A formula for disaster: a unified approach to elliptic curve special-point-based attacks

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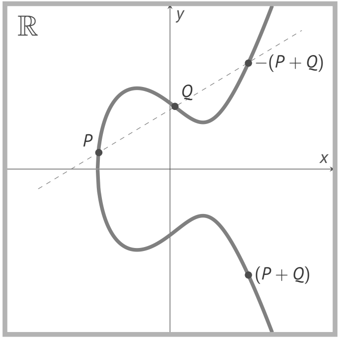
What is ECC?

Elliptic Curve Cryptography

Based on the discrete log: given P and $[k]P$, recover the private key k



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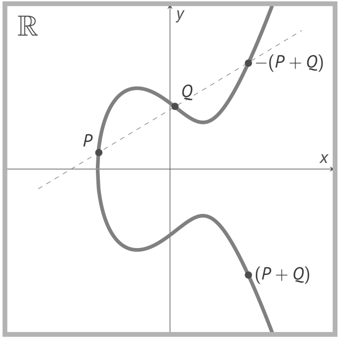

Textbook affine addition: $P = (X_1 : Y_1 : 1), Q = (X_2 : Y_2 : 1) \implies P + Q = (X_3 : Y_3 : 1)$,
where $X_3 = \lambda^2 - X_1 - X_2$, $Y_3 = \lambda(X_1 - X_3) - Y_1$, $\lambda = \frac{Y_1 - Y_2}{X_1 - X_2}$.

Implementing ECC

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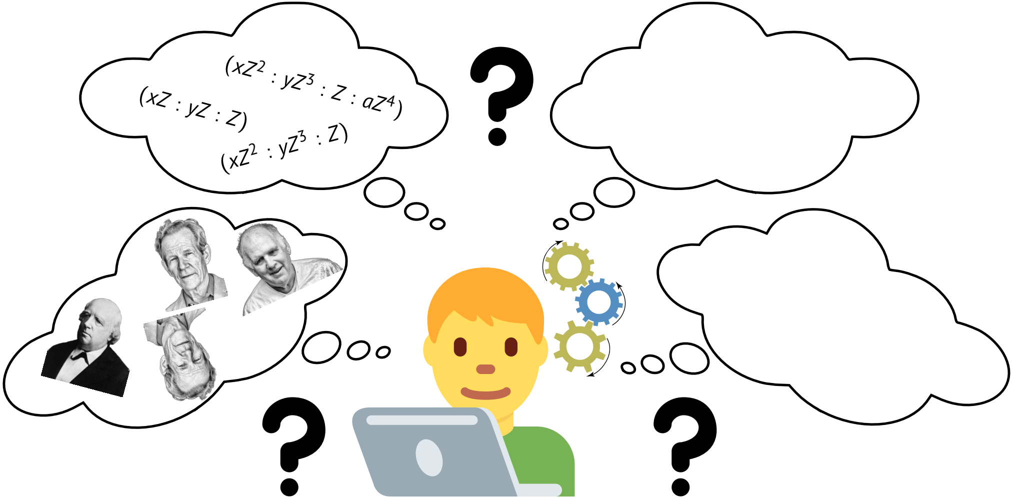

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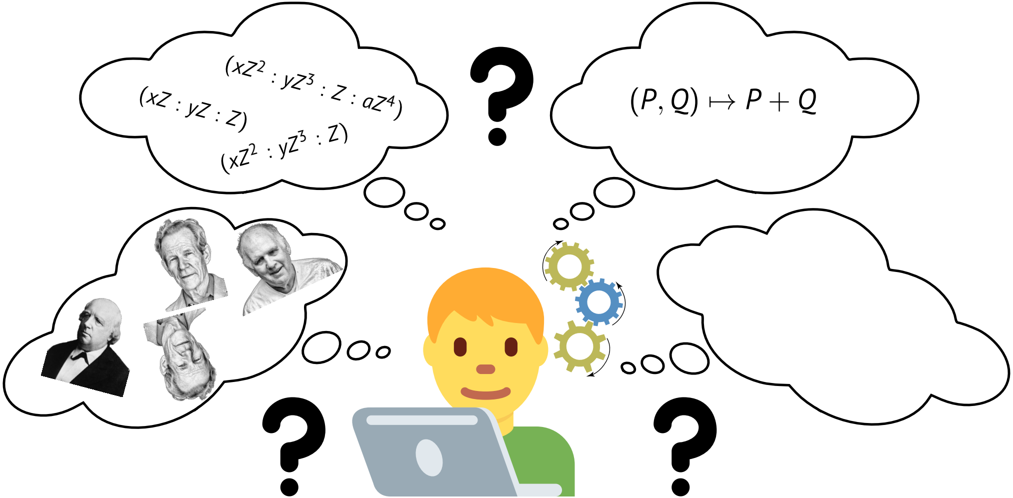

