DualRing: Generic Construction of Ring Signatures with Efficient Instantiations

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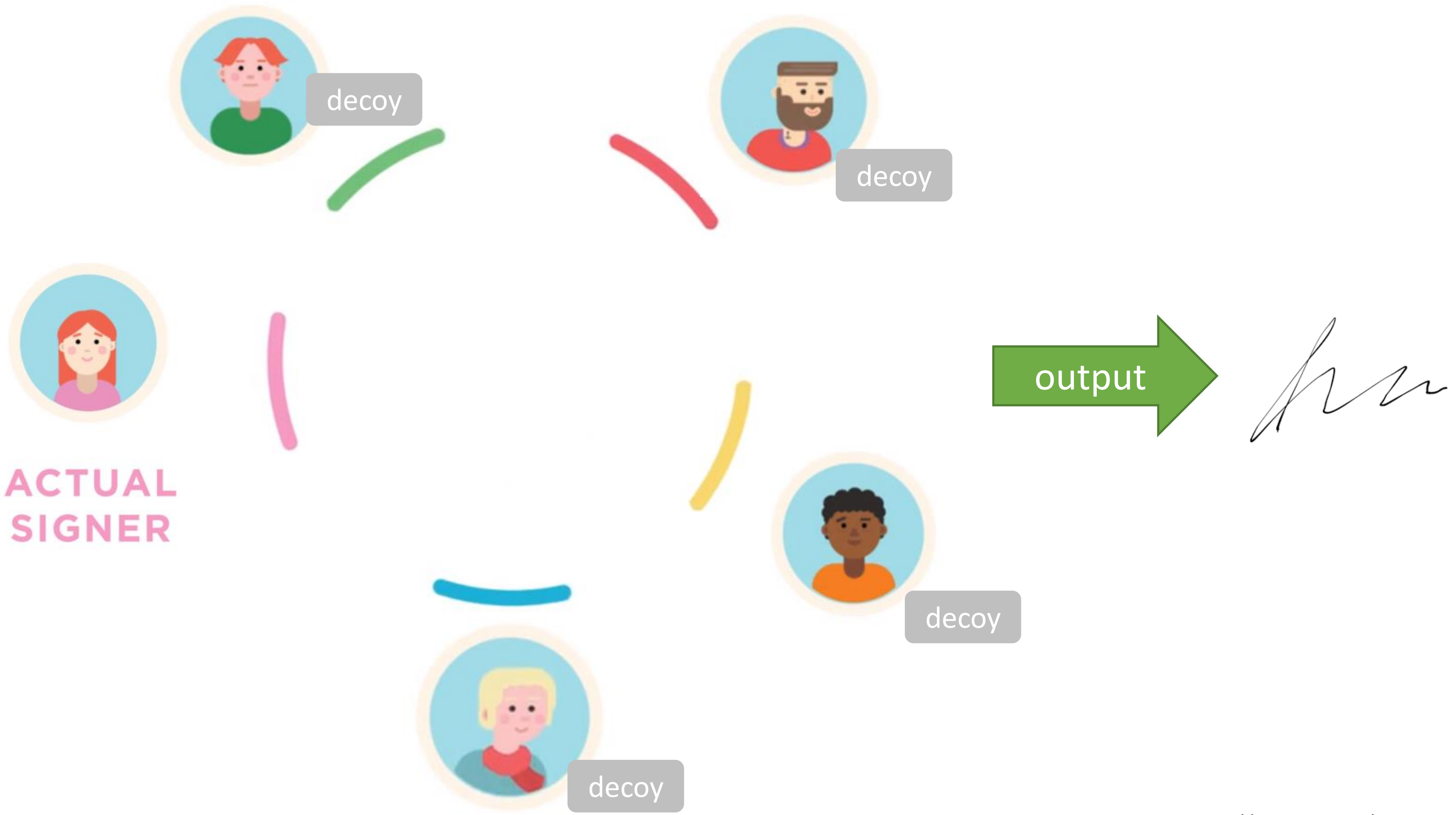
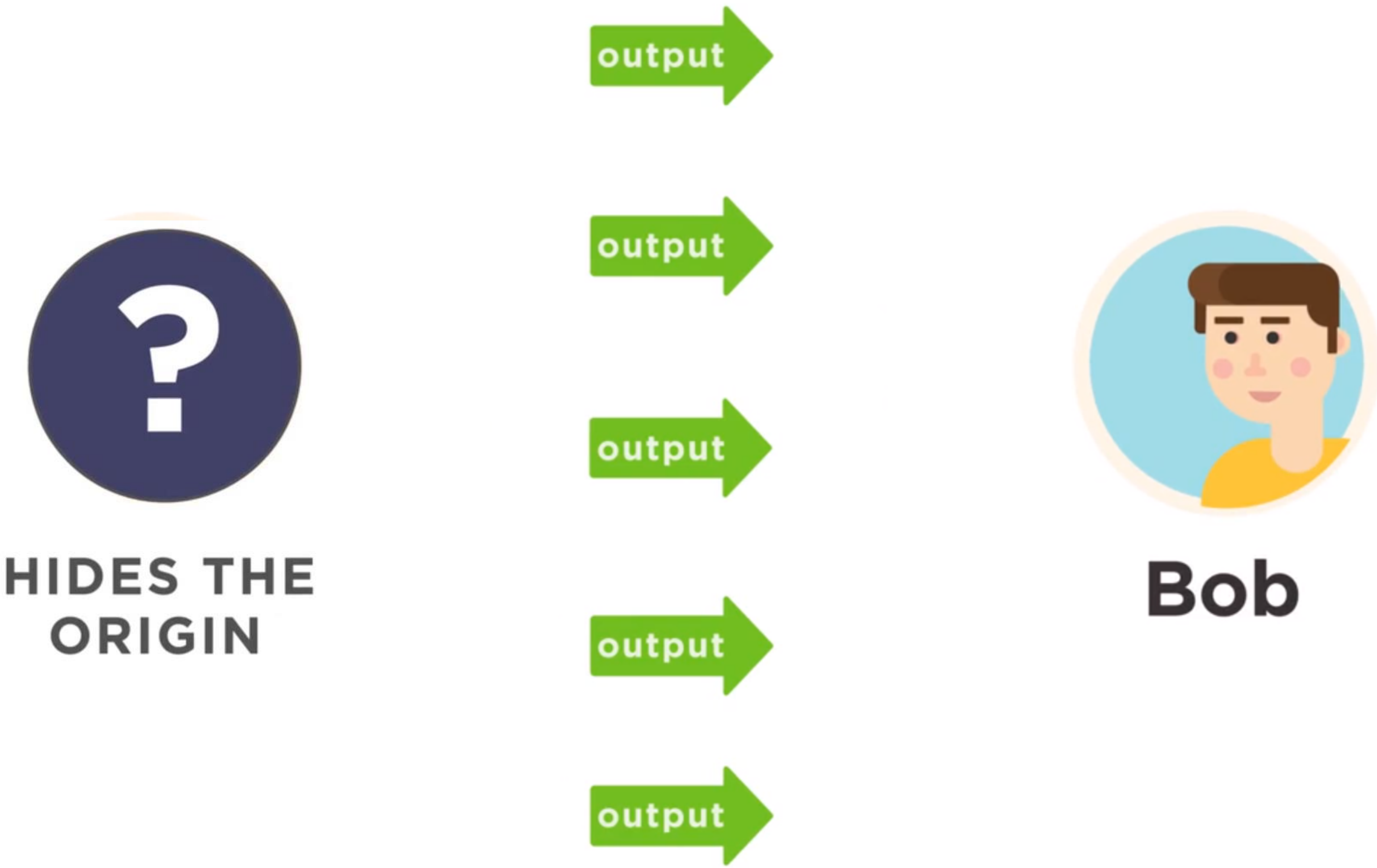


