Quantum Collision Attacks on Reduced SHA-256 and SHA-512

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@Crypto 2021

Results

- First dedicated quantum collision attacks on SHA-2
  - 38-step attack on SHA-256 & 39-step attack on SHA-512
  - Classical collision attacks: 31-step for SHA-256 & 27-step for SHA-512
  - Still far from full-step attacks (64 steps / 80 steps)

- We convert classical semi-free-start collisions on 38-step SHA-256 & 39-step SHA-512 into collisions in the quantum setting

- Our attacks are valid in the setting of time-space tradeoff
  - Invalid in other quantum settings

Basics of
Classical Collision Attacks

Valid Classical Collision Attacks

- Generic Attack: Birthday Attack (Time $2^{n/2}$)
- A dedicated attack is valid iff $\boldsymbol{T < 2^{n/2}}$

The number of attacked steps

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Input

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

Step 8

Step 9

Step 10

Output

The number of attacked steps

Input

Step 1

Step 2

Step 3

Step 4

Step 5

Step 6

Step 7

Step 8

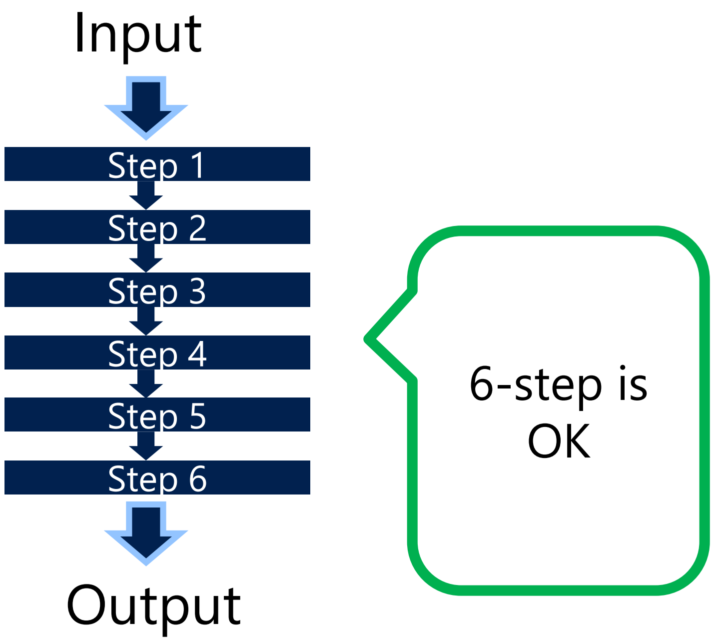
Step 9

Step 10

Output

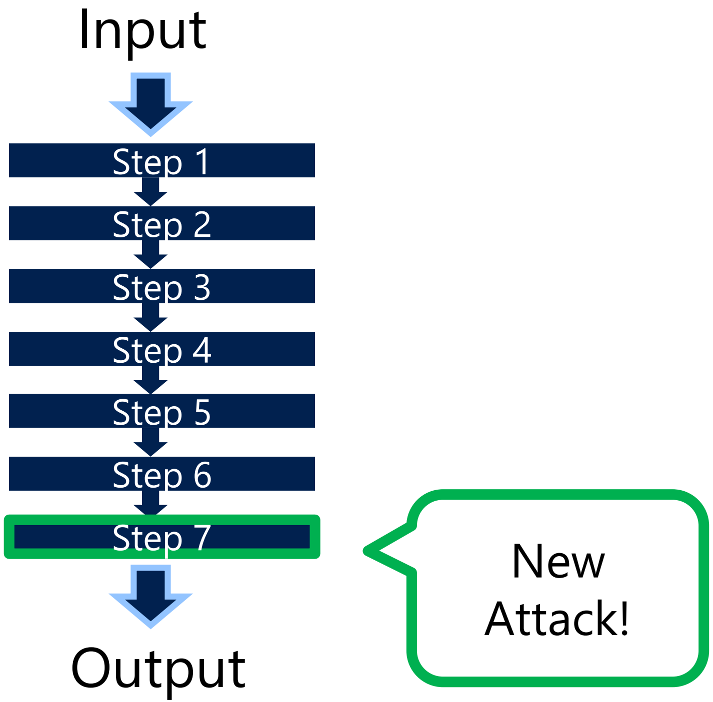
10-step is hard to break…

The number of attacked steps



- When an original primitive is hard to break, usually symmetric-key cryptanalysts try to break its reduced-step variants

The number of attacked steps



- What is important: <u>**How many steps can we break?**</u>
  (rather than the actual complexity)

Valid Classical Collision Attacks

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- Generic Attack: Birthday Attack (Time $2^{n/2}$)
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- Basic approach: Differential cryptanalysis
- A suitable differential trail of which probability is p

  $\rightarrow$ Collision attack of time $T = 1/p$

The differential trail leads to a valid attack only if

$$\boldsymbol{p > 2^{-n/2}}$$

Some Observations on Dedicated Quantum Collision Attacks at Eurocrypt 2020 [HY20]

[HY20] Akinori Hosoyamada, Yu Sasaki: Finding Hash Collisions with Quantum Computers by Using Differential Trails with Smaller Probability than Birthday Bound. Eurocrypt 2020.

Generic Quantum Collision Attacks

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Three settings depending on available computational resources

1. Small quantum computer + Large qRAM

Best algorithm: BHT ($T = 2^{n/3}$ & qRAM $2^{n/3}$) [BHT98]

2. Efficiency is measured by Time-Space tradeoff (No qRAM)

Quantum computer of size S + Classical computer of size S

Best algorithm: Parallel rho (Tradeoff $T = 2^{n/2}/S$) [Ber09]

3. Small quantum computer + Large classical memory (No qRAM)

Best algorithm: CNS ($T = 2^{2n/5}$, $2^{n/5}$classical memory) [CNS17]

[BHT98] Gilles Brassard, Peter Høyer, Alain Tapp: Quantum Cryptanalysis of Hash and Claw-Free Functions. LATIN 1998

[Ber09] D. J. Bernstein: Cost analysis of hash collisions: Will quantum computers make SHARCS obsolete?. SHARCS 2009.

[CNS17] A. Chailloux, M. Naya-Plasencia, A. Schrottenloher: An efficient quantum collision search algorithm and implications on symmetric cryptography. Asiacrypt 2017.

Generic Quantum Collision Attacks

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Quantum speed-up for generic collision attack is always <u>less-than-quadratic</u>

[BHT98] Gilles Brassard, Pe
[Ber09] D. J. Bernstein: Co
[CNS17] A. Chailloux, M. N ... ic cryptography. Asiacrypt 2017.

Speed-up for Differential Cryptanalysis

Very roughly speaking, the time to find a collision with a differential path of prob. p is

Classical… $T = 1/p$

Quantum… $T = \sqrt{1/p}$ (with the Grover search)[KLLN16]

<u>Quadratic</u> speed-up for Differential Cryptanalysis

[KLLN16] M. Kaplan, G. Leurent, A. Leverrier, M. Naya-Plasencia: Improved rebound attack on the finalist Grostl. *IACR Trans. Symmetric Cryptol.*, 2016(1) pp. 71-94, 2016.

Our Observation @ EC2020: Speed-up gap

Attack	Speed-up
Generic Collision	Less-than-Quadratic
Differential Cryptanalysis	Quadratic

Differential cryptanalysis becomes relatively stronger in the quantum setting

The validity condition $p > 2^{-n/2}$ can be relaxed

Example: Small quantum computer + Large qRAM

- Generic algorithm (BHT): $T = 2^{n/3}$

- Differential cryptanalysis: $T = \sqrt{1/p}$

- Collision attack based on differential cryptanalysis is valid only if

$$\sqrt{1/p} < 2^{n/3} \iff \boldsymbol{p > 2^{-2n/3}}$$

Relaxed from the classical condition $p > 2^{-n/2}$
p may lead to a valid attack even if $2^{-n/2} \geq p$

Example: Time-Space Tradeoff

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- Generic algorithm (parallel rho): $T = 2^{n/2}/S$
- Differential cryptanalysis: $T = \sqrt{1/p}$
- Collision attack based on differential cryptanalysis that requires space S is valid only if

$$\sqrt{1/p} < 2^{n/2}/S \Leftrightarrow \boldsymbol{p > 2^{-n} \cdot S^2}$$

p may lead to a valid attack even if p is very close to 2^{-n}

Results @ EC2020

- The condition for p is relaxed → dedicated quantum collision attacks can reach more steps than classical attacks
  - We indeed showed dedicated quantum collision attacks on AES-MMO and Whirlpool that break more steps than classical attacks