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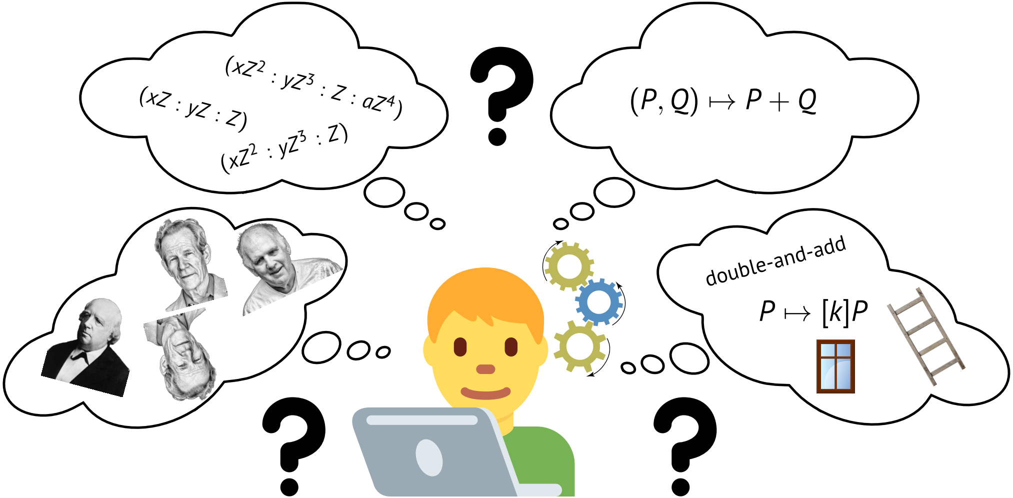

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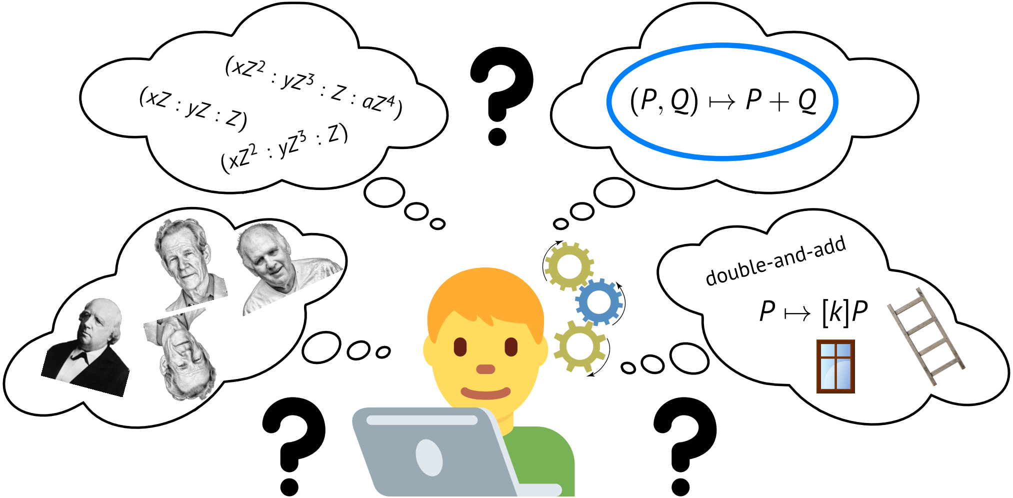

Formula example: add-2007-bl

Input: $P = (X_1 : Y_1 : Z_1), \quad Q = (X_2 : Y_2 : Z_2)$

Output: $P + Q = (X_3 : Y_3 : Z_3)$

$$
\begin{aligned}
U_1 &= X_1 \cdot Z_2 \\
U_2 &= X_2 \cdot Z_1 \\
S_1 &= Y_1 \cdot Z_2 \\
S_2 &= Y_2 \cdot Z_1 \\
ZZ &= Z_1 \cdot Z_2 \\
T &= U_1 + U_2 \\
TT &= T^2 \\
M &= S_1 + S_2 \\
t_0 &= ZZ^2 \\
t_1 &= a \cdot t_0 \\
t_2 &= U_1 \cdot U_2
\end{aligned}
$$