Source: Monero Brasil. https://vimeo.com/233677706

Introduction



Source: Monero Brasil. https://vimeo.com/233677706

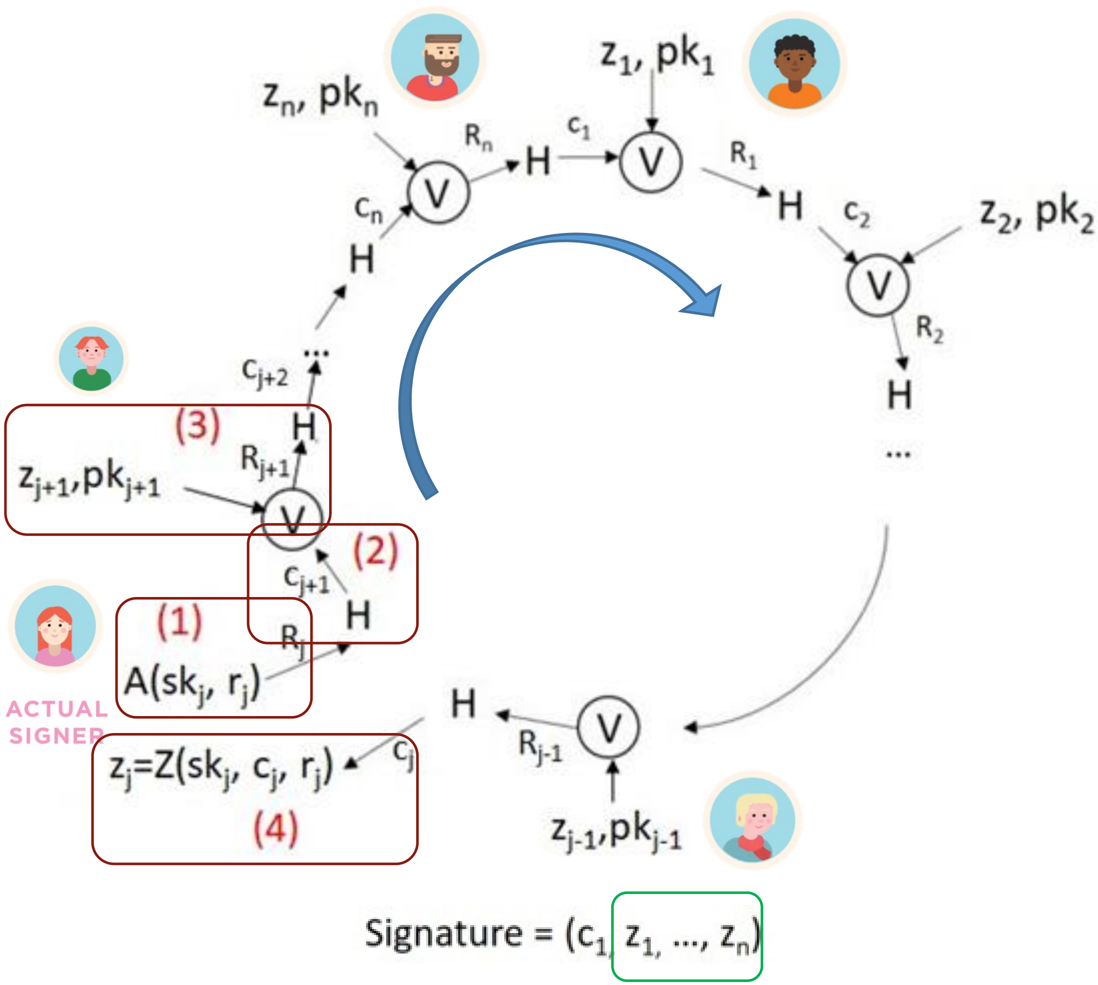
Classical Ring Signature

- [RST01], [AOS02]
- Recall: Type-T (Three-move) Signature (e.g. Schnorr)
  1. a commit function A, which outputs a commitment R
     - $A(r) \rightarrow R := g^r$
  2. a hash function H, which outputs a challenge c
     - $H(M, R) \rightarrow c$
  3. a response function Z, which outputs a response z
     - $Z(r, c, sk) \rightarrow z = r + c \cdot sk$
  4. a verification function V, which reconstruct R from (c,z) and then runs H to check if c is correct
     - $V(pk, c, z) \rightarrow R = g^z \cdot pk^{-c},\ c = H(M, R)$

[RST01] Rivest, R.L., Shamir, A., Tauman, Y.: How to leak a secret. In: ASIACRYPT 2001.
[AOS02] Abe, M., Ohkubo, M., Suzuki, K.: 1-out-of-n signatures from a variety of keys. In: ASIACRYPT 2002.

