2,4,6,8,10,11,12,13,15,16,19,23,25,27,28,31,34, 36,38,39,40,41,43,45,47,49,50,58,62,64,74,77,79	$1 \oplus k_{19} \oplus k_{34} \oplus k_{51}$

A Practical Key-Recovery Attack on 805-Round Trivium

Table: Linear superpolies of 805-round Trivium (incomplete)

cube indies	superpolies
0,2,4,6,8,10,12,13,15,17,19,21,23,25,26,27,28,29,31, 34,38,39,40,41,43,45,47,49,50,58,62,64,70,74,77,79	k_{21}
1,2,4,6,8,10,11,12,13,15,17,19,21,23,25,26,27,28,29, 31,34,36,38,39,40,41,43,47,49,50,58,62,70,74,77,79	$1 \oplus k_{29}$
0,1,2,4,6,8,10,11,13,15,17,19,21,23,25,26,27,29,32,34, 36,38,39,42,43,45,47,48,50,52,53,57,59,69,71,75,76,79	$k_{31} \oplus k_{46} \oplus k_{56}$
0,1,2,4,6,8,10,13,15,17,19,21,23,25,26,28,29,32,34, 36,38,39,41,42,45,47,48,50,52,57,59,69,71,75,76,79	$k_{17} \oplus k_{32}$
0,1,2,4,6,8,10,11,13,15,17,19,21,23,25,26,27,29,32,34, 36,38,39,42,43,45,47,48,50,52,53,57,59,69,71,76,79	$1 \oplus k_{33}$
0,1,2,4,6,8,10,11,13,15,17,19,21,23,25,26,27,29,32, 34,36,39,41,42,43,45,47,48,50,52,57,59,69,71,76,79	k_{34}
0,1,2,4,6,8,10,11,13,15,17,19,21,23,25,26,27,29,32, 34,36,38,39,41,42,43,45,47,50,52,53,57,69,71,75,79	k_{36}
0,1,2,4,6,8,10,12,13,15,17,19,21,23,25,26,27,28, 29,31,34,36,39,40,41,43,47,49,50,62,64,70,77,79	k_{40}
0,1,2,4,6,8,10,11,13,15,17,19,21,23,26,27,28,31, 34,36,38,40,41,43,45,47,49,50,58,62,64,70,77,79	k_{42}

A Practical Key-Recovery Attack on 805-Round Trivium

Table: Linear superpolies of 806-round Trivium (incomplete)

cube indices	superpoly
0,1,3,5,7,9,10,11,12,14,15,18,20,22,24,27,28,30, 33,35,37,39,40,42,44,46,48,49,57,61,63,73,76,78	$k_{14} \oplus k_{44}$
0,1,3,5,7,9,10,11,12,14,15,18,20,22,24,26,28,30, 33,35,37,39,40,42,44,46,48,49,57,61,63,73,76,78	k_{15}
0,1,3,5,7,9,10,11,12,14,15,18,20,22,24,26,28,30, 33,35,37,39,40,42,44,46,48,49,57,61,63,76,78	$1 \oplus k_{17}$
0,1,3,5,7,9,10,11,12,14,16,18,20,22,24,25,26,27,28, 30,33,35,37,38,39,40,42,46,48,49,57,61,69,73,76,78	$1 \oplus k_{28}$
0,1,3,5,7,9,10,11,12,14,15,16,18,20,22,24,25,26,27, 28,30,33,35,37,38,39,40,42,46,48,49,57,61,63,76,78	k_{32}
0,1,3,5,7,9,10,11,12,14,15,18,20,22,24,26,27,28, 33,35,37,39,40,42,44,46,48,49,57,61,63,73,76,78	k_{33}
0,3,5,7,9,11,14,15,18,20,22,24,25,26,27,30,33,35, 37,39,40,42,44,46,48,49,57,61,63,69,73,76,78	k_{41}
0,1,3,5,7,9,10,11,12,14,15,18,20,22,24,26,27,28, 30,33,35,37,40,42,44,46,48,49,57,61,63,73,76,78	k_{42}
1,3,5,7,9,10,11,12,14,16,18,20,22,24,25,26,27,28, 30,33,35,37,38,39,40,42,46,48,49,57,61,63,73,76,78	k_{44}

A Summery of Key-Recovery Attacks on Trivium

Attack type	# of rounds	Off-line phase		On-line phase	Total time	ref.
		cube size	# of key bits			
Practical	672	12	63	2^{17}	$2^{18.56}$	[DS09]
	709	22-23	79	< 2	$2^{29.14}$	[MS11]
	767	28-31	35	2^{45}	$2^{45.00}$	[DS09]
	784	30-33	42	2^{38}	2^{39}	[FV13]
	805	32-38	42	2^{38}	$2^{41.40}$	this report
Not practical	799	32-37	18	2^{62}	$2^{62.00}$	[FV13]
	802	34-37	8	2^{72}	$2^{72.00}$	[YT18]
	805	28	7	2^{73}	$2^{73.00}$	[LYWL18]
	806	34-37	16	2^{64}	2^{64}	this report
	835	35	5	2^{75}	$2^{75.00}$	[LYWL18]
	832	72	1	2^{79}	$2^{79.01}$	[WHG+19, TIHM17a, TIHM18]
	832	72	> 1	2^{79}	$< 2^{79.01}$	[YT20]
	840	78	1	2^{79}	$2^{79.58}$	[HLM+20b]
	840	75	3	2^{77}	$2^{77.32}$	[HSWW20]
	841	78	1	2^{79}	$2^{79.58}$	[HLM+20b]
	841	76	2	2^{78}	$2^{78.58}$	[HSWW20]
	842	78	1	2^{79}	$2^{79.58}$	[HLM+20a]
	842	76	2	2^{79}	$2^{78.58}$	[HSWW20]

A candidate cube for 810-Round Trivium

- We construct a candidate cube I_9 for 810-round Trivium of size 43, and we find two subcubes of size 42 with linear superpolies.

$$I_9 = \{v_2, v_6, v_8, v_{10}, v_{11}, v_{15}, v_{19}, v_{21}, v_{25}, v_{29}, v_{30}, v_{32}, v_{34}, v_{36},$$

$$v_{39}, v_{41}, v_{43}, v_{45}, v_{50}, v_0, v_{75}, v_{12}, v_4, v_{14}, v_{20}, v_{22}, v_{16}, v_{27}, v_{23},$$

$$v_{72}, v_{52}, v_{55}, v_{60}, v_{37}, v_{79}, v_{62}, v_{64}, v_{47}, v_{54}, v_{69}, v_{71}, v_{18}, v_{53}\}$$

Conclusions

- A new algorithm to construct good cubes and its application to 805-round Trivium.
  - A Heuristic Algorithm to Construct Candidate Cubes.
  - The Preference Bit and an Algorithm to Predict It.
  - The Improved Möbius Transformation.
- The new algorithm could also be applied to Trivium-like ciphers.

Thanks for Your Attention!

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