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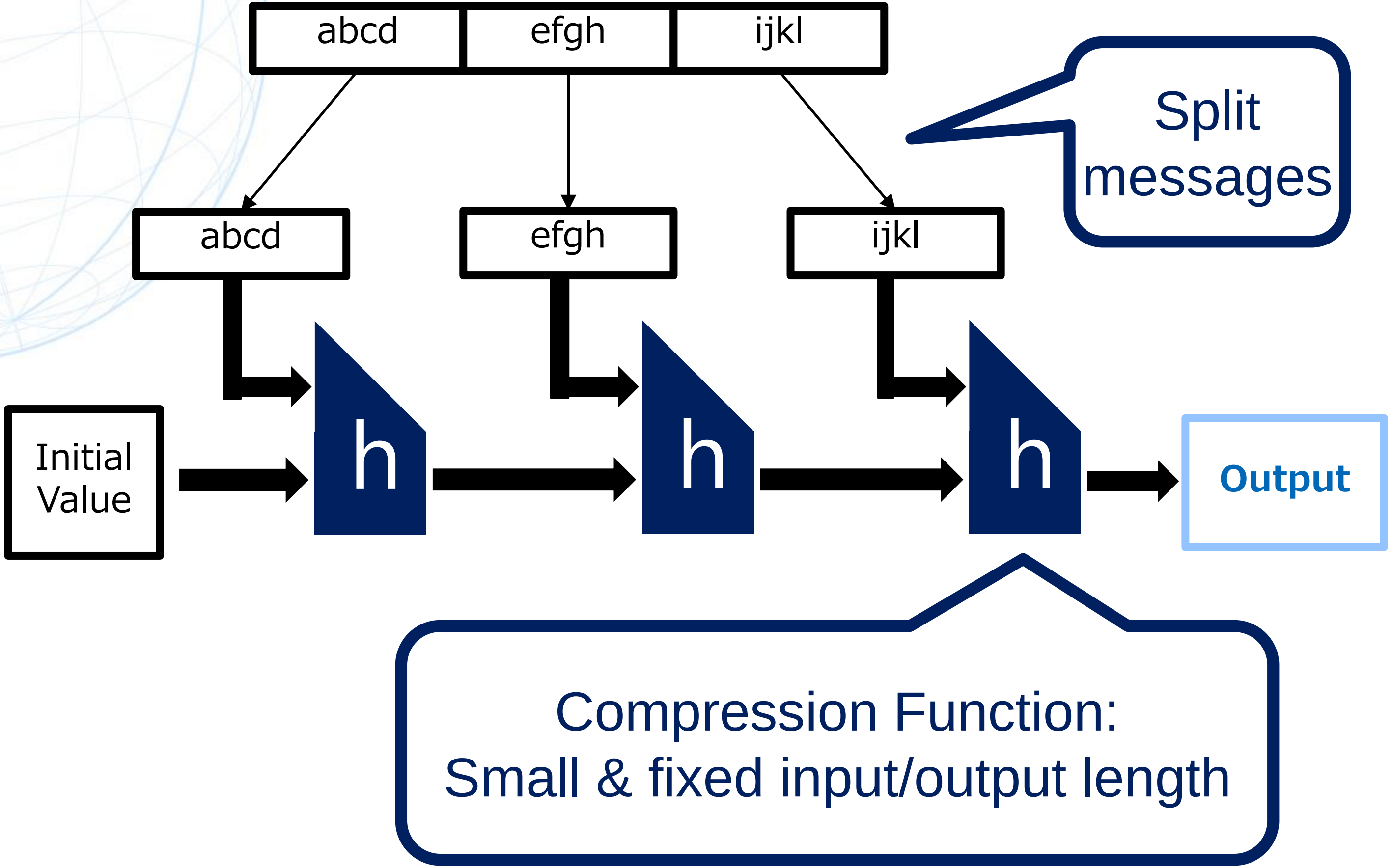
Q. Can we similarly extend the number of attacked steps of SHA-2 in the quantum setting??

Basics of SHA-2

SHA-2

- Current most popular hash function family standardized by NIST
- Consists of several functions:
  - SHA-224, SHA-256, SHA-384, SHA-512, SHA-512/224, SHA-512/256
  - SHA-224 is a truncated version of SHA-256
  - SHA-384, SHA-512/224, SHA-512/256 are truncated versions of SHA-512
- Davies-Meyer + Merkle-Damgaard