Classical Ring Signature

- [AOS02]



1) Signer picks r_j to generate R_j via the commit function A
2) Signer uses R_j to compute the (j+1)-th challenge c_{j+1} by the hash function H
3) Pick a random response z_{j+1} and the *j+1*-th user pk_{j+1}, signer reconstruct the R_{j+1} using the verify function V and then generate c_{j+2} by H.
   - A ring is then formed sequentially.
4) The last step is to compute z_j from sk_j, c_j, r_j using the response function Z

Current Ring Signature Schemes

Accumulator-Based

- √ Constant signature size
- X Trusted setup: RSA/pairing-based
- X Large signature size: lattice-based (~ several MBs)

ZK Proof-Based

- √ O(log n) size
- One-out-of-many proof/Bulletproof: DL-based [GK15], lattice-based [ESLL19]

[GK15] Groth, J., Kohlweiss, M.: One-out-of-many proofs: Or how to leak a secret and spend a coin. In: EUROCRYPT 2015.
[ESLL19] Esgin, M.F., Steinfeld, R., Liu, J.K., Liu, D.: Lattice-based zero-knowledge proofs: New techniques for shorter and faster constructions and applications. In: CRYPTO 2019.

Can we do better?

- DL-based:

Ring Signatures	# elements in signature		Signature Size (Bytes)				
	$\mathbb{G}$	$\mathbb{Z}_p$	$n=2$	$n=8$	$n=64$	$n=2048$	$n=4096$
LPQ@ESORICS18	$4\log n + 2$	$5\log n + 4$	480	1070	1946	3114	3406
GK@Eurocrypt15	$4\log n$	$3\log n + 1$	260	716	1400	2540	2768
BCC+@ESORICS15	$\log n + 12$	$\frac{3}{2}\log n + 6$	669	831	1074	1479	1560
YSLAEZ@FC20	$2\log n + 7$	7	521	653	851	1181	1247
LRR+@CCS19	$2\log(n+2) + 4$	5	424	523	721	1051	1117
DualRing-EC	$2\log n + 1$	3	195	327	525	855	921

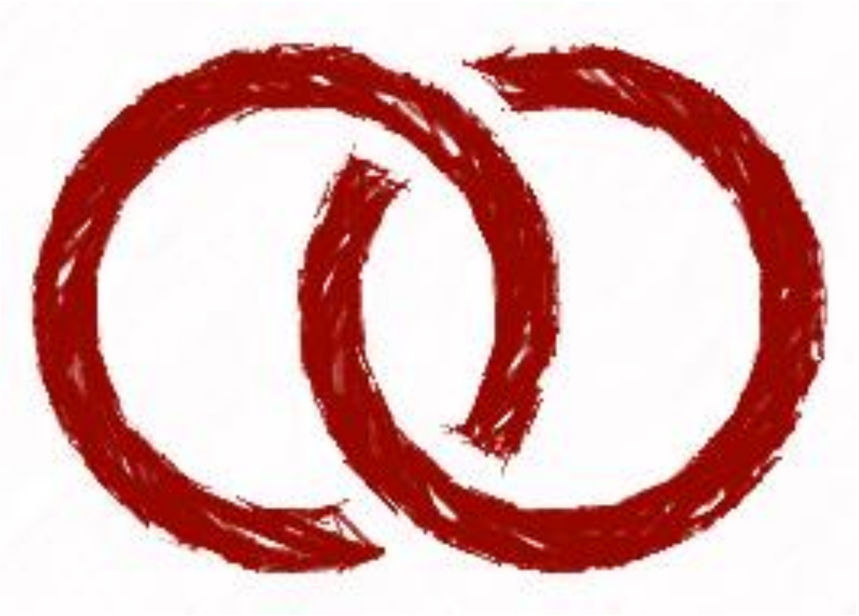
- Lattice-based:

Ring Signatures	Signature Size (Bytes)						Assumption
	$n=2$	$n=8$	$n=64$	$n=1024$	$n=2048$	$n=4096$	
LAZ@ACNS19	2532	10128	81024	1296384	2592768	6564888576	NTRU
BKP@Asiacrypt20	49000	50000	52000	54000	54500	55000	M-LWE+M-SIS
EZSLL@CCS19	18000	19000	31000	48000	53000	59000	M-LWE+M-SIS
DualRing-LB	4480	4630	6020	31160	55500	106570	M-LWE+M-SIS

DualRing: Generic construction from the classical ring approach!

