Batching CSIDH Group Actions using AVX-512

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CSIDH

Commutative Supersingular Isogeny Diffie-Hellman – CSIDH

- It is a non-interactive key exchange protocol
- Allows for full public-key validation
- Drop-in PQ replacement for ECDH
- Based on the action of an ideal class group on a set of supersingular ECs
- The execution time is prohibitively high for many real-world applications

The CSIDH Group Action

Setting:

- Small, odd primes $\ell_1, \ldots, \ell_n$ s.t. $p = 4 \cdot \ell_1 \cdots \ell_n - 1$ is prime
- $E_A : y^2 = x^3 + Ax^2 + x$ supersingular elliptic curve over $\mathbb{F}_p$ (s.t. $A^2 \neq 4$)
- Hence $\#E(\mathbb{F}_p) = p + 1 = 4 \cdot \ell_1 \cdots \ell_n$

CSIDH group action:

- Compute the action of an ideal $\mathfrak{a} = \mathfrak{l}_1^{e_1} \cdots \mathfrak{l}_n^{e_n}$ on E_A.
- $\mathfrak{l}_1, \ldots \mathfrak{l}_n$: prime ideals, $e_1, \ldots e_n$: small exponents chosen from some $[-b, b]$
- Equiv. compute curve E' as the image of the isogeny ϕ of degree $\ell_1^{e_1} \cdots \ell_n^{e_n}$.
- CSIDH key feature: break into smaller isogenies:

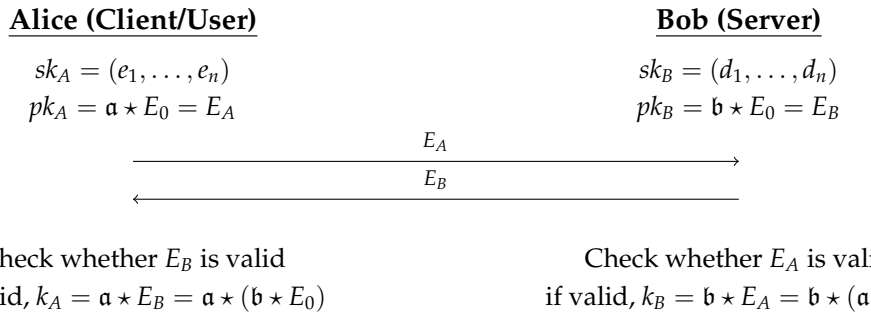
$$\begin{array}{lcl} |e_1| & \text{isogenies of degree} & \ell_1 \\ \vdots & \vdots & \vdots \\ |e_n| & \text{isogenies of degree} & \ell_n \end{array}$$

- Each isogeny using Vélu formulæ [Vél71] with $O(\ell_i \log p^2)$ bit operations

The CSIDH protocol

CSIDH parameters:

- Public params: prime $p = 4 \cdot \ell_1 \cdots \ell_n - 1$, starting curve: $E_0 : y^2 = x^3 + x$
- Secret keys: exponents $(e_1, \ldots, e_n)$, $(d_1, \ldots, d_n)$ chosen randomly from $[-b, b]^n$



Curves k_A and k_B are isomorphic

Security of CSIDH

Original CSIDH-512 instantiation [CLM+18]:

- $p = 4 \cdot \ell_1 \cdots \ell_{74} - 1$ and starting curve $E_0/\mathbb{F}_p : y^2 = x^3 + x$
- $\ell_1, \ldots, \ell_{73}$: first odd primes from $\ell_1 = 3$ and $\ell_{74} = 587$
- Secret exponents $(e_1, \ldots, e_{74})$ sampled from $[-5, 5]^{74}$
- Conjectured CSIDH-512 achieves NIST PQ security level 1
- Security based on the asymptotic complexity of Kuperberg's algorithm

Peikert [Pei20]:

- Prime p should be significantly larger (e.g. 4096-bits [CCJR20])

Constant-time implementation is needed:

- For computing the codomain curve $\mathfrak{a} \star E$

Constant-time Implementation/Optimizations

Bernstein, Lange, Martindale, and Panny [BLMP19]