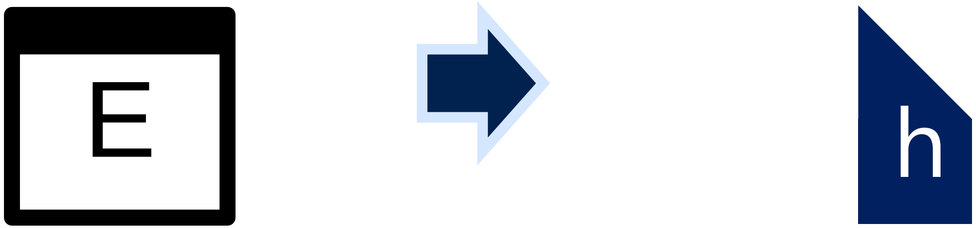
Merkle-Damgaard construction



How to make compression functions

Block cipher

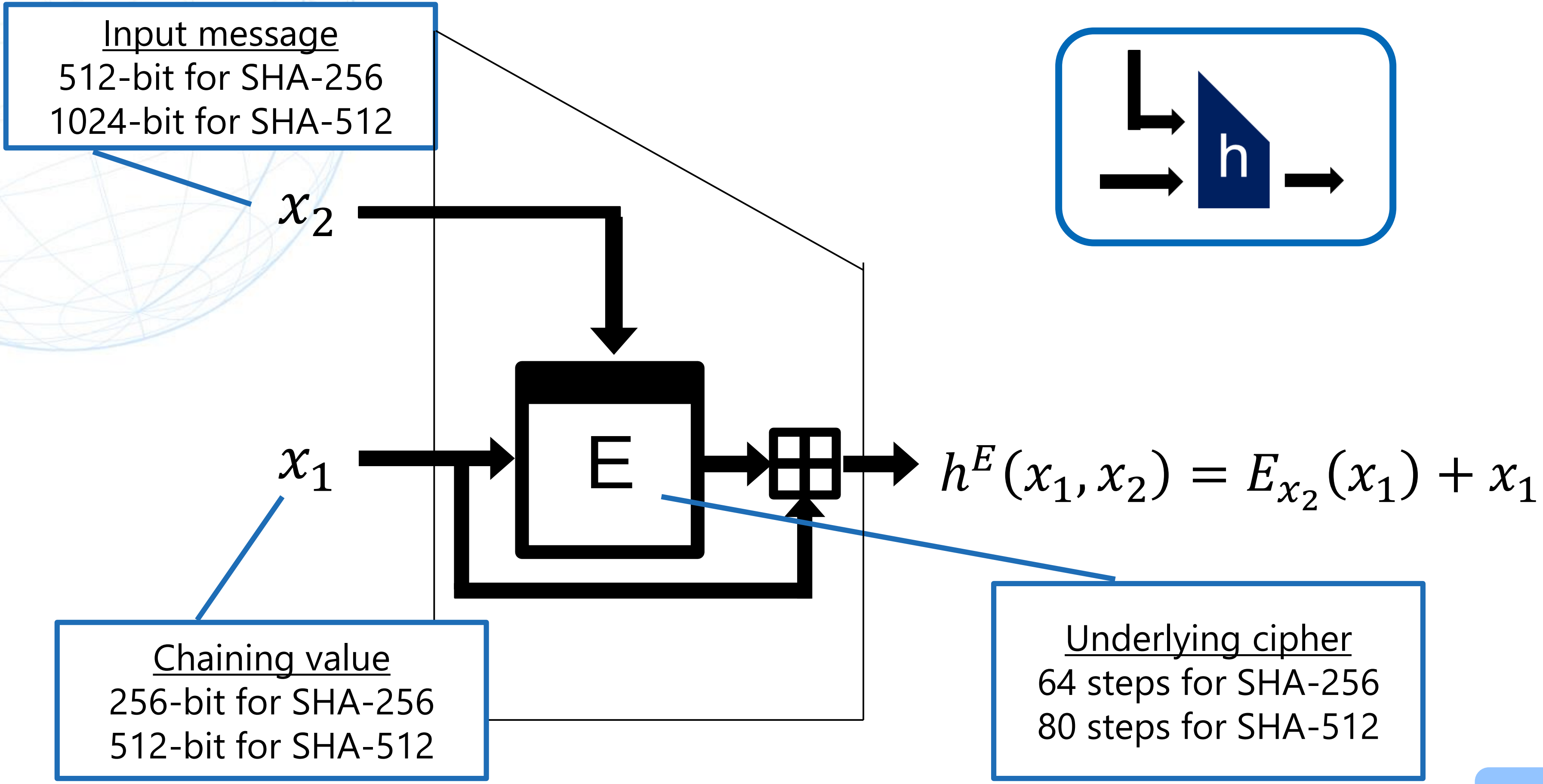
Compression Function



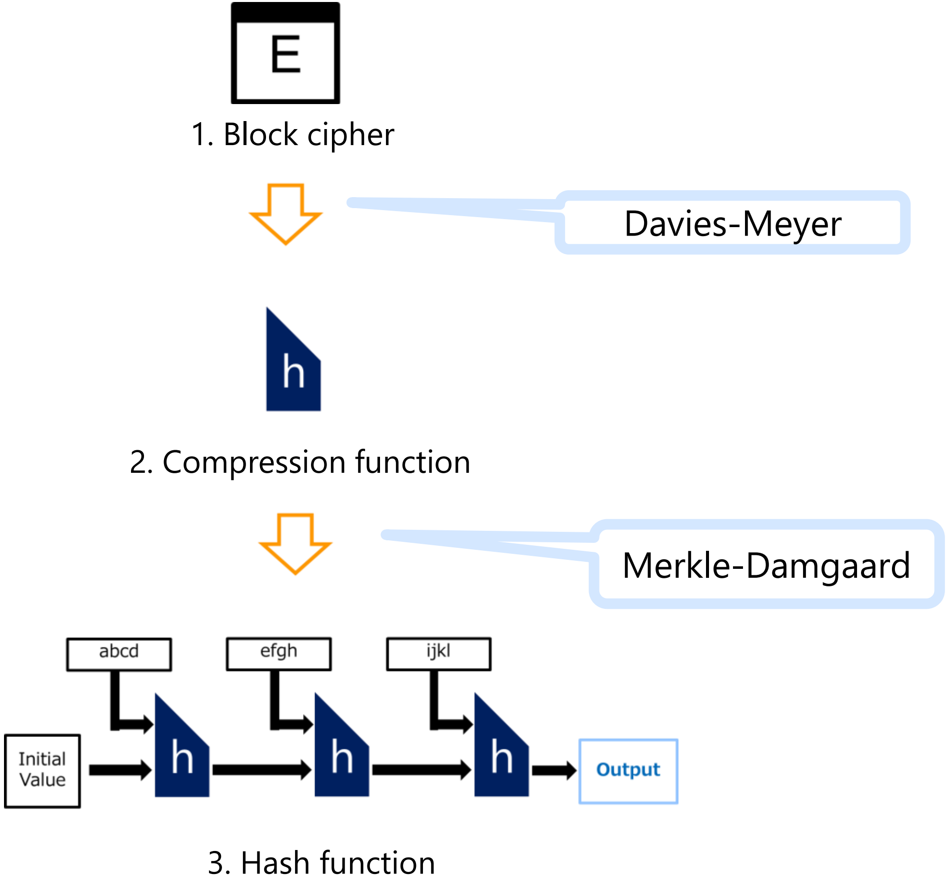
SHA-2

<u>Davies-Meyer Construction</u>,
Matyas–Meyer–Oseas (MMO) Construction,
Miyaguchi-Preneel (MP) Construction,
etc...

Davies-Meyer Construction



Construction of SHA-2: Summary



Semi-Free-Start Collision

Collision of a Hash Function

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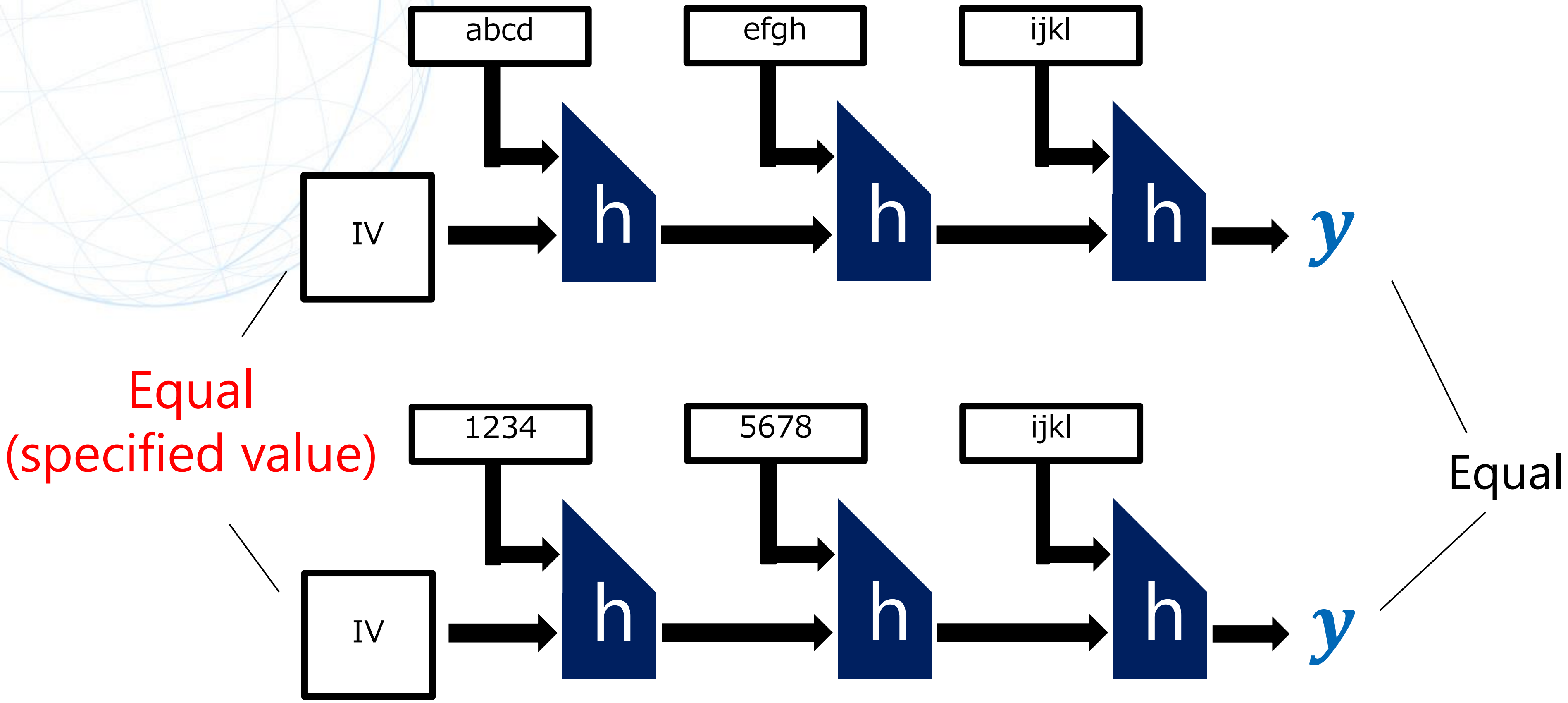
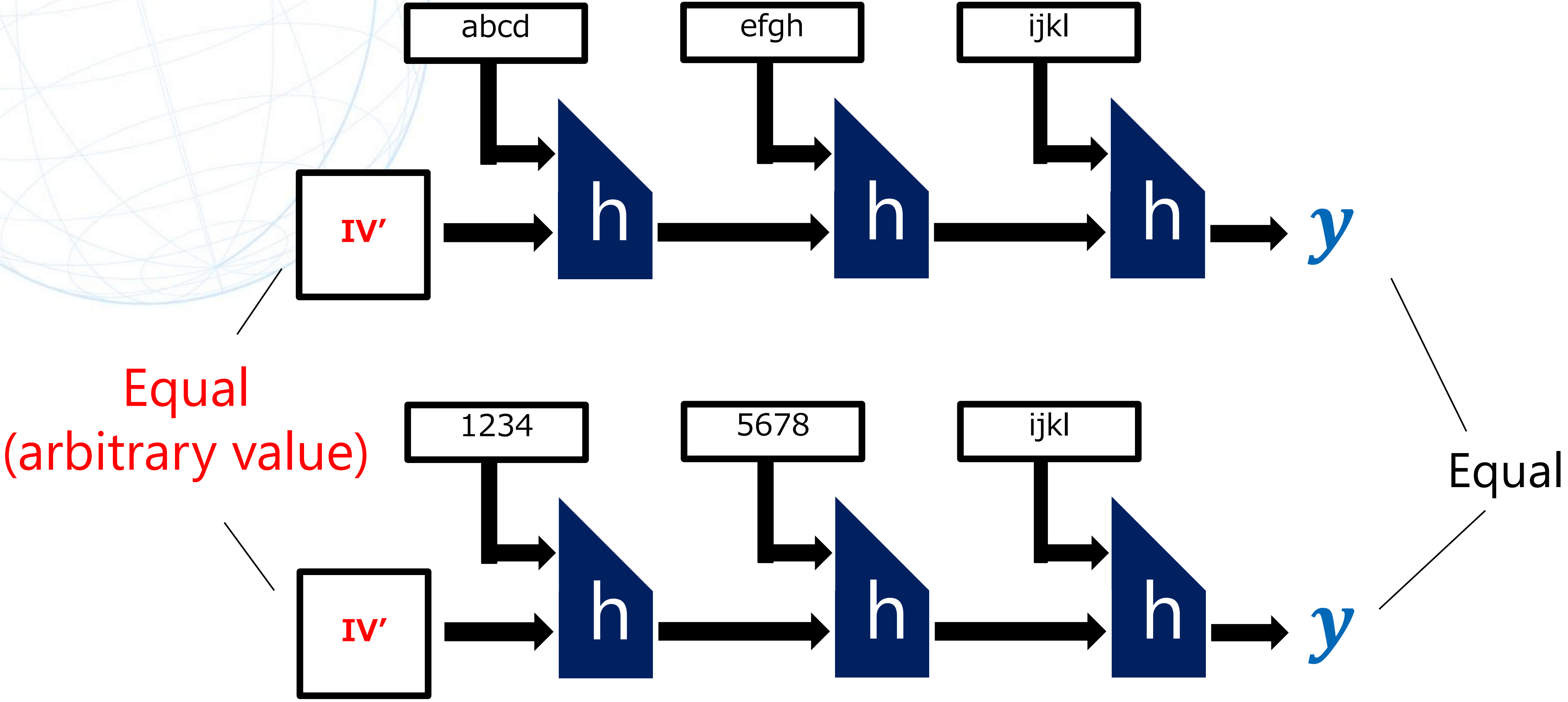