DualRing

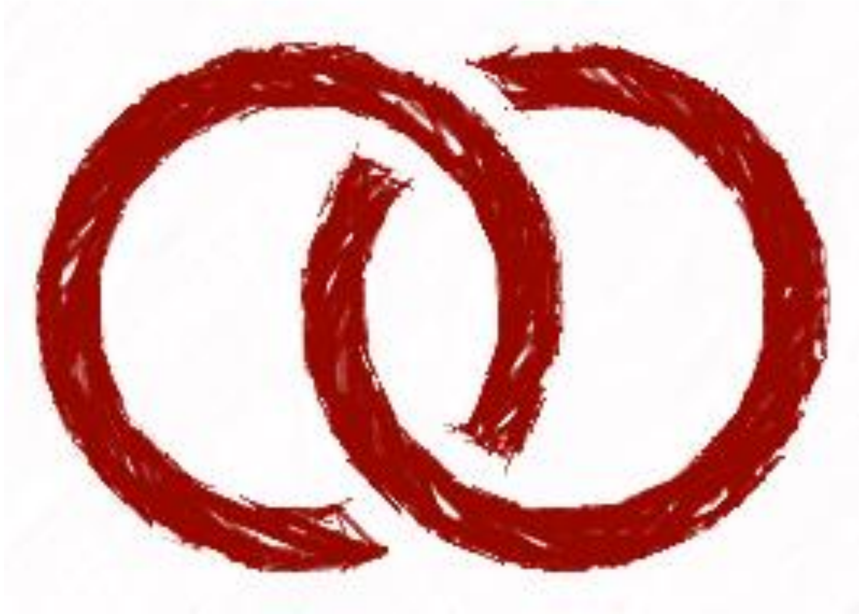
High level idea



Overview of DualRing

- Revisit the classical ring approach and make it shorter (i.e., O(log n) size)
  - [AOS02] includes the hash function H in the ring structure
    → difficult to shorten the signature
- Goal: formation of rings with simple algebraic operation
  - Ring structure provides anonymity as [RST01], [AOS02]
  - Simple algebraic operations (i.e., without H) allow compression

Ring of
commitments



Ring of
challenges

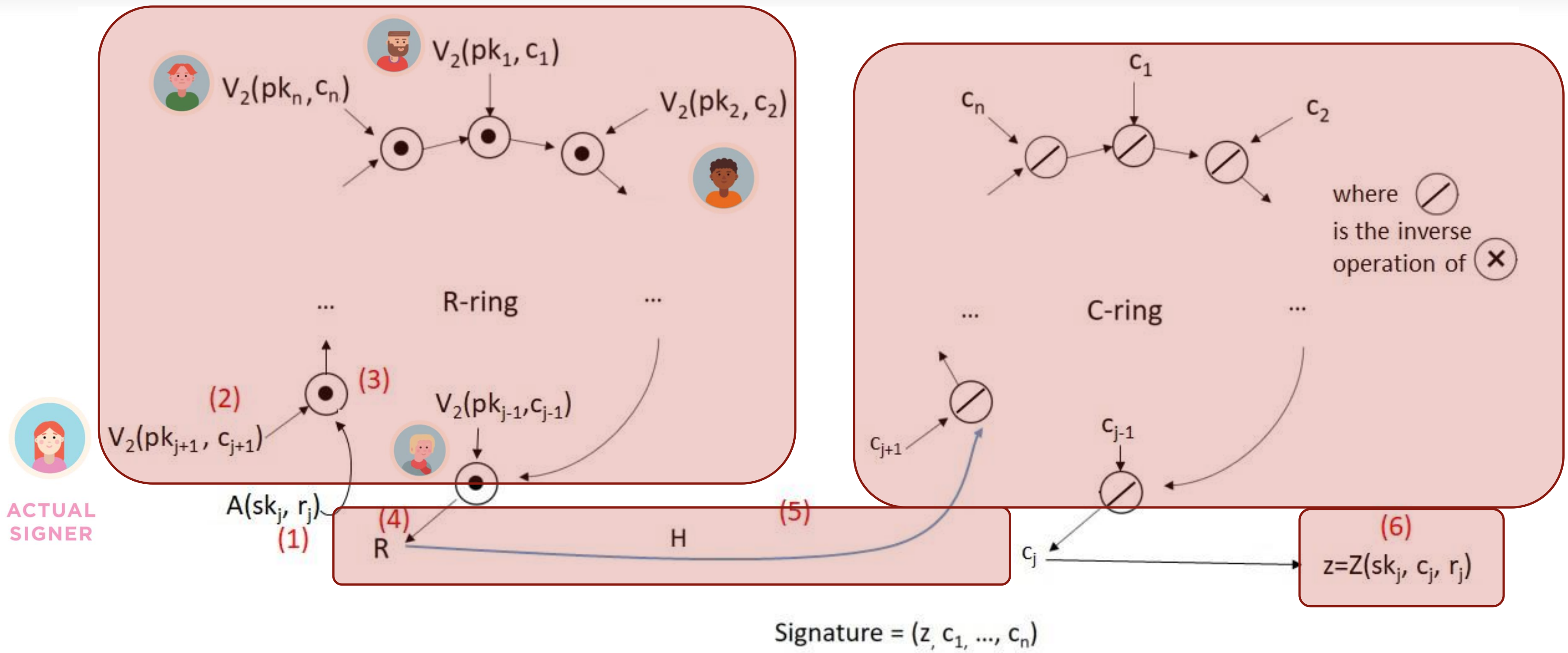
Building Block

- Type-T* canonical identification (Three-move)
  - The verify function V can be written as:
    $V(pk, c, z) = V_1(z) \odot V_2(pk, c)$ $\qquad V(pk, c, z) \rightarrow R = g^z \cdot pk^c$
  - V_1 is additive/multiplicative homomorphic
  - Given sk and c, there exists a function $T(sk, c) \rightarrow z'$ such that $\mathrm{V}_1(\mathrm{z}') = \mathrm{V}_2(pk, c)$
  - The challenge space is a group
  - Has special soundness*

> - Schnorr identification, identification scheme from Katz-Wong signature, Chaum-Pedersen identification and Okamoto-Schnorr identification
> - GQ identification
> - Lattice-based identification from "Fiat-Shamir with Aborts"

* In the paper, we use a new notion called *special impersonation* to deal with the "knowledge gap" in the lattice-based setting