- 1st constant-time implementation, dummy computations, failure probability

Jalali, Azarderakhsh, Kermani, and Jao [JAKJ19]

- large amount of dummy computations, failure probability

Meyer, Campos, and Reith [MCR19]: MCR-style

- only positive secret exponents $e_i \in [0, b_i]$, dummy isogenies
- Elligator 2 map for sampling random points on curve
- SIMBA-m-μ: splits $S = \{1, \ldots, n\}$ into m disjoint subsets

Onuki, Aikawa, Yamazaki, and Takagi [OAYT19]: OAYT-style

- Improves on MCR-style by 29.03%
- two-point approach
- allows for negative secret exponents as well, thus $e_i \in [-b_i, b_i]$

Constant-time Implementation/Optimizations

Cervantes-Vázquez, Chenu, Chi-Domínguez, De Feo, Rodríguez-Henríquez, and Smith [CCC+19]: improve MCR-and OAYT-style algorithms

- Efficient formulas in the twisted Edwards model
- for isogeny computations/evaluations and point doubling/addition
- differential addition chains instead of Montgomery ladder: 25% speedup
- Dummy-Free-style group action: resistant to fault injections

Hutchinson, LeGrow, Koziel, and Azarderakhsh [HLKA20]

- optimal strategies for choosing b_i, the ordering of l_i and SIMBA parameters

Chi-Domínguez and Rodríguez-Henríquez [CR20]

- not based on SIMBA approach
- generalized the computational strategies used in SIKE implementation

Banegas, Bernstein, Campos, Chou, Lange, Meyer, Smith and Sotáková [BBC+21]: CTIDH

- watch this talk also at CHES 2021!

Our Software – CSIDH-512 Group Action Evaluation

High-throughput implementation (batched, 8 parallel instances)

- Speeds up server-side TLS processing.
- Minimizes the latency of a single CSIDH-based signature, e.g. CSI-FiSh [BKV19] and SeaSign [DG19].

Low-latency implementation (1 instance)

- Accelerates CSIDH on the TLS client side.

Includes AVX-512F and AVX-512IFMA two different versions.

Target x64 software:

- OAYT and Dummy-Free style implementations of Cervantes-Vázquez, Chenu, Chi-Domínguez, De Feo, Rodríguez-Henríquez, and Smith [CCC+19]. *Latincrypt'19* (SIMBA, two-point approach, twisted Edwards model, differential addition chains)

AVX-512

Single Instruction Multiple Data – SIMD

- 512-bit vectors
- core extension AVX-512 Foundation – AVX-512F (32-bit vector multiplier)
- 64-bit $\times$ 8

Example: r = `_mm512_mul_epu32`(a, b)

$$< a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7 > \times < b_0, b_1, b_2, b_3, b_4, b_5, b_6, b_7 >$$
$$\Downarrow$$
$$< a_0 \times b_0, a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4, a_5 \times b_5, a_6 \times b_6, a_7 \times b_7 >$$

AVX-512IFMA (`vpmadd52luq` and `vpmadd52huq`)

"Two new instructions for big number multiplication for acceleration of RSA vectorized SW and other Crypto algorithms (Public key) performance". – *Intel*

Integer Fused Multiply-Add:

1. $a \times b \to t$ (52×52)-bit $\to$ 104-bit
2. $t_L / t_H + c \to r$ 52-bit + 64-bit $\to$ 64-bit

Intel hardware support:

- Palm Cove: Cannon Lake
- Sunny Cove: Ice Lake, Ice Lake-SP
- Willow Cove: Tiger Lake
- Cypress Cove: Rocket Lake

We target the Intel Ice Lake processor (Sunny Cove microarchitecture)

- Intel Core i3-1005G1 CPU clocked at 1.2 GHz.

