

Non-Interactive Batch Arguments for NP from Standard Assumptions



Arka Rai Choudhuri

Johns Hopkins University



Zhengzhong Jin

Johns Hopkins University

Abhishek Jain

Johns Hopkins University

Non-Interactive Batch Arguments for NP

CRS



C, x_1, \dots, x_k



C, x_1, \dots, x_k

Non-Interactive Batch Arguments for NP

CRS



C, x_1, \dots, x_k

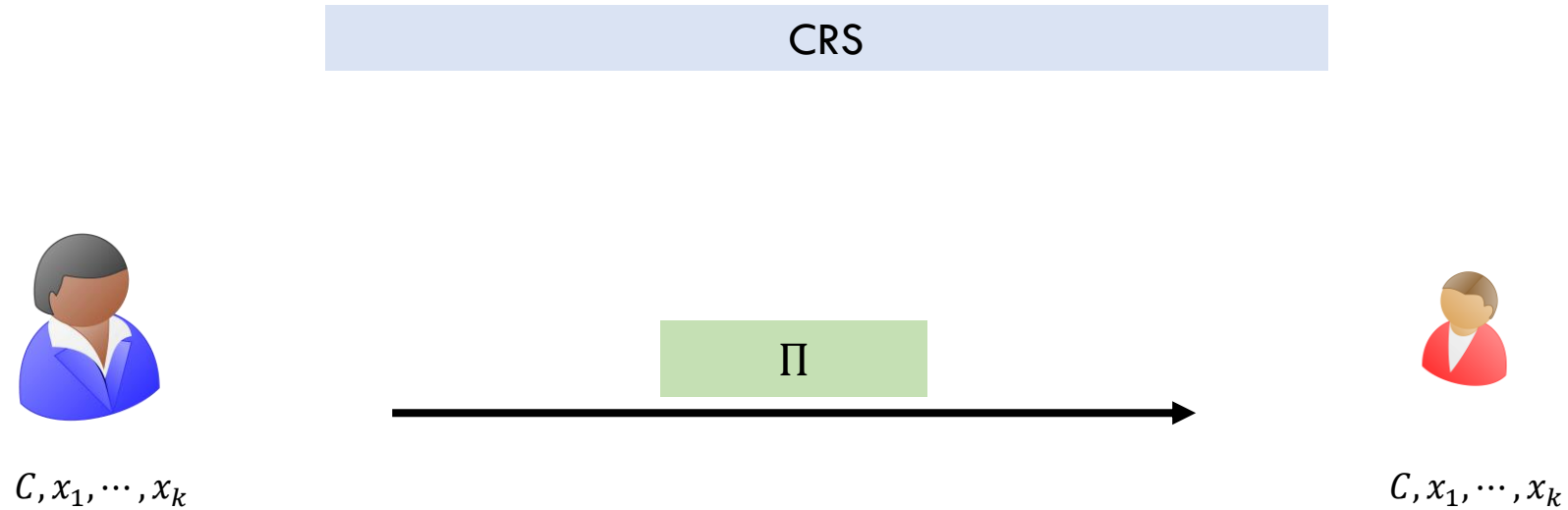


C, x_1, \dots, x_k

$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$

$\forall i \in [k], (C, x_i) \in \text{SAT}$

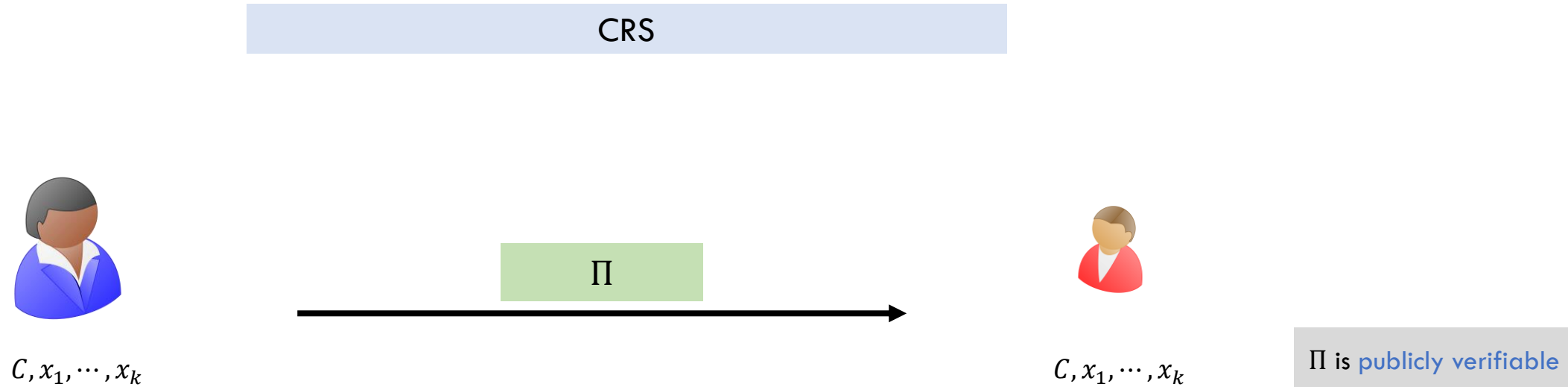
Non-Interactive Batch Arguments for NP



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

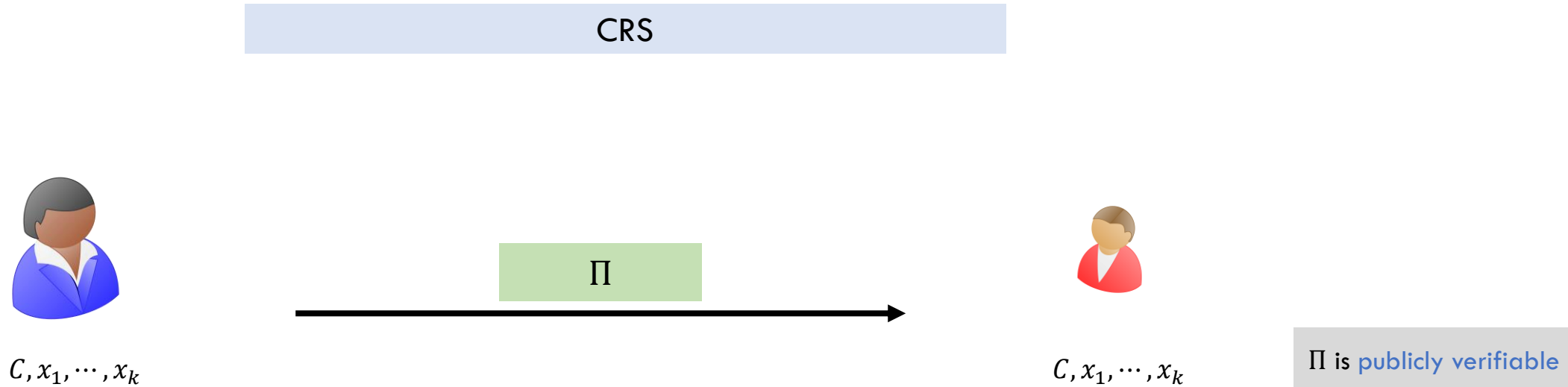
Non-Interactive Batch Arguments for NP



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Non-Interactive Batch Arguments for NP



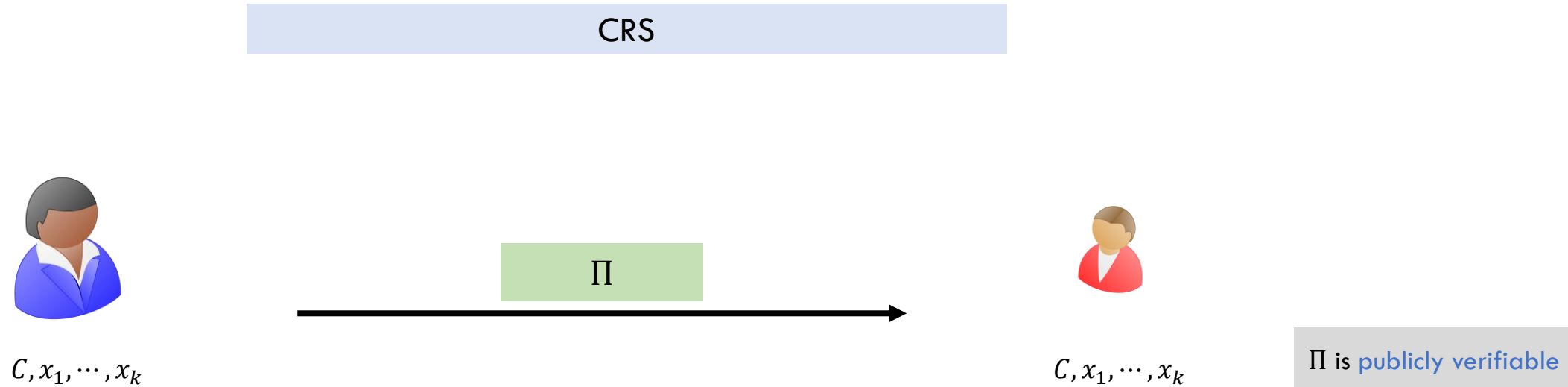
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

