

Probing Security through Input-Output Separation and Revisited Quasilinear Masking

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Matthieu Rivain and Damien Vergnaud

— CHES 2021 —



Introduction

- What is this about ?
 - Security against [side-channel attacks](#)
 - [Masking](#) schemes
 - Formal proofs through [probing security](#)
- Our contributions
 - New masking composition approach:

IOS refresh gadget + probing-secure gadgets
⇒ region probing security of the composition
 - Quasilinear IOS refresh gadget (variant of [BPCZ, CHES'16])
 - Quasilinear masking scheme (improved version of [GJR, AC'18])

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 - weaker than composition properties*
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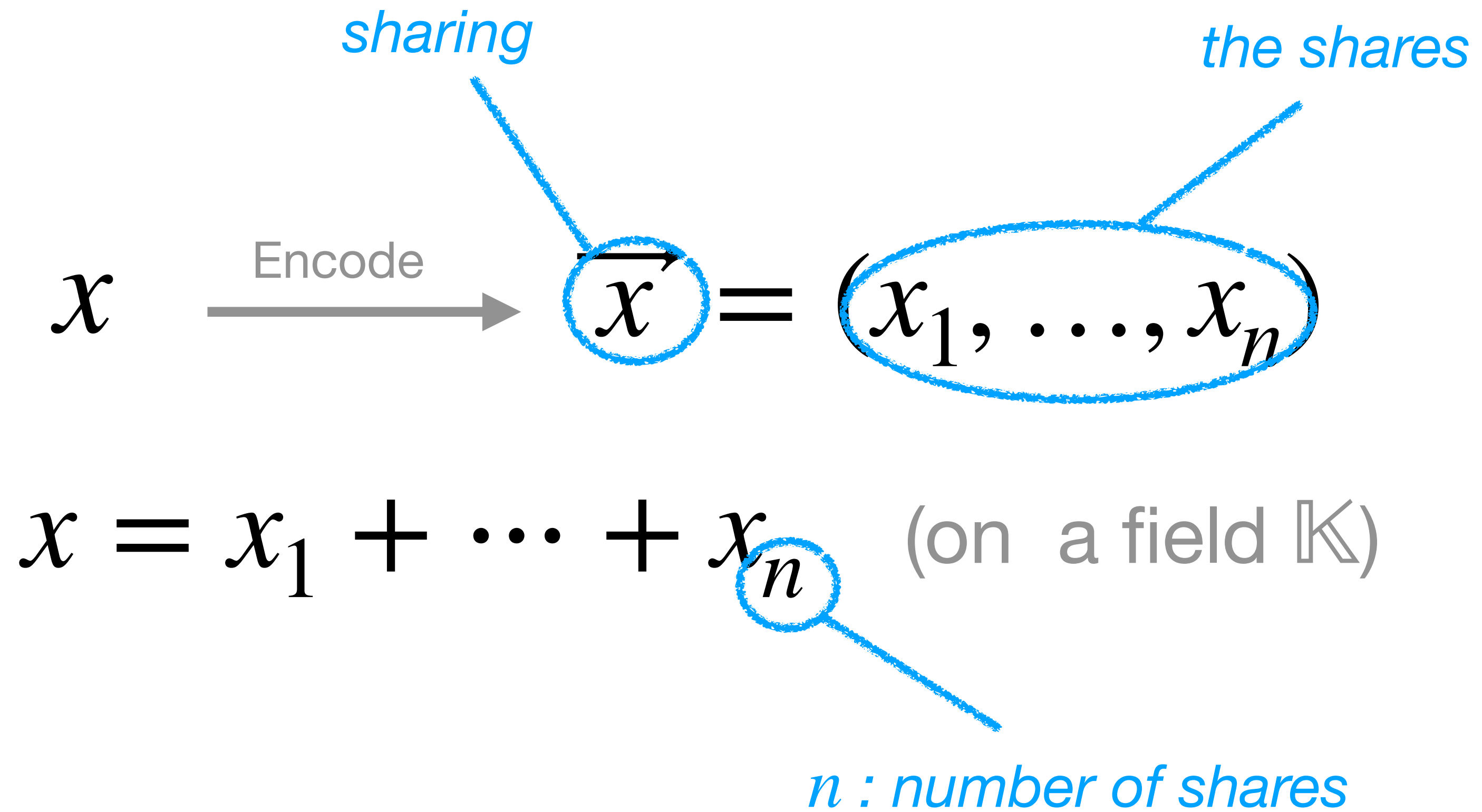
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 - ⇒ **region probing security** of the composition *stronger than probing security*
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Masking

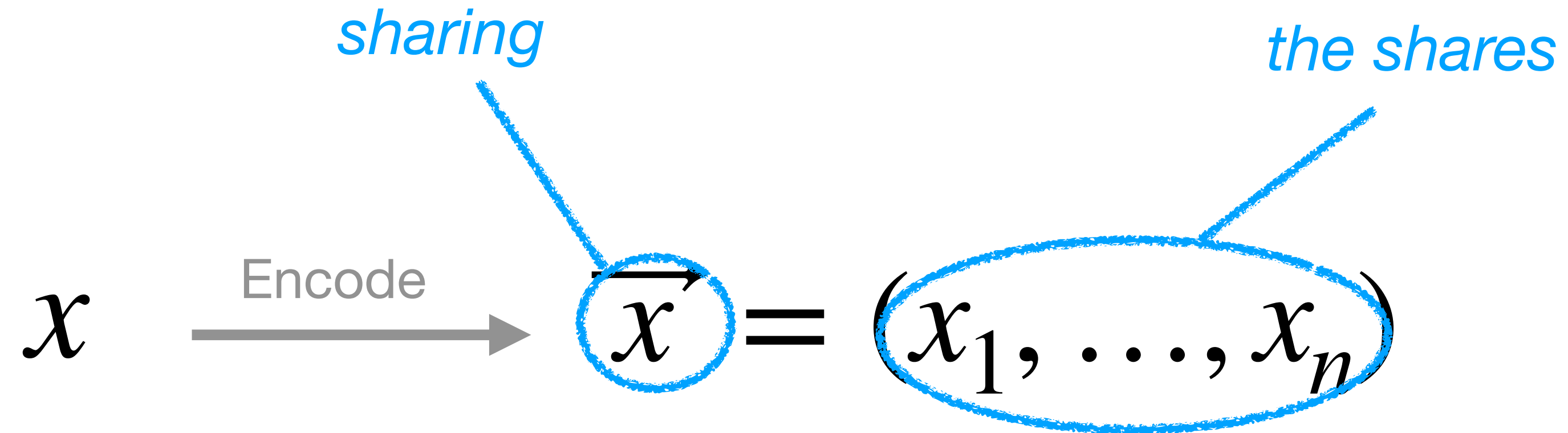
$$x \xrightarrow{\text{Encode}} \overrightarrow{x} = (x_1, \dots, x_n)$$

$$x = x_1 + \dots + x_n \quad (\text{on a field } \mathbb{K})$$

Masking



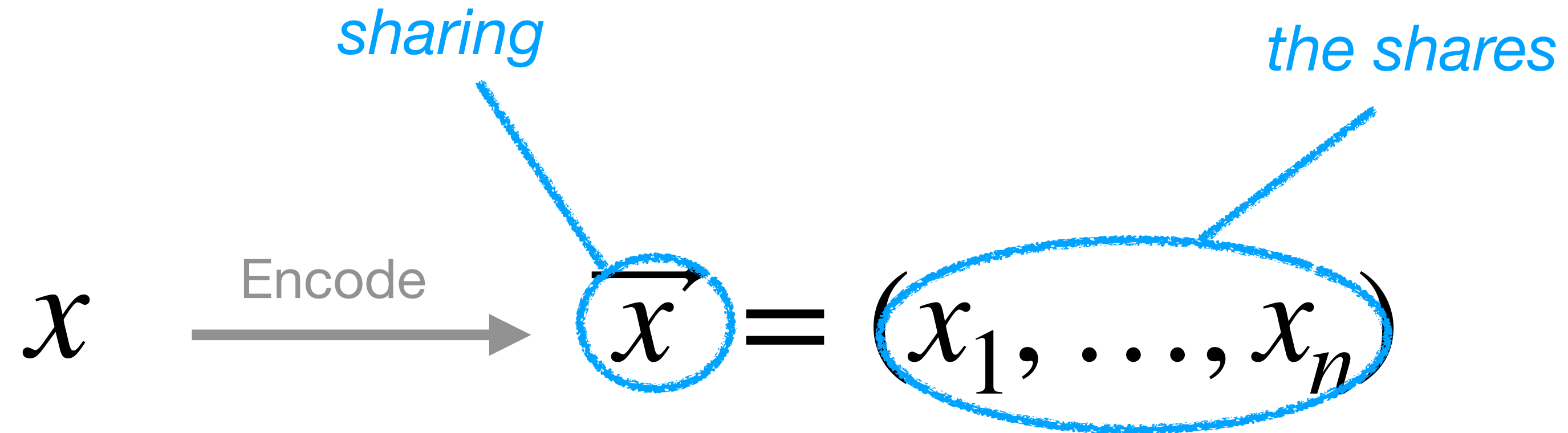
Masking



In this work:

$$\begin{aligned} x &= v_1 \cdot x_1 + \dots + v_n \cdot x_n \\ &= \langle \vec{v}, \vec{x} \rangle \quad (\text{on a field } \mathbb{K}) \end{aligned}$$

Masking



In this work:

$$x = v_1 \cdot x_1 + \dots + v_n \cdot x_n$$

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constant coefficients

sharing

Masking

$$x \xrightarrow{\text{Encode}} \vec{x} = (x_1, \dots, x_n)$$

sharing (points to \vec{x})

the shares (points to (x_1, \dots, x_n))

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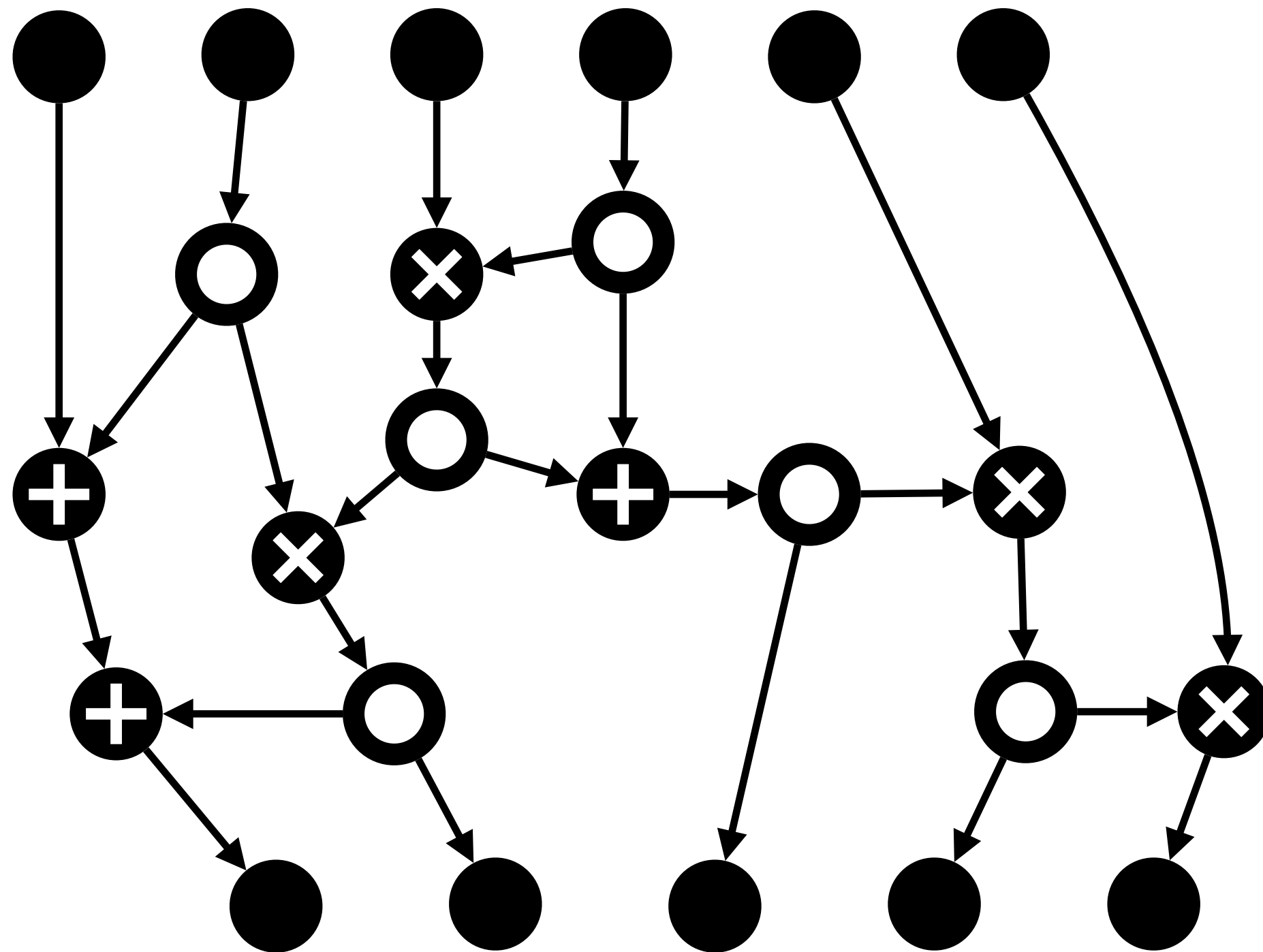
constant coefficients (points to \vec{v})

sharing (points to \vec{x})