Original CSIDH Group Action [CLM+18]

In: secret exp. $(e_1,\ldots,e_n)$, s.t. $e_i \in [-b_i, b_i]$; starting curve E_A
Out: target curve $E_B = [\mathfrak{l}_1^{e_1} \cdots \mathfrak{l}_n^{e_n}] * E_A$

01 $E_B \leftarrow E_A$
02 **while** some $e_j \neq 0$:
03 $(P_0, P_1) \leftarrow \mathtt{Elligator}(E_B)$ //$P_0 \in E[\pi - 1]$ and $P_1 \in E[\pi + 1]$
04 $(T_0, T_1) \leftarrow ([4]P_0, [4]P_1)$ //$\mathrm{ord}(T_0) = \mathrm{ord}(T_1) = (p+1)/4 = \ell_1 \cdots \ell_n$
05 **for** $i = 1,\ldots,n$:
06 $s \leftarrow \mathrm{sign}(e_i)$ //$s = 0$, if $e_i < 0$; $s = 1$, if $e_i \geq 0$
07 $\mathtt{cswap}(T_0, T_1, 1 - s)$ // swap if $s = 0 \Rightarrow e_i < 0$
08 $R \leftarrow [\ell_{i+1} \cdots \ell_n] T_0$ //$\mathrm{ord}(R) = \ell_i$
09 $T_1 \leftarrow [\ell_i] T_1$ //$\mathrm{ord}(T_1) = \ell_{i+1} \cdots \ell_n$
10 **if** $R \neq \infty$ **then**:
11 $E_C, \phi \leftarrow \{\mathtt{isogeny}; \mathtt{evaluation}; \mathtt{update}\}$
12 $E_B \leftarrow E_C$, $e_i \leftarrow e_i + (-1)^s$ // update curve and secret exp.
13 $\mathtt{cswap}(T_0, T_1, 1 - s)$ // back to the initial point ordering
14 **return** E_B

Original CSIDH Group Action [CLM+18]

The algorithm is not constant-time, which depends on $e_i \in [-b_i, b_i]$.

$$\left.\begin{array}{lcll} \text{isog. of deg: } \ell_1 & \to & |e_1| \le b_i & \text{times} \\ \text{isog. of deg: } \ell_2 & \to & |e_2| \le b_i & \text{times} \\ \vdots & \vdots & \vdots & \vdots \\ \text{isog. of deg: } \ell_{74} & \to & |e_{74}| \le b_i & \text{times} \end{array}\right\} \Rightarrow \#\text{isog} = \sum_{i=1}^{74} |e_i|$$

OAYT-style CSIDH [OAYT19]: computes b_i isogenies instead of $|e_i|$ for each ℓ_i
$\Rightarrow |e_i|$ "real" + $(b_i - |e_i|)$ "dummy" isogenies:

$$\left.\begin{array}{lcllll} b_1 \text{ isog. of deg: } \ell_1 & \to & |e_1| \text{ "real"} & + & (b_1 - |e_1|) \text{ "dummy"} \\ b_2 \text{ isog. of deg: } \ell_2 & \to & |e_2| \text{ "real"} & + & (b_2 - |e_2|) \text{ "dummy"} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{74} \text{ isog. of deg: } \ell_{74} & \to & |e_{74}| \text{ "real"} & + & (b_{74} - |e_{74}|) \text{ "dummy"} \end{array}\right\} \Rightarrow \#\text{isog} = \sum_{i=1}^{74} b_i = 404$$

OAYT-Style CSIDH Group Action [OAYT19, CCC+19]

In: secret exp. $(e_1,\ldots,e_n)$, s.t. $e_i \in [-b_i, b_i]$; starting curve E_A
Out: target curve $E_B = [\mathfrak{l}_1^{e_1} \cdots \mathfrak{l}_n^{e_n}] * E_A$

```
01 E_B ← E_A, #isog ← 0
02 while some b_j ≠ 0:
03   (P_0, P_1) ← Elligator(E_B); (T_0, T_1) ← ([4]P_0, [4]P_1)
04   for i = 1,...,n:
06     s ← sign(e_i)                          //s = 0, if e_i < 0; s = 1, if e_i ≥ 0
07     cswap(T_0, T_1, 1 − s)                        // swap if s = 0 ⇒ e_i < 0
08     R ← [ℓ_{i+1} ··· ℓ_n]T_0                                  //ord(R) = ℓ_i
09     T_1 ← [ℓ_i]T_1                                  //ord(T_1) = ℓ_{i+1} ··· ℓ_n
10     if R ≠ ∞ then:
11       w ← 1 − isequal(e_i, 0)                 //w = 0, if e_i = 0; w = 1, if e_i ≠ 0
12       E_C, φ ← {isogeny; evaluation; update}
13       cswap(E_B, E_C, w)                      // update curve if w = 1 ⇒ e_i ≠ 0
14       e_i ← e_i + (−1)^s · w, b_i − −
15     cswap(T_0, T_1, 1 − s)                  // back to the initial point ordering
16 return E_B
```

