

## On Tight Quantum Security of HMAC and NMAC in the Quantum Random Oracle Model

**Akinori Hosoyamada** (NTT Corporation / Nagoya University)

Tetsu Iwata (Nagoya University)

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## Introduction

## **Types of MACs**

- Block cipher based
  - CBC-MAC, PMAC,...
- Wegman-Carter & polynomial
  - GMAC, Poly1305,...
- Hash based
  - HMAC/NMAC, keyed-sponge,...



## HMAC (Hash-based MAC)



- Most basic approach to convert Merkle-Damgård hash  $\rightarrow$  MAC
- Standardized in FIPS PUB 198 / Used in TLS, SSH, IPsec,...



### NMAC



• Two-key variant of HMAC



## **Security of HMAC/NMAC**



• Security against classical attacks:

Tight security bound... $O(2^{n/2})$  (n: output length) [GPR14]

• Security against quantum query attacks:

Secure up to  $O(2^{n/5})$  (or  $O(2^{n/8})$ ) queries<sup>\*1</sup> (in the standard model) [SY17]

Trivial attack... $O(2^{n/3})$  queries

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## **Security of HMAC/NMAC**



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#### Q. Can we show the tight quantum security bound?

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### Results



- Tight quantum security bound in the QROM (*h* : QRO)
- HMAC/NMAC are indistinguishable from a RF against quantum query attacks as long as  $(q_h + Q) \cdot \ell^{\frac{3}{5}} \ge 2^{n/3}$ 
  - $-q_h$ : max. num. of quantum queries to h
  - *Q*: max. num. of quantum queries to the keyed oracle of HMAC/NMAC
  - $-\ell$ : maximum message length
- Tight when  $\ell$  is not exponentially large
- Compressed oracle technique
- Hardest part: Proving the prob. of a bad event is low
  - We introduce a new idea to capture the uncertainty of outputs of a random function that the adversary cannot observe



# **Rough Overview of the Proof**

Game G<sub>0</sub> (NMAC)











### Game G<sub>0</sub> (NMAC)











Game G<sub>1</sub>









 $h(K_1,\cdot) \Rightarrow f_1$  $h(K_2,\cdot) \Rightarrow f_{out}$ 

![](_page_11_Picture_6.jpeg)

## Game G<sub>1</sub>

![](_page_12_Picture_1.jpeg)

![](_page_12_Figure_2.jpeg)

![](_page_12_Figure_3.jpeg)

![](_page_12_Picture_4.jpeg)

![](_page_12_Picture_5.jpeg)

![](_page_13_Figure_0.jpeg)

 $f_{out} \circ f_1 \Rightarrow g_1$ 

![](_page_13_Picture_2.jpeg)

**Game**  $G'_1$ 

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![](_page_14_Figure_2.jpeg)

![](_page_14_Picture_3.jpeg)

![](_page_15_Figure_0.jpeg)

![](_page_15_Figure_1.jpeg)

 $h(f_1(\cdot),\cdot) \Rightarrow f_2(\cdot,\cdot)$ 

![](_page_15_Picture_3.jpeg)

![](_page_15_Picture_5.jpeg)

Game G<sub>2</sub>

![](_page_16_Picture_1.jpeg)

![](_page_16_Figure_2.jpeg)

![](_page_16_Figure_3.jpeg)

![](_page_16_Figure_4.jpeg)

![](_page_16_Picture_5.jpeg)

![](_page_17_Figure_0.jpeg)

 $f_{out} \circ f_2 \Rightarrow g_2$ 

![](_page_17_Picture_2.jpeg)

![](_page_18_Picture_1.jpeg)

- From  $G_i$  to  $G'_i$ :  $f_{out} \circ f_i \Rightarrow g_i$
- From  $G'_i$  to  $G_{i+1}$ :  $h(f_i(\cdot), \cdot) \Rightarrow f_{i+1}(\cdot, \cdot)$
- $f_{out}, f_1, f_2, ..., g_1, g_2, ...$  are independent random functions
- $G'_{\ell}$ : The ideal game (random function)

![](_page_19_Picture_1.jpeg)

- From G<sub>i</sub> to G'<sub>i</sub>: f<sub>out</sub> ∘ f<sub>i</sub> ⇒ g<sub>i</sub> The hardest part
  From G'<sub>i</sub> to G<sub>i+1</sub>: h(f<sub>i</sub>(·),·) ⇒ f<sub>i+1</sub>(·,·)
  f<sub>out</sub>, f<sub>1</sub>, f<sub>2</sub>, ..., g<sub>1</sub>, g<sub>2</sub>, ... are independent random functions
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![](_page_20_Picture_0.jpeg)

# The hardest part: Classical proof

## The hardest part (simplified)

![](_page_21_Picture_1.jpeg)