Construction

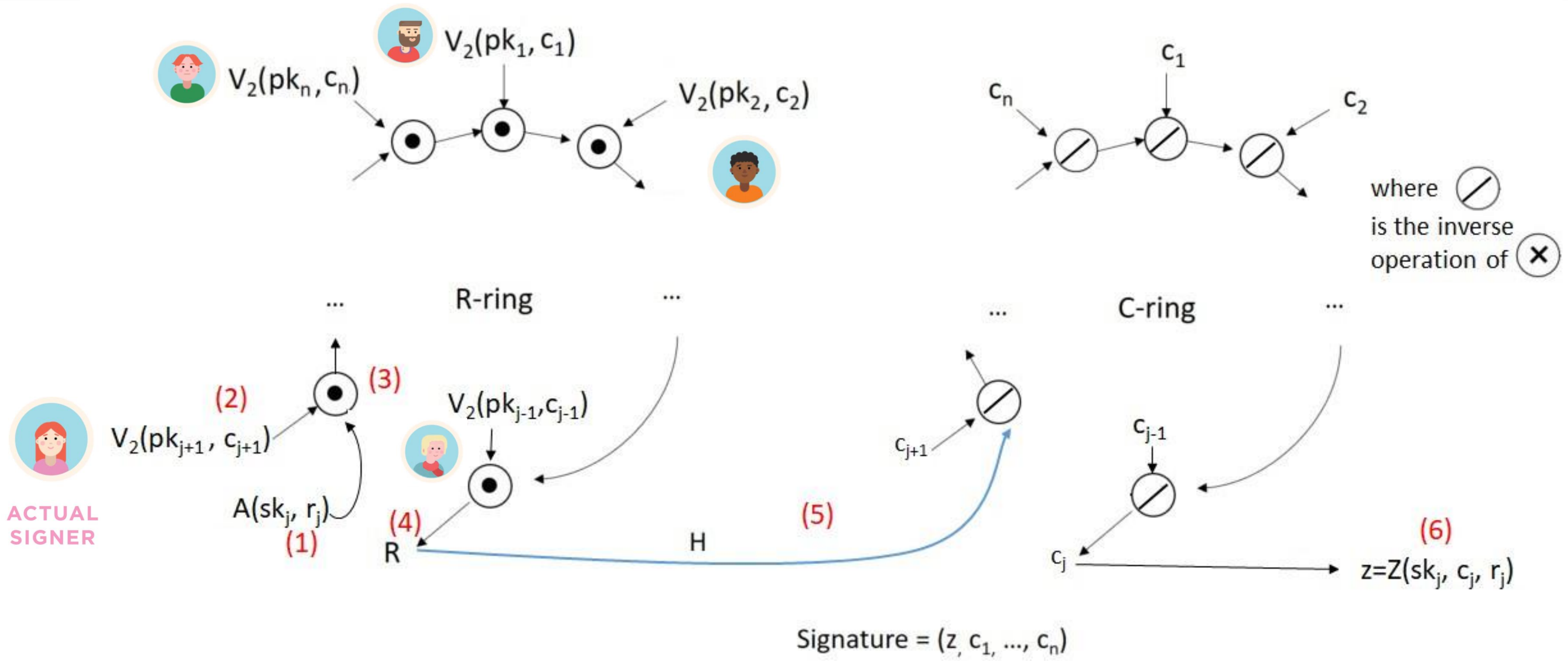


1) Signer picks r_j
2) Signer picks random challenges $c_1, \dots, c_{j-1}, c_{j+1}, \dots, c_n$
3) Form an R-ring by:

R= $A(r_j) \odot V_2(pk_{j+1}, c_{j+1}) \dots \odot V_2(pk_n, c_n) \odot V_2(pk_1, c_1) \dots \odot V_2(pk_{j-1}, c_{j-1})$

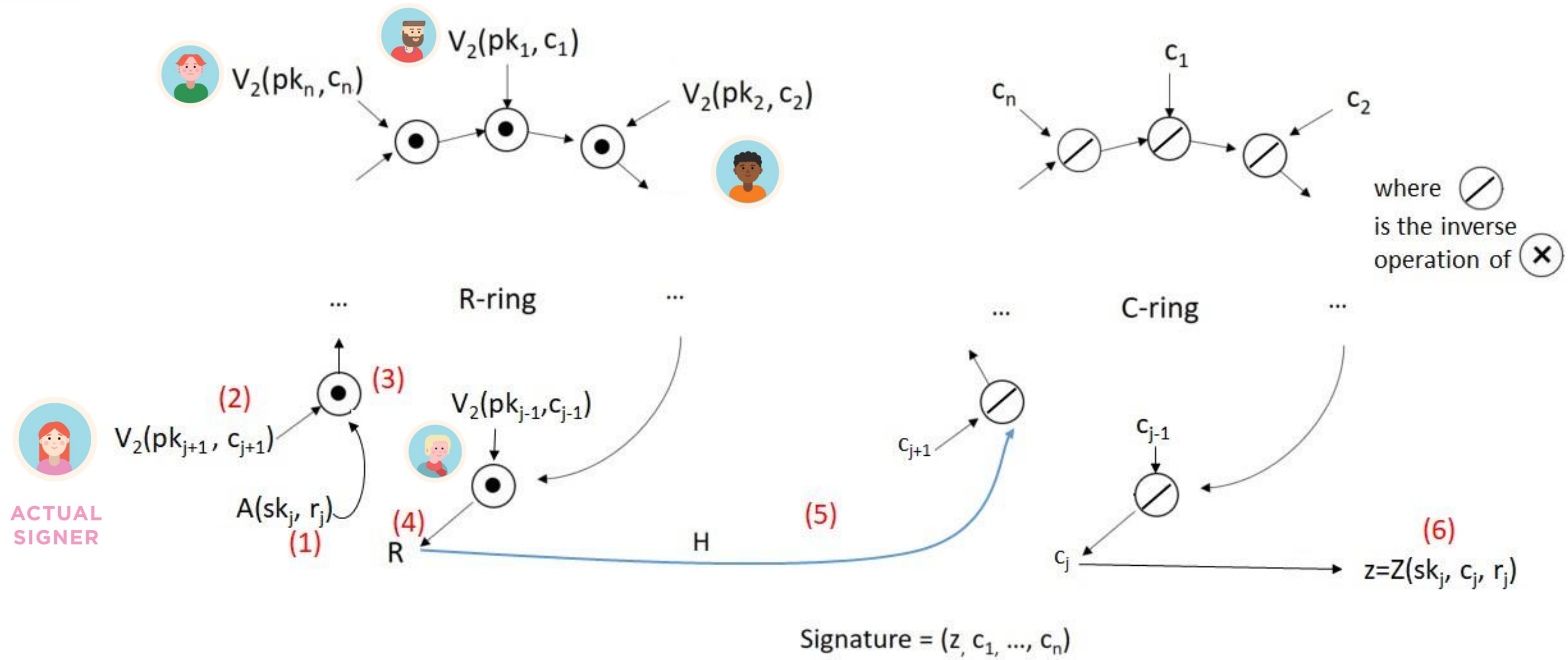
4) Use R to compute c = H(R, {pk}, m)
5) Form C-ring, by computing c_j where:
$$c = c_1 \otimes \cdots \otimes c_n$$
6) Compute $z = Z(sk_j, c_j, r_j)$