Semi-Free-Start Collision of a Hash Function



Collision and Semi-Free-Start Collision

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- Collision

  IVs are equal to the specified value

- Semi-Free-Start Collision

  IVs are the same but not equal to the specified value

Previous Work on SHA-256

Previous Classical Work on SHA-256

- Mendel et al. showed
  - 31-step collision attack on SHA-256
  - 38-step semi-free-start collision attack on SHA-256

- The attacks are based on differential cryptanalysis
  - Differential characteristic, (some parts of) conforming message pairs / internal states are searched simultaneously with automated tools
  - Characteristic is very complicated

[MNS13] Florian Mendel and Tomislav Nad and Martin Schläffer: Improving Local Collisions: New Attacks on Reduced SHA-256 (Eurocrypt 2013)

The 31-step characteristic by Mendel et al.

i	ΔA_i	ΔE_i	ΔW_i
-4	---- ---- ---- ---- ---- ---- ---- ----	---- ---- ---- ---- ---- ---- ---- ----	
-3	---- ---- ---- ---- ---- ---- ---- ----	---- ---- ---- ---- ---- ---- ---- ----	
-2	---- ---- ---- ---- ---- ---- ---- ----	---- ---- ---- ---- ---- ---- ---- ----	
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6	unnn n--- ---- ---- ---- ---- ---- --0-	n-n1 0111 n--u 11u0 0n10 u1n- nn1n -1uu	nn1- n--- nu-n n--1 u--0 -un0 --n0 -nn-
7	---- ---- ---- ---- ---n ---- ---- n-0u	101u 0nn1 0-11 011u -n11 1n11 0un1 -nnn	00nn 0n10 1-n1 nnn1 u0nn -n01 1u-1 n0--
8	---- ---- ---- ---- ---- ---- ---- ----	1-uu 1111 0--0 u101 10n- 1010 1010 -0n0	0001 u000 1-00 0nuu un1n 01nn -01n uuuu
9	---- ---- ---- ---- ---- ---- ---- ----	1011 00uu 1111 11nu 1110 01-- 0111 10nn	---- -1-- ---- ---u n--- 0--- --11 un--
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11	---- ---- ---- ---- ---- ---- ---- ----	0101 00u0 nu1u uuuu u100 1000 000n 1u10	---- ---- ---- ---- ---- ---- ---- ----
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13	---- ---- ---- ---- ---- ---- ---- ----	---1 01-1 1-1- ---- 1--- ---- ---0 -0--	---- ---- ---- ---- ---- ---- ---- ----
14	---- ---- ---- ---- ---- ---- ---- ----	---1 00-- -001 1111 u--- ---- 1--- -u--	---- ---- ---- ---- ---- ---- ---- ----
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The 31-step characteristic by Mendel et al.

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Conditions for Internal states

Conditions for Message words

i	ΔA_i								ΔE_i								ΔW_i							
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	nnn	n---	----	----	----	----	----	--0-	n-n1	0111	n--u	11u0	0n10	u1n-	nn1n	-1uu	nn1-	n---	nu-n	n--1	u--0	-un0	--n0	-nn-
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8	----	----	----	----	----	----	----	----	1-uu	1111	0--0	u101	10n-	1010	1010	-0n0	0001	u000	1-00	0nuu	un1n	01nn	-01n	uuuu
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Previous Classical Work on SHA-256

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  - 38-step semi-free-start collision attack on SHA-256
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- <u>The 31-step collision attack is mounted by converting 31-step semi-free-start collisions into a collision</u>

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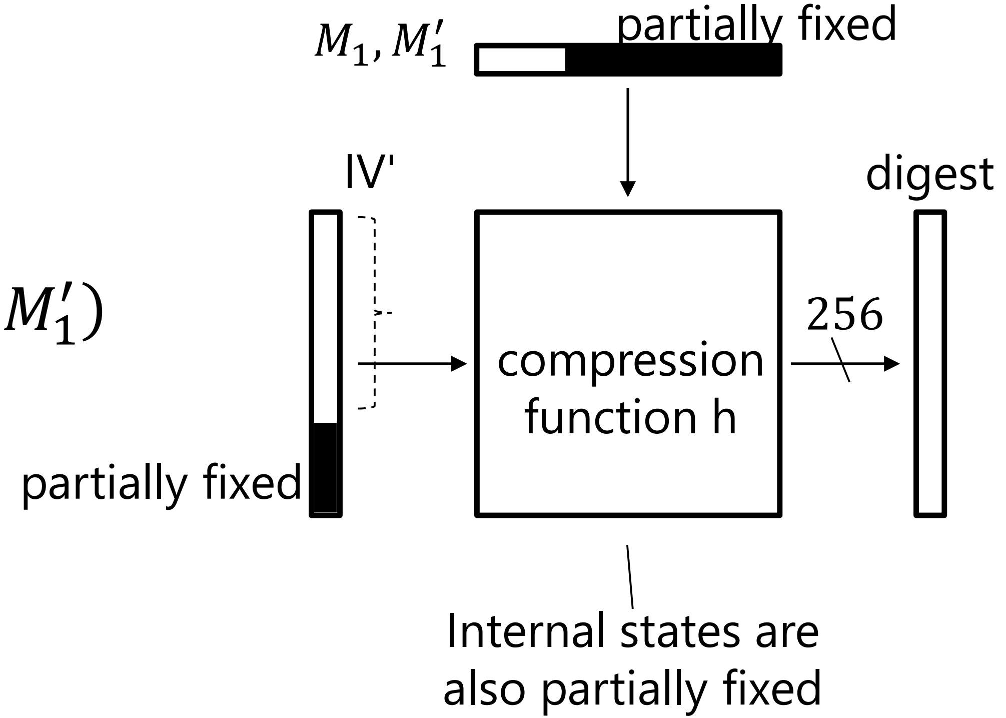
31-Step collision attack on SHA-256 by Mendel et al.

- We can make many <u>semi-free-start collisions</u> of the compression function from the differential characteristic