![](_page_21_Figure_2.jpeg)

f, f', h are independent random functions

![](_page_22_Figure_0.jpeg)

f, g, h are independent random functions

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![](_page_23_Figure_0.jpeg)

f, g, h are independent random functions

![](_page_24_Picture_1.jpeg)

- $(F_1^h, h)$  and  $(F_2, h)$  are indistinguishable if **coll** and **hit** do not occur
- **coll** : a new output of *f* collides with a previous input to h **hit** : a new direct query to h collides with a previous output of *f*

![](_page_24_Figure_4.jpeg)

![](_page_25_Picture_1.jpeg)

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![](_page_25_Figure_4.jpeg)

![](_page_26_Picture_1.jpeg)

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![](_page_26_Figure_4.jpeg)

![](_page_27_Picture_1.jpeg)

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![](_page_27_Figure_4.jpeg)

![](_page_28_Picture_1.jpeg)

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![](_page_28_Figure_4.jpeg)

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![](_page_30_Picture_1.jpeg)

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 $\neg$ (coll  $\lor$  hit)  $\Rightarrow$  outputs of  $F_1^h$  and  $F_2$  seem completely random

![](_page_30_Figure_5.jpeg)

![](_page_31_Picture_1.jpeg)

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- What we have to prove: Pr[coll] and Pr[hit] are small
  - Pr[coll] can easily be upper bounded by using the randomness of f
  - Pr[hit]...the randomness of *f* cannot be directly used and we do not know what the adversary will query
    - $\rightarrow$  some techniques needed (e.g., coefficient-H) to deal with hit

![](_page_32_Picture_0.jpeg)

# **Compressed Oracle Technique**

## **Compressed Oracle Technique**

![](_page_33_Picture_1.jpeg)

- Classical proofs often rely on the fact that queries/answers can be recorded
- However, in the quantum setting "recording queries" in naïve ways disturb quantum states → lots of classical proofs are invalid in the quantum setting
- Zhandry's compressed oracle technique enables us to record queries of random functions to some extent [Zha19]

## **Compressed Oracle Technique (cont'd)**

- The compressed oracle holds databases of queries/answers in quantum superposition
- Quantum states of the adversary and the oracle look like

$$\sum_{a,b,c,x_1,\dots,y_q} \alpha_{a,b,c,x_1,\dots,y_q} |a,b,c\rangle \otimes |(x_1,y_1),\dots,(x_q,y_q)\rangle$$
  
adversary Oracle's database

- It behaves like the classical lazy-sampling (to some extent)
  - A fresh query  $x \rightarrow$  uniform superposition of y is added:  $\sum_{y \sqrt{2^n}} |y\rangle$
  - Sometimes records are forgotten or overwritten

![](_page_35_Picture_0.jpeg)

# **Proof in the Quantum setting**

## The first proof idea in the quantum setting

![](_page_36_Picture_1.jpeg)

The first idea: classical proof idea + compressed oracle

The joint quantum states of

- Adversary A and the oracles  $(F_1^h, h)$ , and
- Adversary A and the oracles  $(F_2, h)$

will be indistinguishable as long as coll and hit do not happen in the databases & for *A*'s query

What we have to prove: "Pr[coll]" and "Pr[hit]" are small

- Pr[coll] can easily be upper bounded by using the randomness of f
- Pr[hit]... the randomness of f cannot be directly used and we do not know what the adversary will query <u>and classical proof techniques cannot be used</u>

 $\rightarrow$  **<u>new technique</u>** needed to deal with <u>hit</u>

hit...the event that the adversary A succeeds to guess an output of f

![](_page_37_Figure_2.jpeg)

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 $F_1^h$ 

![](_page_38_Picture_2.jpeg)

![](_page_38_Picture_3.jpeg)

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Suppose the adversary A makes a fresh query (u,v) to  $F_1^h$ 

![](_page_39_Figure_2.jpeg)

![](_page_39_Figure_3.jpeg)

![](_page_39_Picture_4.jpeg)

NTT

## Suppose the adversary A makes a fresh query (u,v) to $F_1^h$ 1. $\alpha = f(u)$ is sampled

![](_page_40_Figure_2.jpeg)

![](_page_40_Figure_3.jpeg)

Database for f $|(u, \alpha)\rangle$ 

![](_page_40_Picture_6.jpeg)

![](_page_41_Picture_1.jpeg)

- Suppose the adversary A makes a fresh query (u,v) to  $F_1^h$ 1.  $\alpha = f(u)$  is sampled
- 2.  $w = h(\alpha, v)$  is sampled and returned to A

![](_page_41_Figure_4.jpeg)

![](_page_41_Figure_5.jpeg)

Database for f Database for h  $|(u, \alpha)\rangle \otimes |(\alpha||v, w)\rangle$ 

![](_page_42_Picture_1.jpeg)