\vec{v} -sharing of x

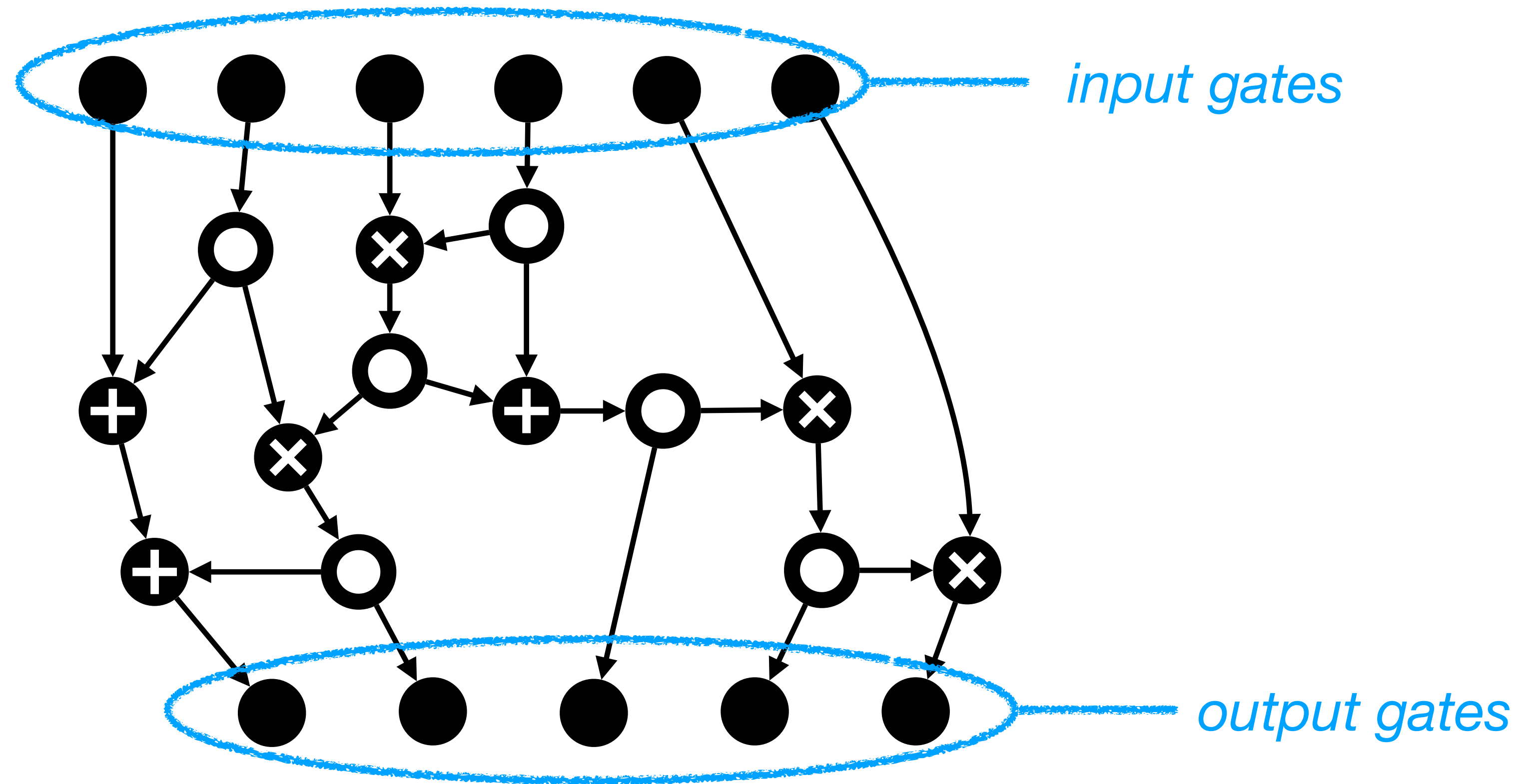
Circuit model

Crypto computation modelled as an arithmetic circuit on \mathbb{K}



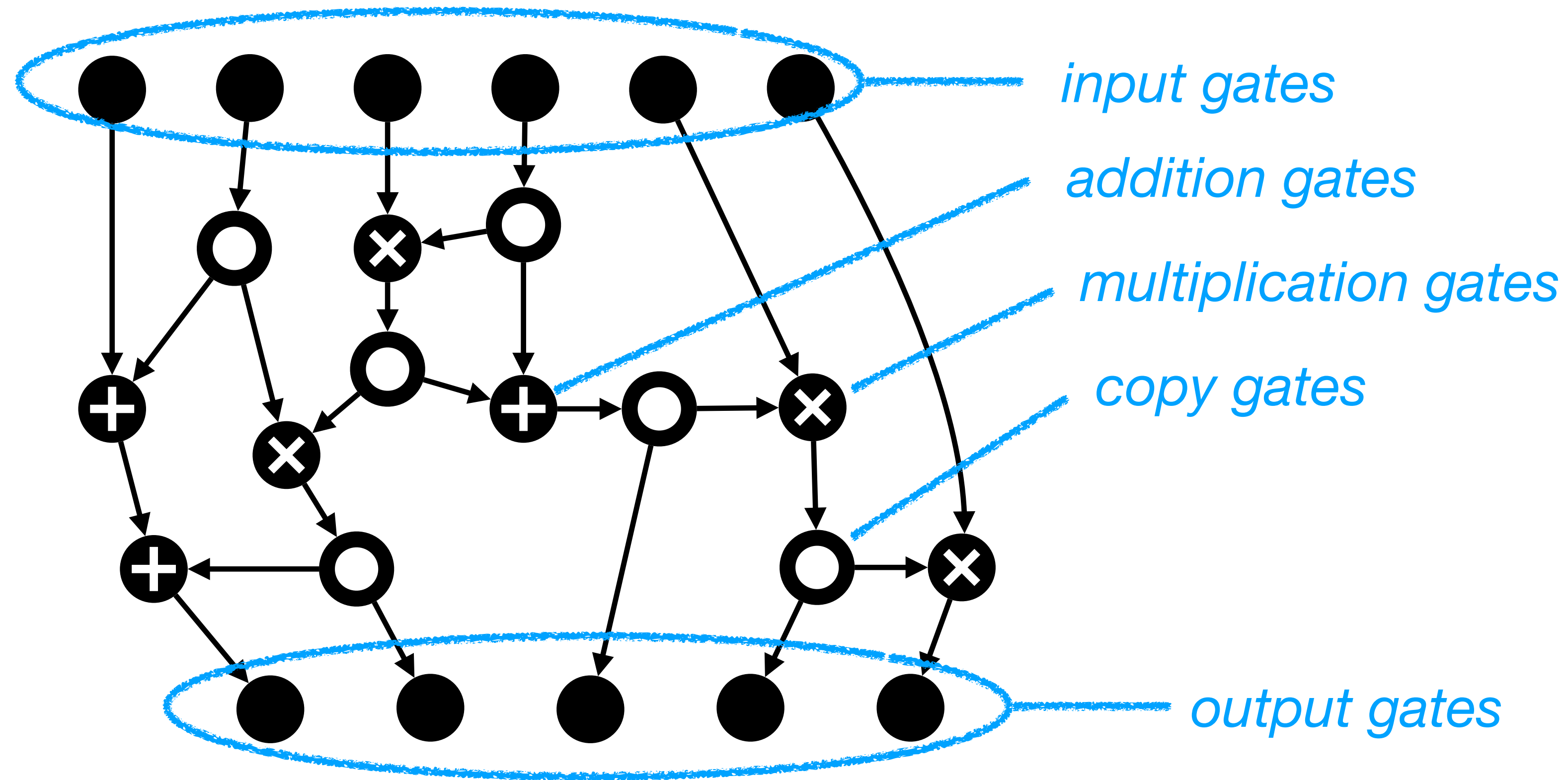
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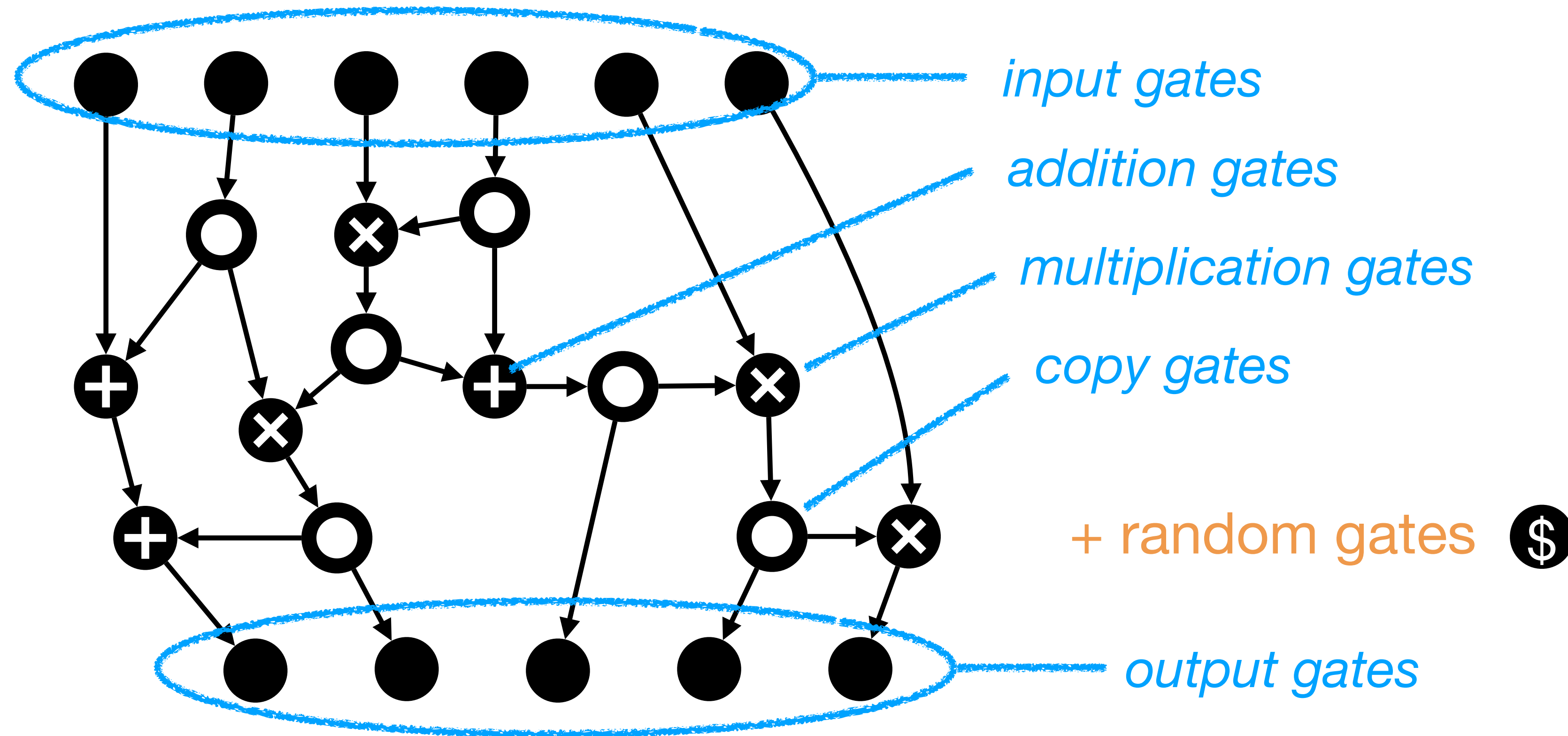
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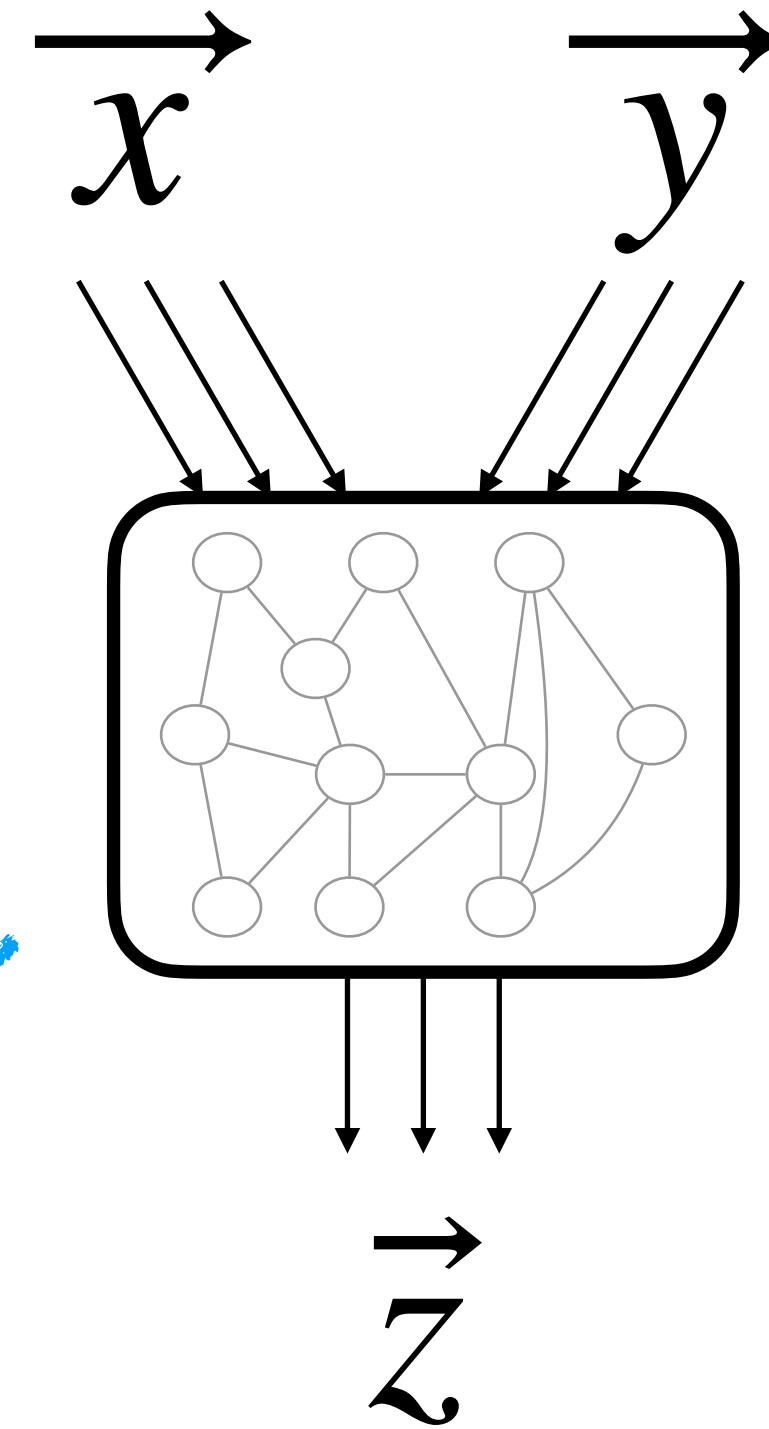
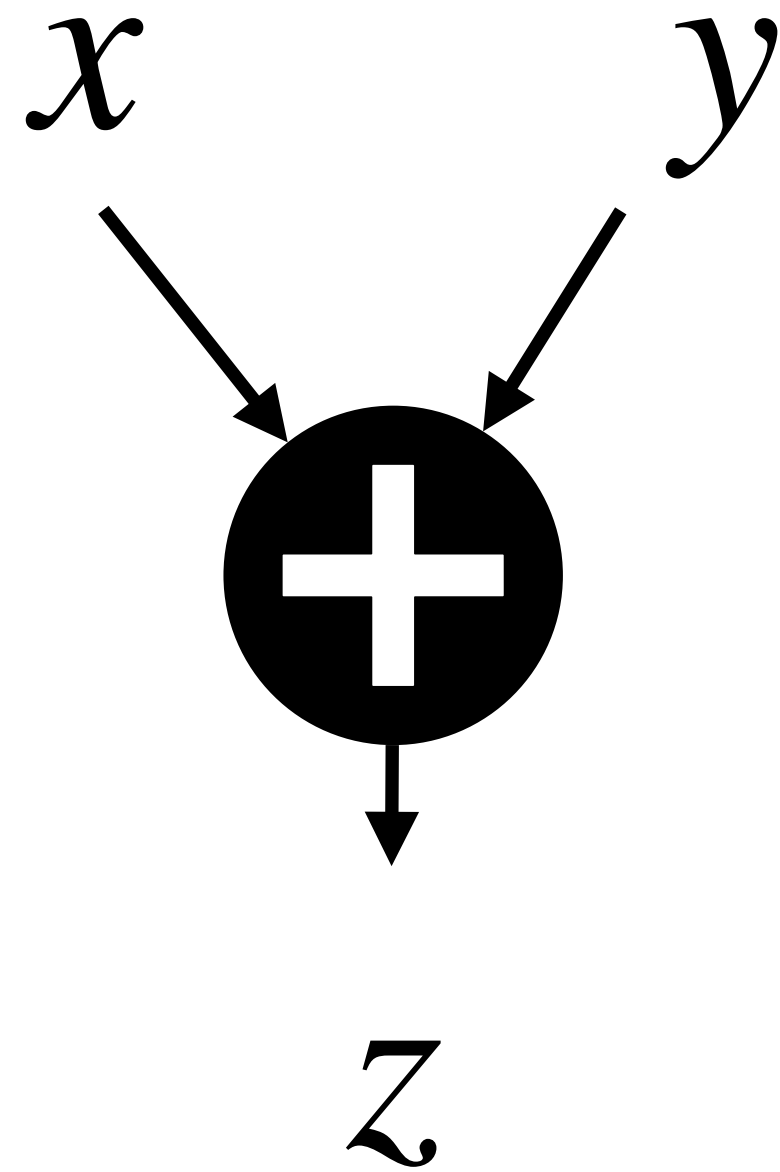


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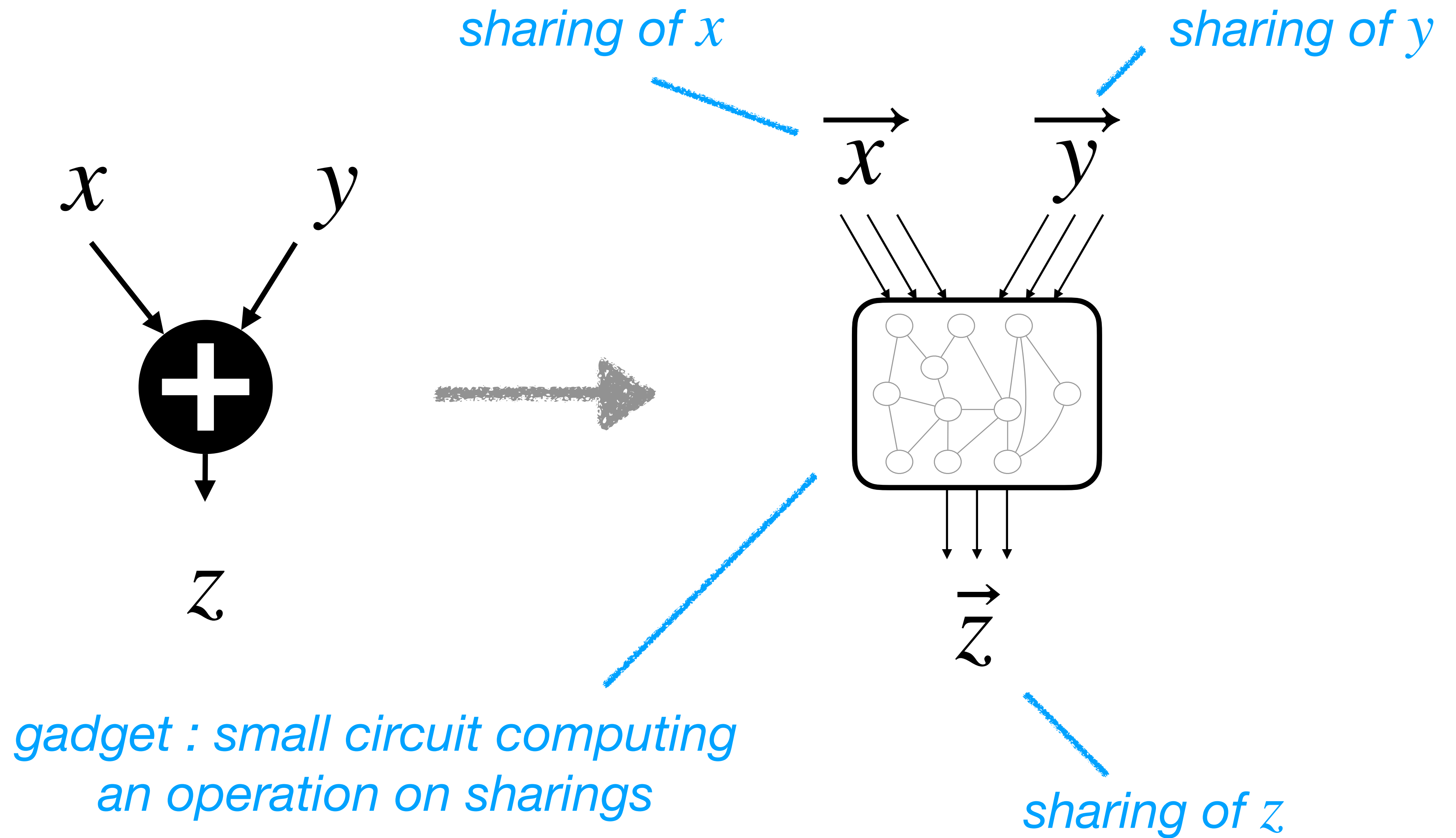


Gadgets

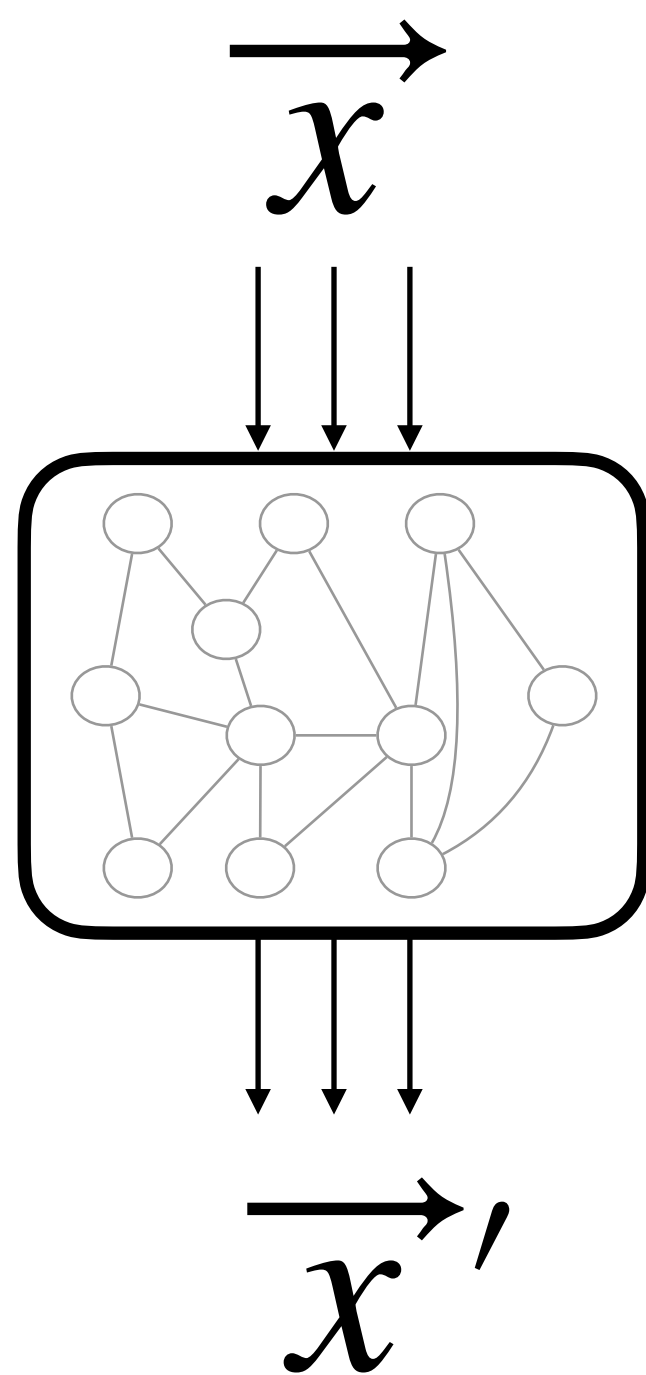


*gadget : small circuit computing
an operation on sharings*

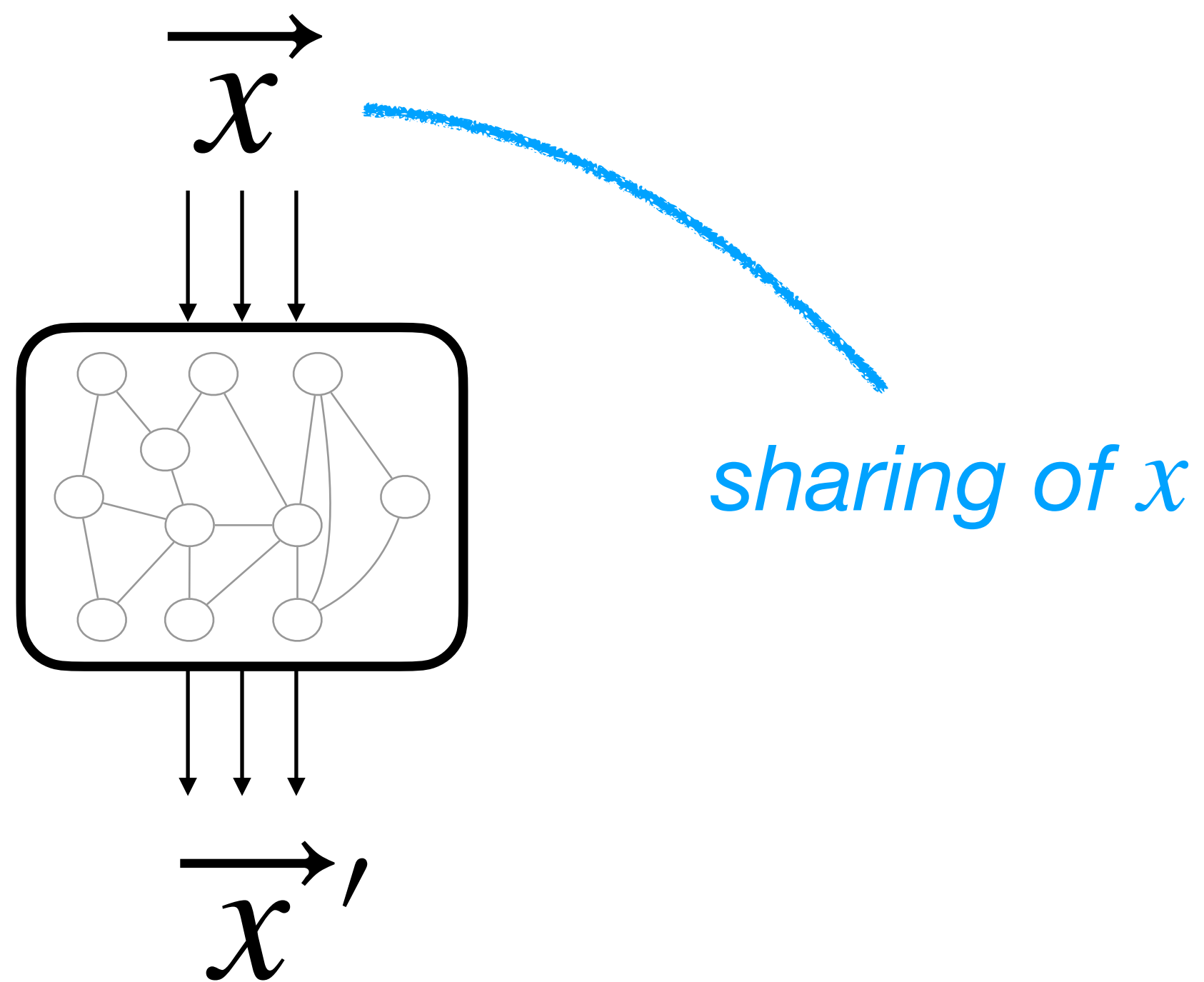
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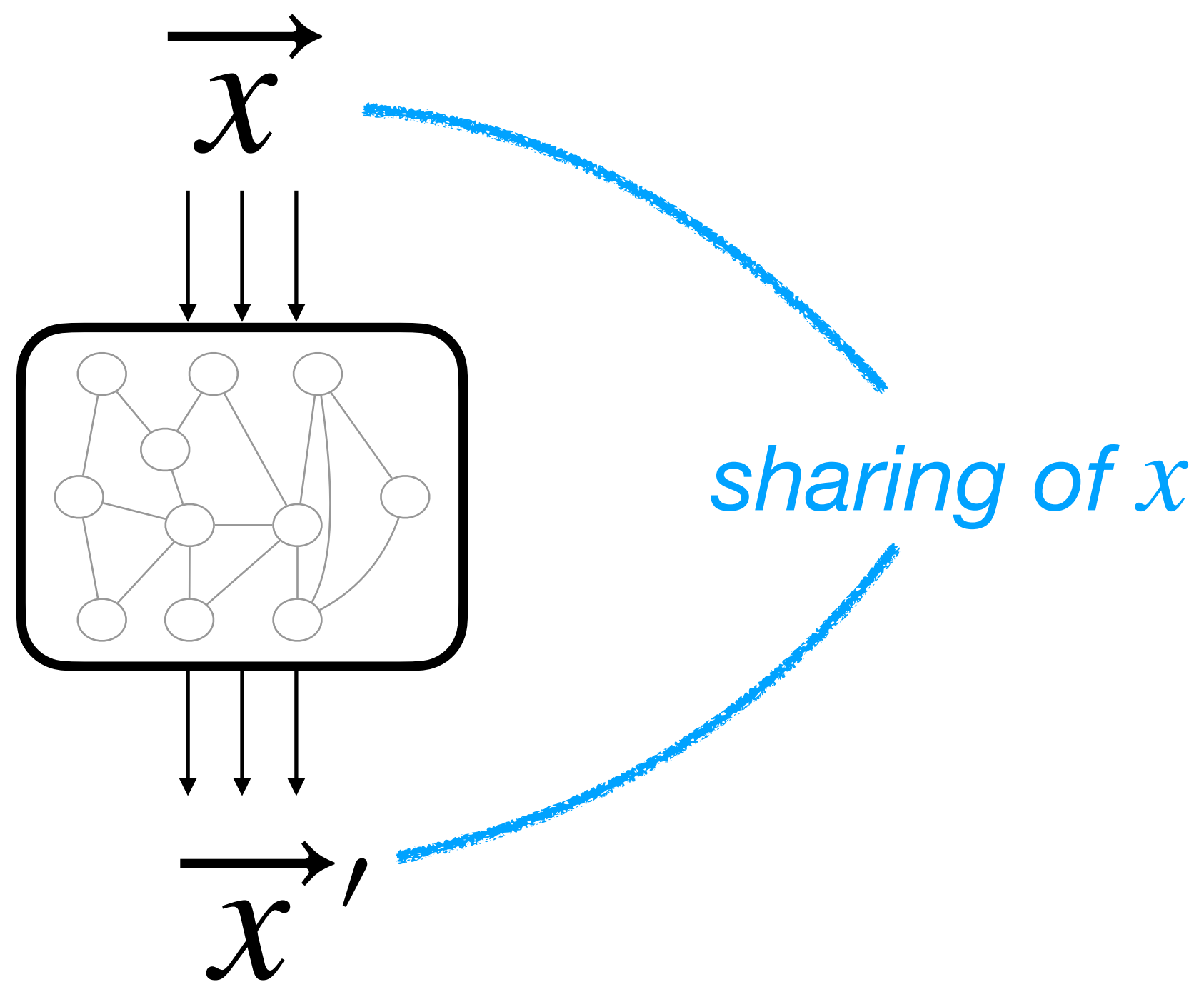
Refresh gadgets