Construction



- Verification:
  - $R = V_1(z) \odot \bigodot_{i=1}^{n} V_2(pk_i, c_i)$
  - Check if $c = H(R, \{pk\}, M) = c_1 \otimes \cdots \otimes c_n$

Security

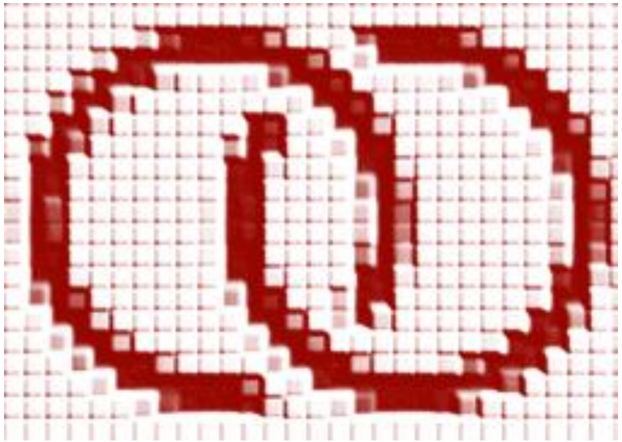


Anonymity:
- Provided by the ring structure in the ROM as [RST01], [AOS02]

Unforgeability:
- Reduced to the security of Type-T* canonical identification in the ROM

Instantiations

ECC-based and lattice-based



DualRing-EC

- DualRing only gives a O(n)-size generic ring signature $(z, c_1, \ldots, c_n)$
- In the DL-based setting, we have

  $c = c_1 + c_2 + \ldots + c_n \quad \text{mod p}$

- We propose a (non-interactive) *sum argument of knowledge (NISA)* to compress the challenges
  - Similar to the Bulletproof (for inner product relation), our NISA saves about half of the computation due to the simplicity of the sum relation.

Algorithm 4: NISA

Procedure NISA.$\textsc{Proof}(\{\mathsf{param}, \mathbf{g}, P, c\}, \mathbf{a})$:
  Run protocol $\textsc{Pf}$ on input $(\mathbf{g}, u^{H'_Z(P,u,c)}, \mathbf{a}, \mathbf{1}^n)$;

Procedure $\textsc{Pf}(\mathbf{g}, \hat{u}, \mathbf{a}, \mathbf{b})$:
  // Assume **L**, **R** are initially empty, but maintains its memory throughout the recurrsion. n is the length of vector **a** and **b**.
  if $n = 1$ **then**
    Output $\pi = (\mathbf{L}, \mathbf{R}, a, b)$.
  else
    Compute $n' = \frac{n}{2}$, $c_L = \langle \mathbf{a}_{[:n']}, \mathbf{b}_{[n':]} \rangle \in \mathbb{Z}_p$, $c_R = \langle \mathbf{a}_{[n':]}, \mathbf{b}_{[:n']} \rangle \in \mathbb{Z}_p$;
    $L = \mathbf{g}_{[n':]}^{\mathbf{a}_{[:n']}} \hat{u}^{c_L} \in \mathbb{G}$ and $R = \mathbf{g}_{[:n']}^{\mathbf{a}_{[n':]}} \hat{u}^{c_R} \in \mathbb{G}$;
    Add L to **L** and R to **R** and compute $x = H_Z(L, R)$;
    Compute $\mathbf{g}' = \mathbf{g}_{[:n']}^{x^{-1}} \circ \mathbf{g}_{[n':]}^{x} \in \mathbb{G}^{n'}$, $\mathbf{a}' = x \cdot \mathbf{a}_{[:n']} + x^{-1} \cdot \mathbf{a}_{[n':]} \in \mathbb{Z}_p^{n'}$ and $\mathbf{b}' = x^{-1} \cdot \mathbf{b}_{[:n']} + x \cdot \mathbf{b}_{[n':]} \in \mathbb{Z}_p^{n'}$;
    Run protocol $\textsc{Pf}$ on input $(\mathbf{g}', \hat{u}, \mathbf{a}', \mathbf{b}')$;

Procedure NISA.$\textsc{Verify}(\mathsf{param}, \mathbf{g}, P, c, \pi = (\mathbf{L}, \mathbf{R}, a, b))$:
  $P' = P \cdot u^{c \cdot H'_Z(P,u,c)}$;
  Compute for all $j = 1, \ldots, \log_2 n$: $x_j = H_Z(L_j, R_j)$;
  Compute for all $i = 1, \ldots, n$:
  $y_i = \prod_{j \in [\log_2 n]} x_j^{f(i,j)}, f(i,j) = \begin{cases} 1 & \text{if } (i-1)\text{'s } j\text{-th bit is } 1 \\ -1 & \text{otherwise} \end{cases};$
  Set $\mathbf{y} = (y_1, \ldots, y_n)$, $\mathbf{x} = (x_1, \ldots, x_{\log_2 n})$;
  if $\mathbf{L}^{\mathbf{x}^2} P' \mathbf{R}^{\mathbf{x}^{-2}} = \mathbf{g}^{a \cdot \mathbf{y}} u^{ab \cdot H'_Z(P,u,c)}$ **then**
    Output 1
  Output 0