$$h(\mathrm{IV}', M_1) = h(\mathrm{IV}', M_1')$$



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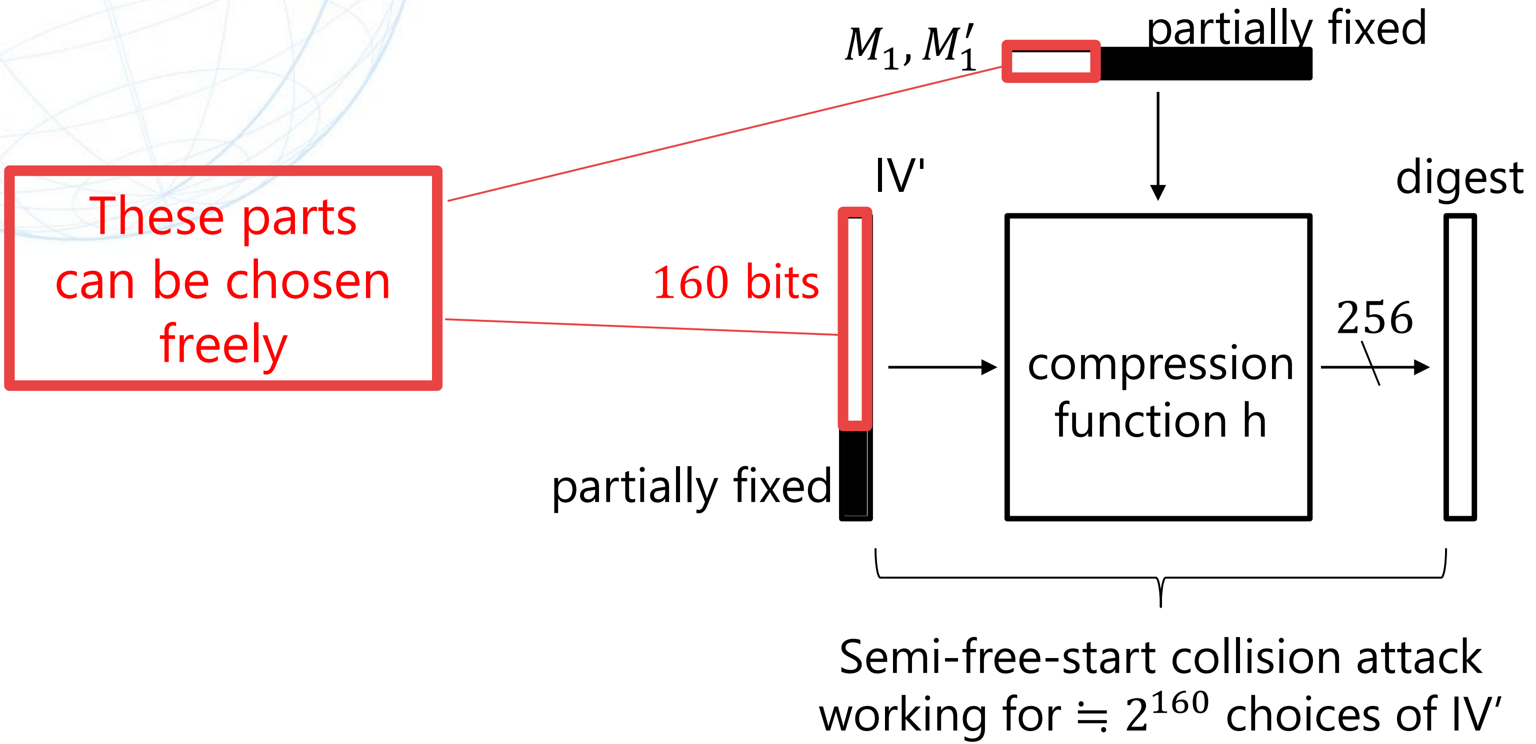
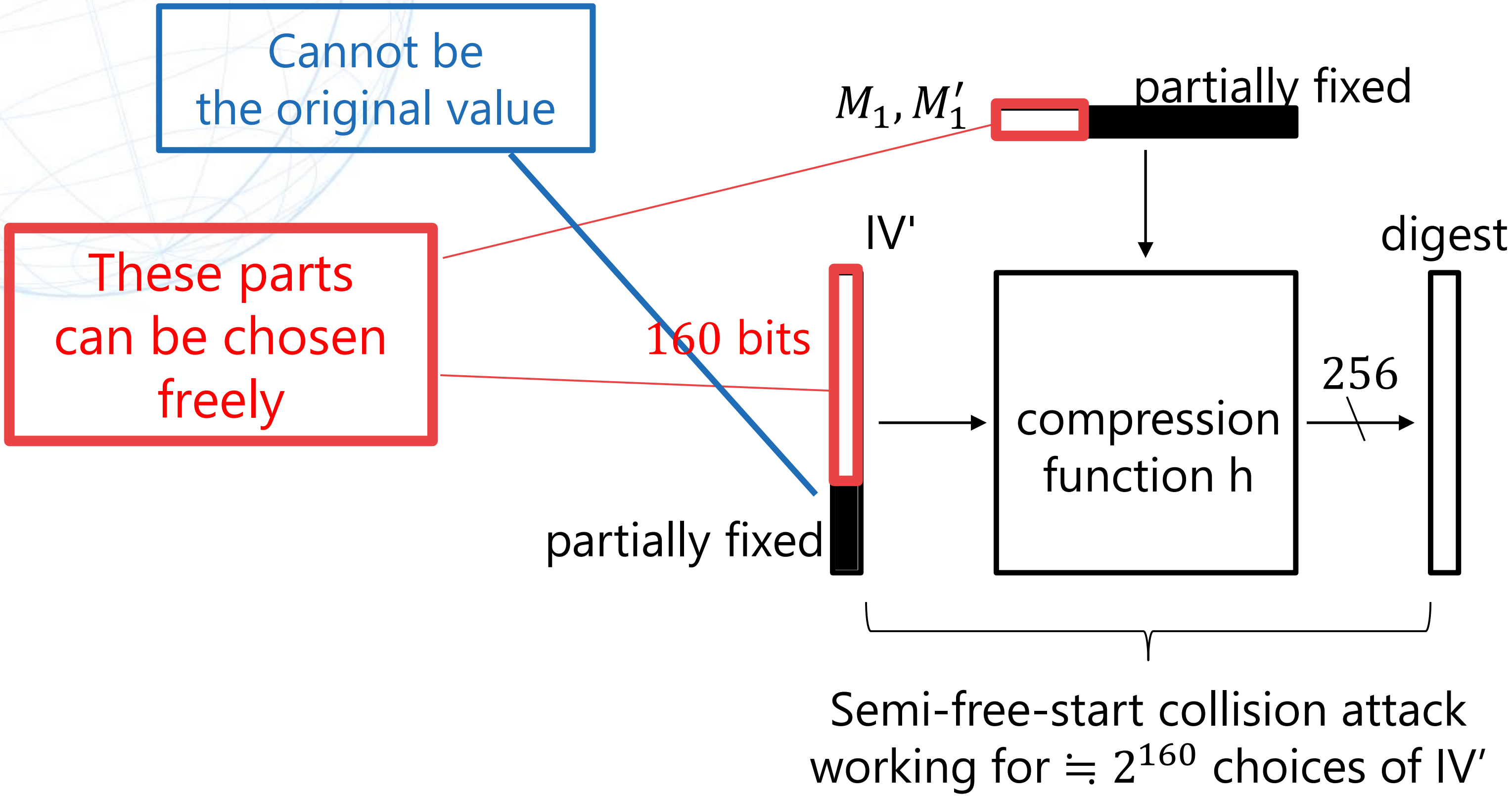


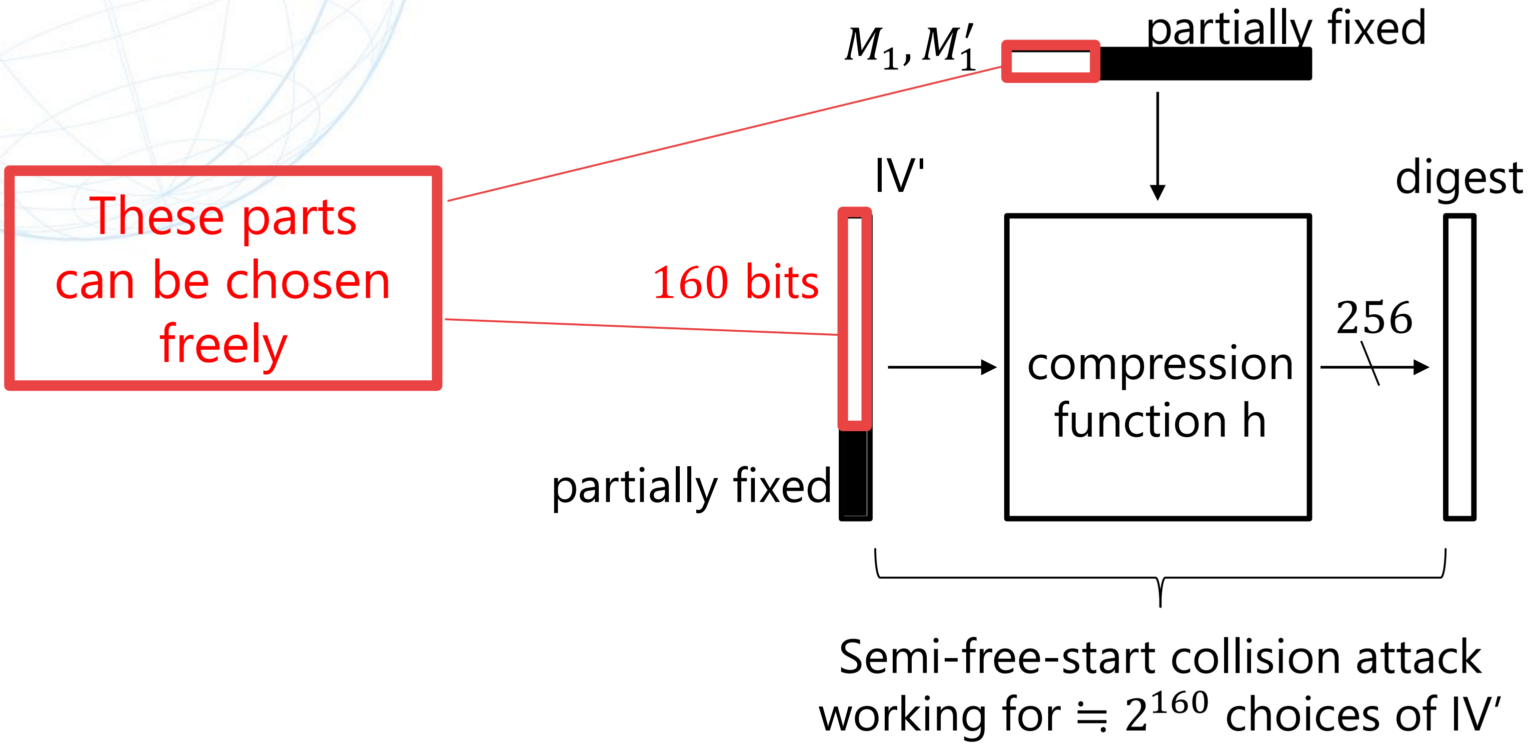
31-Step collision attack on SHA-256 by Mendel et al.

- However, IV’ is not equal to the original IV…



31-Step collision attack on SHA-256 by Mendel et al.