Obstacles to Batching CSIDH

SIMD requires the parallel group action instances to possess the same operation (instruction) sequence.

The kernel generator R might be a point at infinity in some instances.

- Leads to different operation sequences in eight parallel instances.
- $\text{Prob}_{R=\infty} = 1/l_i$, considerably high when the prime $l_i = 3, 5, 7, \ldots$

group action #1 ($R \neq \infty$)

```
09 ...
10 if R ≠ ∞ then:
11   w ← 1 − isequal(e_i, 0)
12   E_C, φ ← {isog; eval; update}
13   cswap(E_B, E_C, w)
14   e_i ← e_i + (−1)^s · w,  b_i − −
15 cswap(T_0, T_1, 1 − s)
```

group action #2 ($R = \infty$)

```
09 ...
10 if R ≠ ∞ then:
11   w ← 1 − isequal(e_i, 0)
12   E_C, φ ← {isog; eval; update}
13   cswap(E_B, E_C, w)
14   e_i ← e_i + (−1)^s · w,  b_i − −
15 cswap(T_0, T_1, 1 − s)
```

Not batching-friendly!

The Extra-Dummy Batching Method

Extra dummy isogeny computations for $R = \infty$ [BLMP19, JAKJ19].

In: secret exp. $(e_1, \ldots, e_n)$, s.t. $e_i \in [-b_i, b_i]$; starting curve E_A; $(c_1, \ldots, c_n)$

Out: target curve $E_B = [\mathfrak{l}_1^{e_1} \cdots \mathfrak{l}_n^{e_n}] * E_A$

01 $E_B \leftarrow E_A$, #isog $\leftarrow 0$, each $g_j \leftarrow b_j + c_j$

02 **while** some $g_j \neq 0$:

03 $(P_0, P_1) \leftarrow \text{Elligator}(E_B)$; $(T_0, T_1) \leftarrow ([4]P_0, [4]P_1)$

04 **for** $i = 1, \ldots, n$:

06 $s \leftarrow \text{sign}(e_i)$ //$s = 0$, if $e_i < 0$; $s = 1$, if $e_i \geq 0$

07 $\text{cswap}(T_0, T_1, 1 - s)$ // swap if $s = 0 \Rightarrow e_i < 0$

08 $R \leftarrow [\ell_{i+1} \cdots \ell_n]T_0$ //ord(R) $= \ell_i$

09 $T_1 \leftarrow [\ell_i]T_1$ //ord(T_1) $= \ell_{i+1} \cdots \ell_n$

10 $f \leftarrow 1 - \text{isinfinity}(R)$ //$f = 0$, if $R = \infty$; $f = 1$, if $R \neq \infty$

11 $w \leftarrow 1 - \text{isequal}(e_i, 0)$ //$w = 0$, if $e_i = 0$; $w = 1$, if $e_i \neq 0$

12 $E_C, \phi \leftarrow \{\text{isogeny}; \text{evaluation}; \text{update}\}$

13 $\text{cswap}(E_B, E_C, w\&f)$ // update curve if $w\&f = 1 \Rightarrow e_i \neq 0$ & $R \neq \infty$

14 $e_i \leftarrow e_i + (-1)^s \cdot (w\&f)$, $g_i - -$

15 $\text{cswap}(T_0, T_1, 1 - s)$ // back to the initial point ordering

16 **return** E_B

The Extra-Dummy Batching Method

$$\left.\begin{array}{llllll} \text{isog of deg} & \ell_1 & \to & |e_1| \text{ “real”} & + & (b_1 + c_1 - |e_1|) \text{ “dummy”} \\ \text{isog of deg} & \ell_2 & \to & |e_2| \text{ “real”} & + & (b_2 + c_2 - |e_2|) \text{ “dummy”} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \text{isog of deg} & \ell_{74} & \to & |e_{74}| \text{ “real”} & + & (b_{74} + c_{74} - |e_{74}|) \text{ “dummy”} \end{array}\right\} \Rightarrow \#\text{isog} = \sum_{i=1}^{74} (b_i + c_i)$$