DualRing-EC

- Combining DualRing with NISA, we get the shortest ring signature

Algorithm 5: DualRing-EC

Procedure $\textsc{Setup}(\lambda)$:
- $\mathsf{param}' \leftarrow \textsc{DualRing.Setup}(\lambda)$;
- pick a generator $u \leftarrow_s \mathbb{G}$;
- return $\mathsf{param} = (\mathsf{param}', u)$;

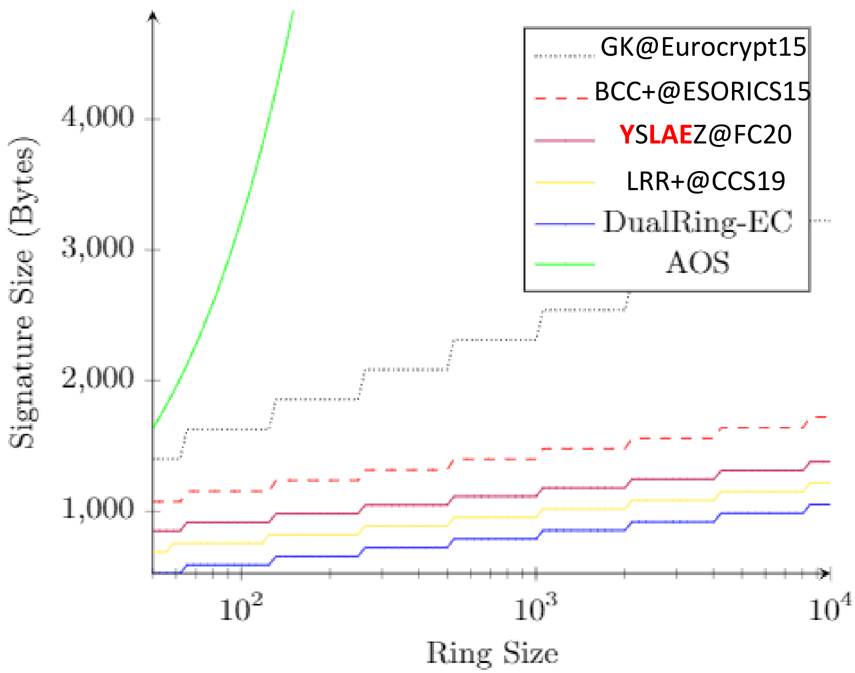
Procedure $\textsc{Sign}(\mathsf{param}, m, \mathbf{pk}, sk_j)$:
- $(c_1, \ldots, c_n, z) \leftarrow \textsc{DualRing.Sign}(\mathsf{param}, m, \mathbf{pk}, sk_j)$;
- `// (c, R) is computed in` $\textsc{DualRing.Sign}$
- $\mathbf{a} \leftarrow (c_1, \ldots, c_n)$;
- $P = R \cdot (V_1(z))^{-1}$;
- $\pi \leftarrow \text{NISA.}\textsc{Proof}(\{\mathsf{param}, \mathbf{pk}, u, P, c\}, \mathbf{a})$;
- return $\sigma = (z, P, \pi)$;

Procedure $\textsc{KeyGen}(\mathsf{param})$:
- return $(\mathsf{pk}, \mathsf{sk}) \leftarrow \textsc{DualRing.KeyGen}(\mathsf{param}')$;

Procedure $\textsc{Verify}(\mathsf{param}, m, \mathbf{pk}, \sigma)$:
- parse $\sigma = (z, P, \pi)$;
- $R = P \cdot V_1(z)$;
- $c = H_Z(m, \mathbf{pk}, R)$;
- **if** $0 \leftarrow \text{NISA.}\textsc{Verify}(\mathsf{param}, \mathbf{pk}, u, P, c)$ **then**
  - return 0;
- return 1;

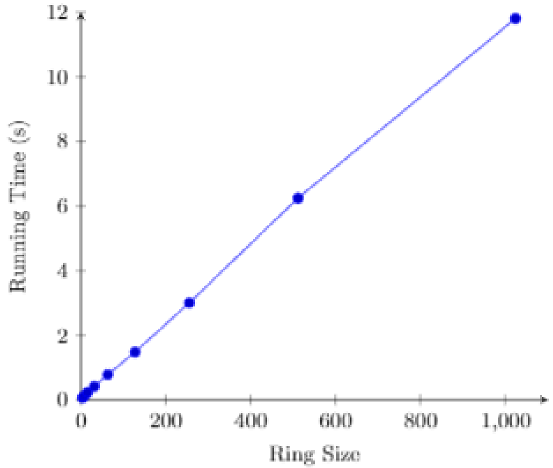
DualRing-EC

- Signature size

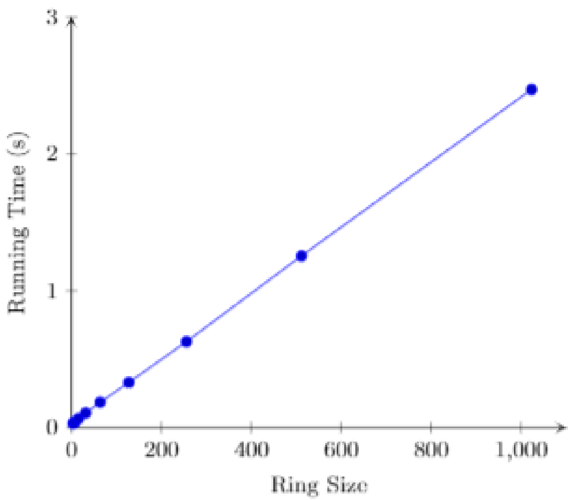


Intel Core i5 2.3GHz, 8GB RAM with MacOS 10.
Python, using the P256 curve in the fastecdsa library