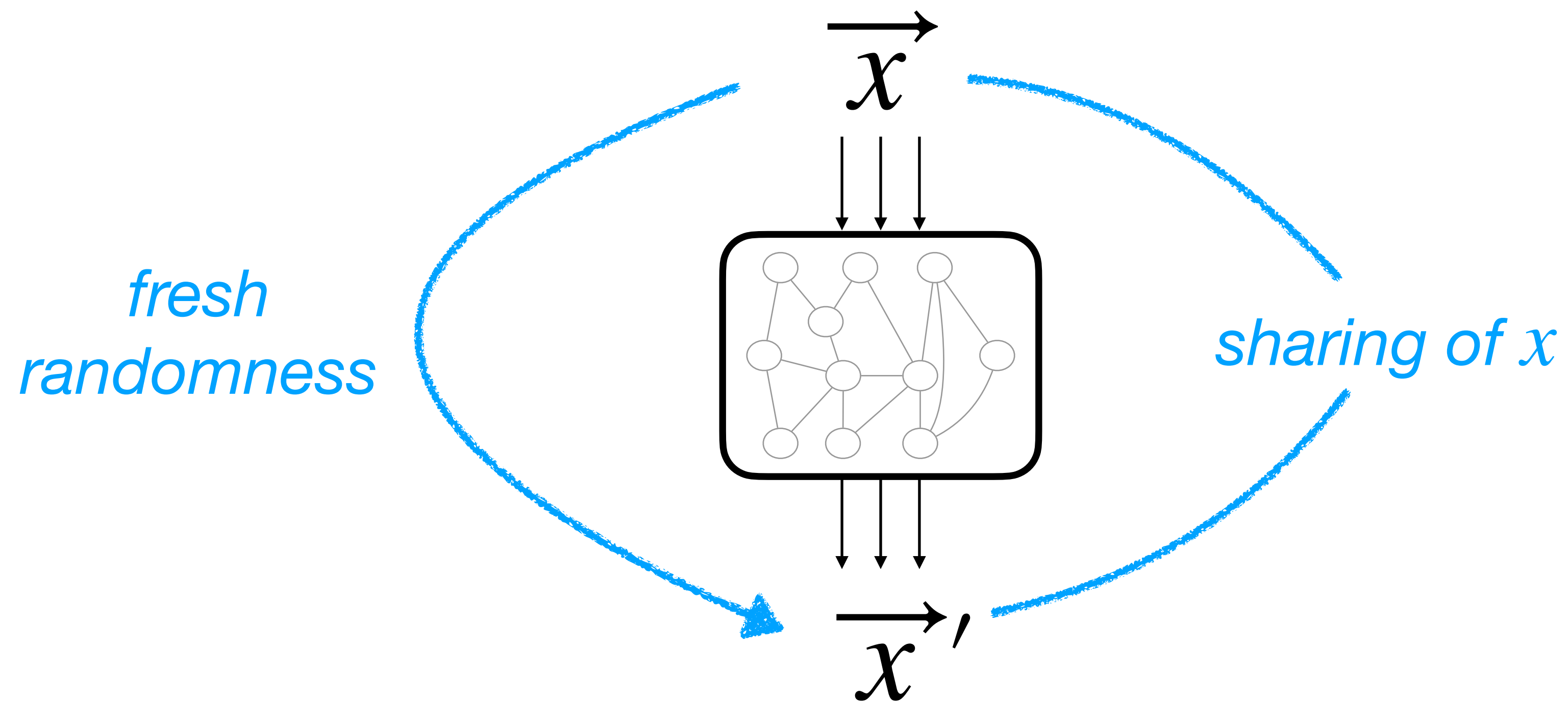
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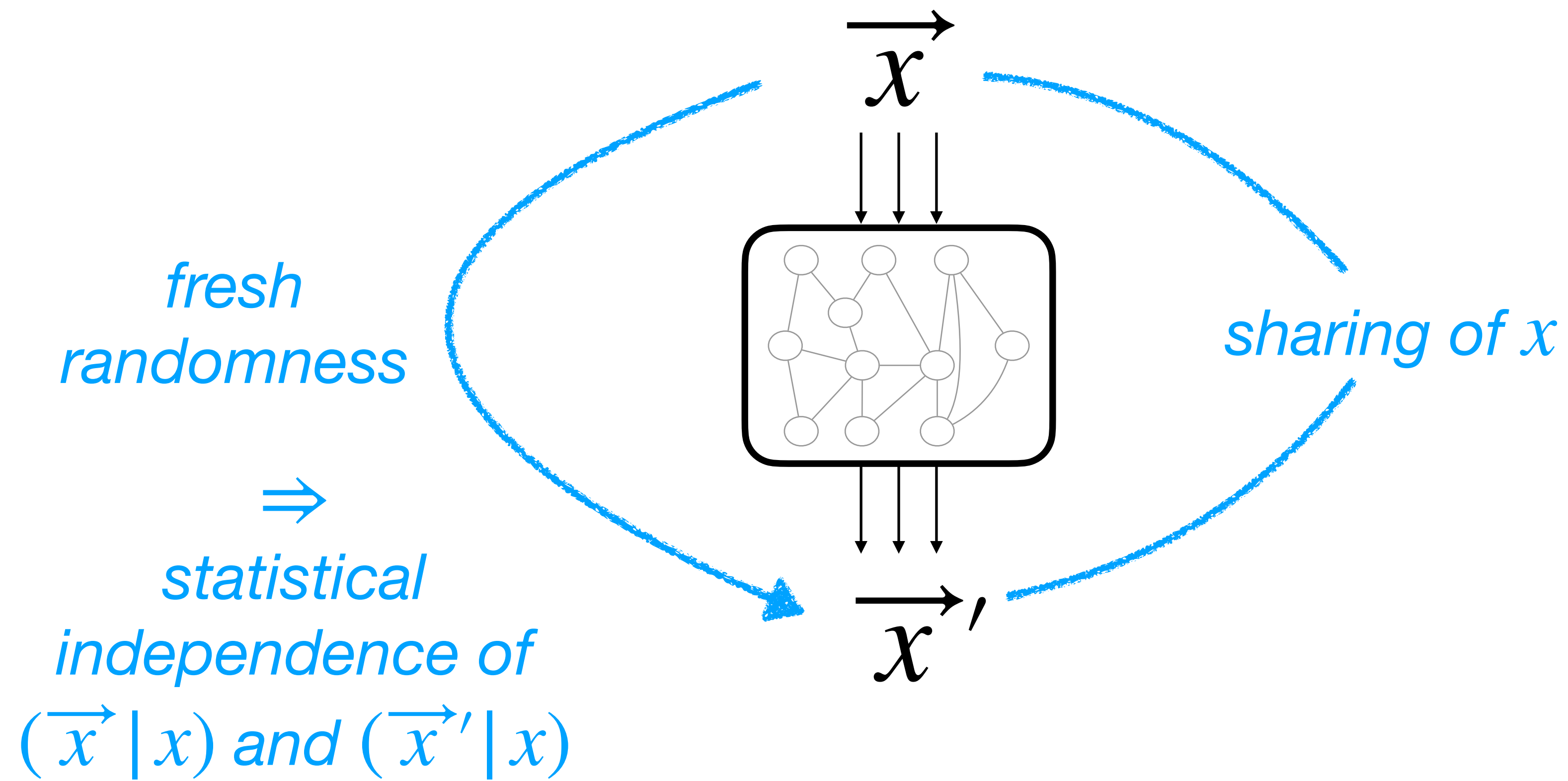
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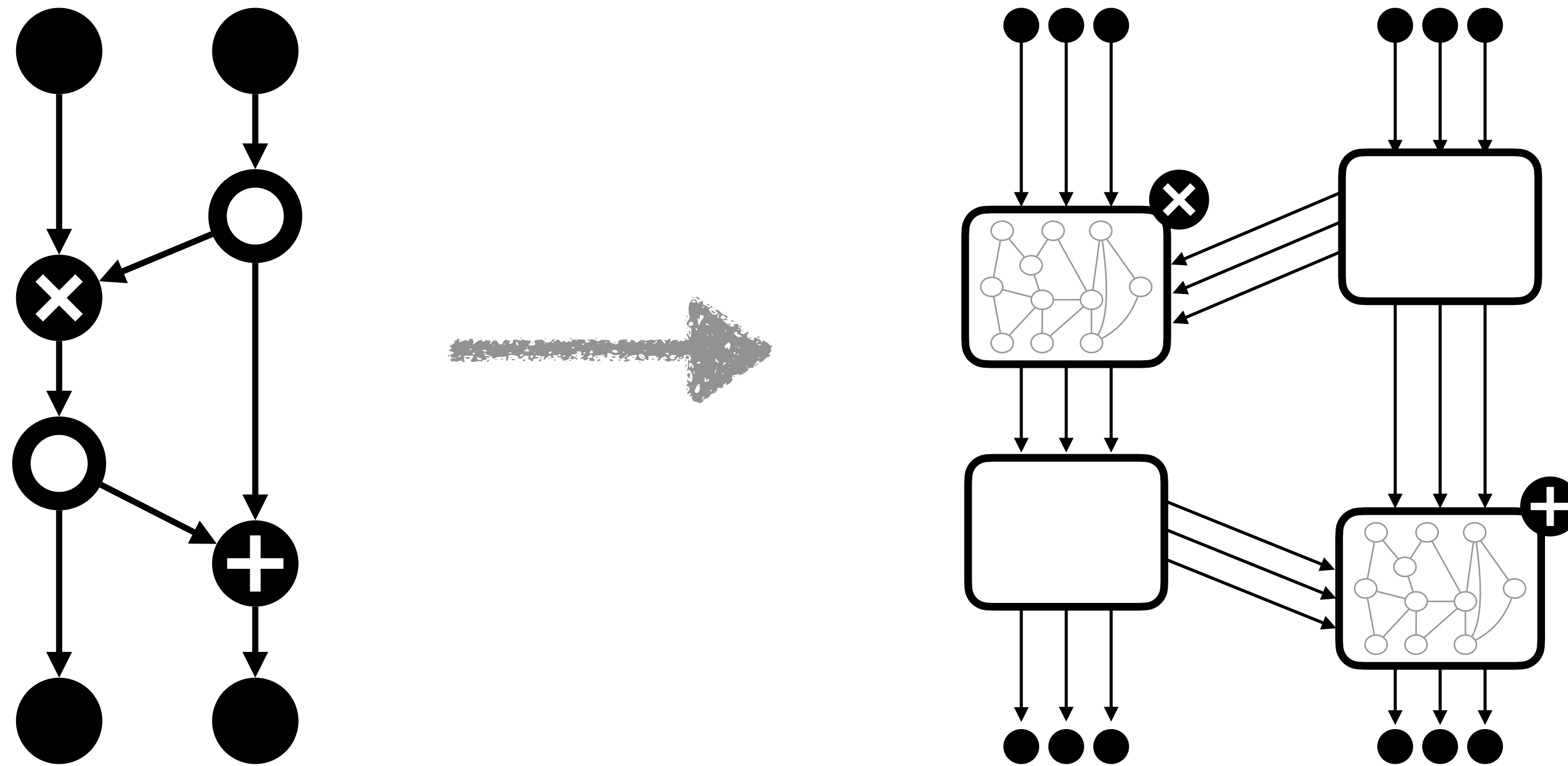
Refresh gadgets



Standard circuit compiler

wire $\rightarrow n$ wires (sharing)

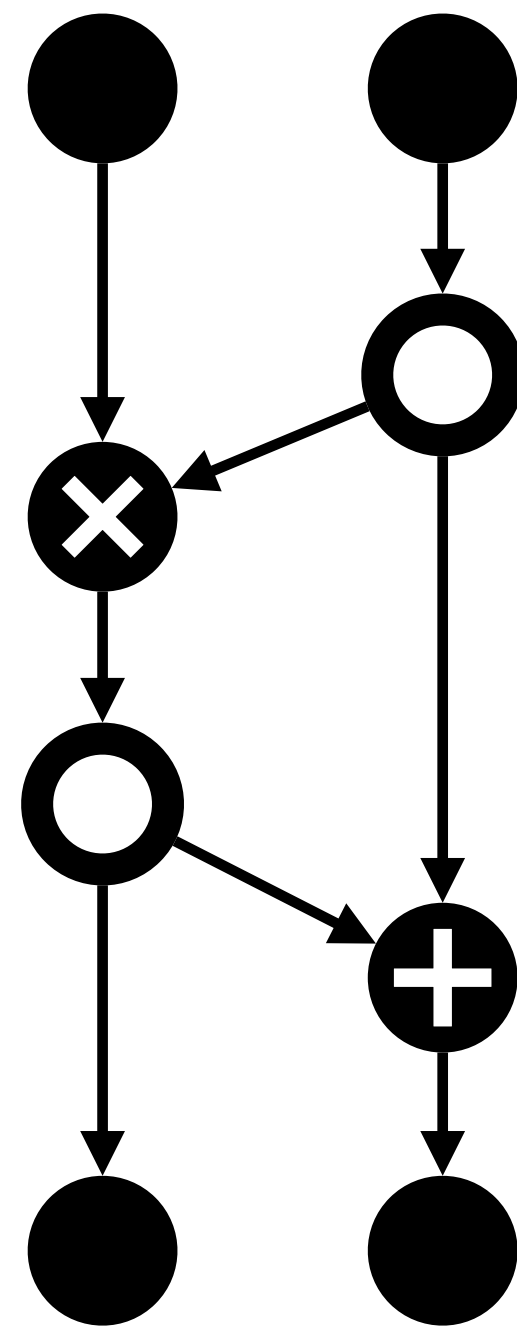
gate \rightarrow gadget



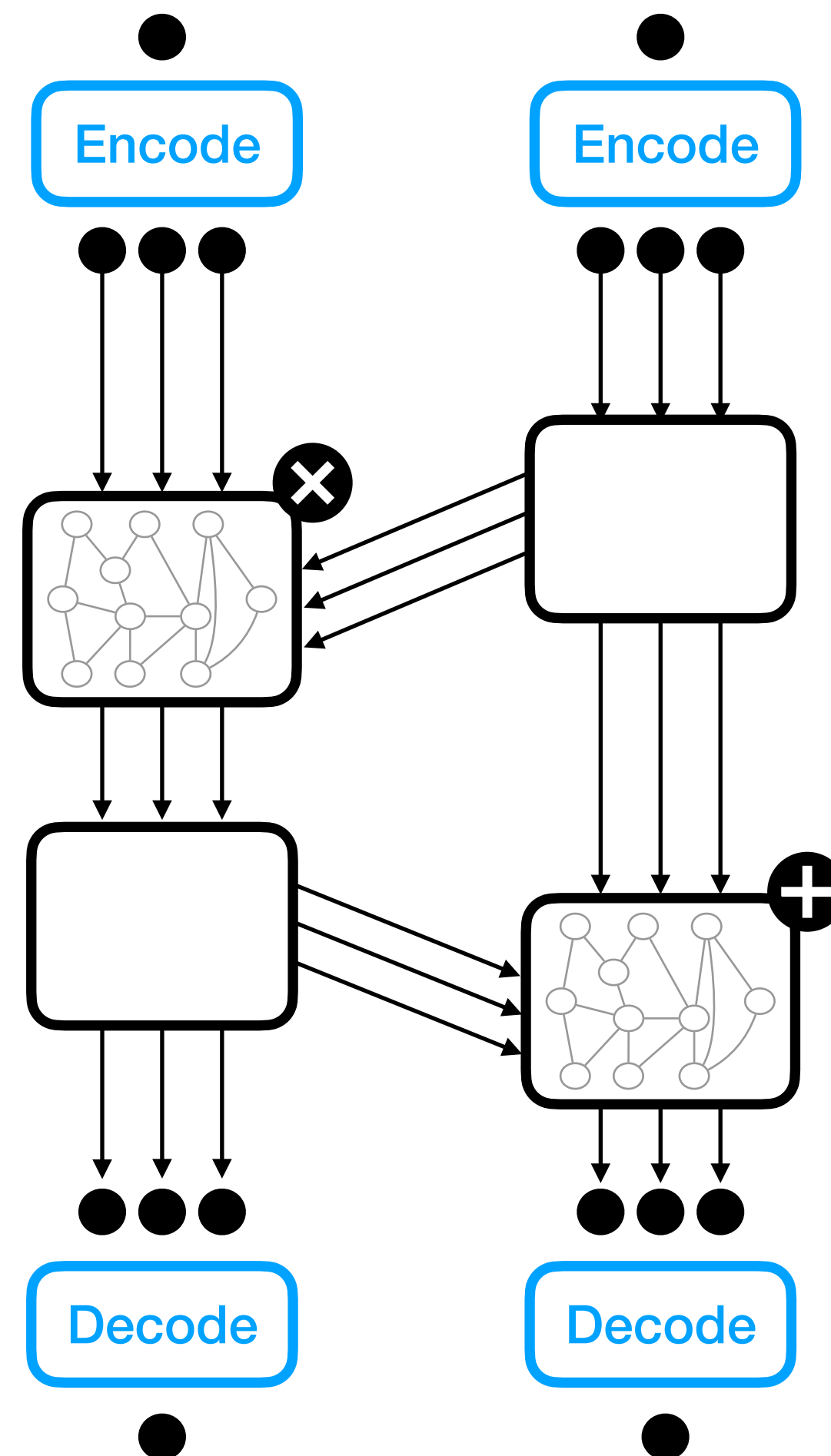
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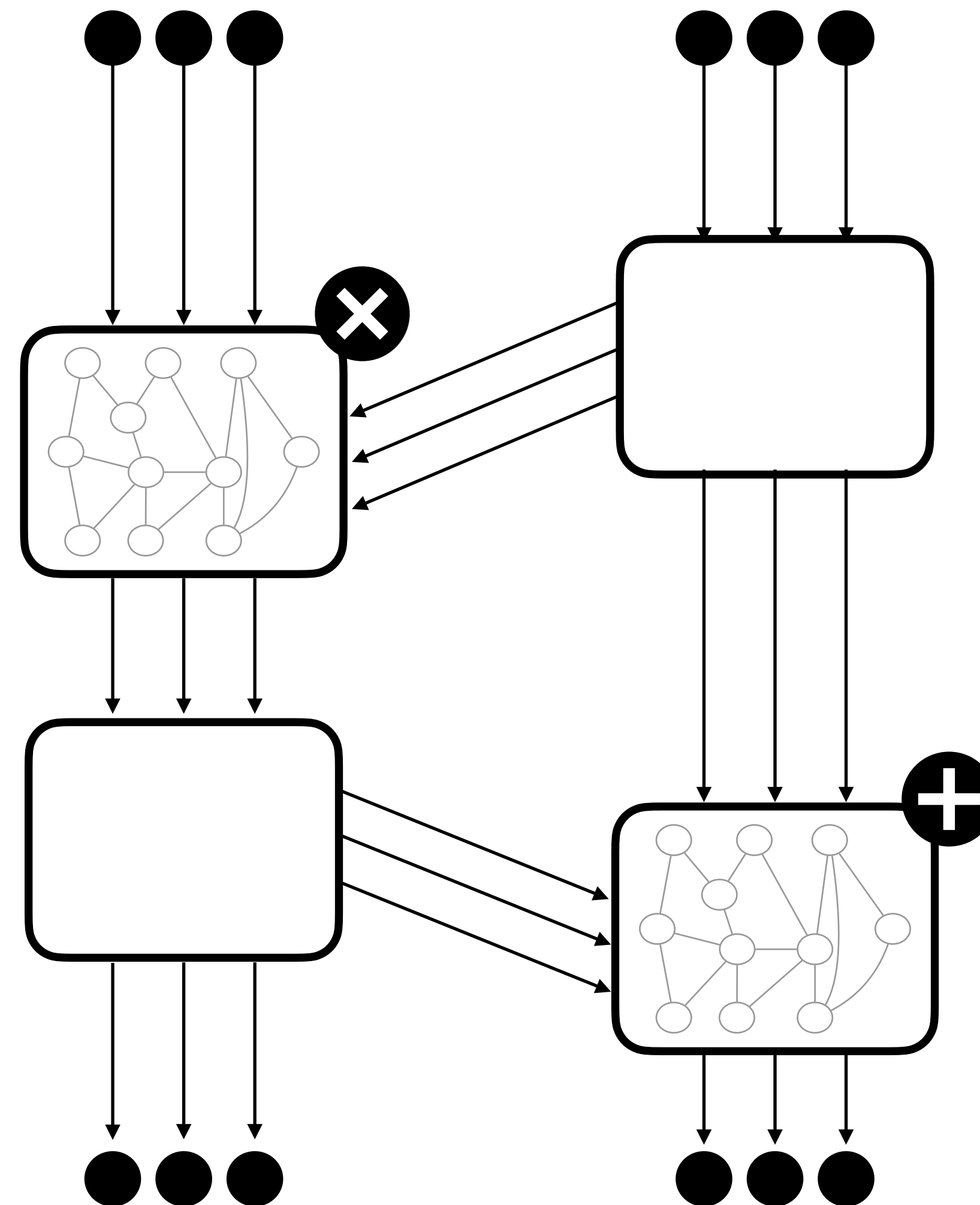


*functional
equivalence*



Standard circuit compiler ...