No PPT  can produce accepting Π if

$$\exists i^* \in [k], (C, x_{i^*}) \notin \text{SAT}$$

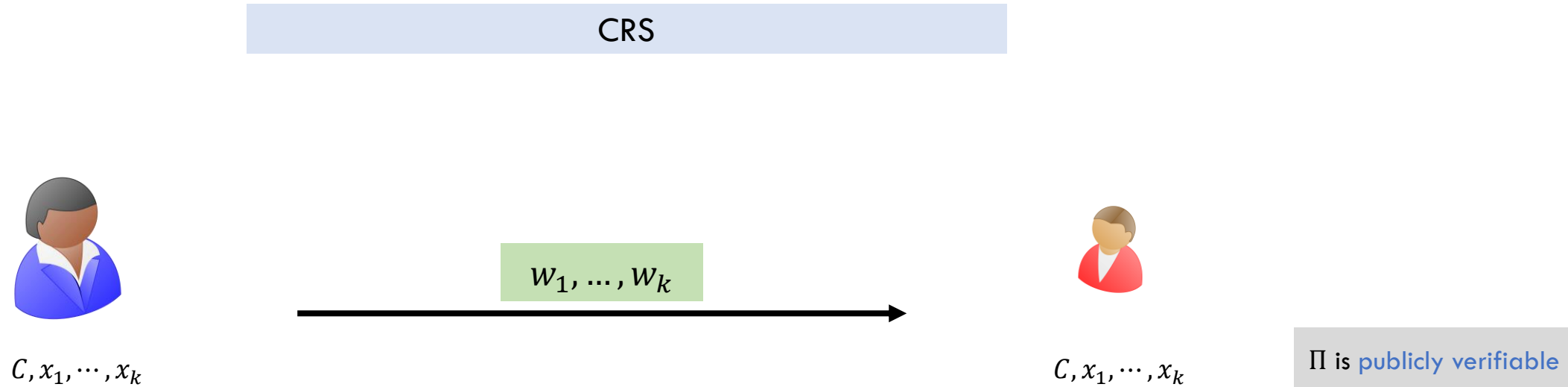
Non-Interactive Batch Arguments for NP



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

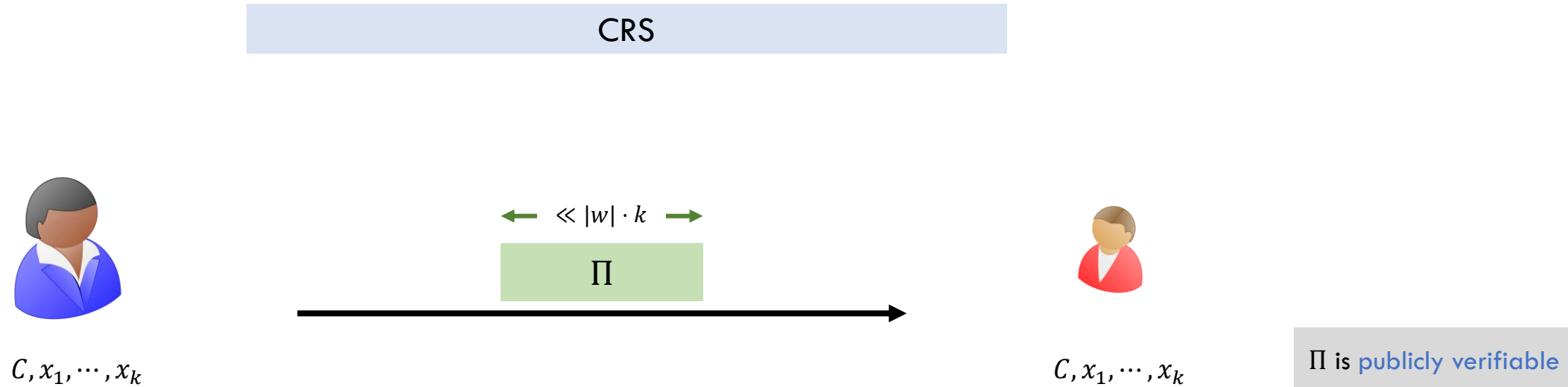
Non-Interactive Batch Arguments for NP



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

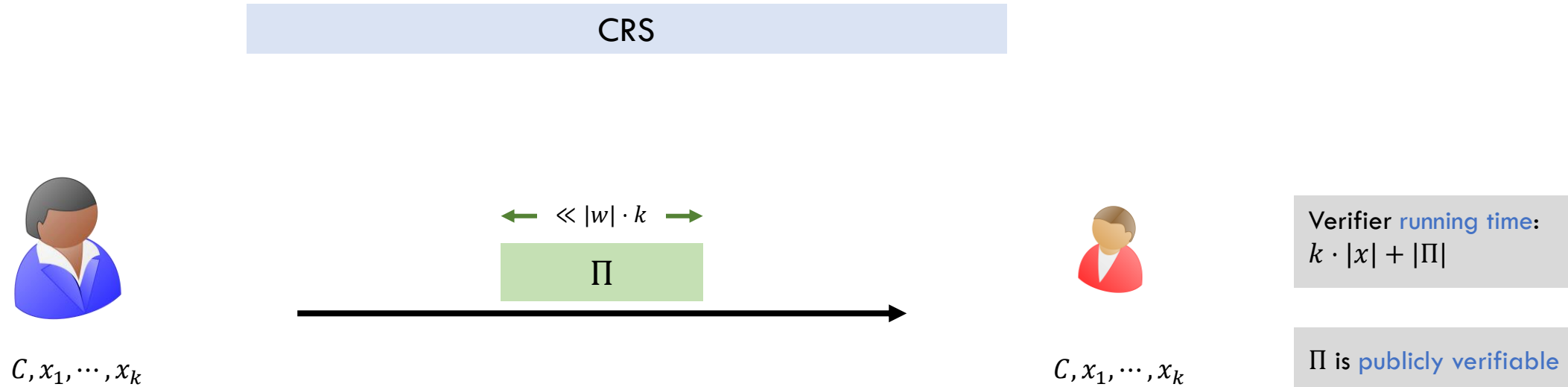
Non-Interactive Batch Arguments for NP



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Non-Interactive Batch Arguments for NP



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Prior Works

Prior Works

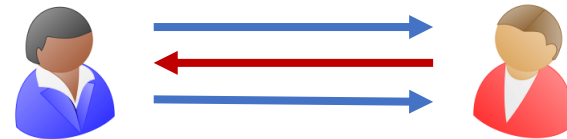
Interactive batch proofs

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]

Prior Works

Interactive batch proofs

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]



Secure against
unbounded cheating
prover.

Prior Works

Interactive batch proofs for UP

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]



UP – each statement has a **unique witness**.

Prior Works

Interactive batch proofs for UP

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]

Succinct Non-interactive Arguments (SNARGs) for NP

[Micali'94, Damgård-Faust-Hazay'12, Bitansky-Canetti-Chiesa-Tromer'13, Bitansky-Canetti-Chiesa-Goldwasser-Lin-Rubinfeld-Tromer'16]

SNARGs

$$|\Pi| \ll |w|$$

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Prior Works

Interactive batch proofs for UP

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]

Succinct Non-interactive Arguments (SNARGs) for NP

[Micali'94, Damgård-Faust-Hazay'12, Bitansky-Canetti-Chiesa-Tromer'13, Bitansky-Canetti-Chiesa-Goldwasser-Lin-Rubinfeld-Tromer'16]

SNARGs

$$|\Pi| \ll |w|$$

$$\text{SAT}^{\otimes k} = \{(C, x_1, \dots, x_k) \mid \forall i \in [k], (C, x_i) \in \text{SAT}\}$$

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Prior Works

Interactive batch proofs for UP

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]

SNARGs for NP from **Non-falsifiable assumptions** / **Random oracle model**

[Micali'94, Damgård-Faust-Hazay'12, Bitansky-Canetti-Chiesa-Tromer'13, Bitansky-Canetti-Chiesa-Goldwasser-Lin-Rubinfeld-Tromer'16]

SNARGs

$$|\Pi| \ll |w|$$

$$\text{SAT}^{\otimes k} = \{(C, x_1, \dots, x_k) \mid \forall i \in [k], (C, x_i) \in \text{SAT}\}$$

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Prior Works

Interactive batch proofs for UP

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]

SNARGs for NP from Non-falsifiable assumptions/ Random oracle model

[Micali'94, Damgård-Faust-Hazay'12, Bitansky-Canetti-Chiesa-Tromer'13, Bitansky-Canetti-Chiesa-Goldwasser-Lin-Rubinfeld-Tromer'16]

Designated Verifier non-interactive batch arguments for NP

[Brakerski-Holmgren-Kalai'17, Brakerski-Kalai'20]

Prior Works

Interactive batch proofs for UP

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]

SNARGs for NP from Non-falsifiable assumptions/ Random oracle model

[Micali'94, Damgård-Faust-Hazay'12, Bitansky-Canetti-Chiesa-Tromer'13, Bitansky-Canetti-Chiesa-Goldwasser-Lin-Rubinfeld-Tromer'16]

Designated Verifier from **standard assumptions**

[Brakerski-Holmgren-Kalai'17, Brakerski-Kalai'20]

Prior Works

Interactive batch proofs for UP

[Reingold-Rothblum-Rothblum'16, Reingold-Rothblum-Rothblum'18, Rothblum-Rothblum'20]

SNARGs for NP from Non-falsifiable assumptions/ Random oracle model

[Micali'94, Damgård-Faust-Hazay'12, Bitansky-Canetti-Chiesa-Tromer'13, Bitansky-Canetti-Chiesa-Goldwasser-Lin-Rubinfeld-Tromer'16]

Designated Verifier from standard assumptions

[Brakerski-Holmgren-Kalai'17, Brakerski-Kalai'20]

Non-interactive batch arguments for NP from new **non-standard assumption**

[Kalai-Paneth-Yang'19]

Falsifiable assumption
on groups with bilinear
maps.