$$
\begin{aligned}
t_3 &= TT - t_2 \\
R &= t_3 + t_1 \\
F &= ZZ \cdot M \\
L &= M \cdot F \\
LL &= L^2 \\
t_4 &= T + L \\
t_5 &= t_4^2 \\
t_6 &= t_5 - TT \\
G &= t_6 - LL \\
t_7 &= R^2 \\
t_8 &= 2 \cdot t_7
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W &= t_8 - G \\
t_9 &= F \cdot W \\
X_3 &= 2 \cdot t_9 \\
t_{10} &= 2 \cdot W \\
t_{11} &= G - t_{10} \\
t_{12} &= 2 \cdot LL \\
t_{13} &= R \cdot t_{11} \\
Y_3 &= t_{13} - t_{12} \\
t_{14} &= F^2 \\
t_{15} &= F \cdot t_{14} \\
Z_3 &= 4 \cdot t_{15}
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Goal: classify all exceptional cases in EFD formulas

EFD addition formulas

Model	Coordinates	(x, y) representation	Number of formulas
short Weierstrass	projective	$(xZ : yZ : Z)$	21
	Jacobian	$(xZ^2 : yZ^3 : Z)$	36
	modified	$(xZ^2 : yZ^3 : Z : aZ^4)$	4
	w12 with $b = 0$	$(xZ : yZ^2 : Z)$	2
	xyzz	$(xZ^2 : yZ^3 : Z^2 : Z^3)$	6
	xz	$(xZ : Z)$	22
Montgomery	xz	$(xZ : Z)$	8
twisted Edwards	projective	$(xZ : yZ : Z)$	3
	extended	$(xZ : yZ : xyZ : Z)$	18
	inverted	$\left(\frac{Z}{x} : \frac{Z}{y} : Z\right)$	3
Edwards	projective	$(xZ : yZ : Z)$	12
	inverted	$\left(\frac{Z}{x} : \frac{Z}{y} : Z\right)$	6
	yz	$\left(yZ\sqrt{d} : Z\right)$	6
	yzsquared	$\left(y^2Z\sqrt{d} : Z\right)$	6

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- Both fuzzing and manual analysis

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- All 111 addition and 42 differential addition EFD formulas analyzed
- All exceptional points completely classified
- New family of exceptional points found for `add-2007-bl`:

$$P = (X_1 : Y_1 : 1) \text{ and } Q = (X_2 : -Y_1 : 1) \text{ with } X_1 \neq X_2,$$

Formula example: `add-2007-bl` (exceptions)

Input: $P = (X_1 : Y_1 : Z_1), \quad Q = (X_2 : Y_2 : Z_2)$

Output: $P + Q = (X_3 : Y_3 : Z_3)$

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$$Z_3 = 4 \cdot Z_2^3 \cdot Z_1^3 \cdot (Y_2 \cdot Z_1 + Y_1 \cdot Z_2)^3$$

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- Common assumptions:
  - Scalar multiplication side-channel oracle
  - Static private key k (e.g., in ECDH, X25519)
  - If k' is a binary prefix of k, then $[k']P$ appears during $[k]P$ computation

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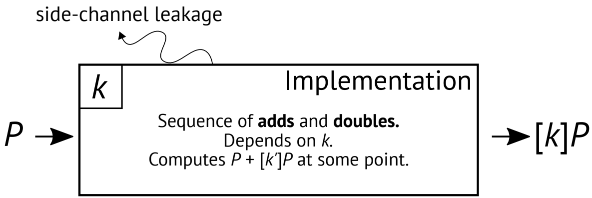

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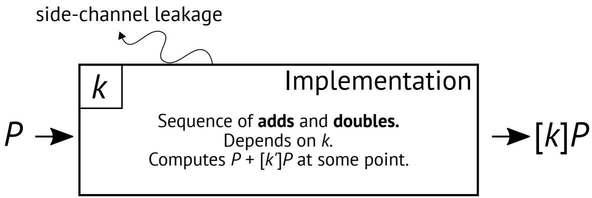

1. Guess k' - a prefix of k

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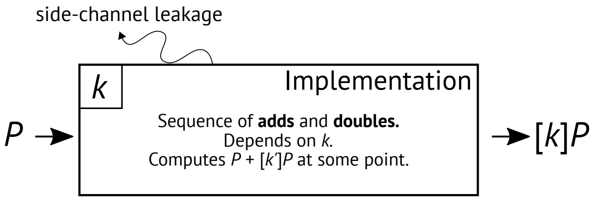