Problem:

- Failure probability $\text{Prob}_{\text{fail}}$ when $\#\text{dummy} > (b_i + c_i - |e_i|)$ for at least one $i = 1, \ldots, 74 \Leftrightarrow$ when too many $R = \infty$.
- $\text{Prob}_{\text{fail}} < 2^{-32}$ needs ~ 900 isogenies (originally 404).

Solution:

- Hybrid mode!

Hybrid Mode

Batched component.

- Incomplete batched implementation performing 8 instances.
- Computes the main bulk of the group action evaluation.

Unbatched component.

- Latency-optimized implementation accelerating a single group action evaluation such as [CCC+19] and our low-latency AVX-512 implementation.
- Perform the remaining computation for each instance.

$$\langle \hat{B}^{(1)}, \ldots, \hat{B}^{(8)} \rangle, \langle \hat{\boldsymbol{b}}^{(\mathbf{1})}, \ldots, \hat{\boldsymbol{b}}^{(\mathbf{8})} \rangle, \langle \boldsymbol{e'}^{(\mathbf{1})}, \ldots, \boldsymbol{e'}^{(\mathbf{8})} \rangle \leftarrow \texttt{batched}(\langle A^{(1)}, \ldots, A^{(8)} \rangle, \boldsymbol{b}, \langle \boldsymbol{e}^{(\mathbf{1})}, \ldots, \boldsymbol{e}^{(\mathbf{8})} \rangle)$$

$$\Downarrow$$

$$B^{(k)} \leftarrow \texttt{unbatched}(\hat{B}^{(k)}, \hat{\boldsymbol{b}}^{(\boldsymbol{k})}, \boldsymbol{e'}^{(\boldsymbol{k})}) \text{ for } k = 1, \ldots, 8$$

$$\Downarrow$$

$$B^{(1)}, \ldots, B^{(8)}$$

$$\Rightarrow \#\text{isog} = \sum_{i=1}^{74} b_i + \sum_{i=1}^{74} \hat{b}_i =\sim 414$$

The Extra-Dummy Batching Method (Batched Component)

In: secret exp. $(e_1,\ldots,e_n)$, s.t. $e_i \in [-b_i, b_i]$; starting curve E_A;
Out: target curve $E_B = [\mathfrak{l}_1^{e_1}\cdots\mathfrak{l}_n^{e_n}] * E_A$; secret exp. $(e'_1,\ldots,e'_n)$; bound $(\hat{b}_1,\ldots,\hat{b}_n)$

$E_B \leftarrow E_A$, $\#\text{isog} \leftarrow 0$, $(e'_1,\ldots,e'_n) \leftarrow (e_1,\ldots,e_n)$, each $\hat{b}_j \leftarrow b_j$
while some $b_j \neq 0$:
  $(P_0, P_1) \leftarrow \text{Elligator}(E_B)$; $(T_0, T_1) \leftarrow ([4]P_0, [4]P_1)$
  for $i = 1,\ldots,n$:
    $s \leftarrow \text{sign}(e'_i)$ //$s=0$, if $e'_i < 0$; $s=1$, if $e'_i \geq 0$
    $\text{cswap}(T_0, T_1, 1-s)$ // swap if $s = 0 \Rightarrow e'_i < 0$
    $R \leftarrow [\ell_{i+1}\cdots\ell_n]T_0$ //$\text{ord}(R) = \ell_i$
    $T_1 \leftarrow [\ell_i]T_1$ //$\text{ord}(T_1) = \ell_{i+1}\cdots\ell_n$
    $f \leftarrow 1 - \text{isinfinity}(R)$ //$f=0$, if $R=\infty$; $f=1$, if $R\neq\infty$
    $w \leftarrow 1 - \text{isequal}(e'_i, 0)$ //$w=0$, if $e'_i = 0$; $w=1$, if $e'_i \neq 0$
    $E_C, \phi \leftarrow \{\text{isogeny}; \text{evaluation}; \text{update}\}$
    $\text{cswap}(E_B, E_C, w\&f)$ // update curve if $w\&f = 1 \Rightarrow e'_i \neq 0$ & $R \neq \infty$
    $e'_i \leftarrow e'_i + (-1)^s \cdot (w\&f)$, $b_i --$, $\hat{b_i} \leftarrow \hat{b_i} - f$
    $\text{cswap}(T_0, T_1, 1-s)$ // back to the initial point ordering
return E_B, $(e'_1,\ldots,e'_n)$, $(\hat{b}_1,\ldots,\hat{b}_n)$