Do there exists **non-interactive batch arguments** for
NP based on **standard assumptions**?

Our Result

Theorem

Assuming QR + (LWE/sub-exp DDH) there exists a non-interactive batch argument for NP where

$$|\Pi| = \tilde{O}(|C| + \sqrt{k|C|})$$

QR – Quadratic residuosity, LWE – Learning with Error, DDH – Decisional Diffie-Hellman

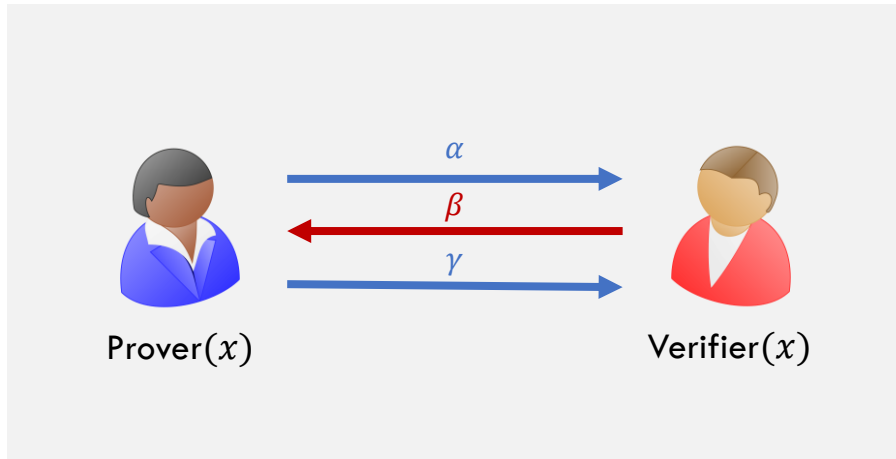
$SAT = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$

$\forall i \in [k], (C, x_i) \in SAT$

Key Insights

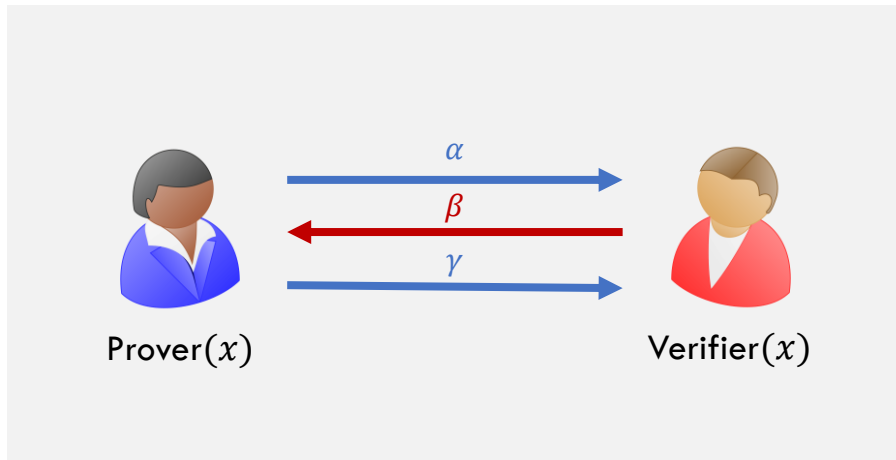
Fiat-Shamir (FS) Methodology

Fiat-Shamir (FS) Methodology



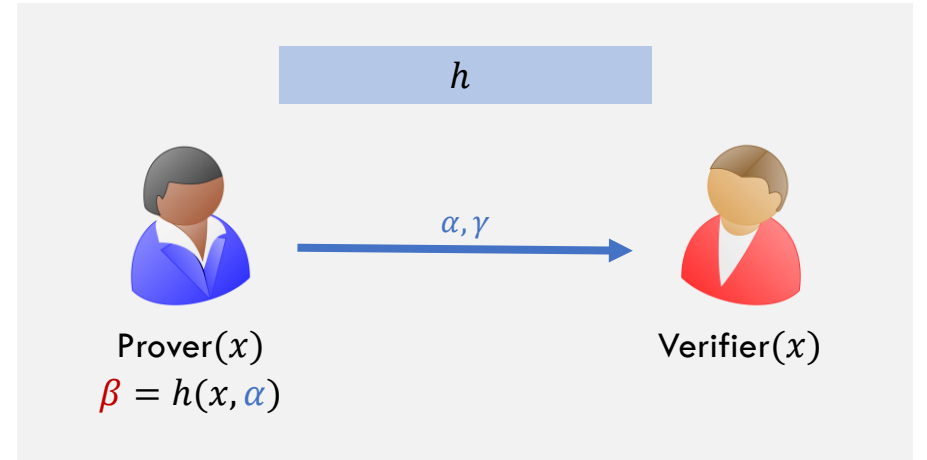
β is a random string

Fiat-Shamir (FS) Methodology

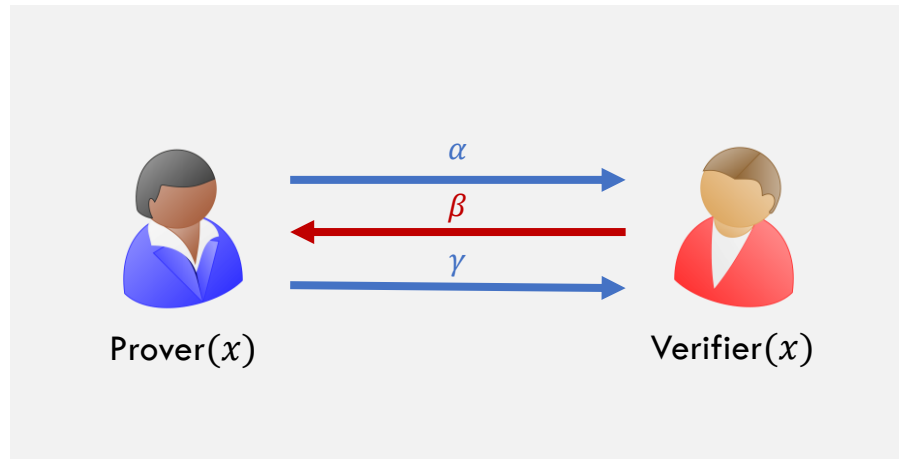


β is a random string

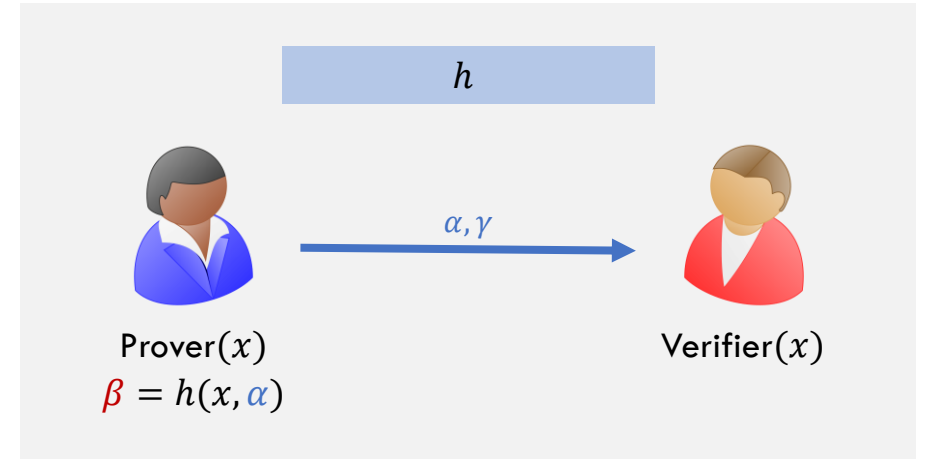
→
[Fiat-Shamir'86]