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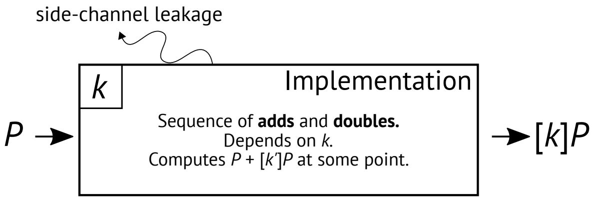

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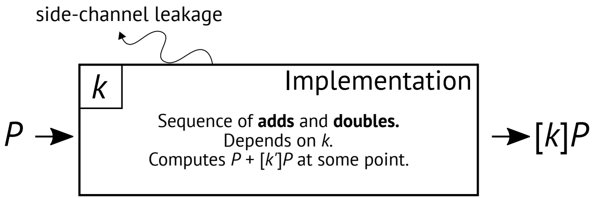

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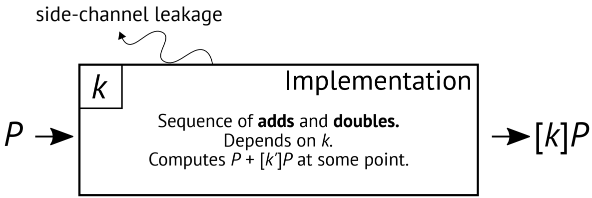

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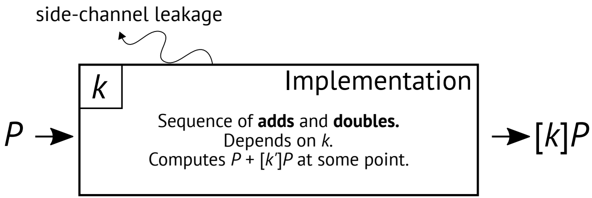

1. Guess k' - a prefix of k
2. Construct a point P s.t. $P + [k']P$ fails
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4. Verify the guess k' using a side channel: error
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The unified scenario - ZVP

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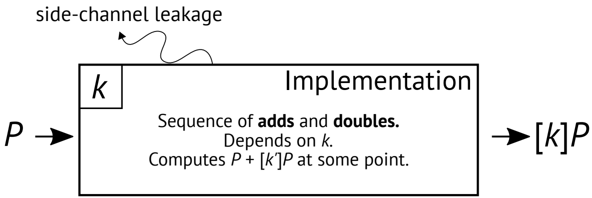

1. Guess k' - a prefix of k
2. Construct a point P s.t. $P + [k']P$ forces an intermediate $f = 0$
3. Input P to the implementation
4. Verify the guess k' using a side channel: intermediate 0 detection
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Formula example: `add-2007-bl` (ZVP)

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The DCP

The dependent coordinates problem (DCP)

Given $k' \in \mathbb{Z}$, an elliptic curve E over $\mathbb{F}_p$ and a polynomial f, find $P, Q \in E(\mathbb{F}_p)$ such that

$$Q = [k']P \quad \text{and} \quad f(P, Q) = 0.$$

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- Solving DCP $\rightarrow$ constructing oracle $\rightarrow$ private key recovery
- For EPA, take $f = Z_3$
- For ZVP, take f an intermediate expression
- For RPA, take $f = X_3$ or $f = Y_3$

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- Hard in general: capturing $Q = [k']P$ does not scale - k' up to ≈ 20 bits
- Solved new cases (e.g., $k' \equiv l/m \pmod{n}$ with $|l|, |m|$ small)

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Given $k' \in \mathbb{Z}$, an elliptic curve E over $\mathbb{F}_p$ and a polynomial f, find $P, Q \in E(\mathbb{F}_p)$ such that

$$Q = [k']P \quad \text{and} \quad f(P, Q) = 0.$$

- Hard in general: capturing $Q = [k']P$ does not scale - k' up to ≈ 20 bits
- Solved new cases (e.g., $k' \equiv l/m \pmod{n}$ with $|l|, |m|$ small)
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  - Some ZVP cases - new adaptations to window methods, simulated attack against `add-2016-rcb`

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- Be explicit about assumptions, document them!

Additional materials

Thanks for your attention!

Tooling, analysis, demos and more: crocs.fi.muni.cz/public/papers/formulas_asiacrypt21

References

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