... with full
refreshing

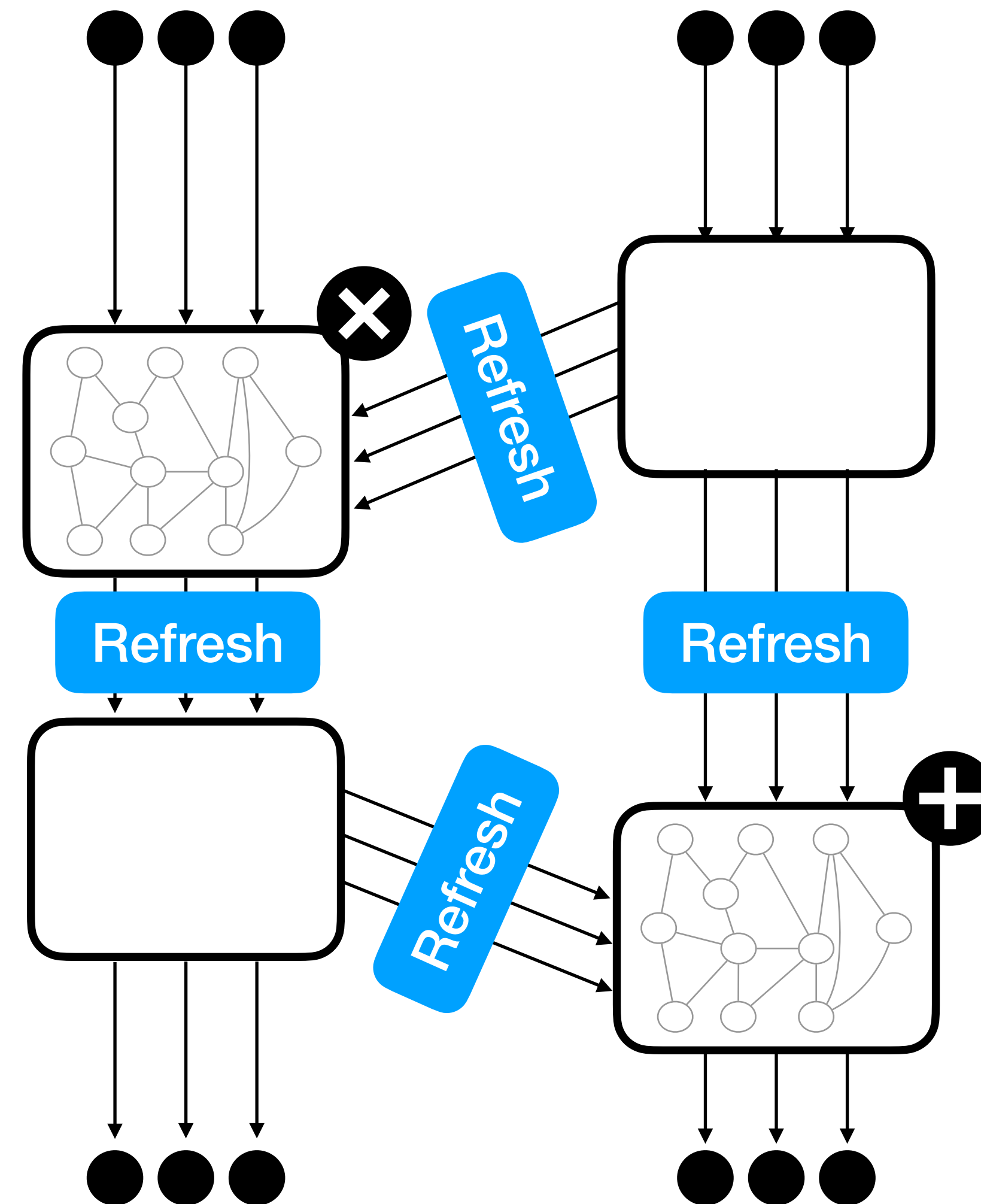


Standard circuit compiler ...

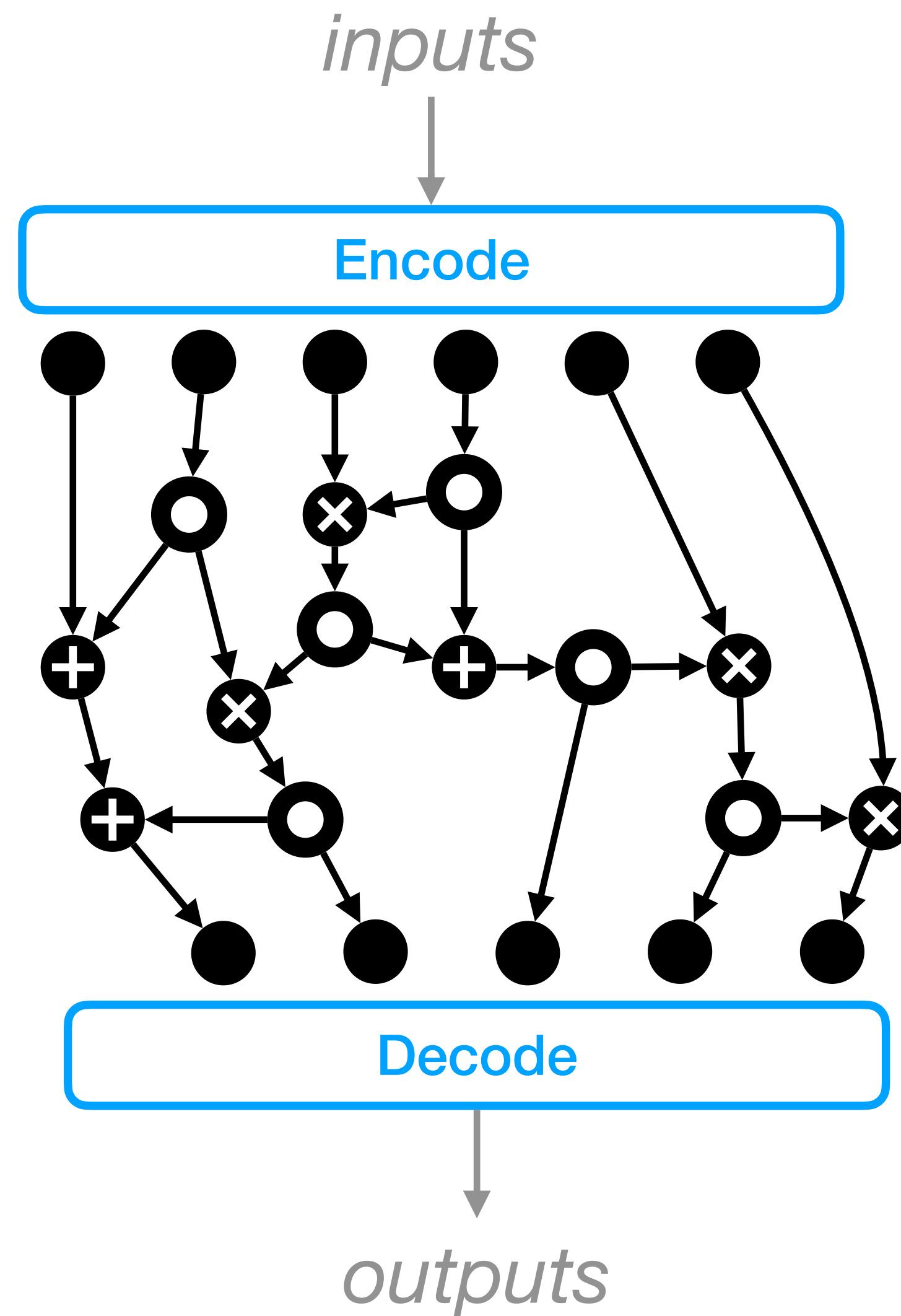
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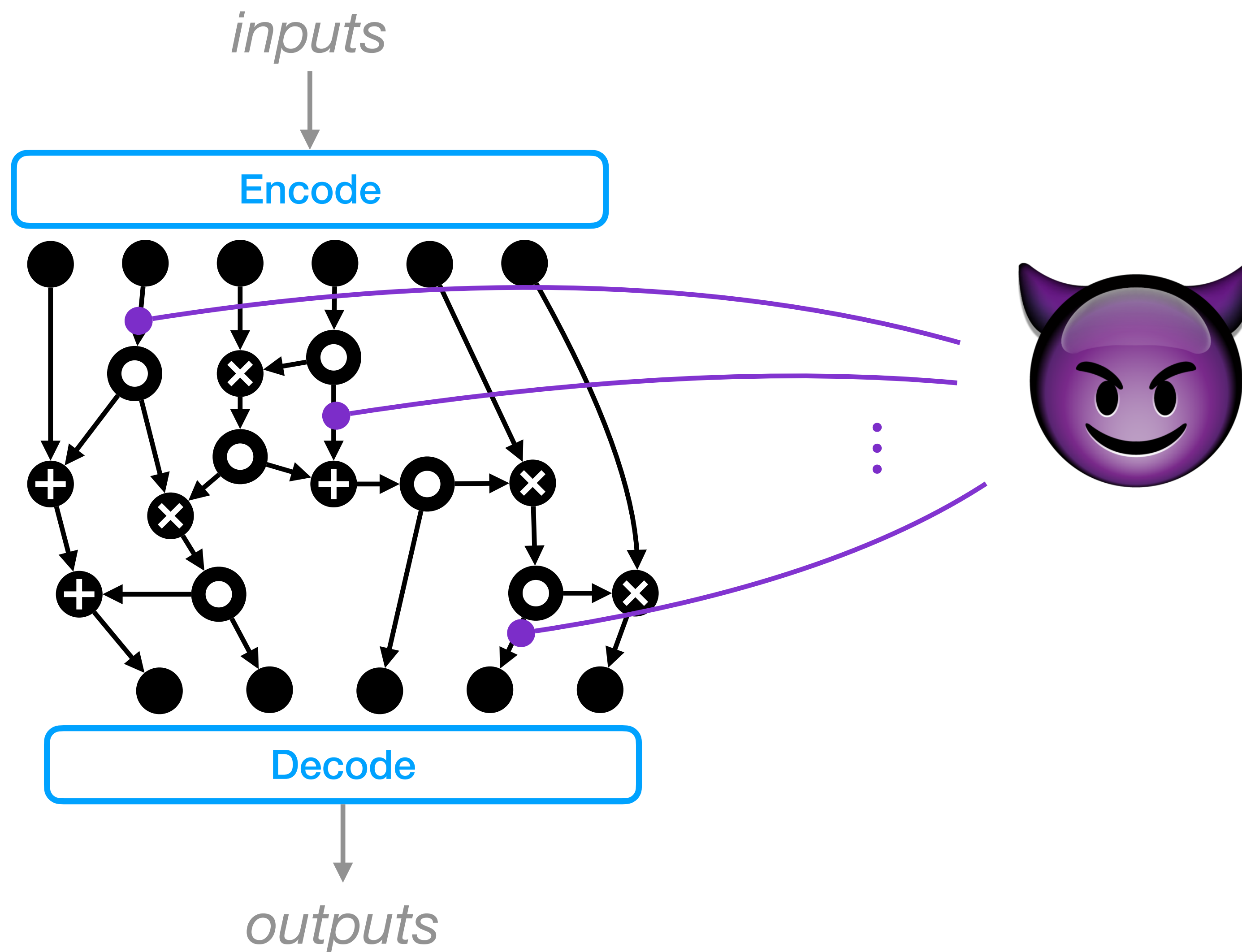
*introduce a refresh gadget
between any two gadgets*



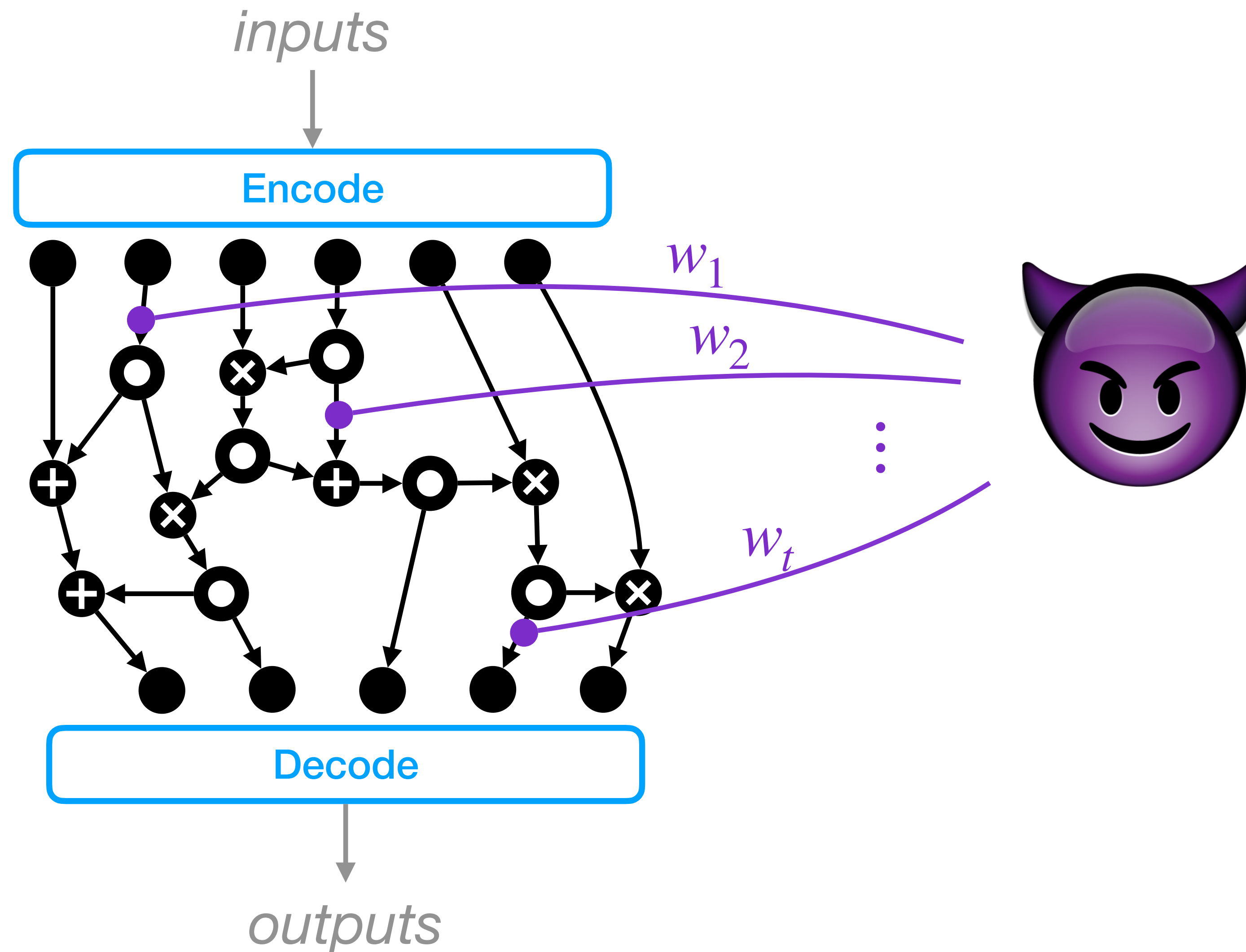
Probing security



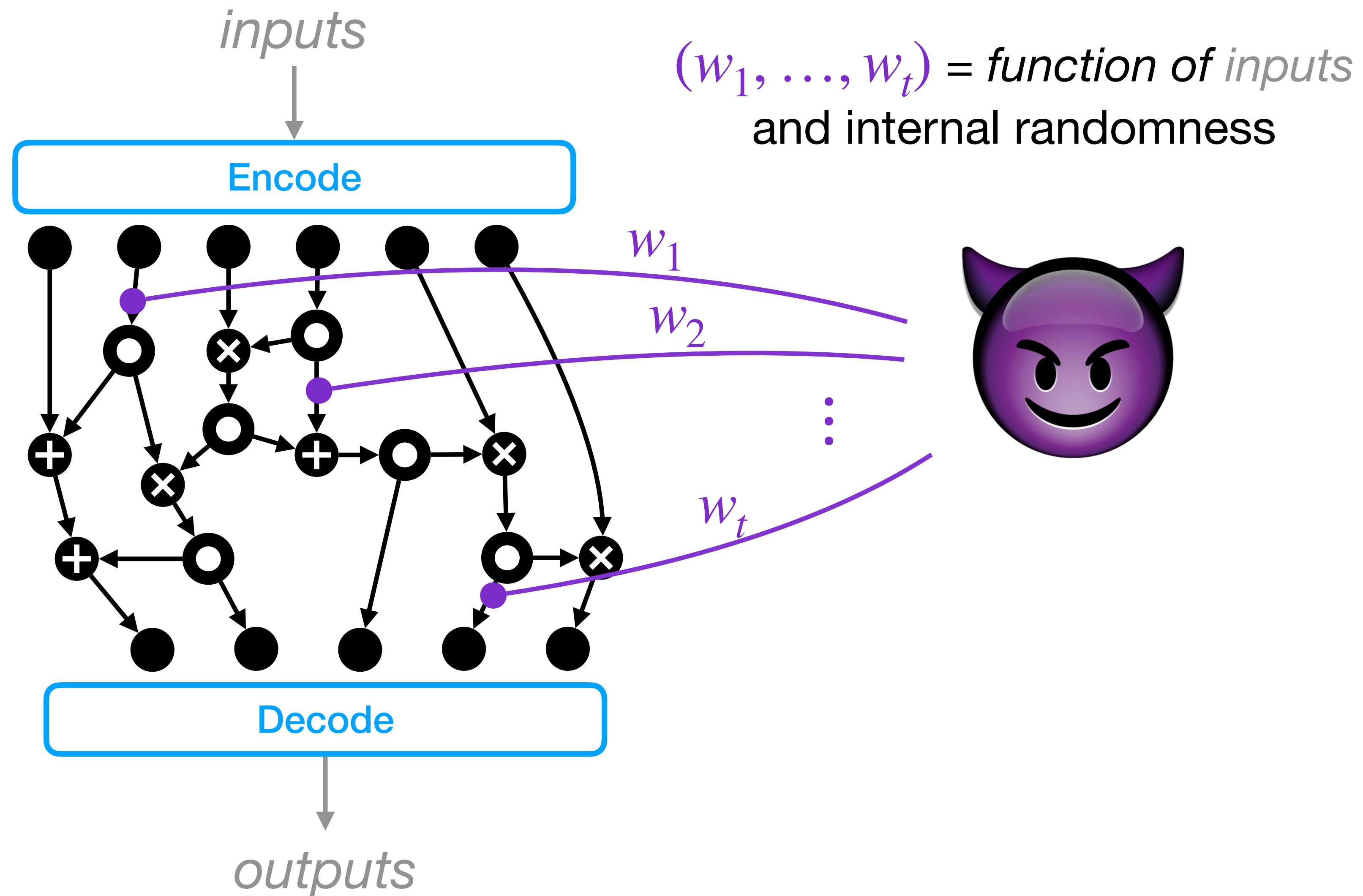
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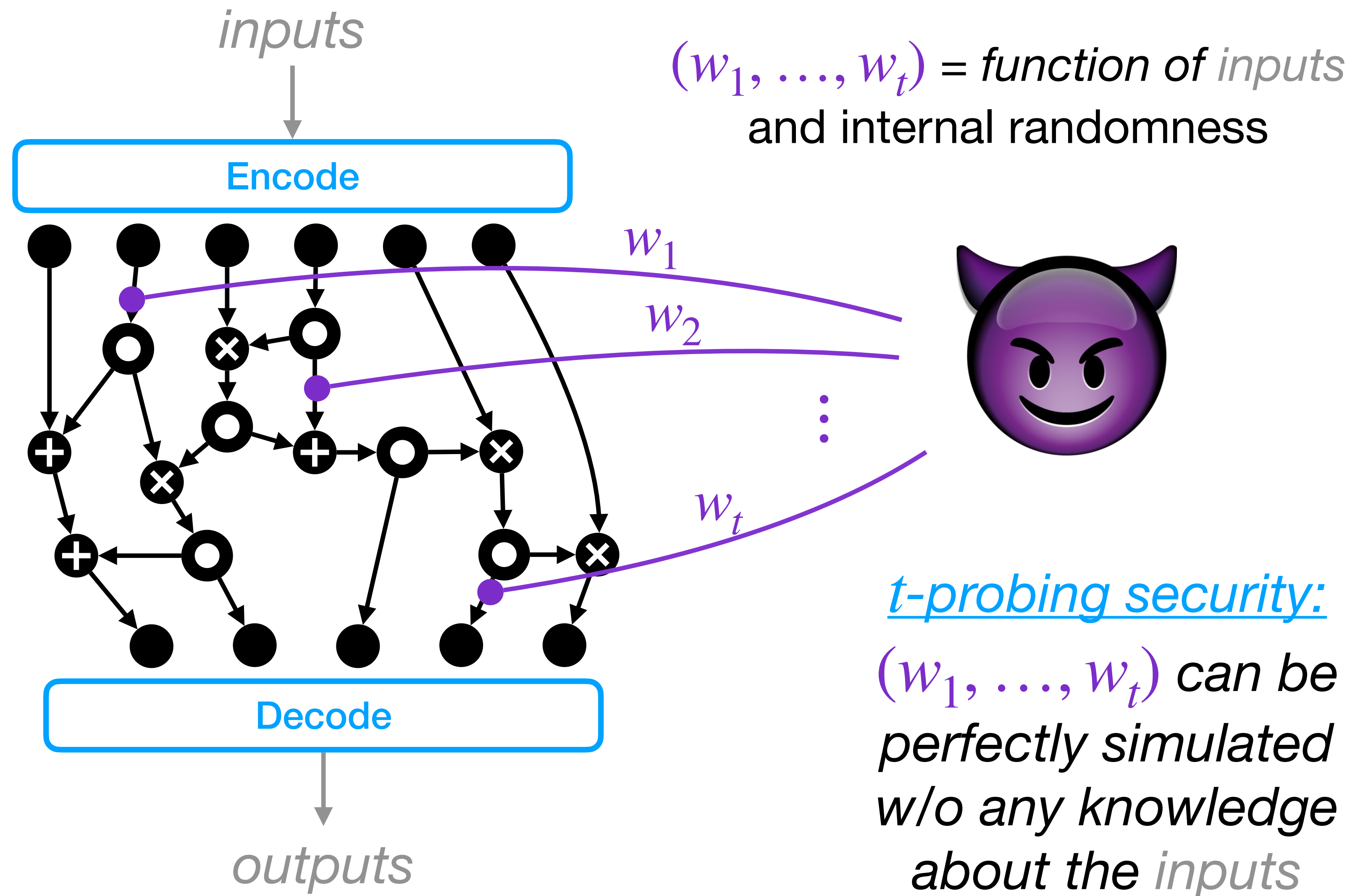
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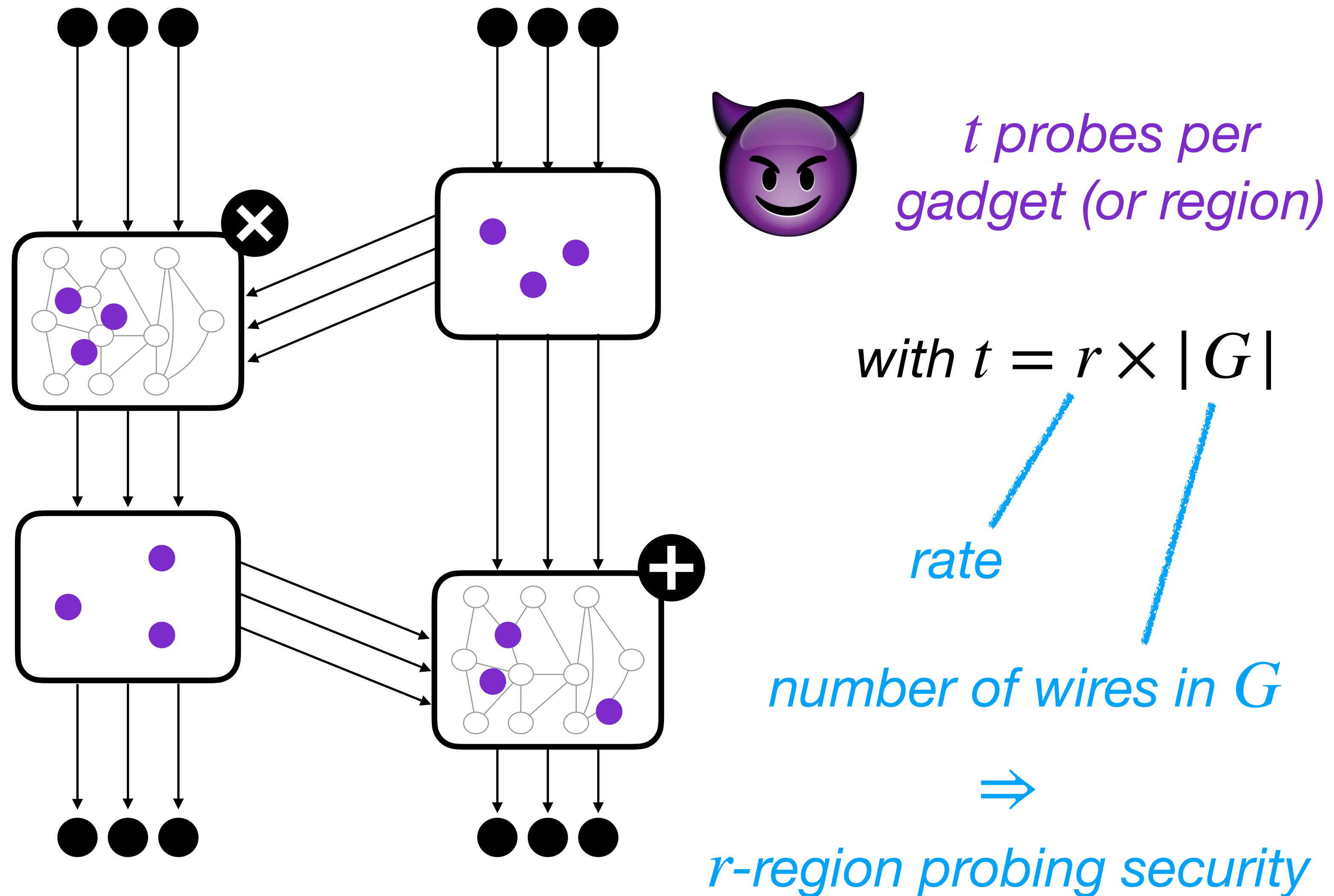
Probing security