- Convert the semi-free-start collision into a 2-block collision by using the degrees of freedom



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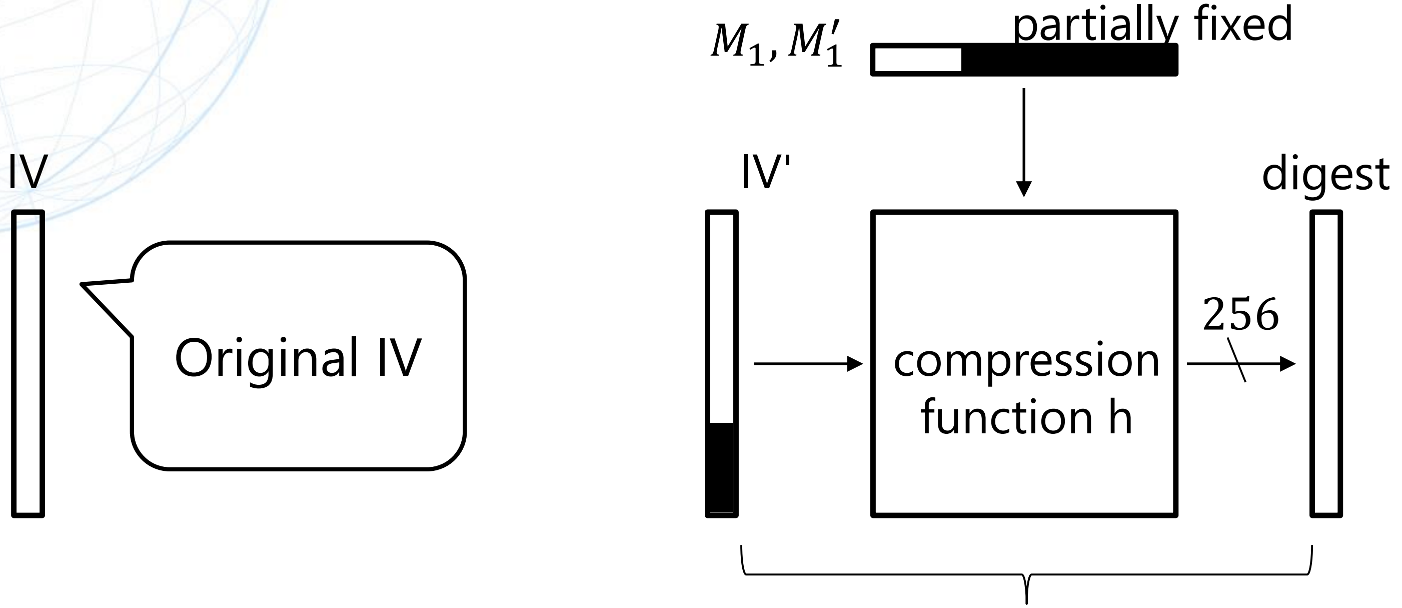
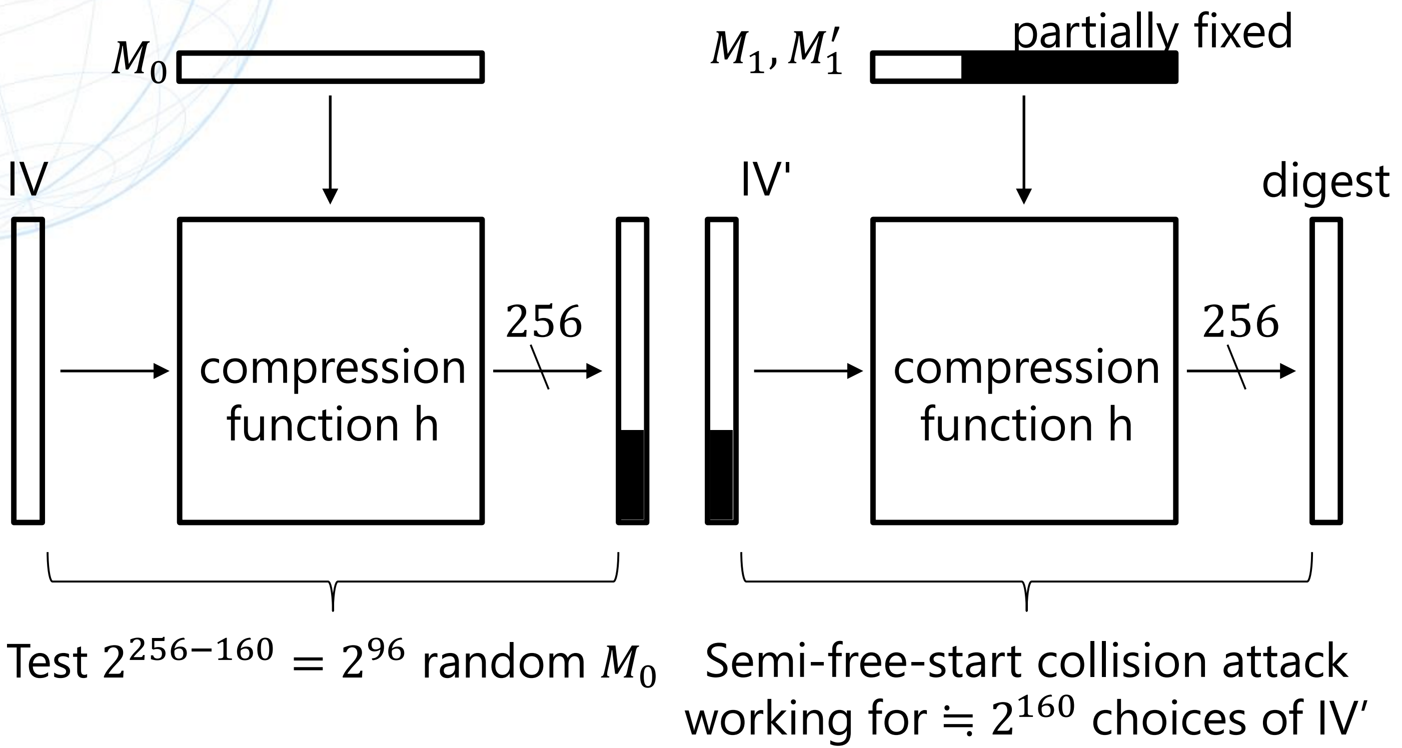


Semi-free-start collision attack working for ≒ 2^{160} choices of IV′

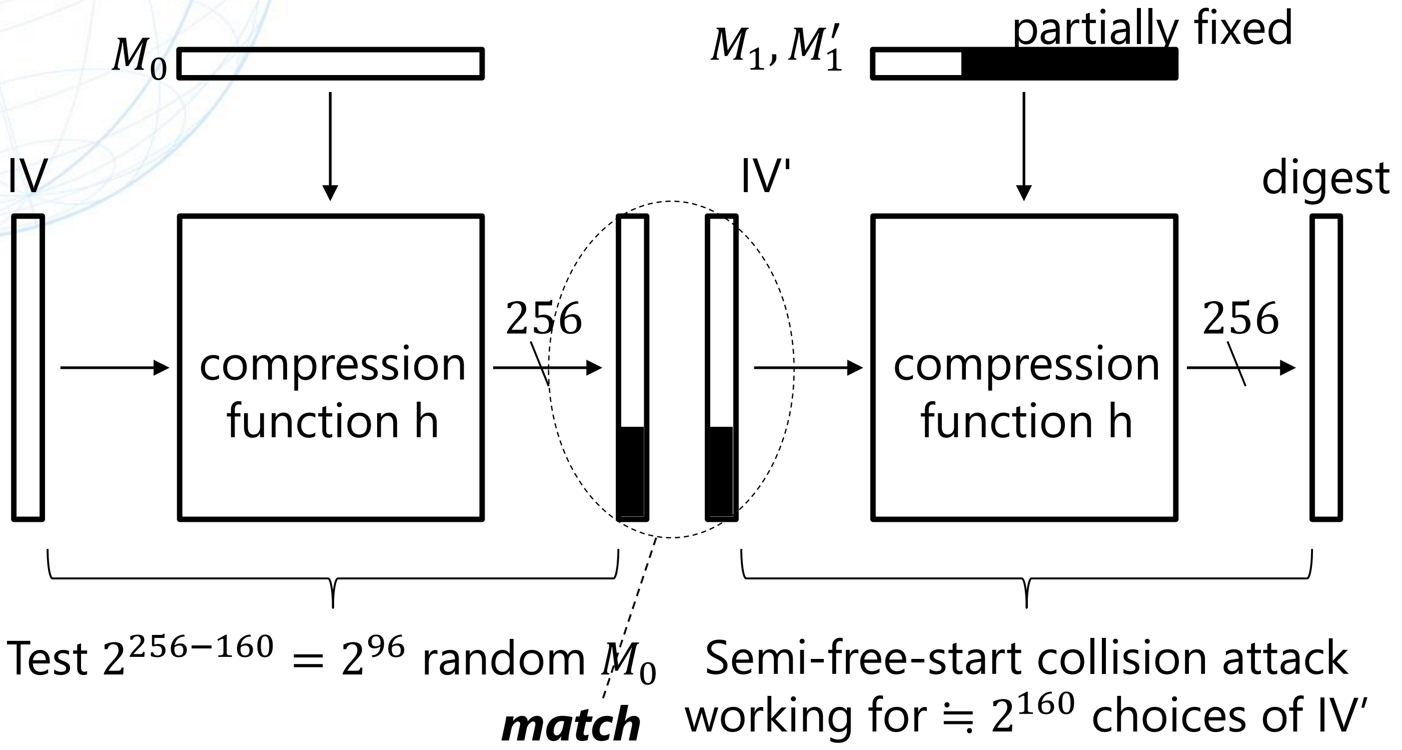
31-Step collision attack on SHA-256 by Mendel et al.