Fiat-Shamir (FS) Methodology



[Fiat-Shamir'86]

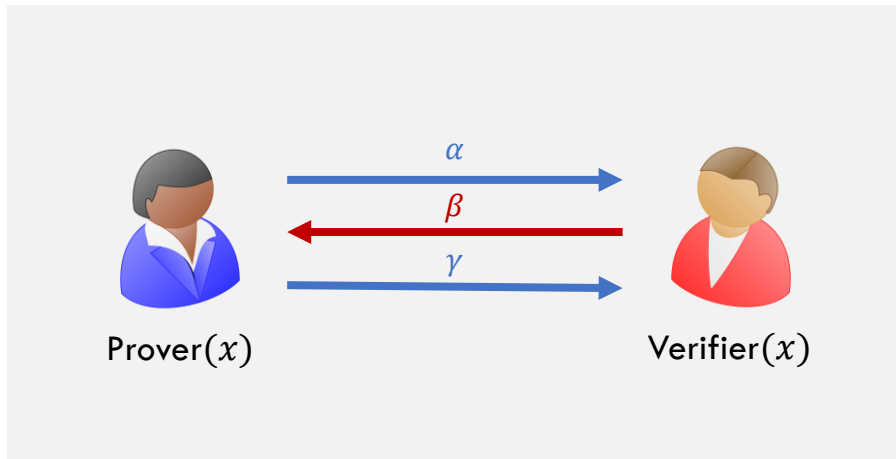


β is a random string

FS methodology is secure for certain protocols under a variety of assumptions (via [correlation intractable hash functions](#))

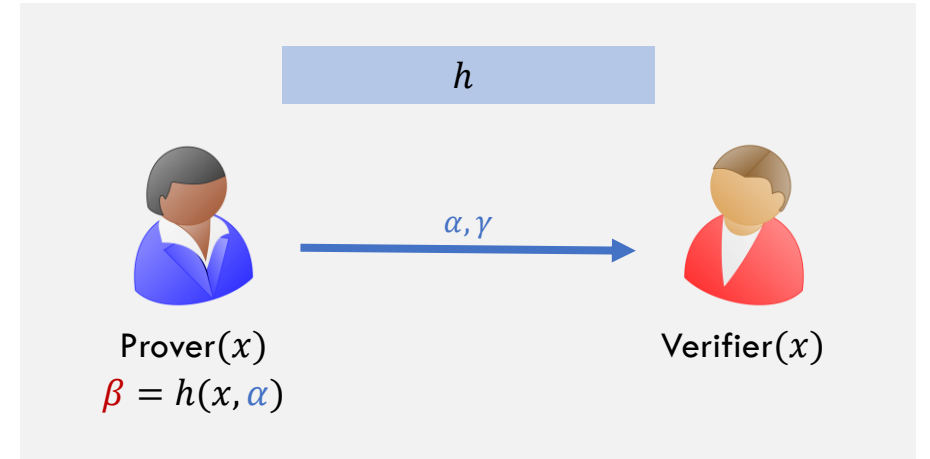
[Kalai-Rothblum-Rothblum'17, Canetti-Chen-Reyzin-Rothblum'18, Holmgren-Lombardi'18, Canetti-Chen-Holmgren-Lombardi-Rothblum-Rothblum-Wichs'19, Peikert-Sheihian'19, Brakerski-Koppula-Mour'20, Couteau-Katsumata-Ursu'20, Jain-Jin'21, Jawale-Kalai-Khurana-Zhang'21, Holmgren-Lombardi-Rothblum'21]

Fiat-Shamir (FS) Methodology



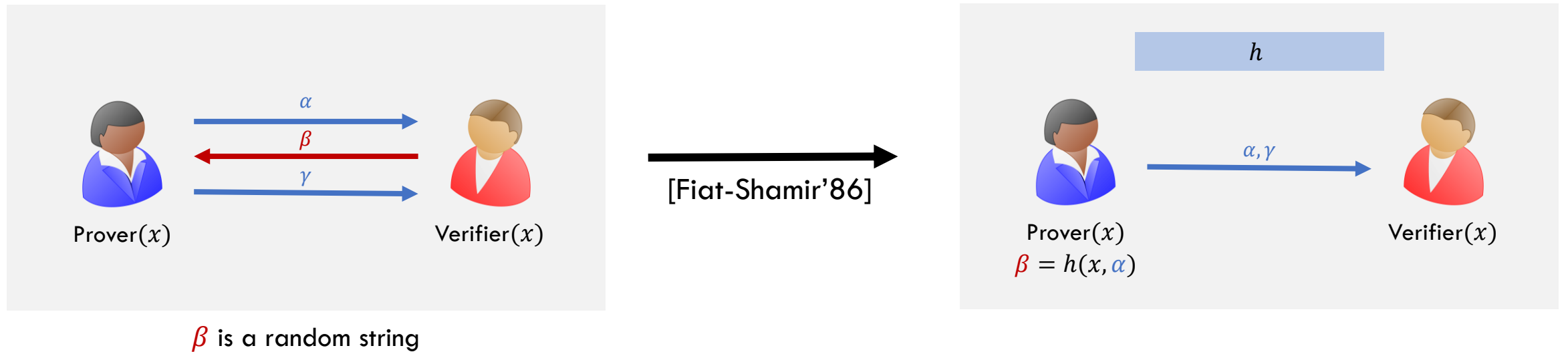
β is a random string

[Fiat-Shamir'86]



FS methodology is secure for certain protocols under a variety of assumptions (via [correlation intractable hash functions](#))

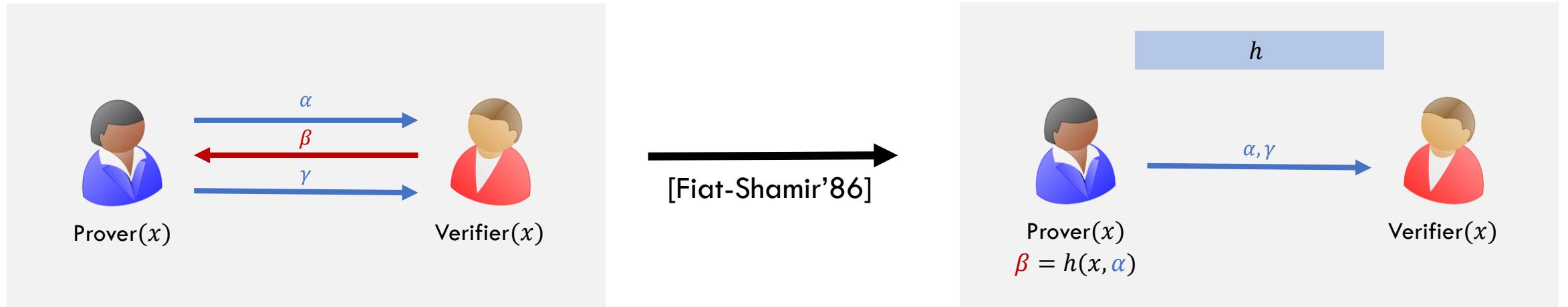
Fiat-Shamir (FS) Methodology



FS methodology is secure for certain protocols under a variety of assumptions (via [correlation intractable hash functions](#))

Proven secure if starting with [statistically secure interactive protocols](#) (interactive proofs).

Fiat-Shamir (FS) Methodology



β is a random string

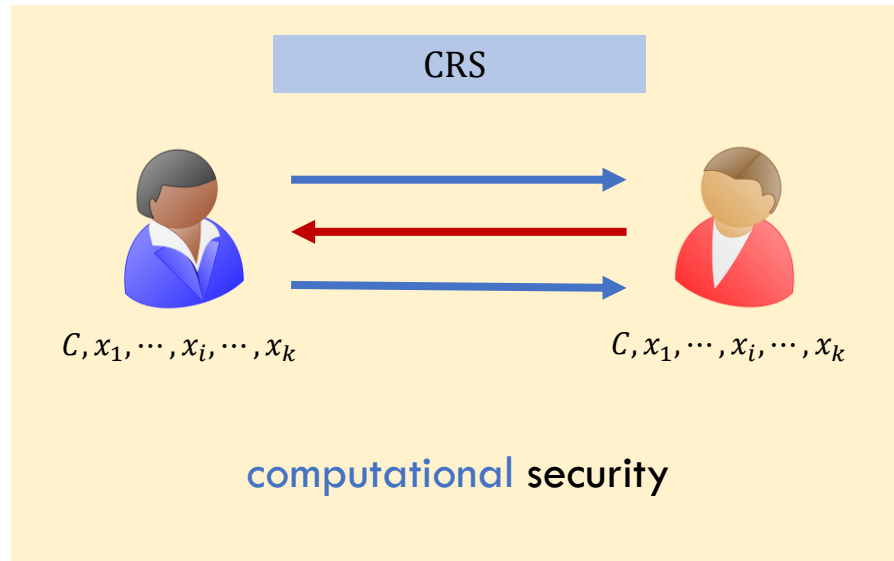
FS methodology is secure for certain protocols under a variety of assumptions (via [correlation intractable hash functions](#))

Proven secure if starting with [statistically secure interactive protocols](#) (interactive proofs).