Probing security



Region probing security



Why region probing security?

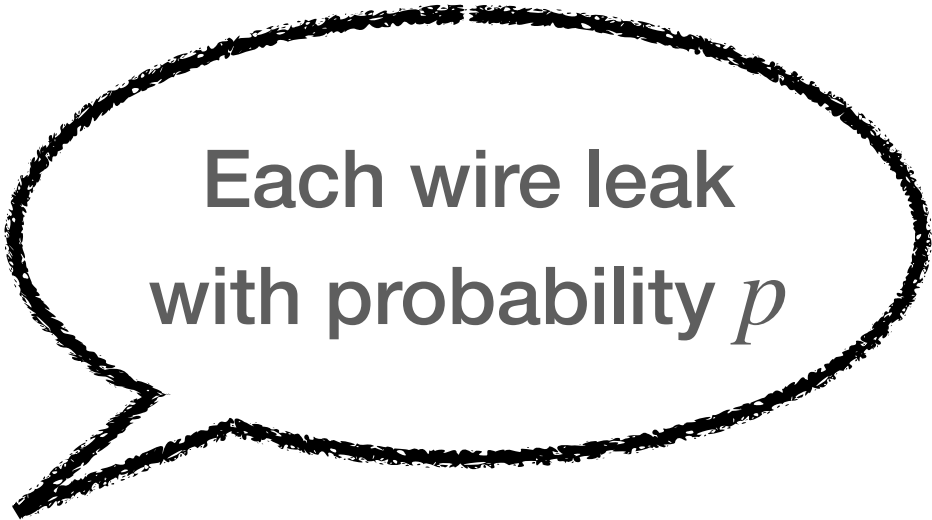
r -region probing security

\Rightarrow p -random probing security

\Rightarrow δ -noisy leakage security

Why region probing security?

r -region probing security



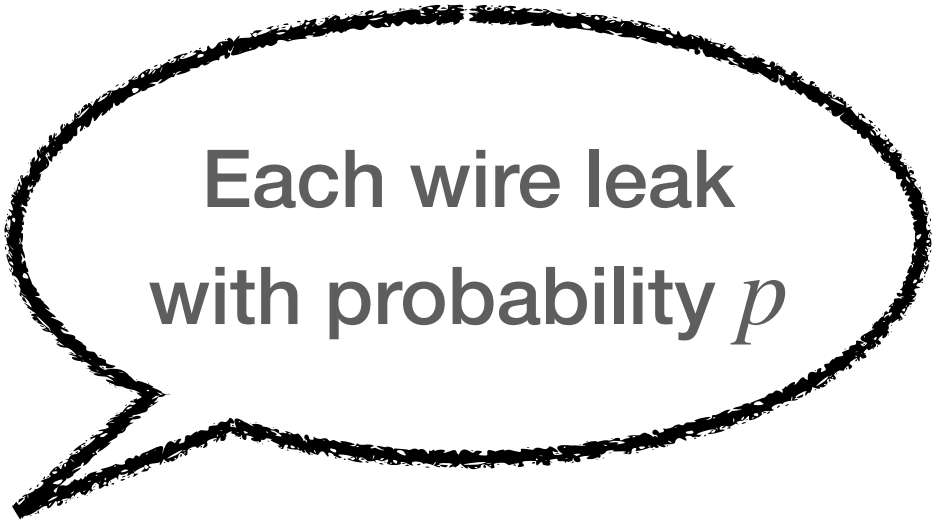
Each wire leak
with probability p

\Rightarrow p -random probing security

\Rightarrow δ -noisy leakage security

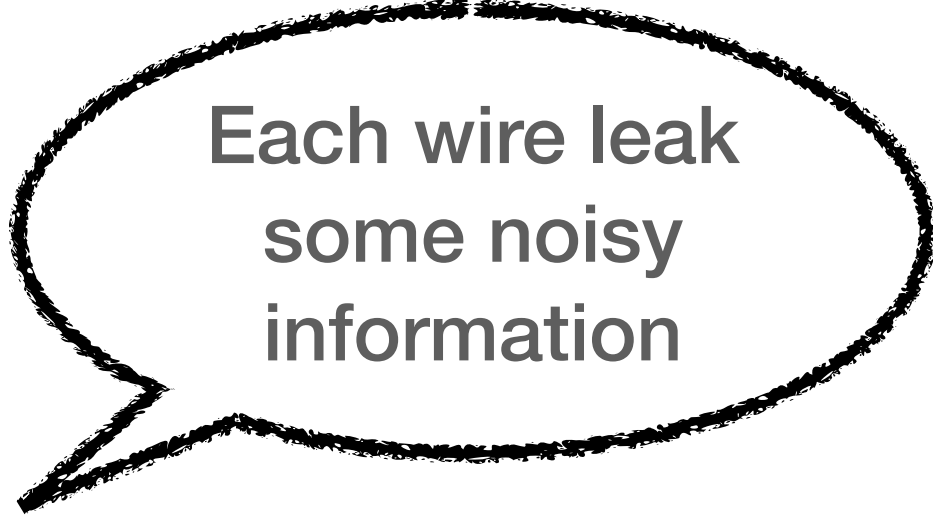
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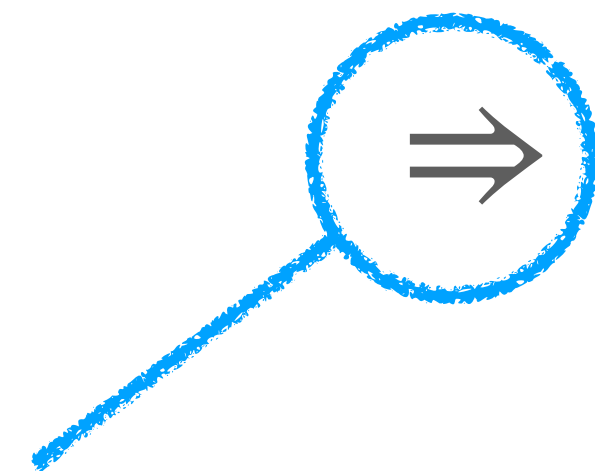
Each wire leak
some noisy
information

\Rightarrow δ -noisy leakage security

Why region probing security?

r -region probing security

Each wire leak
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\Rightarrow

p -random probing security

$p \approx r$

Each wire leak
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Chernoff bound

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 $\delta \approx p \approx r$

Duc-Dziembowski-Faust [EC'14]

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Duc-Dziembowski-Faust [EC'14]

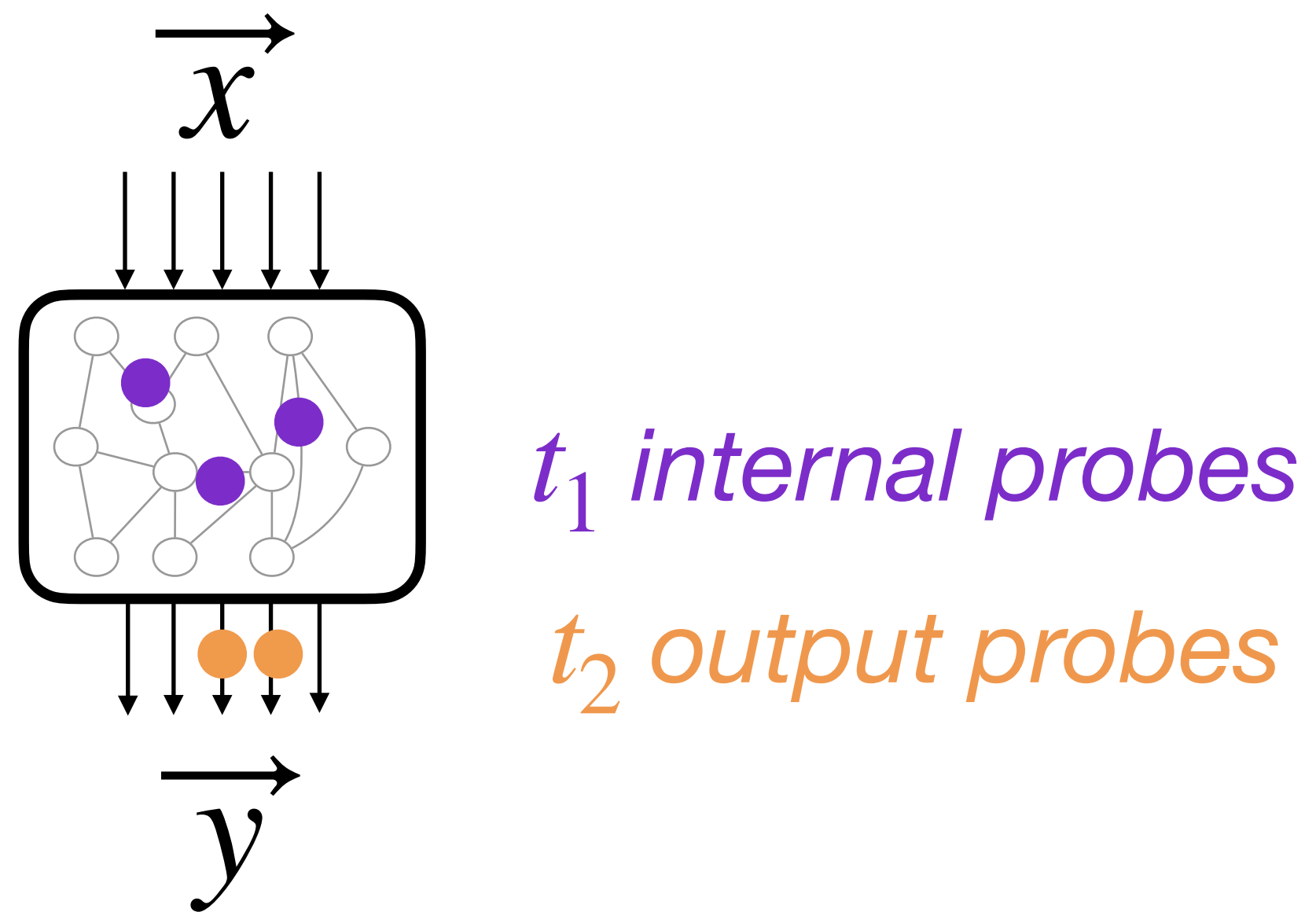
*more realistic to capture
power and EM leakages*

Composition

- Use gadgets achieving **composition properties** (stronger than PS)
- Obtain the (region) PS for the composition

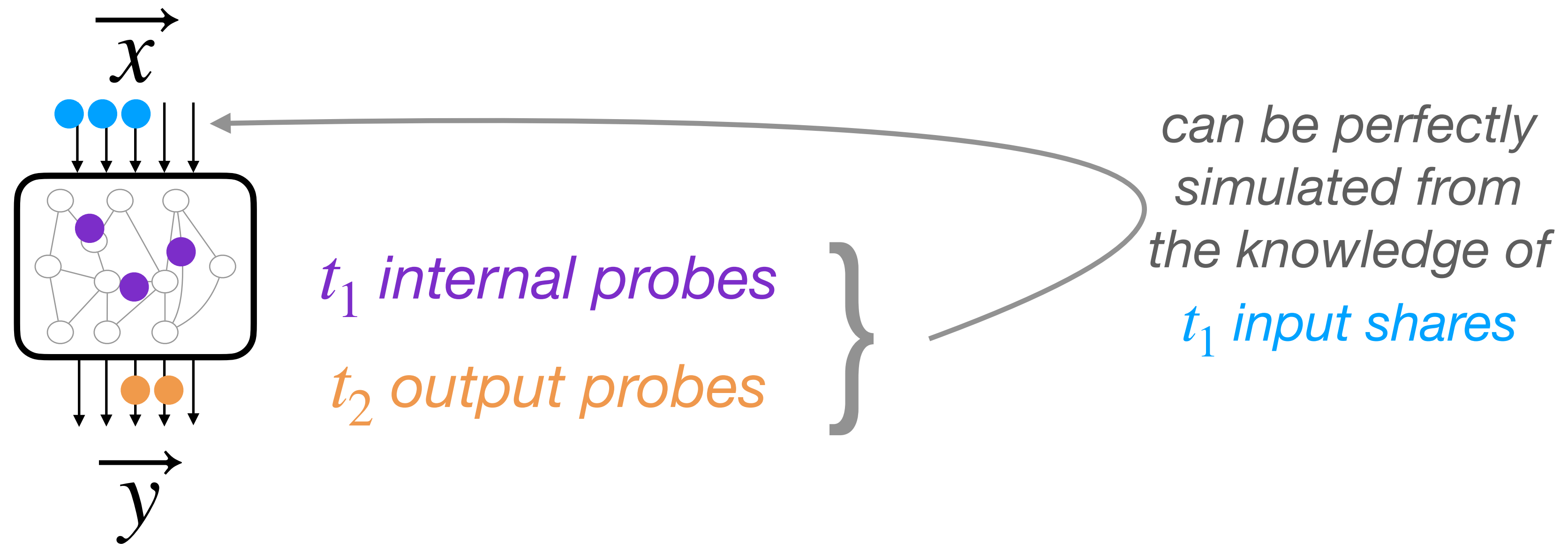
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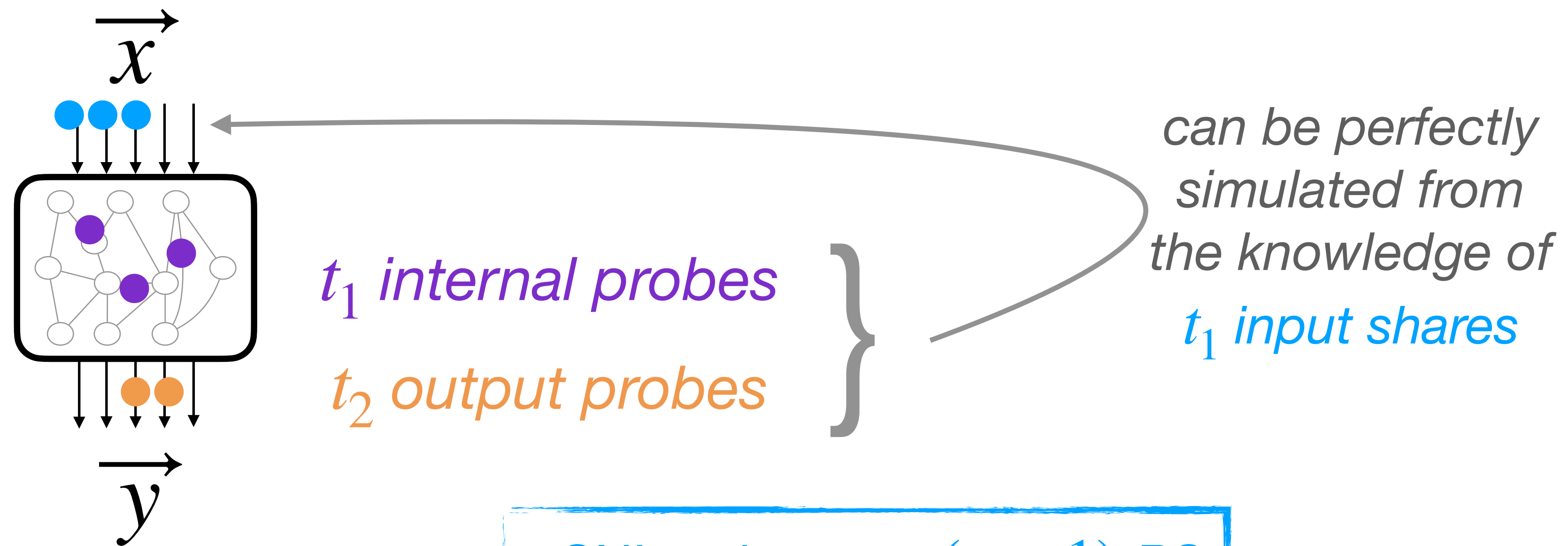
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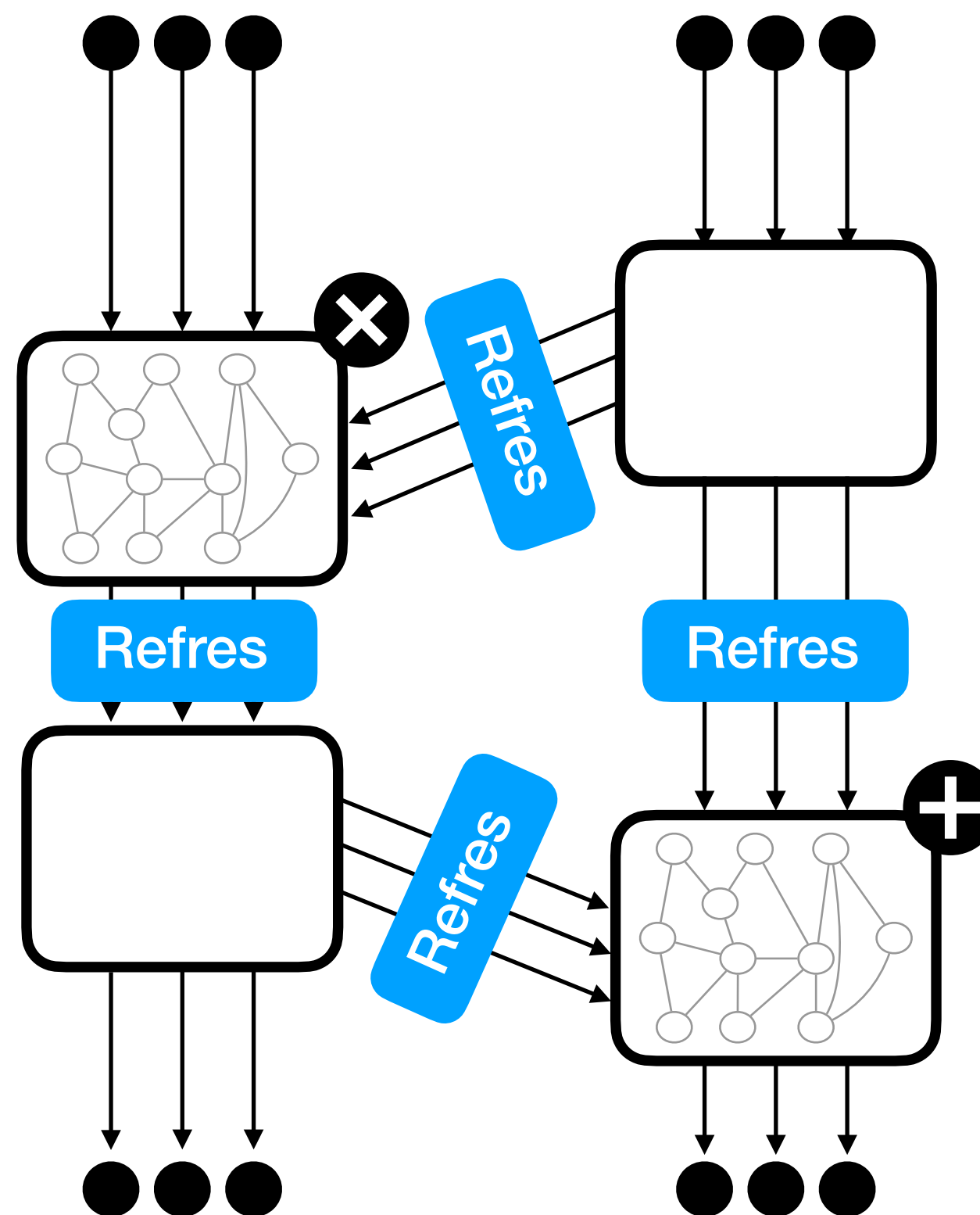
SNI gadgets $\Rightarrow (n - 1)$ -PS
 \Rightarrow region PS