The Extra-Infinity Batching Method

group action #1 ($R \neq \infty$)

```
09 ...
10 if R ≠ ∞ then:
11   w ← 1 − isequal(e_i, 0)
12   E_C, φ ← {isog; eval; update}
13   cswap(E_B, E_C, w)
14   e_i ← e_i + (−1)^s · w,  b_i − −
15 else:
16   T_0 ← [ℓ_i]T_0
17 cswap(T_0, T_1, 1 − s)
```

group action #2 ($R = \infty$)

```
09 ...
10 if R ≠ ∞ then:
11   w ← 1 − isequal(e_i, 0)
12   E_C, φ ← {isog; eval; update}
13   cswap(E_B, E_C, w)
14   e_i ← e_i + (−1)^s · w,  b_i − −
15 else:
16   T_0 ← [ℓ_i]T_0
17 cswap(T_0, T_1, 1 − s)
```

If $R = \infty$ in at least one instance

$\Rightarrow$ execute the "else"-branch in all group actions!

The ℓ_i-torsion parts of some points $T_0^{(k)}$ have not vanished.

The Extra-Infinity Batching Method

$$inf \leftarrow \texttt{isinfinity}(R^{(1)}) \mid \ldots \mid \texttt{isinfinity}(R^{(8)})$$
$$\Downarrow$$

$inf = 0$,	$R^{(k)} \neq \infty$ for all eight instances,	goto "if"-branch
$inf = 1$,	$R^{(k)} = \infty$ in at least one instance,	goto "else"-branch

Problem:

- Still #isog = 404 but more infinity-related computations
  - $T_0 \leftarrow [\ell_i]T_0$ in the "else"-branch.
  - $\texttt{R} \leftarrow [\ell_{i+1} \cdots \ell_n]\texttt{T}_0$, $\texttt{T}_1 \leftarrow [\ell_i]\texttt{T}_1$ in the inner "for" loop.
  - $(\texttt{P}_0, \texttt{P}_1) \leftarrow \texttt{Elligator}(\texttt{E}_B)$, $(\texttt{T}_0, \texttt{T}_1) \leftarrow ([4]\texttt{P}_0, [4]\texttt{P}_1)$ in the outer "while" loop.
- $\text{Prob}_{inf=1} = 1 - (1 - 1/\ell_i)^8$, quite high when ℓ_i is 3, 5, 7...

Solution:

- Hybrid mode!

Hybrid Mode

Divide small primes into two subsets

$\Rightarrow \boldsymbol{\ell} = (\ell_1, \ell_2, \ldots, \ell_{74}) \rightarrow \boldsymbol{\ell}_{\text{unbatch}} = (3, 5, 7, 11, 13, 17, 19)$, $\boldsymbol{\ell}_{\text{batch}} = \complement_{\boldsymbol{\ell}} \boldsymbol{\ell}_{\text{unbatch}}$

$\Rightarrow \boldsymbol{b} = (b_1, b_2, \ldots, b_{74}) \rightarrow \boldsymbol{b}_{\text{unbatch}}, \boldsymbol{b}_{\text{batch}}$