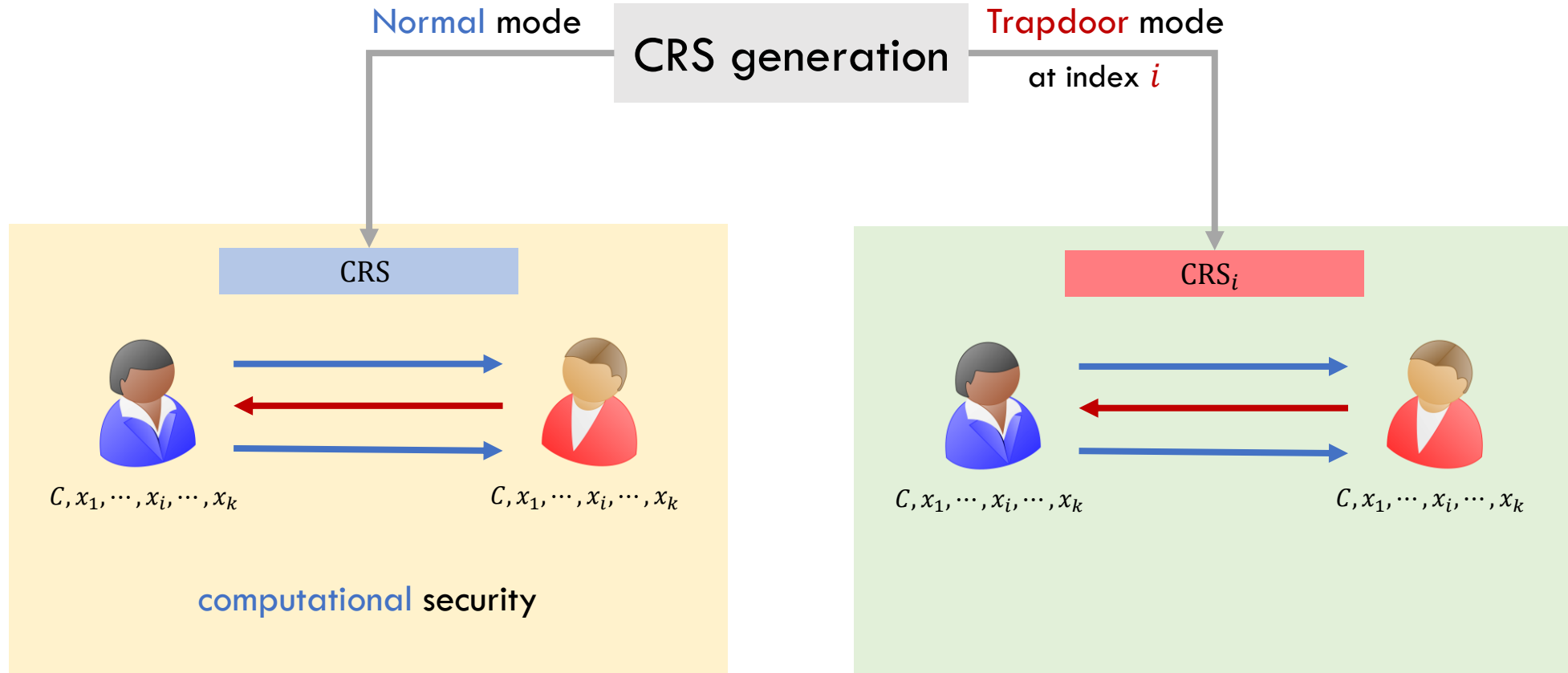
[No known interactive proofs](#) for batch NP.