- Running Time



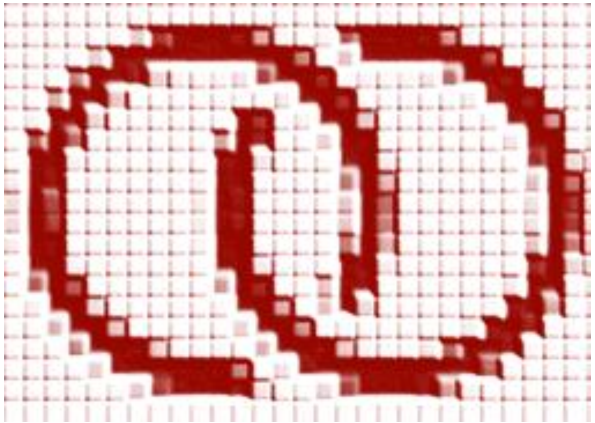
(a) Running time of Sign.



(b) Running time of Verify.

DualRing-LB

- Unlike DualRing-EC, no efficient ZK proof for DualRing in lattice-based setting
- Interestingly, DualRing is already highly efficient in the lattice-based setting:
  - DualRing-LB consists of single response and n challenges.
  - The size of a challenge (around 256 bits) in lattice-based identification is often much smaller than the size of a response (around a few KB).
  - → obtain a compact lattice-based ring signature even without requiring a lattice-based sum argument.



DualRing-LB

- We give a Type-T* canonical identification from M-LWE/SIS, using rejection sampling technique.
- Concrete parameter settings are calculated in the paper.

Ring Signatures	Signature Size (Bytes)						Assumption
	$n=2$	$n=8$	$n=64$	$n=1024$	$n=2048$	$n=4096$	
LAZ@ACNS19	2532	10128	81024	1296384	2592768	6564888576	NTRU
BKP@Asiacrypt20	49000	50000	52000	54000	54500	55000	M-LWE+M-SIS
EZSLL@CCS19	18000	19000	31000	48000	53000	59000	M-LWE+M-SIS
DualRing-LB	4480	4630	6020	31160	55500	106570	M-LWE+M-SIS

O(log n)

O(n)

Shortest for the ring size between 4 - 1946.
→ Useful in practice (e.g., Monero), where the ring size is not very large

	n=32	n=1024	n=32768	
LNS@Crypto21	15960	17270	18730	M-LWE+M-SIS

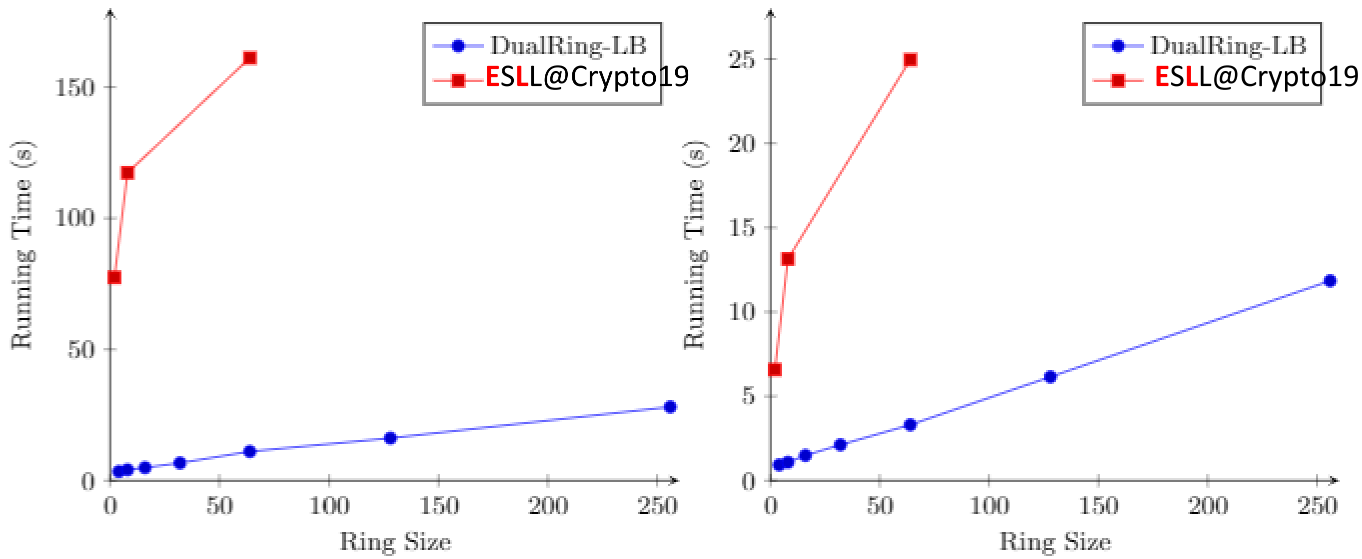
O(log n)

→ DualRing-LB is the shortest for ring size from 4 – 452. (no figure for comparison between 453-505 from [LNS])

[LNS]: Lyubashevsky, Nguyen, Seiler. SMILE: Set Membership from Ideal Lattices with Applications to Ring Signatures and Confidential Transactions. To appear in CRYPTO 2021.

DualRing-LB

- More importantly, much faster in computation
  - Computation in the challenge space is fast



(a) Running Time of Sign.   (b) Running Time of Verify.

Fig. 5: Lattice-based ring signatures

Intel Core i5 2.3GHz, 8GB RAM with MacOS 10.
Python, using the NTT transform in the sympy library

Conclusion

- Propose a generic construction of ring signature scheme using a dual ring structure.
- Instantiated in the DL-setting, it is the shortest ring signature scheme without using trusted setup.
- Instantiated in M-LWE/SIS, we have the shortest ring signature for ring size between 4 and ~500.

Thank you!