$\Rightarrow \boldsymbol{e} = (e_1, e_2, \ldots, e_{74}) \rightarrow \boldsymbol{e}_{\text{unbatch}}, \boldsymbol{e}_{\text{batch}}$

$$\langle \hat{B}^{(1)}, \hat{B}^{(2)}, \ldots, \hat{B}^{(8)} \rangle \leftarrow \texttt{batched}(\langle A^{(1)}, A^{(2)}, \ldots, A^{(8)} \rangle, \boldsymbol{b}_{\text{batch}}, \langle \boldsymbol{e}_{\text{batch}}^{(1)}, \boldsymbol{e}_{\text{batch}}^{(2)}, \ldots, \boldsymbol{e}_{\text{batch}}^{(8)} \rangle)$$

$$\Downarrow$$

$$B^{(k)} \leftarrow \texttt{unbatched}(\hat{B}^{(k)}, \boldsymbol{b}_{\text{unbatch}}, \boldsymbol{e}_{\text{unbatch}}^{(k)}) \text{ for } k = 1, \ldots, 8$$

$$\Downarrow$$

$$B^{(1)}, B^{(2)}, \ldots, B^{(8)}$$

Analysis

Example: n of the eight instances meeting $R = \infty$ in an iteration.

Extra-dummy

- Completes the computation in this iteration of the "for" loop.
- Later computes n "compensatory" isogenies in the unbatched component.

Extra-infinity

- Perform infinity-related computations.
  $\Rightarrow$ needed by n instances, not needed by $8 - n$ instances.

Conclusion

- n is small, extra-dummy better
- n is close to 8, extra-infinity better

The Combined Batching Method

Set a new variable:

$$n_{\text{inf}} = \texttt{isinfty}(R^{(1)}) + \ldots + \texttt{isinfty}(R^{(8)})$$

Set a precomputed threshold:

$$n_{\text{thld}} = 3$$

```
03 ...
04 if i = 1,...,n:
09   ...
10   n_inf = Σisinfinity(R^(k))
11   if n_inf ≤ n_thld then:
12     extra-dummy
13   else:
14     extra-infinity
15   cswap(T_0, T_1, 1 − s)
```

High-Throughput Implementation

Class group action: three different batching methods.

Curve arithmetic: according to x64 software [CCC+19] with minor optimizations.

Prime-field operations: (8×1)-way implementation based on the limb-slicing technique [CGT+21].

- $\mathbb{F}_p$ multiplication: experiments with different variants.
- $\mathbb{F}_p$ squaring: classic optimization (compute the repeated partial products only once, due to $a_i a_j = a_j a_i$).

Field Operation	ISA	Integer Mul/Sqr	Reduction	Structure	Latency
Multiplication	IFMA	Product-Scan.	Operand-Scan.	Interleaved	436 cc
Squaring	IFMA	Product-Scan.	Operand-Scan.	Interleaved	344 cc
Multiplication	AVX-512F	Karatsuba	Product-Scan.	Separated	848 cc
Squaring	AVX-512F	Product-Scan.	Product-Scan.	Interleaved	723 cc

Low-Latency Implementation

Serve as the unbatched component in the hybrid mode for high-throughput implementation.

Class group action: according to x64 software [CCC+19].

Curve arithmetic: can be parallelized to 2-way, with the cost $iM + jS \to \frac{i}{2}M^2 + \frac{j}{2}S^2$.

Prime-field operations: (2×4)-way implementation based on the implementation of Orisaka, Aranha, and López [OAL18] (originally designed for SIDH).

- $\mathbb{F}_p$ multiplication: interleaved version based on [OAL18].
- $\mathbb{F}_p$ squaring: classic optimization (compute the repeated partial products only once, due to $a_i a_j = a_j a_i$).