Dual-Mode Interactive Batch Arguments

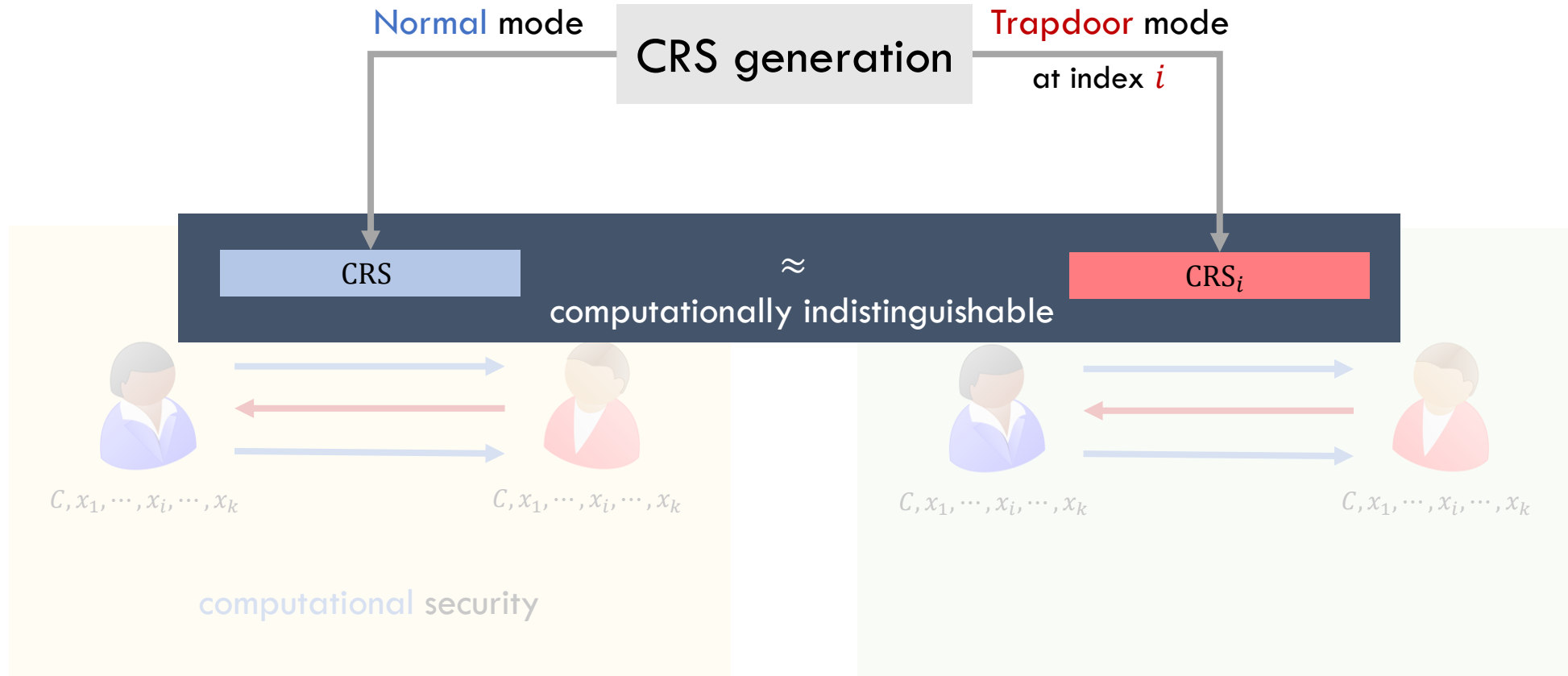
Dual-Mode Interactive Batch Arguments



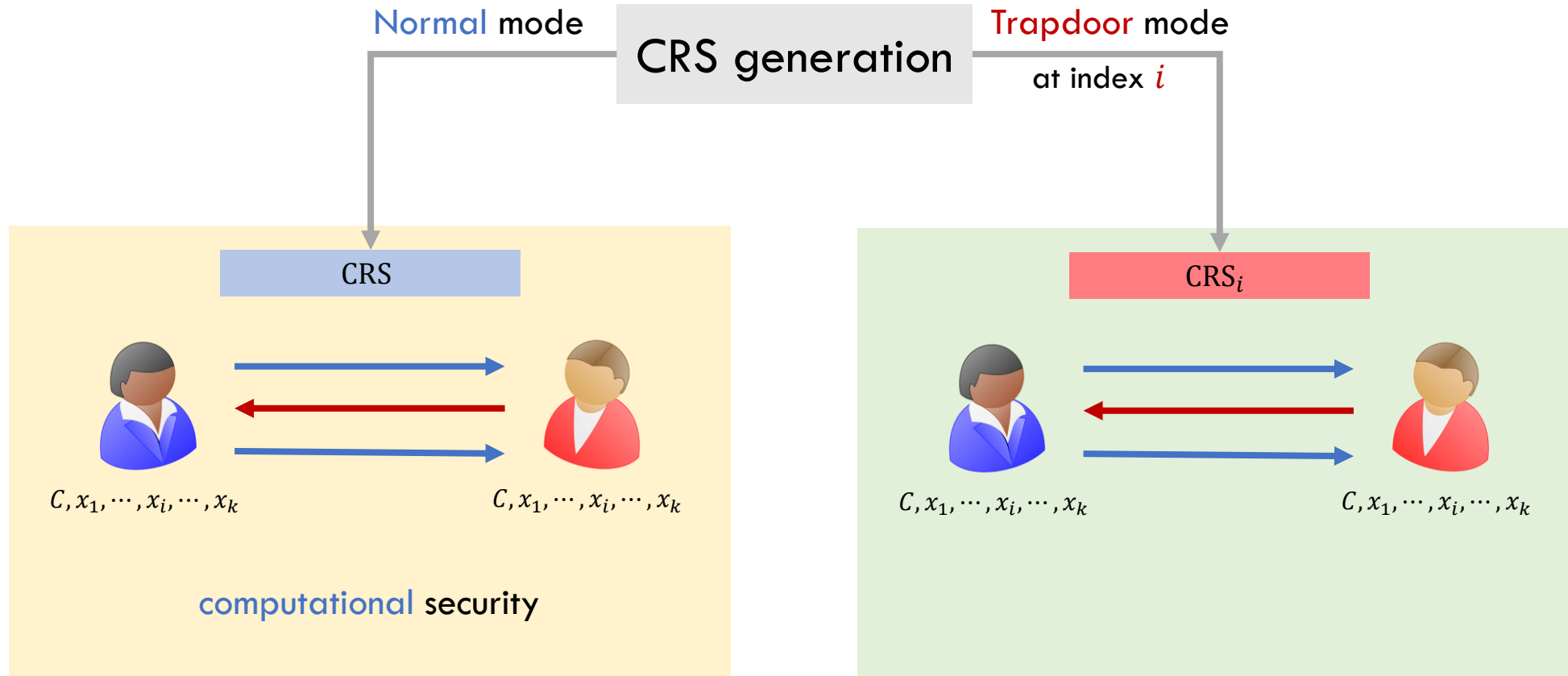
Dual-Mode Interactive Batch Arguments



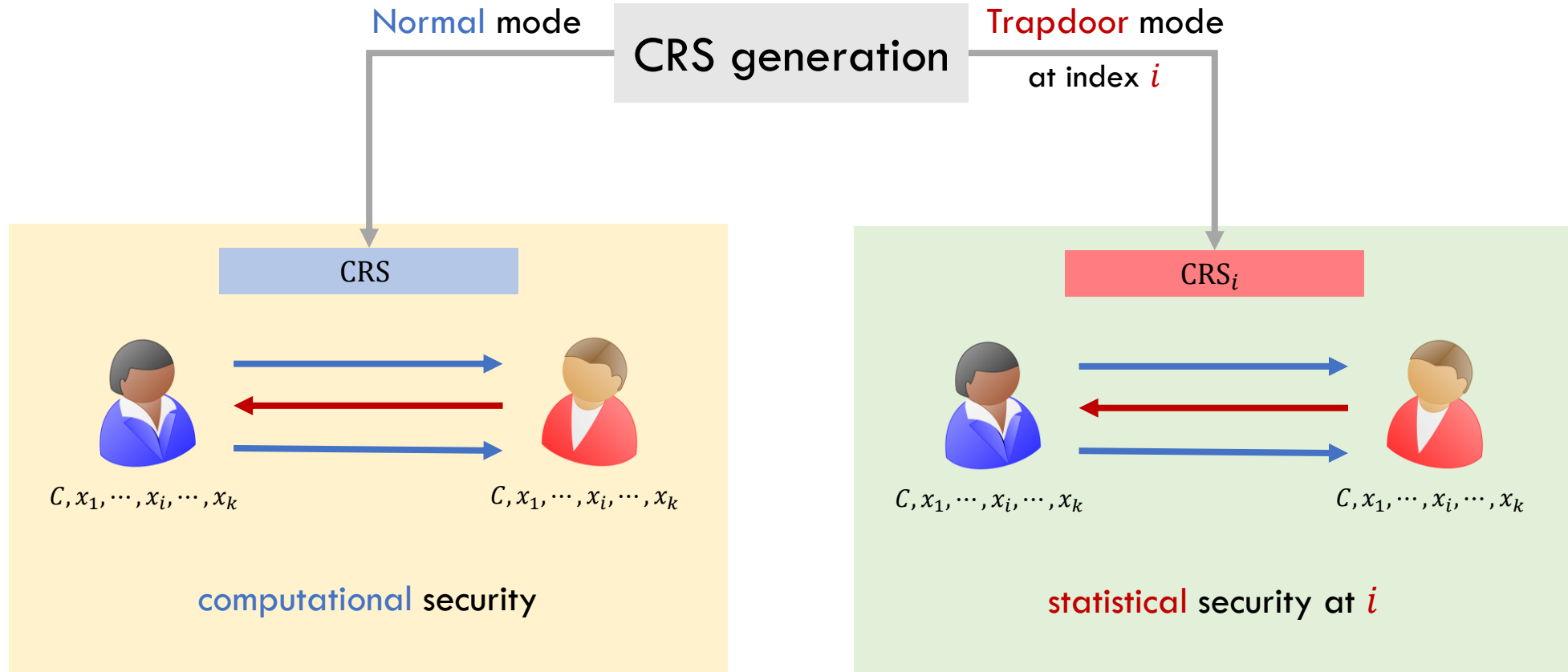
Dual-Mode Interactive Batch Arguments



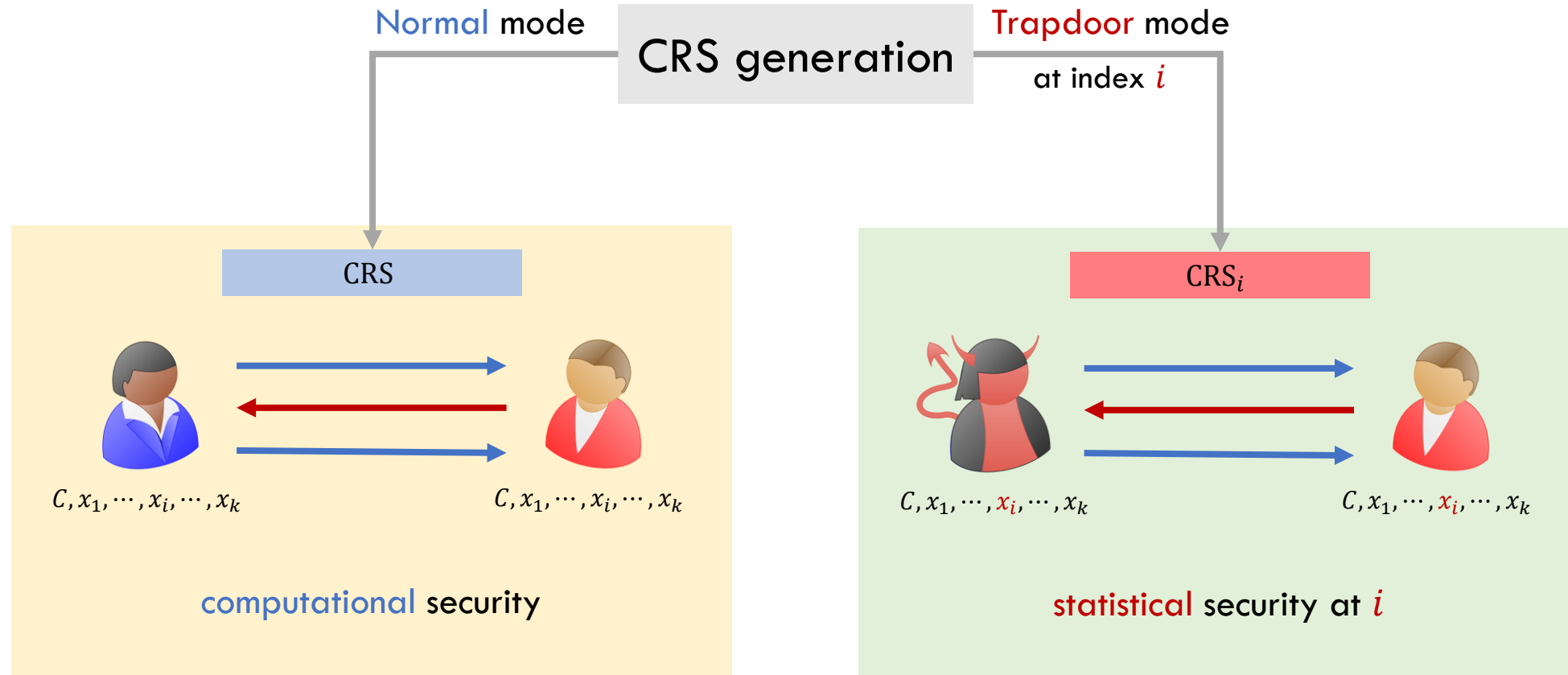
Dual-Mode Interactive Batch Arguments



Dual-Mode Interactive Batch Arguments