Our composition approach

- We only require a composition property for the refresh gadget
- Other gadgets only need to be probing secure

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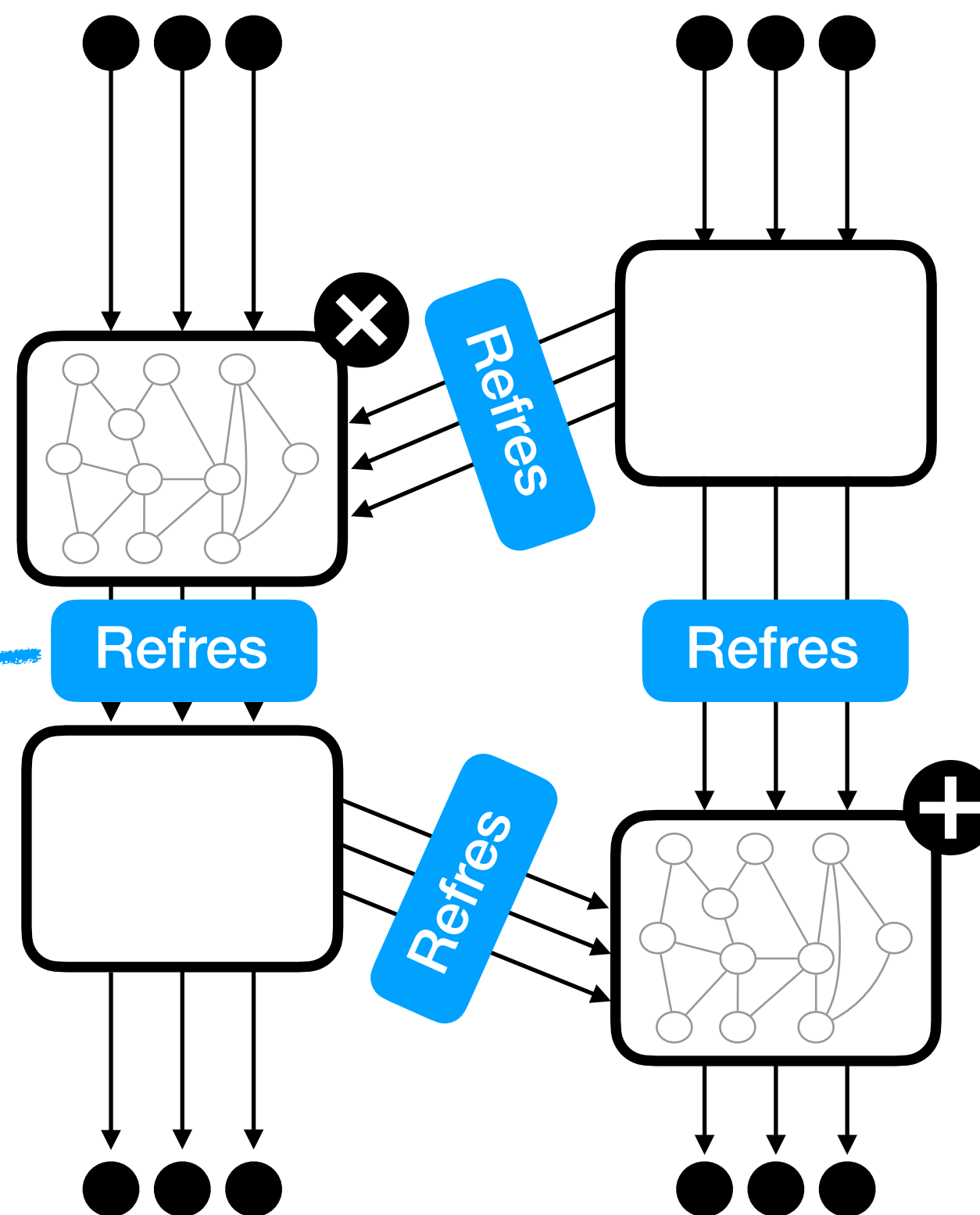
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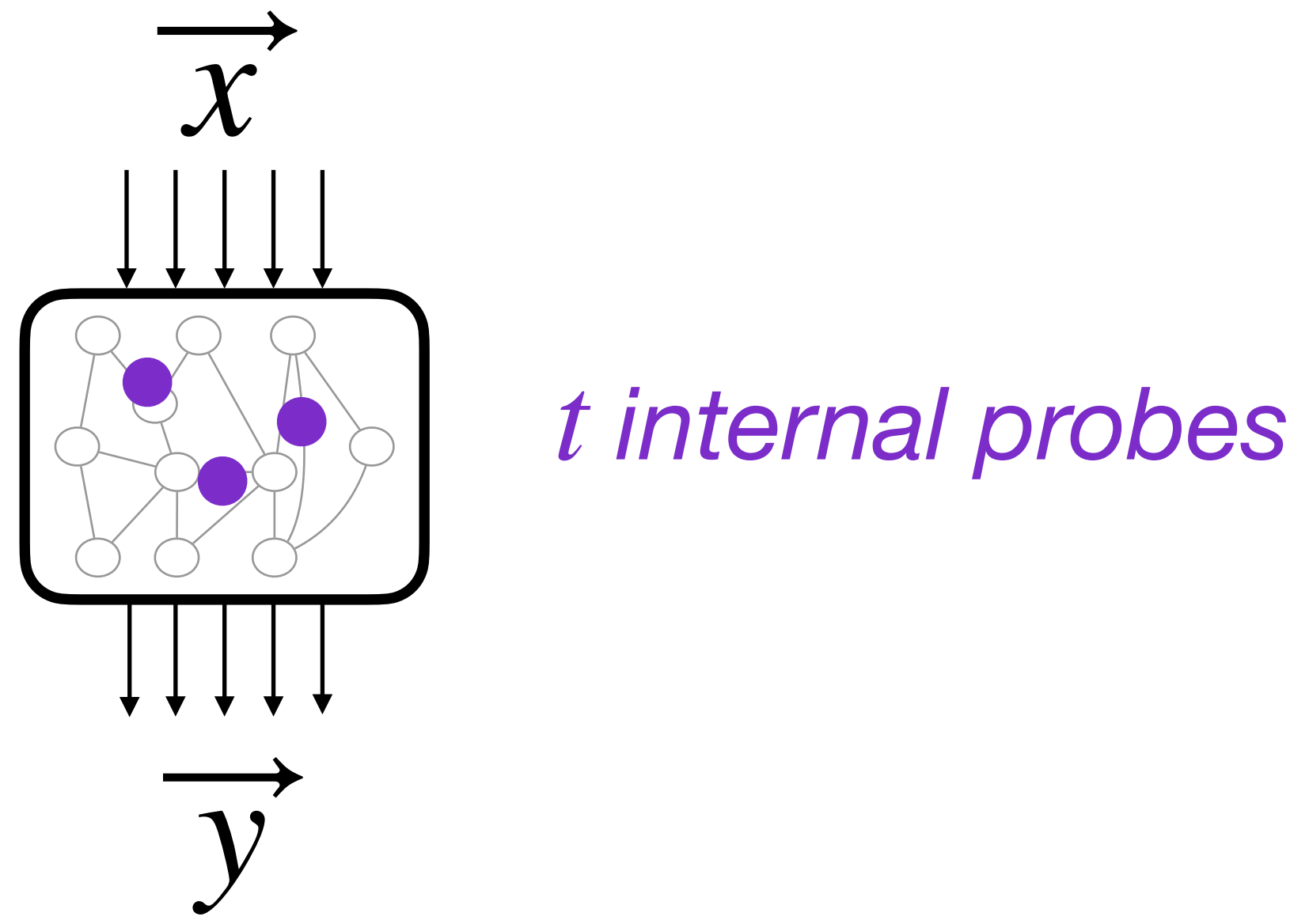
simple probing security

*IOS property
+ uniformity*

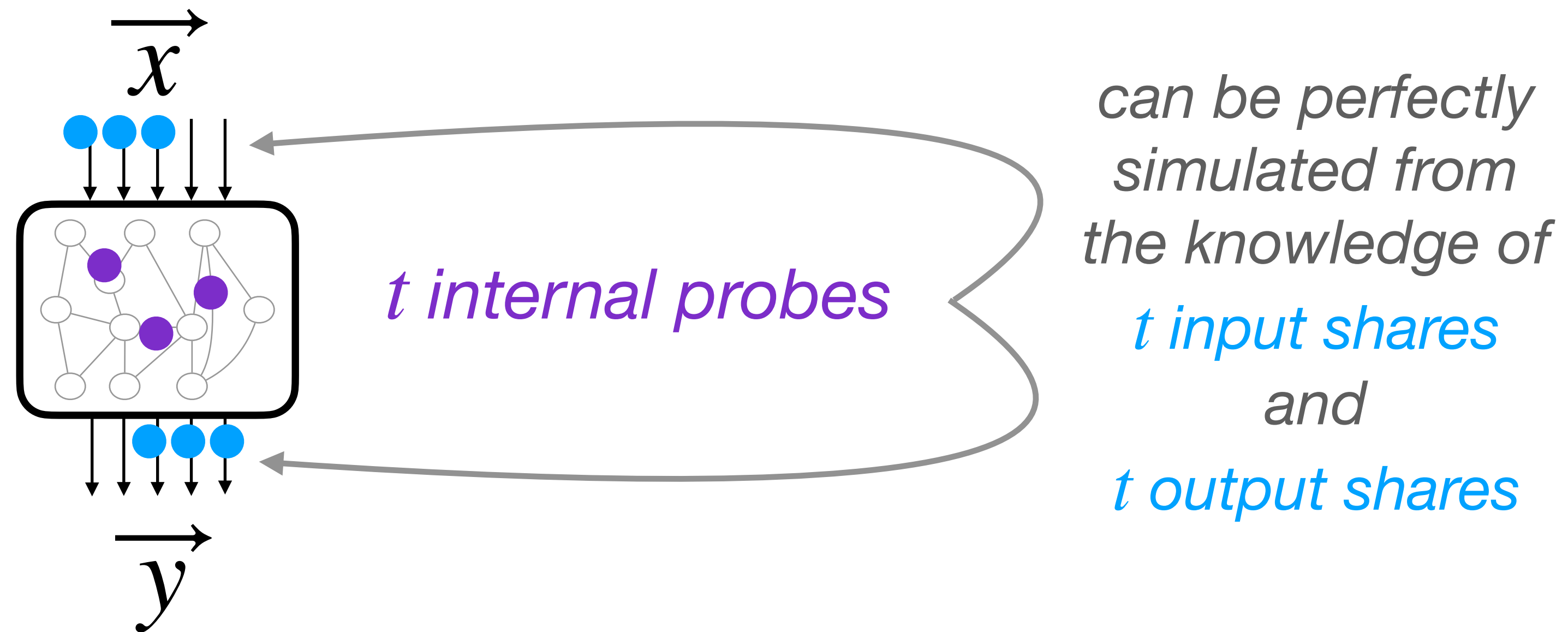
⇒ region probing security



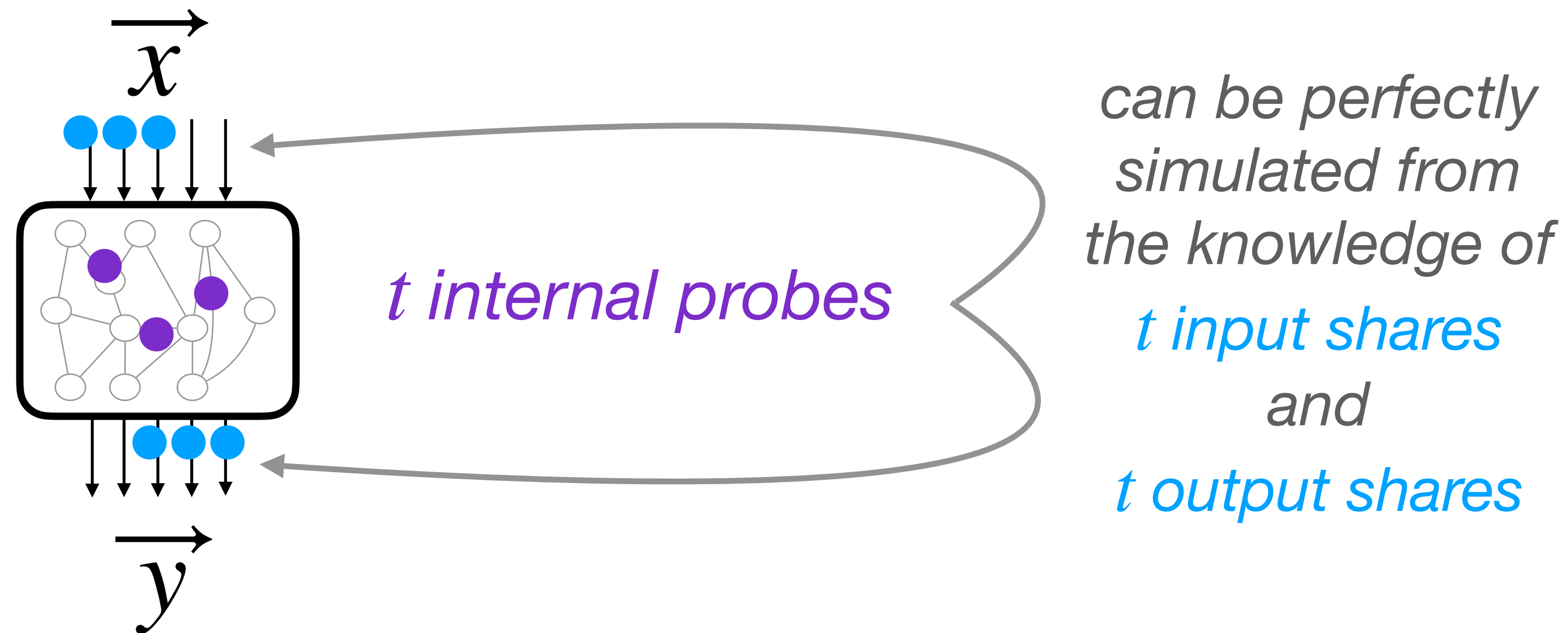
Input-Output Separation (IOS)



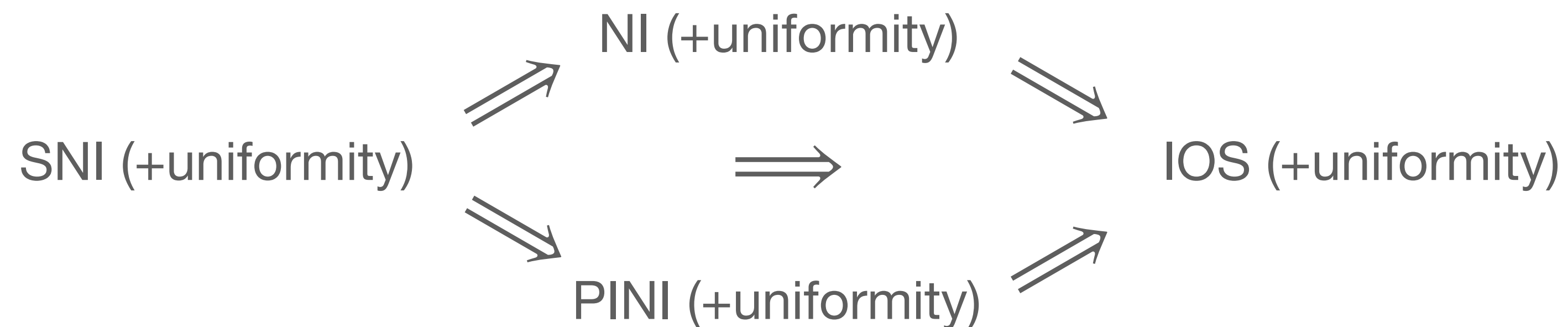
Input-Output Separation (IOS)



Input-Output Separation (IOS)



IOS is weaker than previous composition notions



Composition theorem

t_R probes per refresh gadget
+ t_{op} probes per operation gadget

can be perfectly simulated from

$t_{op} + 3t_R$ probes per operation gadget

IOS refreshing



Composition theorem

t_R probes per refresh gadget
+ t_{op} probes per operation gadget

can be perfectly simulated from

$t_{op} + 3t_R$ probes per operation gadget

can be perfectly simulated

nothing

assuming $(t_{op} + 3t_R)$ -PS
of operation gadgets

IOS refreshing

uniform refreshing

Composition theorem

Obtained rate:

$$\min \left(\frac{t_R}{|G_R|}, \frac{t_{op}}{|G_{op}|} \right)$$

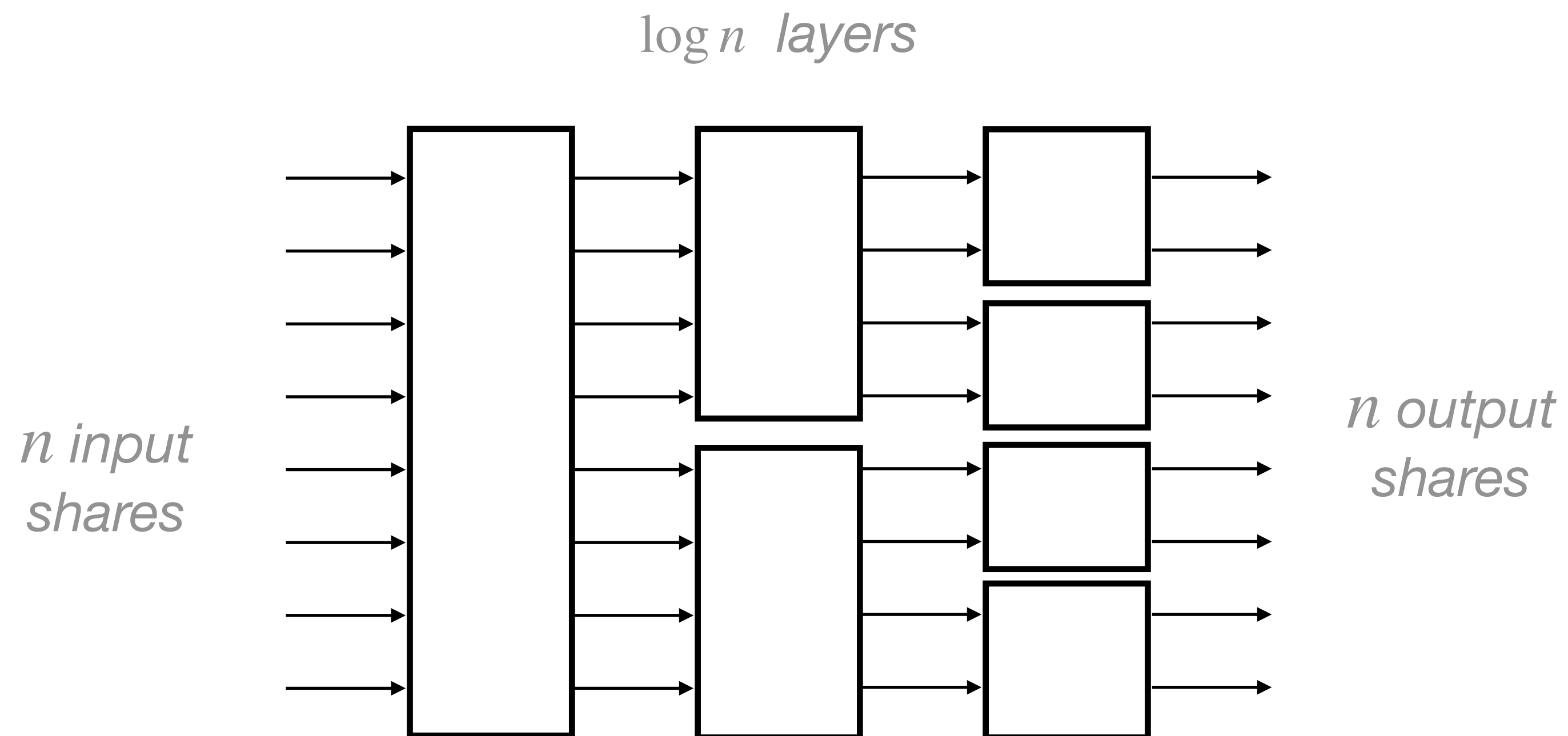
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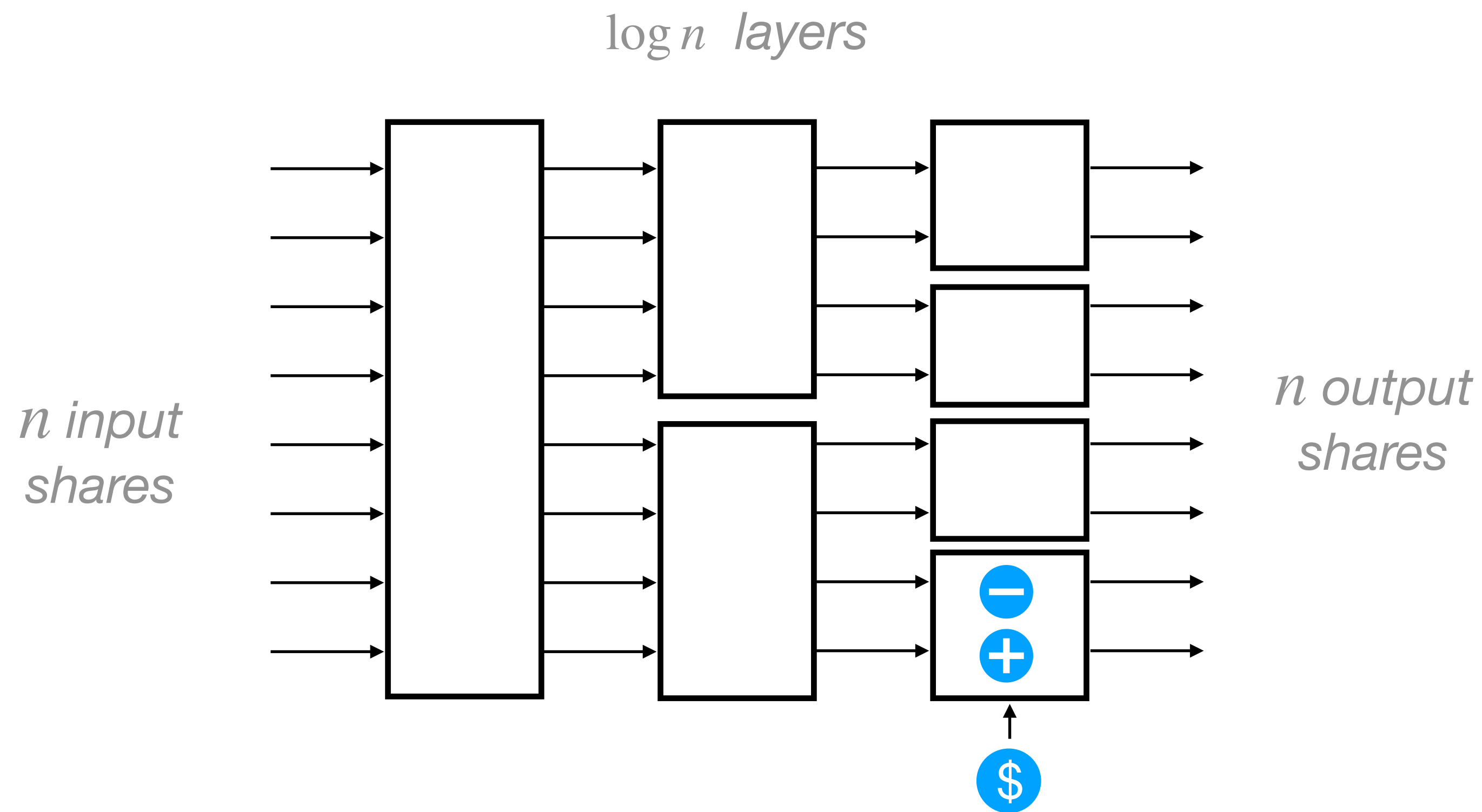
$$\max_{t_R, t_{op}} \min \left(\frac{t_R}{|G_R|}, \frac{t_{op}}{|G_{op}|} \right)$$