Benchmark of CSIDH-512 Group Actions (OAYT)

- Intel Core i3-1005G1 Ice Lake CPU clocked at 1.2 GHz
- Turbo boost was disabled
- GCC 9.3.0

	Reference	ISA	Impl.	#Inst.	Cycles	Cycles/#inst.	Speedup
	[CLM+18][†]	x64	1-way	1	133.7 M	133.7 M	1.52×
	[OAYT19]	x64	1-way	1	248.4 M	248.4 M	0.82×
	[CCC+19]	x64	1-way	1	203.6 M	203.6 M	1.00×
	[CR20]	x64	1-way	1	195.0 M	195.0 M	1.04×
	[HLKA20]	x64	1-way	1	194.7 M	194.7 M	1.05×
This work	Low-Latency	AVX-512F	(2×4)-way	1	232.2 M	232.2 M	0.88×
	Extra-Dummy	AVX-512F	(8×1)-way	8	858.0 M	107.3 M	1.90×
	Extra-Infinity	AVX-512F	(8×1)-way	8	1003.9 M	125.5 M	1.62×
	Combined	AVX-512F	(8×1)-way	8	850.1 M	106.3 M	1.92×
This work	Low-Latency	IFMA	(2×4)-way	1	132.1 M	132.1 M	1.54×
	Extra-Dummy	IFMA	(8×1)-way	8	454.1 M	56.8 M	3.59×
	Extra-Infinity	IFMA	(8×1)-way	8	550.5 M	68.8 M	2.96×
	Combined	IFMA	(8×1)-way	8	446.9 M	55.9 M	3.64×

[†] Not constant-time.

Benchmark of CSIDH-512 Group Actions (Dummy-Free)

- Intel Core i3-1005G1 Ice Lake CPU clocked at 1.2 GHz
- Turbo boost was disabled
- GCC 9.3.0

	Reference	ISA	Impl.	#Inst.	Cycles	Cycles/#inst.	Speedup
	[CCC+19]	x64	1-way	1	433.3 M	433.3 M	1.00×
	[CR20]	x64	1-way	1	394.3 M	394.3 M	1.10×
This work	Low-Latency	AVX-512F	(2×4)-way	1	447.0 M	447.0 M	0.97×
	Extra-Dummy	AVX-512F	(8×1)-way	8	1811.0 M	226.4 M	1.91×
	Extra-Infinity	AVX-512F	(8×1)-way	8	2172.3 M	271.5 M	1.60×
	Combined	AVX-512F	(8×1)-way	8	1801.4 M	225.2 M	1.92×
This work	Low-Latency	IFMA	(2×4)-way	1	253.8 M	253.8 M	1.71×
	Extra-Dummy	IFMA	(8×1)-way	8	967.0 M	120.9 M	3.58×
	Extra-Infinity	IFMA	(8×1)-way	8	1220.5 M	152.6 M	2.84×
	Combined	IFMA	(8×1)-way	8	955.3 M	119.4 M	3.63×

Performance Analysis

The theoretical max speedup of an AVX-512 implementation (compared to x64) of isogeny-based crypto is actually far from $8\times \leftarrow$ multiplier.

High-throughput implementation:

- Example: eight 512-bit integer multiplications
  - x64: $(64 \times 64 \rightarrow 128)$-bit $\Rightarrow 8 \times 8^2 = 512$ mul instructions
  - AVX-512F: eight $(32 \times 32 \rightarrow 64)$-bit $\Rightarrow 16^2 = 256$ vec mul instructions $\Rightarrow 2.0\times$
  - IFMA: eight $(52 \times 52 \rightarrow 104)$-bit $\Rightarrow 2 \times 10^2 = 200$ vec mul instructions $\Rightarrow 2.56+\times$
- $1.92\times$ (AVX-512F) and $3.64\times$ (IFMA) are expected speed-ups.

Performance Analysis

Low-latency implementation:

- AVX-512IFMA implementation of SIKE in [KG19] is 1.72× faster
  ⇒ Our 1.54× (OAYT) and 1.71× (Dummy-Free) speedups correspond to the expected acceleration.

Why less efficient than high-throughput implementation?

- The overheads caused by aligning and blending AVX-512 vectors.
- Some point operations cannot be ideally parallelized to 2-way.
- Some computations in F_p cannot be parallelized in an ideal (2×4)-way.
- Lower instruction-level parallelism due to the fewer limb vectors.

Conclusions

- Vector engines like AVX-512 offer great potential to optimize CSIDH.
- The first vectorized implementation of CSIDH.
- 3.6-fold throughput gain compared to the state-of-the-art x64 implementation.
- Vectorizing methods can also be used for larger primes, and certain parts of the code can be reused.

Thank you!

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