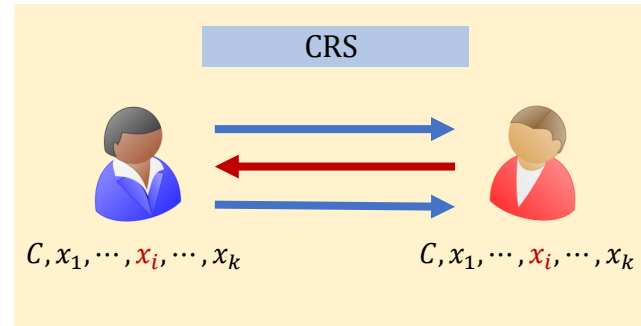


Dual-Mode Interactive Batch Arguments

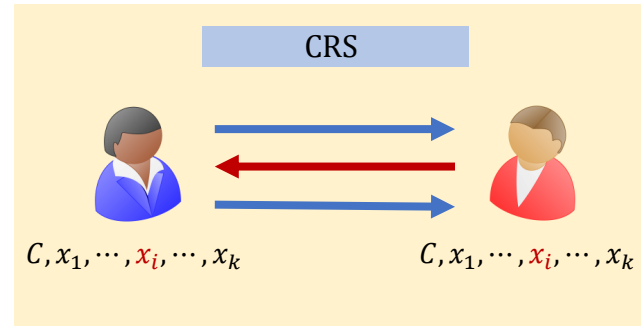


Even **unbounded**  cannot make  accept if $(C, x_i) \notin \text{SAT}$

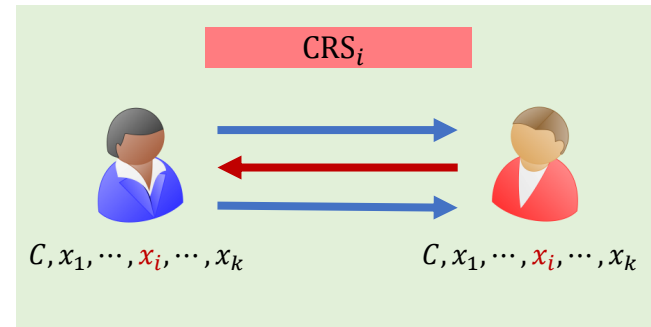
Security Intuition



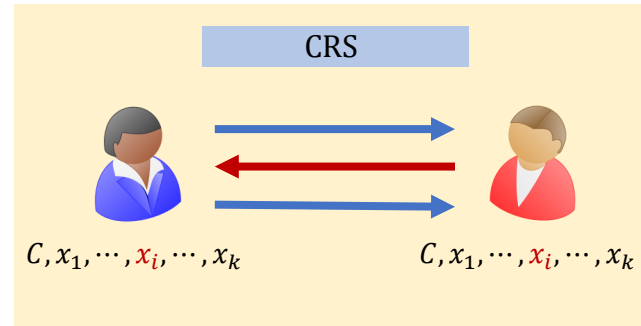
Security Intuition



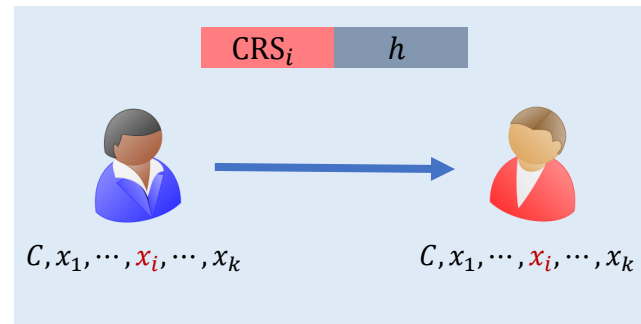
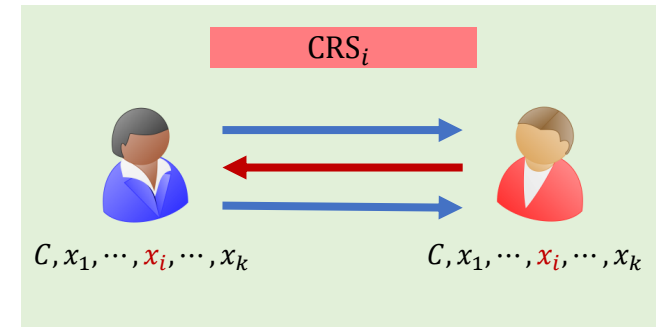
Switch to trapdoor mode at i