with $t_R < n$ and $(t_{op} + 3t_R) \leq t_{PS}$

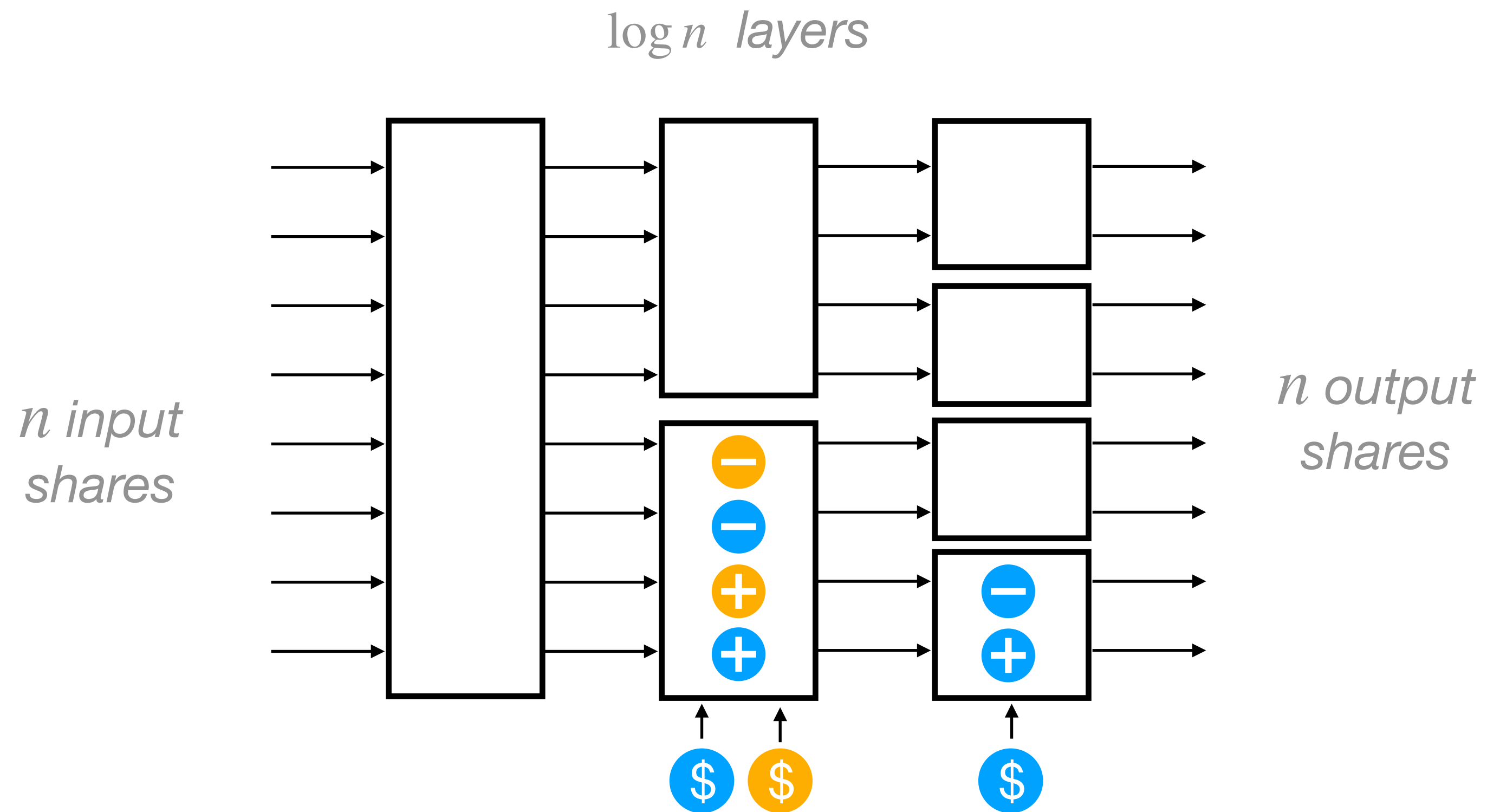
An IOS refresh gadget



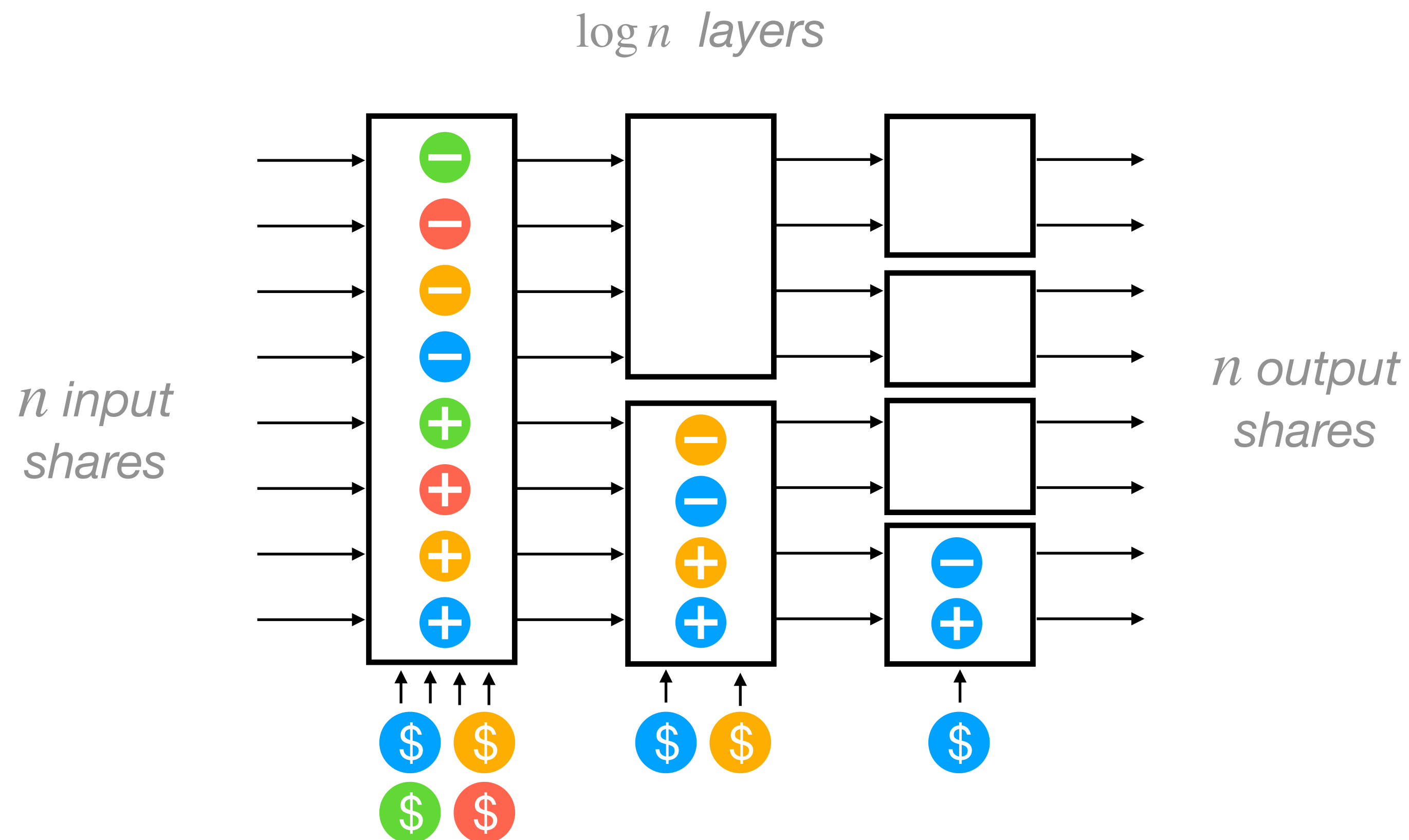
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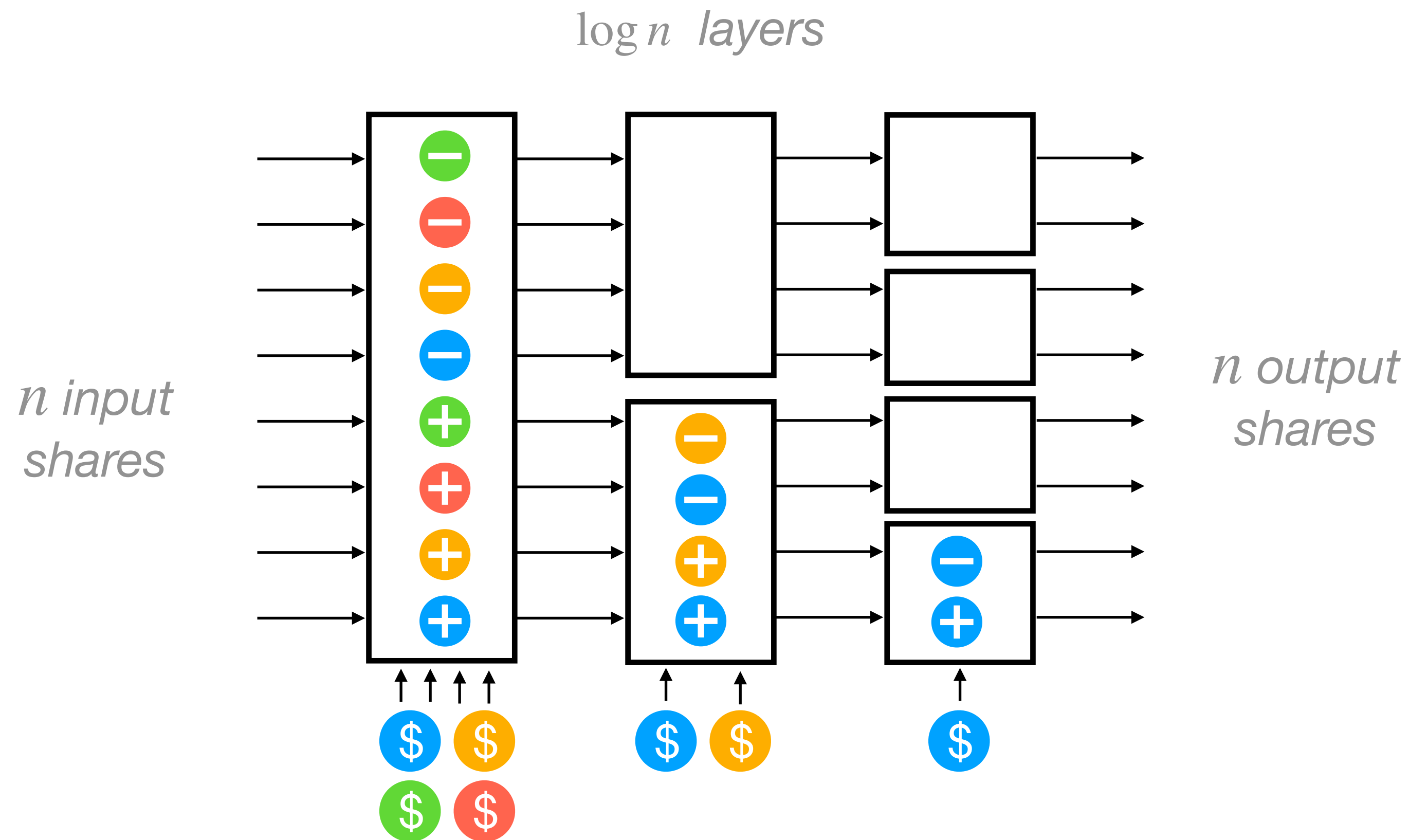
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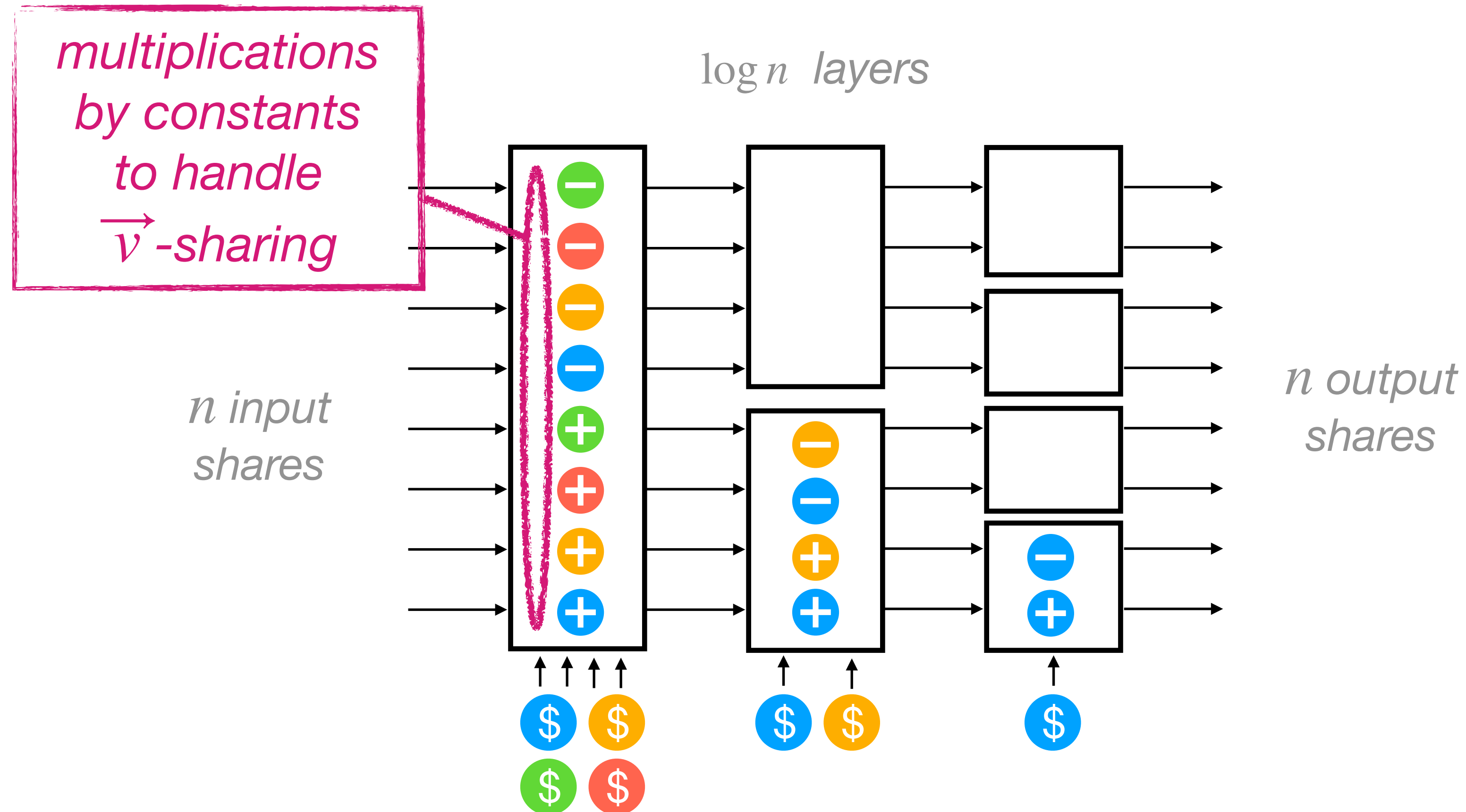


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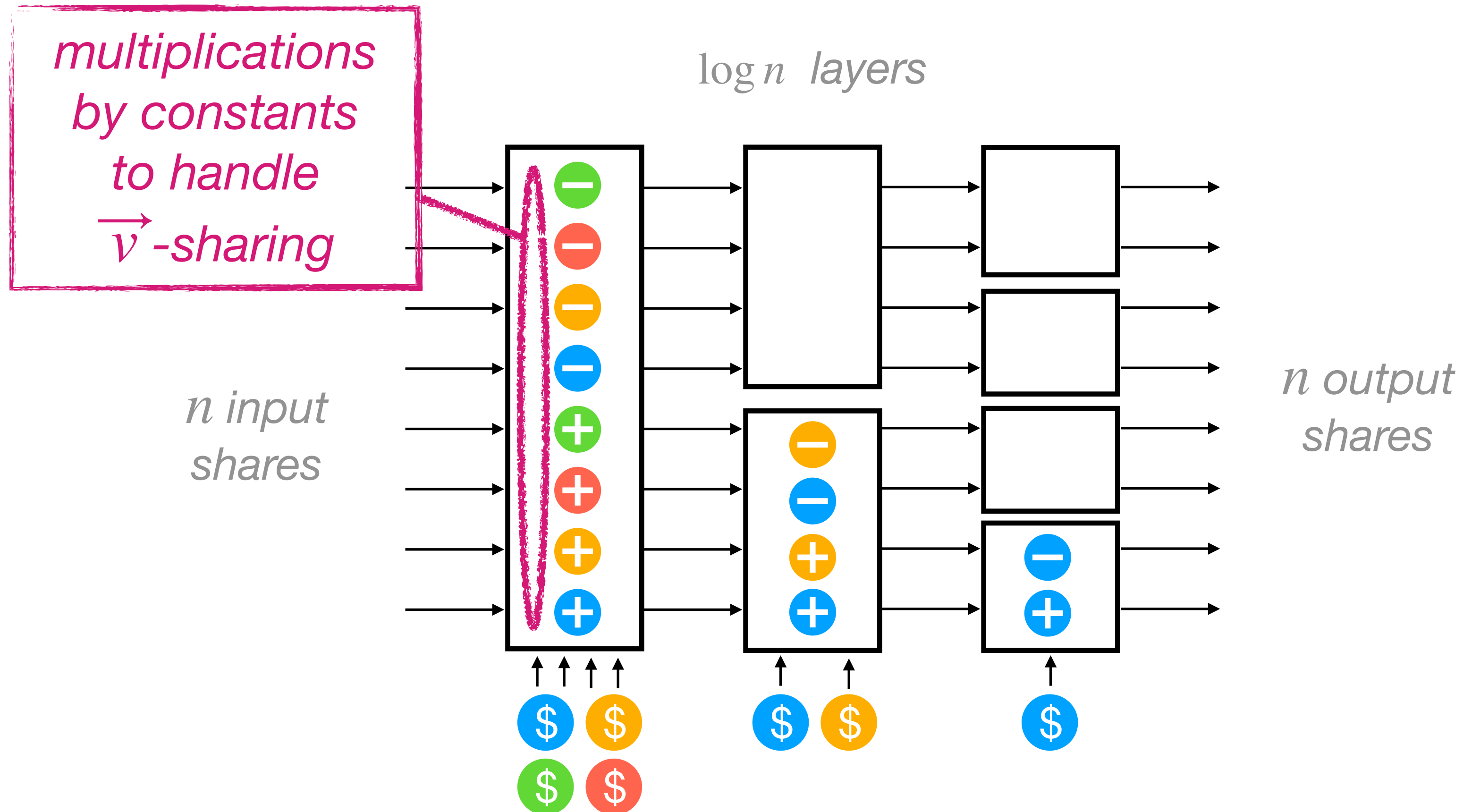
Batistello-Coron-Prouff-Zeitoun
refresh gadget [CHES'16]

An IOS refresh gadget



Batistello-Coron-Prouff-Zeitoun
refresh gadget [CHES'16]

An IOS refresh gadget



Batistello-Coron-Prouff-Zeitoun
refresh gadget [CHES'16]