- Suppose the adversary A makes a fresh query (u,v) to  $F_1^h$ 1.  $\alpha = f(u)$  is sampled
- 2.  $w = h(\alpha, v)$  is sampled and returned to A

![](_page_42_Figure_4.jpeg)

![](_page_43_Picture_1.jpeg)

- Suppose the adversary A makes a fresh query (u,v) to  $F_1^h$
- 1.  $\alpha = f(u)$  is sampled
- 2.  $w = h(\alpha, v)$  is sampled and returned to A
- The adversary knows w but does not know anything about  $\alpha = f(u)$

![](_page_43_Figure_6.jpeg)

![](_page_44_Picture_1.jpeg)

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![](_page_44_Figure_6.jpeg)

We say databases are *equivalent* if they are indistinguishable from the adversary

![](_page_45_Figure_2.jpeg)

NTT

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For arbitrary z (the adversary's guess of an output of f),

 $\Pr[z \text{ is indeed an output of } f] = \frac{\# \{\text{equivalent DBs s.t. } f(u) = z \text{ for some } u\}}{\# \{\text{equivalent DBs}\}}$ 

$$\approx O\left(\frac{i}{2^n}\right)$$
 (after making i queries)

![](_page_46_Figure_5.jpeg)

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$$\approx O\left(\frac{i}{2^n}\right)$$
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 $\Pr[\text{hit occurs at the } i\text{-th query}] \approx O\left(\frac{i}{2^n}\right)$ 

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 $|\phi_{\text{hit}}\rangle \coloneqq$  The vector that corresponds to "hit happens after q queries"  $|\phi_{\text{hit}}^{(i)}\rangle \coloneqq$  The vector that corresponds to "hit happens at the i-th query"

$$\||\boldsymbol{\phi}_{\text{hit}}\rangle\| \le \sum_{1\le i\le q} \left\| |\boldsymbol{\phi}_{\text{hit}}^{(i)}\rangle \right\| \le \sum_{1\le i\le q} \sqrt{\Pr[\text{hit occurs at the }i\text{-th query}]} \le q \cdot O\left(\sqrt{\frac{q}{2^n}}\right) \le O\left(\sqrt{\frac{q^3}{2^n}}\right)$$

The norm of 
$$|\phi_{\rm hit}\rangle$$
 is small as long as  $q \ll 2^{\frac{\mu}{3}}$ 

$$\left(\||\boldsymbol{\phi}_{\rm hit}\rangle\| = \sqrt{\Pr[\rm{hit}]}\right)$$

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### Remarks

![](_page_49_Picture_1.jpeg)

More precisely, we show

- $|\phi_{\rm hit}\rangle$  is small as long as  $q \ll 2^{\frac{\mu}{3}}$
- $|\phi_{\text{coll}}\rangle$  is small as long as  $q \ll 2^{\frac{n}{3}}$
- The quantum states are "equal" (there is an isometry) if  $\neg$  (hit V coll)
- Quantum coefficients for equivalent databases are exactly equal by tracing coefficient of each vector in detail

![](_page_50_Picture_1.jpeg)

- From  $G_i$  to  $G'_i$ :  $f_{out} \circ f_i \Rightarrow g_i$
- From  $G'_i$  to  $G_{i+1}$ :  $h(f_i(\cdot), \cdot) \Rightarrow f_{i+1}(\cdot, \cdot)$
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- $G_0$ : The real game (NMAC)
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![](_page_51_Picture_1.jpeg)

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- From G'<sub>i</sub> to G<sub>i+1</sub>:  $h(f_i(\cdot), \cdot) \Rightarrow f_{i+1}(\cdot, \cdot)$  indistinguishable up to  $2^{n/3}$  queries
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![](_page_52_Picture_1.jpeg)

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![](_page_54_Picture_1.jpeg)

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•  $G'_{\ell}$ : The ideal game (INIVIAC) •  $G'_{\ell}$ : The ideal game (random function) indistinguishable up to  $2^{n/3}$  queries (when  $\ell$  is small)

Proof for HMAC is almost the same

![](_page_55_Picture_0.jpeg)

## Summary

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![](_page_56_Picture_1.jpeg)

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- HMAC/NMAC are indistinguishable from a RF against quantum query attacks as long as  $(q_h + Q) \cdot \ell^{\frac{3}{5}} \ge 2^{n/3}$ 
  - $q_h$ : max. num. of quantum queries to h, Q: max. num. of quantum queries to the keyed oracle of HMAC/NMAC,  $\ell$ : maximum message length
- Tight when  $\ell$  is not exponentially large
- Compressed oracle technique
- Hardest part: Proving the adversary's guess success prob. is low
  - We introduced "equivalent databases" to capture the uncertainty of outputs of a random function that the adversary cannot observe directly

![](_page_56_Picture_9.jpeg)