- When we test $2^{256-160} = 2^{96}$ random M_0,



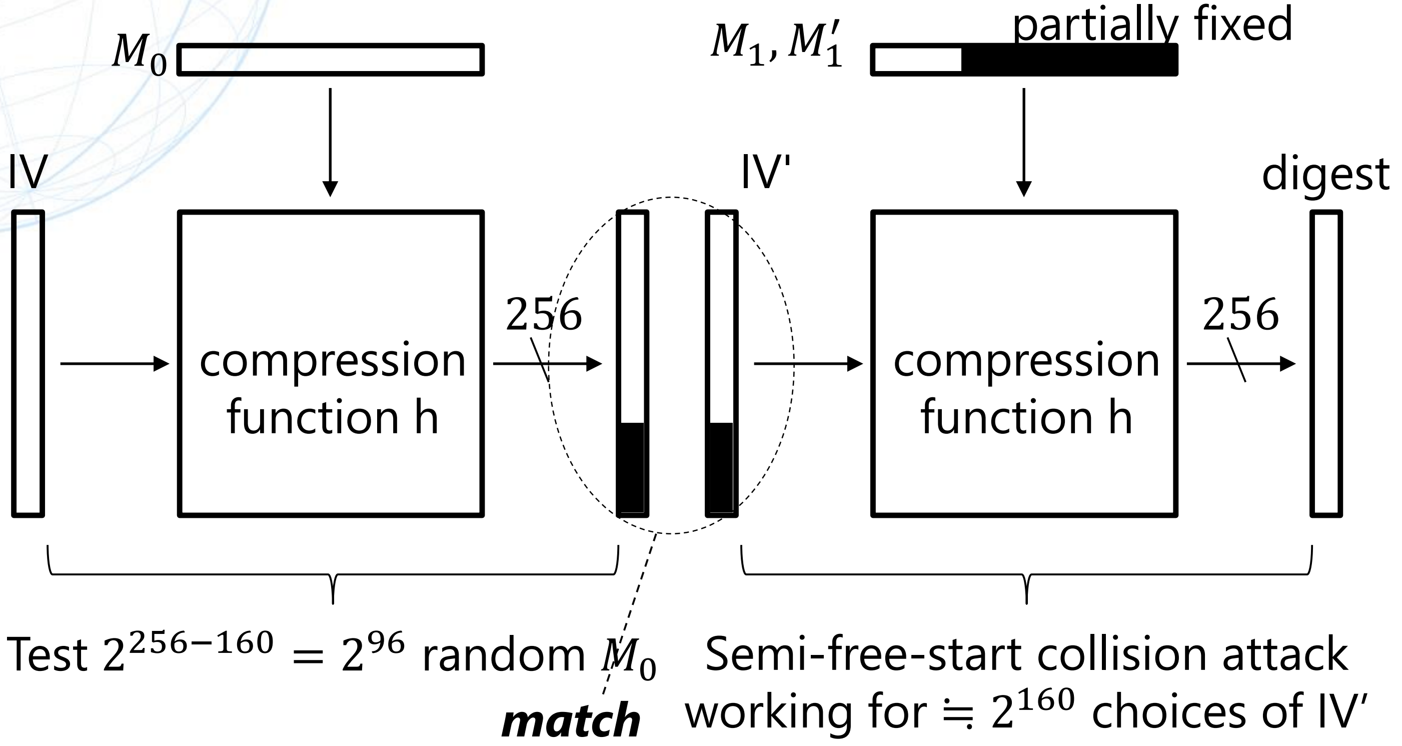
31-Step collision attack on SHA-256 by Mendel et al.

- When we test $2^{256-160} = 2^{96}$ random M_0, one of the outputs will match an IV′ of the second block (among 2^{160} choices of IV′)



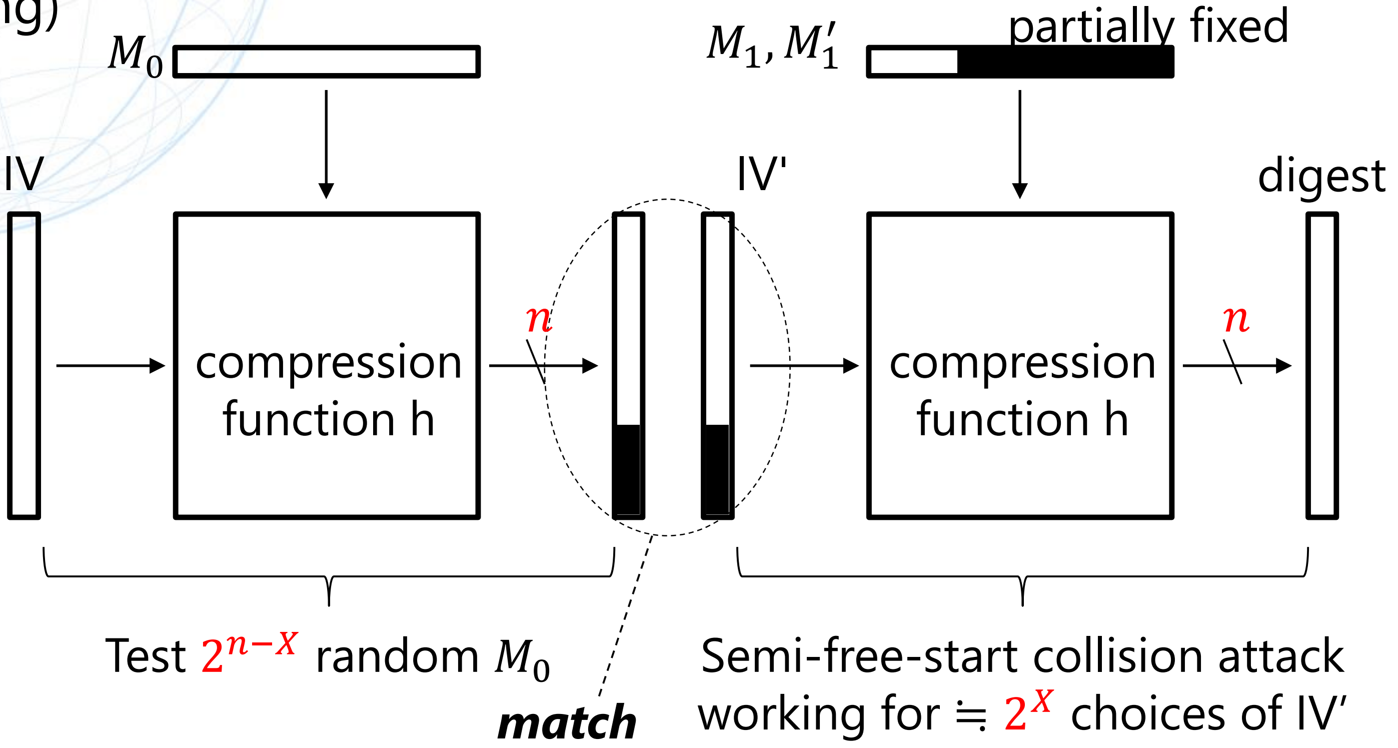
31-Step collision attack on SHA-256 by Mendel et al.

- We can find a 2-block collision in time $2^{96} < 2^{\frac{256}{2}} = 2^{128}$ (actually the attack is more complicated...)



Generalization of the 2-block collision attack

- If we can make many semi-free-start collisions for 2^X choices of IV's, then we can find a 2-block collision in time 2^{n-X} (in the classical setting)



Generalization of the 2-block collision attack

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- If we can make many semi-free-start collisions for 2^X choices of IV's, then we can find a 2-block collision in time 2^{n-X} (in the classical setting)

- The attack is valid only if $2^{n-X} < 2^{n/2}$, i.e., $\boldsymbol{X > n/2}$

- Mendel et al. showed not only the 31-step collision attack but also a 38-step semi-free-start collision attack in the same paper, but it is not converted into a collision attack

  →<u>The parameter X for the 38-step attack is not large enough</u>

Generalization of the 2-block collision attack

NTT

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Idea:

The validity condition may be relaxed in the quantum setting

Conversion of Semi-Free-Start Collisions into Collisions in the Quantum Setting

Generic Quantum Collision Attacks

NTT

Three settings depending on available computational resources

1. Small quantum computer + Large qRAM

Best algorithm: BHT ($T = 2^{n/3}$ & qRAM $2^{n/3}$) [BHT98]

2. Efficiency is measured by Time-Space tradeoff (No qRAM)

Quantum computer of size S + Classical computer of size S

Best algorithm: Parallel rho (Tradeoff $T = 2^{n/2}/S$) [Ber09]

3. Small quantum computer + Large classical memory (No qRAM)

Best algorithm: CNS ($T = 2^{2n/5}, 2^{n/5}$classical memory) [CNS17]

[BHT98] Gilles Brassard, Peter Høyer, Alain Tapp: Quantum Cryptanalysis of Hash and Claw-Free Functions. LATIN 1998

[Ber09] D. J. Bernstein: Cost analysis of hash collisions: Will quantum computers make SHARCS obsolete?. SHARCS 2009.

[CNS17] A. Chailloux, M. Naya-Plasencia, A. Schrottenloher: An efficient quantum collision search algorithm and implications on symmetric cryptography. Asiacrypt 2017.