*Only half of the
layers for IOS*

Quasilinear masking

- We extend the Goudarzi-Joux-Rivain (GJR) scheme [AC'18]
 - complexity $O(n \log n)$ against $O(n^2)$ for many probing secure scheme
 - proof of p -random probing security with $p = O(1/\log n)$
 - defined over fields \mathbb{F}_p with $p = 2^{\lceil \log n \rceil + 1} \alpha + 1$
- Our extension enjoys
 - base field \mathbb{K} of **any form**
 - proof in the (stronger) **r -region probing model** (still with $r = O(1/\log n)$)
 - we **patch a flaw** in the security proof thanks to the IOS approach

Quasilinear masking

- GJR scheme uses \overrightarrow{v} -sharings with

$$\overrightarrow{v} = (1, \omega, \omega^2, \dots, \omega^{n-1})$$

- A sharing of x

$$\overrightarrow{x} = (x_0, x_1, \dots, x_{n-1})$$

satisfies

$$\langle \overrightarrow{v}, \overrightarrow{x} \rangle = \sum_{i=0}^{n-1} x_i \cdot \omega^i = x$$

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*polynomial $P_{\overrightarrow{x}}(\omega)$
shares = coefficients*

Multiplication gadget

- Let \vec{t} such that

$$P_{\vec{t}}(W) = P_{\vec{x}}(W) \cdot P_{\vec{y}}(W)$$

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- Compression:

$$\vec{z} = (t_0, \dots, t_{n-1}) + \omega^n \cdot (t_n, \dots, t_{2n-1})$$

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Multiplication gadget

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*Evaluation-
interpolation
using FFT*



- We get

$2n-1$

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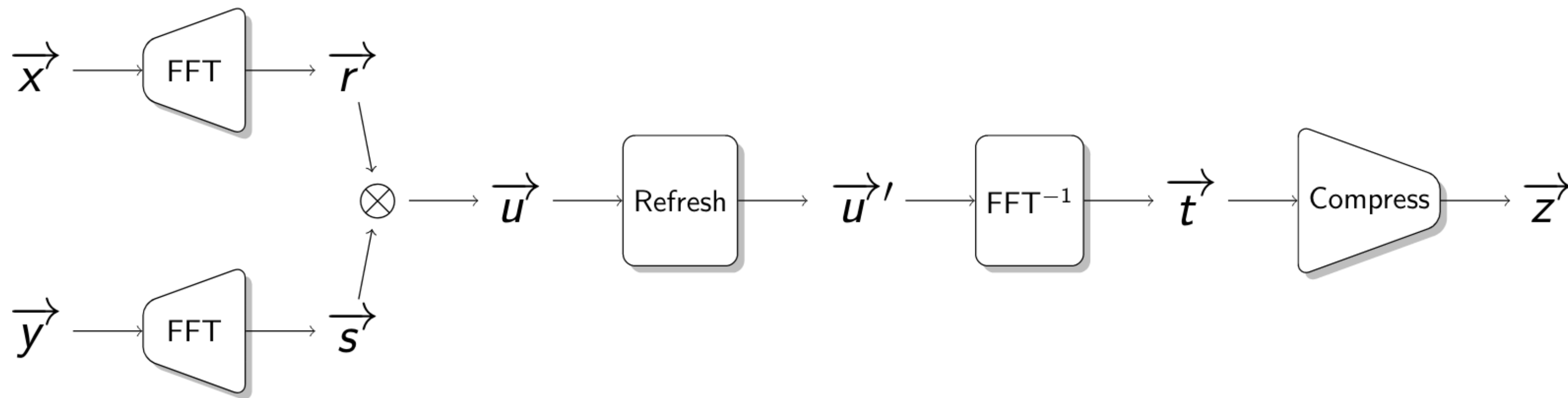
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- Compression:

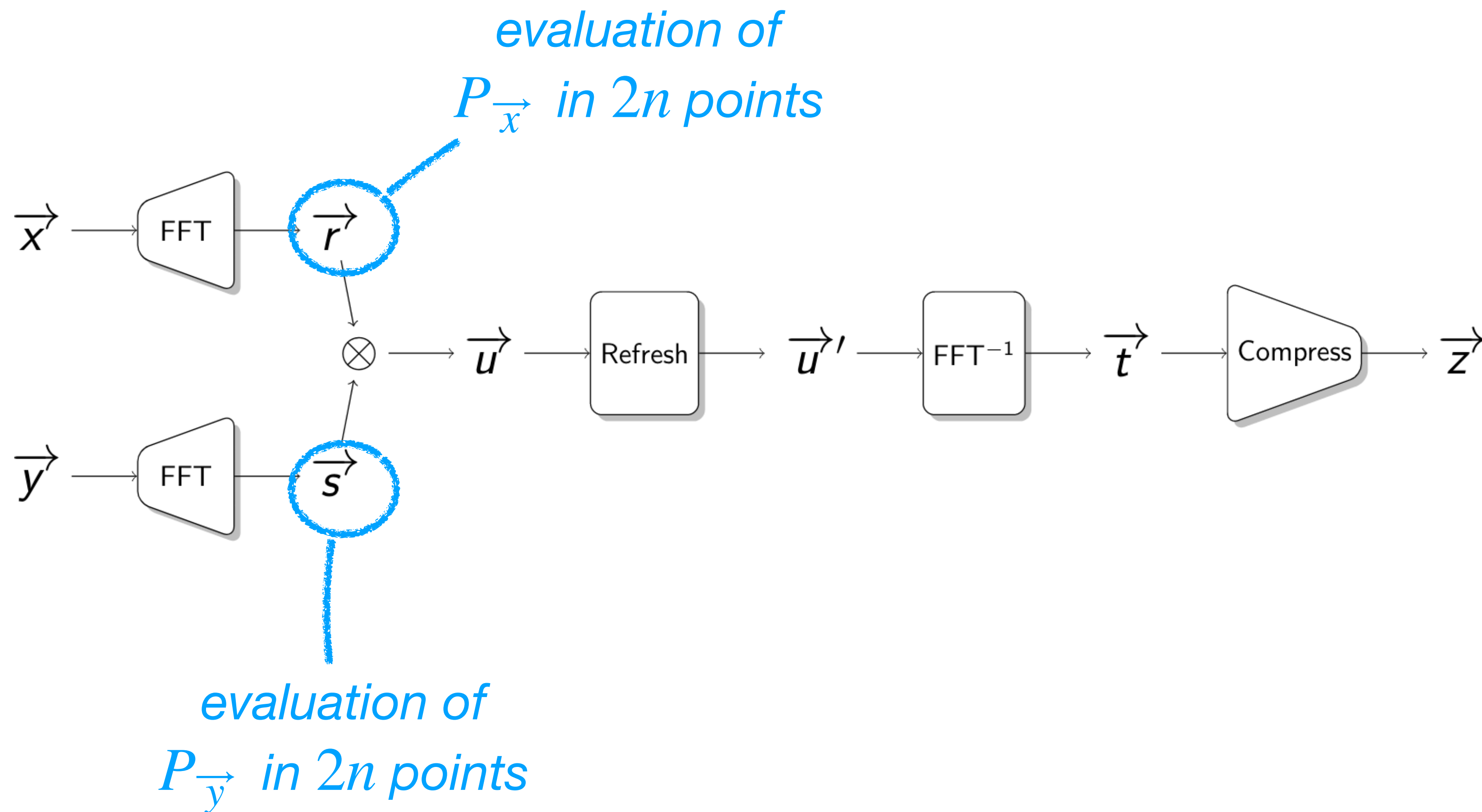
$$\vec{z} = (t_0, \dots, t_{n-1}) + \omega^n \cdot (t_n, \dots, t_{2n-1})$$

$$\sum_{i=0}^{n-1} (t_i + t_{n+i} \omega^n) \omega^i$$

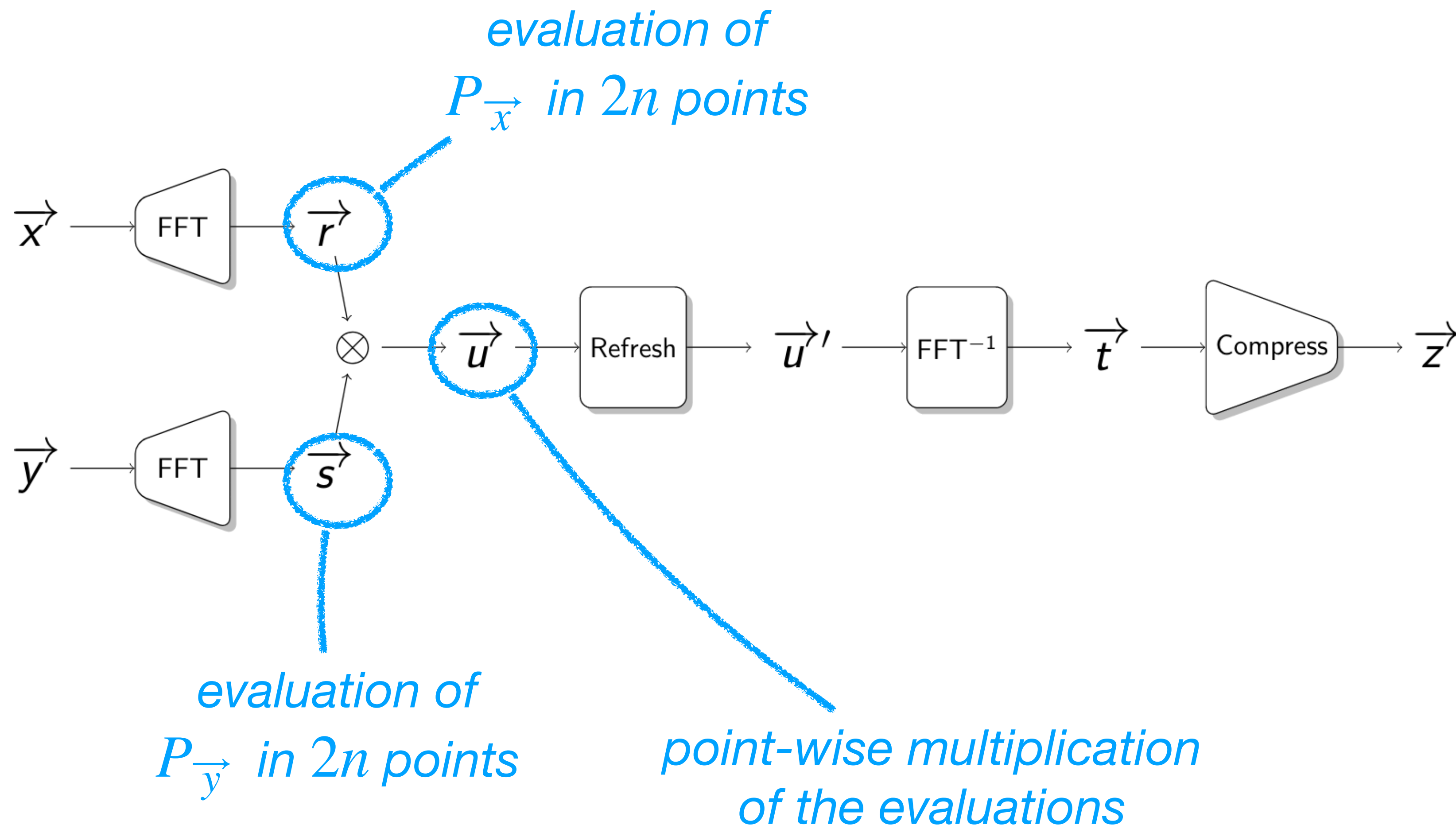
Multiplication gadget



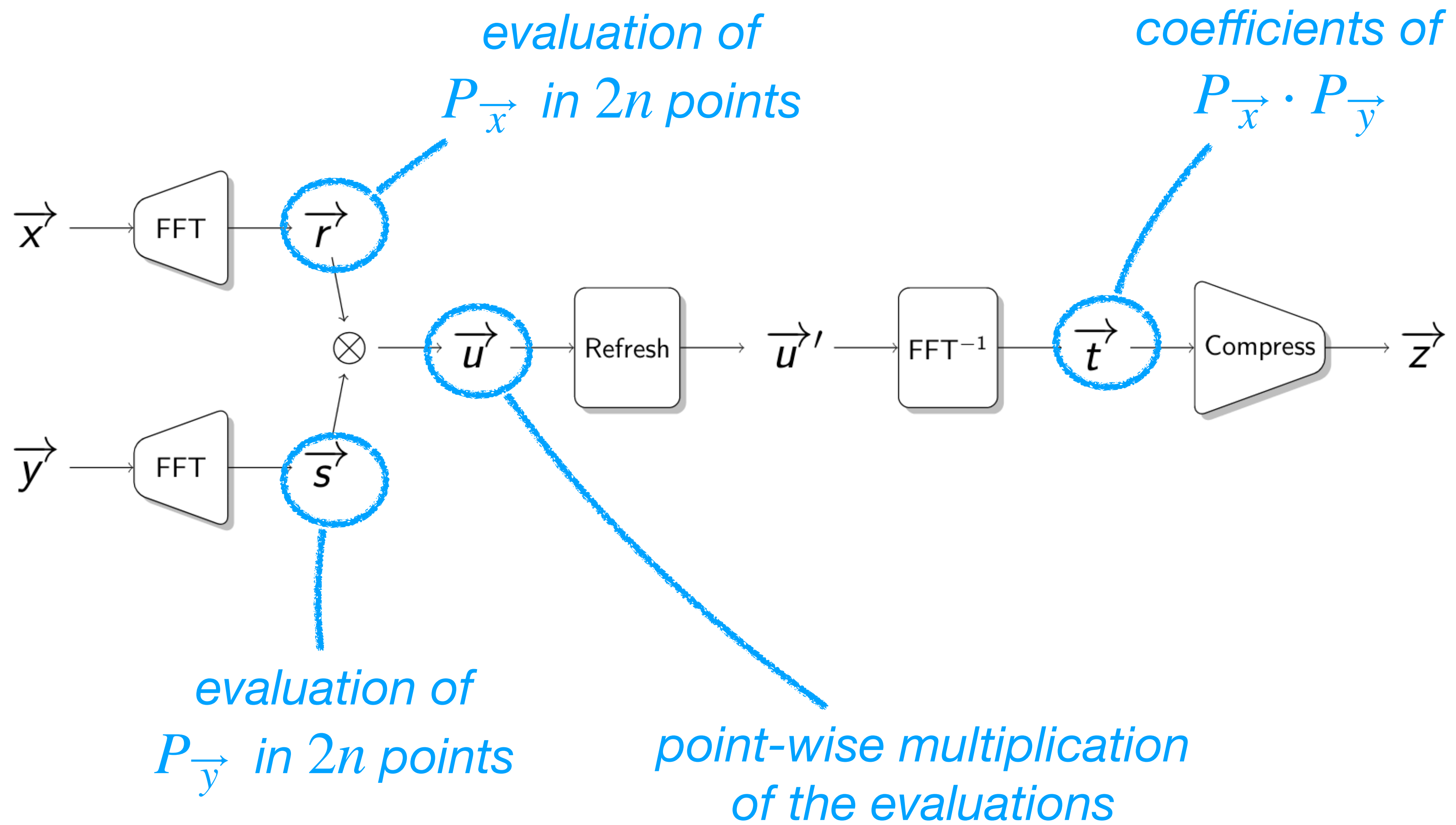
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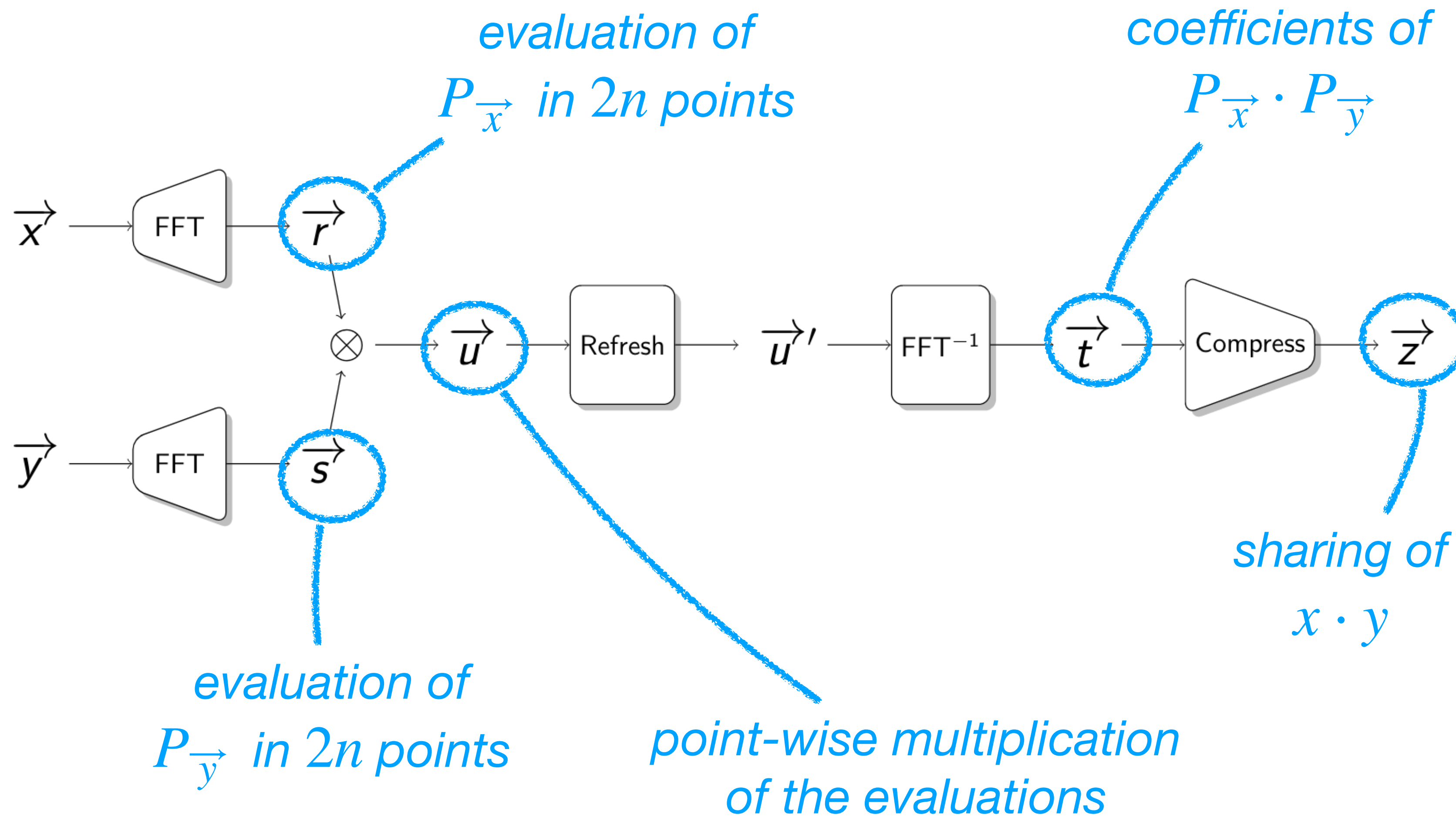
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- We have sharewise addition / subtraction / copy gadgets
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- Multiplication gadgets composed of
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Security reduction: PS FFT \Rightarrow region PS scheme

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$\Rightarrow (\lambda + \log n)$ -bit field elements

- Open problem: probing secure FFT on smaller fields

Application to AES and MiMC

- We apply
 - GJR+ (our variant with IOS composition)
 $\Rightarrow O(n \log n)$ complexity / $O(1/\log n)$ leakage rate
 - ISW+ (ISW mult. & BPCZ refresh)
 $\Rightarrow O(n^2)$ complexity / $O(1/n)$ leakage rate
- To
 - AES: $\mathbb{K} = \mathbb{F}_{256} \Rightarrow$ Gao-Mateer additive FFT
 - MiMC: $\mathbb{K} = \mathbb{F}_p \Rightarrow$ Number Theoretic Transform (NTT)

Application to AES and MiMC

— Results for AES —

n		Mul	Add.	Random
8	Full AES with ISW ⁺	64896	297088	123520
	Full AES with GJR ⁺	157056	257408	110080
	Efficiency ratio (GJR ⁺ /ISW ⁺)	2.43	0.87	0.9
16	Full AES with ISW ⁺	211712	926976	372480
	Full AES with GJR ⁺	396032	683776	286720
	Efficiency ratio (GJR ⁺ /ISW ⁺)	1.88	0.74	0.77
32	Full AES with ISW ⁺	751104	2847232	1077760
	Full AES with GJR ⁺	955904	1725952	706560
	Efficiency ratio (GJR ⁺ /ISW ⁺)	1.28	0.61	0.66
64	Full AES with ISW ⁺	2812928	8991744	3148800
	Full AES with GJR ⁺	2239488	4209664	1679360
	Efficiency ratio (GJR ⁺ /ISW ⁺)	0.8	0.47	0.54
128	Full AES with ISW ⁺	10868736	29820928	9594880
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! The field should be large for GJR+

Application to AES and MiMC

— Results for MiMC —

n		Mul	Add.	Random
8	Full MiMC with ISW ⁺	10416.0	45408.0	17544.0
	Full MiMC with GJR ⁺	40512.0	66128.0	20100.0
	Efficiency ratio (GJR ⁺ /ISW ⁺)	3.89	1.46	1.15
16	Full MiMC with ISW ⁺	41600.0	153056.0	55856.0
	Full MiMC with GJR ⁺	100796.0	165968.0	51872.0
	Efficiency ratio (GJR ⁺ /ISW ⁺)	2.43	1.09	0.93
32	Full MiMC with ISW ⁺	166208.0	513536.0	173984.0
	Full MiMC with GJR ⁺	240812.0	399360.0	127088.0
	Efficiency ratio (GJR ⁺ /ISW ⁺)	1.45	0.78	0.74
64	Full MiMC with ISW ⁺	664320.0	1773696.0	555456.0
	Full MiMC with GJR ⁺	559740.0	933568.0	300864.0
	Efficiency ratio (GJR ⁺ /ISW ⁺)	0.85	0.53	0.55
128	Full MiMC with ISW ⁺	2656000.0	6367744.0	1857664.0
	Full MiMC with GJR ⁺	1275388.0	2136832.0	695104.0
	Efficiency ratio (GJR ⁺ /ISW ⁺)	0.49	0.34	0.38

Thank you for watching!



For any questions:
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