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Our Focus

2. Efficiency is measured by Time-Space tradeoff (No qRAM)

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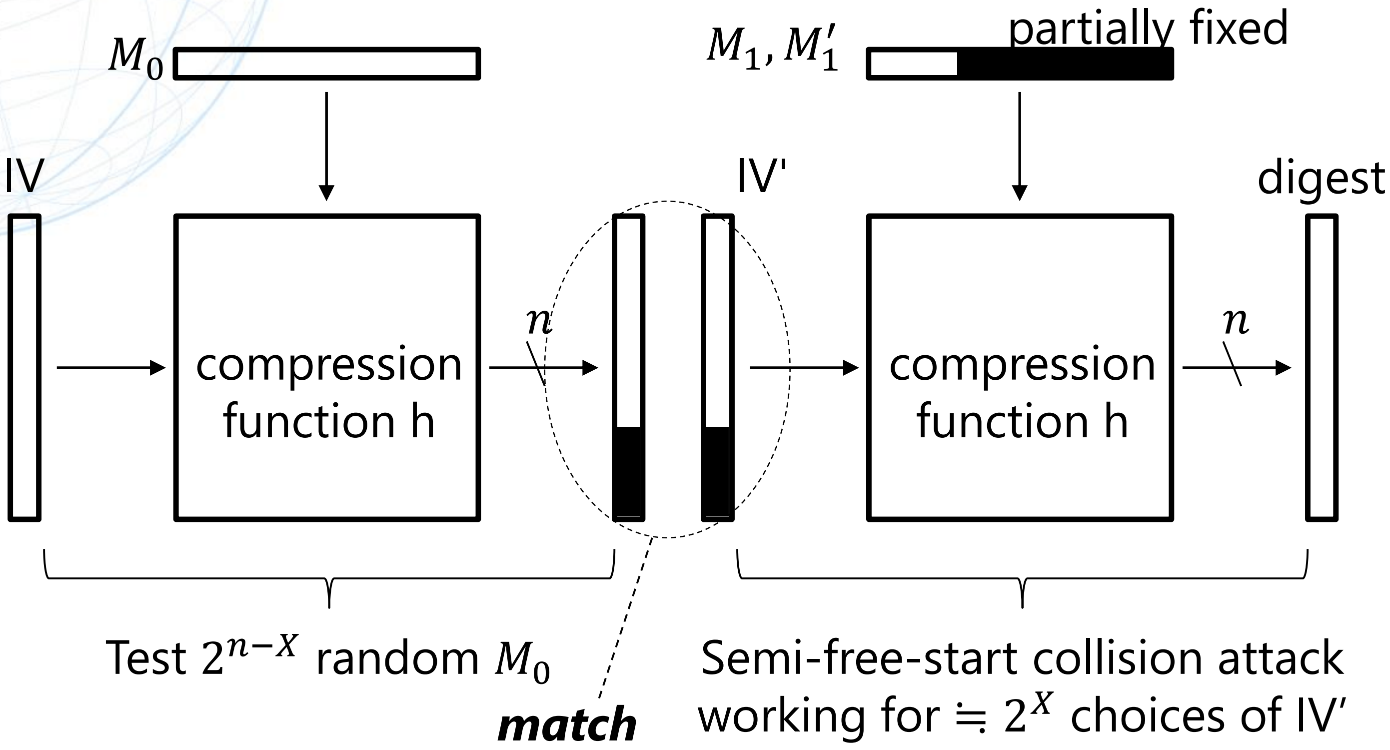
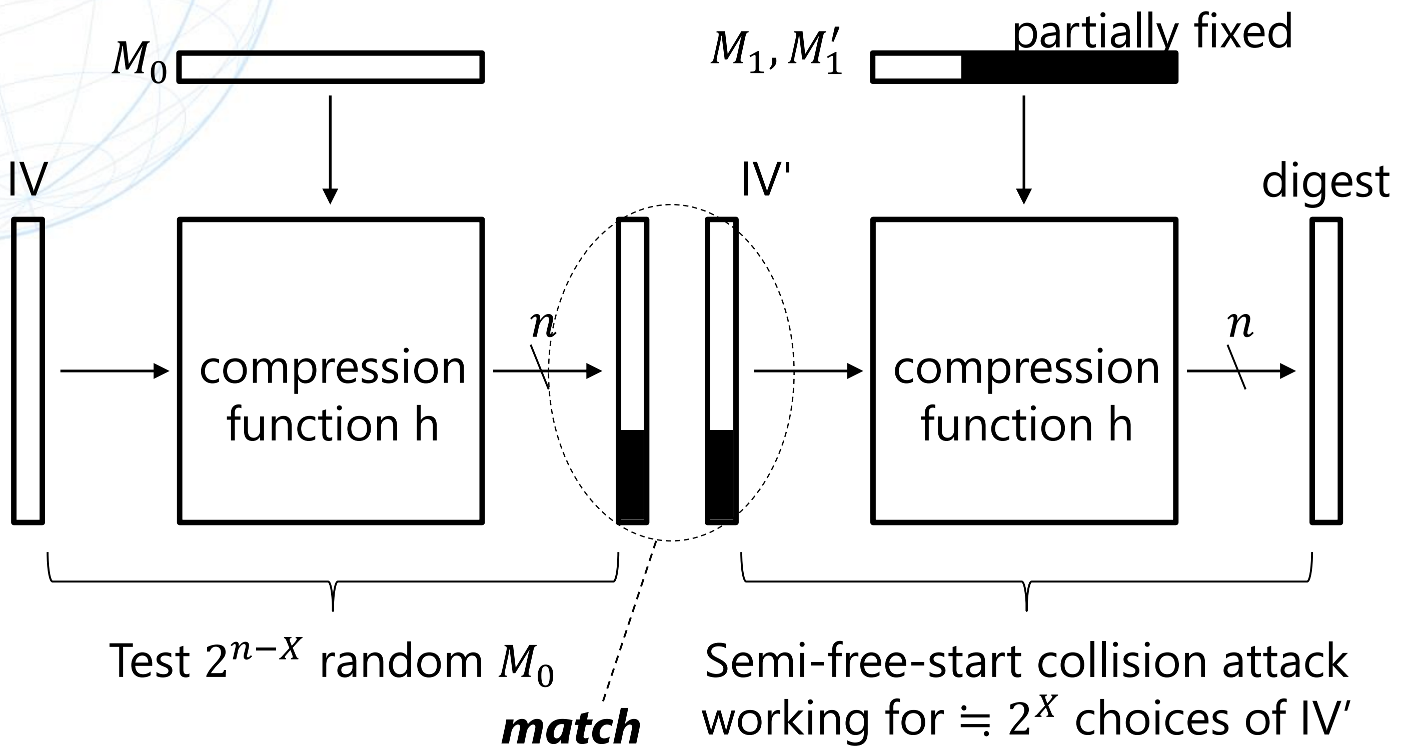
Classical 2-block collision attack

- If we can make many semi-free-start collisions for 2^X choices of IV's, then we can find a 2-block collision in **time 2^{n-X}**



Quantum 2-block collision attack

- If we can make many semi-free-start collisions for 2^X choices of IV's, then we can find a 2-block collision in **time $\sqrt{2^{n-X}}$** (Grover)



Quantum 2-block collision attack

- If S-qubits are available, the attack can be parallelized: $\boldsymbol{T = \sqrt{2^{n-X}/S}}$
- Generic attack… $\boldsymbol{T = \sqrt{2^n}/S}$
- The attack is valid if $\sqrt{2^{n-X}/S} < \sqrt{2^n}/S$, i.e., $\boldsymbol{X > 0}$ (for $S < 2^X$)
- Actually the condition for X will be stronger because here I'm ignoring many things: qubits required to implement Grover, time for sub-procedures, etc.
- Still, the new condition $\boldsymbol{X > 0}$ seems much weaker than $\boldsymbol{X > n/2}$

Main Results

Results on SHA-256 and SHA-512

NTT

- We convert the 38-step semi-free-start collision attack on SHA-256 by Mendel et al. [MNS13] and 39-step semi-free-start collision attack on SHA-512 by Dobraunig et al. [DEM15] into a 2-block collision.
- With some analysis and computer experiments, we confirmed that the attacks are valid in the quantum setting:

Attack Target	Time Complexity	(Generic Complexity)
38-step SHA-256	$2^{121}/\sqrt{S}$ $(2.4 < S < 2^{14})$	$2^{128}/S$
39-step SHA-512	$2^{252.2}/\sqrt{S}$ $(2.5 < S < 2^{7.6})$	$2^{256}/S$

Note: classical best collision attacks are 31-step for SHA-256 and 27-step for SHA-512

Remark: the attacks are invalid in other settings

[MNS13] Florian Mendel and Tomislav Nad and Martin Schläffer: Improving Local Collisions: New Attacks on Reduced SHA-256 (Eurocrypt 2013)
[DEM15] Christoph Dobraunig, Maria Eichlseder, Florian Mendel: Analysis of SHA-512/224 and SHA-512/256. (Asiacrypt 2015)

Summary & Future Directions

Summary & Future Directions

- First dedicated quantum collision attacks on SHA-2
  - 38-step attack on SHA-256 & 39-step attack on SHA-512
  - Classical collision attacks: 31-step for SHA-256 & 27-step for SHA-512
  - Still far from full-step attacks (64 steps / 80 steps)
- We convert classical semi-free-start collisions on 38-step SHA-256 & 39-step SHA-512 into collisions in the quantum setting
- There are many functions which is similar to SHA-2 (RIPEMD-128, RIPEMD-160, SM3, HAS-160, etc……), but so far we haven't found any quantum collision attacks on them: Existing characteristics are not suitable for our idea
- We should revisit differential characteristics search activities
  - Possibility of quantum attacks should be taken into account

Thank you!