Security Intuition



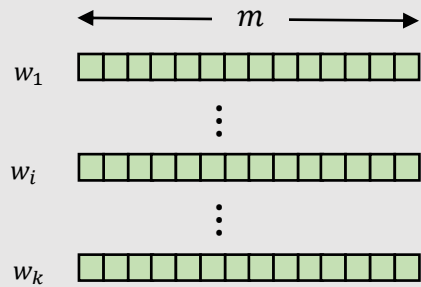
Switch to trapdoor mode at i



Rely on FS transformation

Dual Mode Batch Argument

Protocol Template

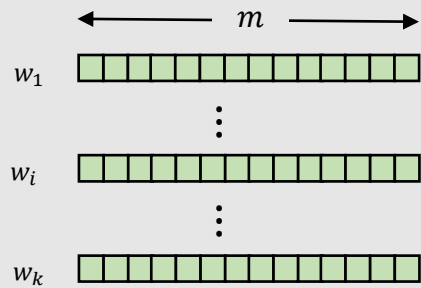


$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Dual Mode Batch Argument

Protocol Template



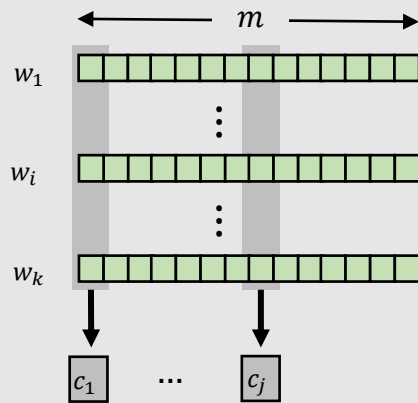
commitment key K

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Dual Mode Batch Argument

Protocol Template

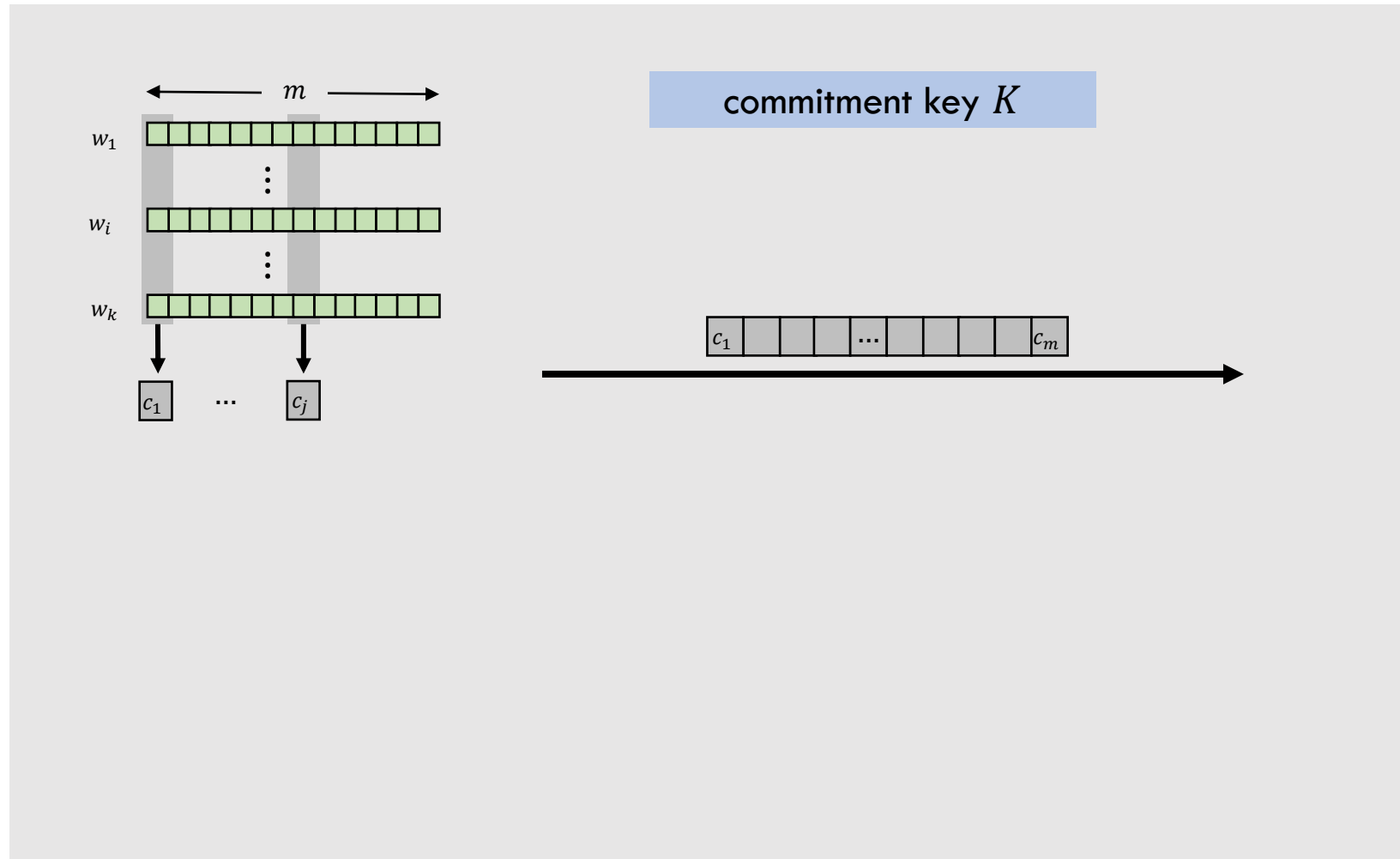


$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Dual Mode Batch Argument

Protocol Template

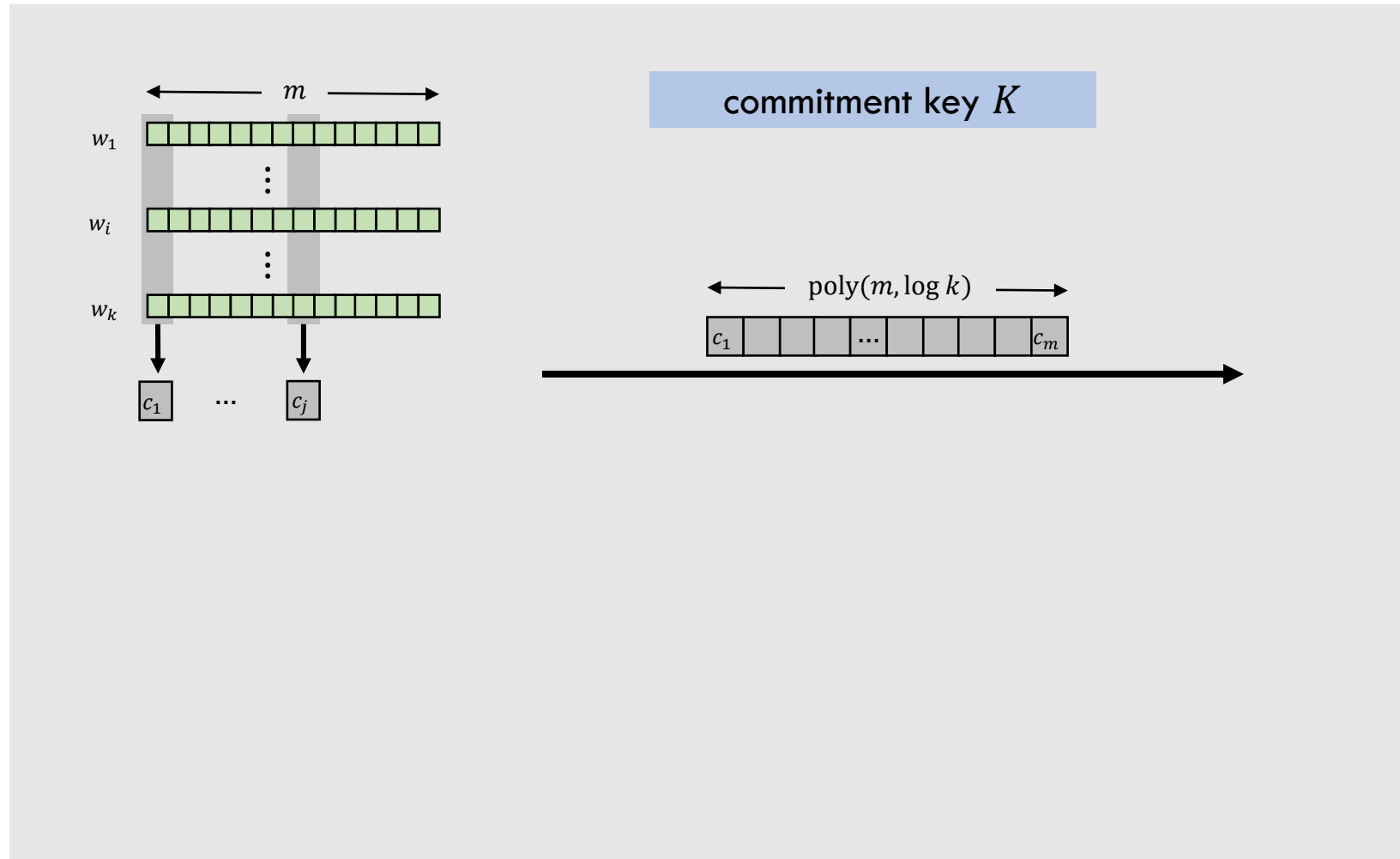


$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Dual Mode Batch Argument

Protocol Template

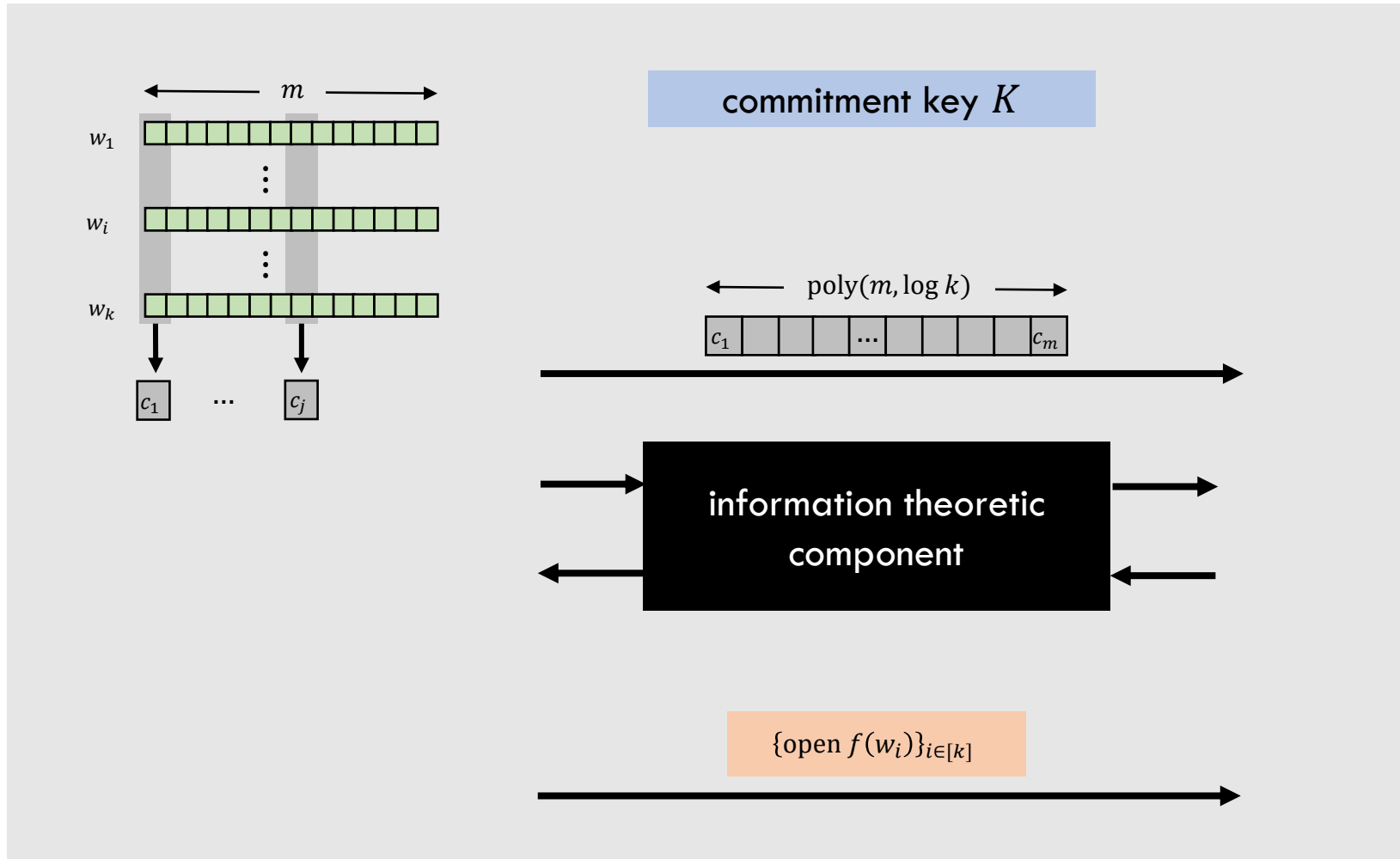


$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Dual Mode Batch Argument

Protocol Template



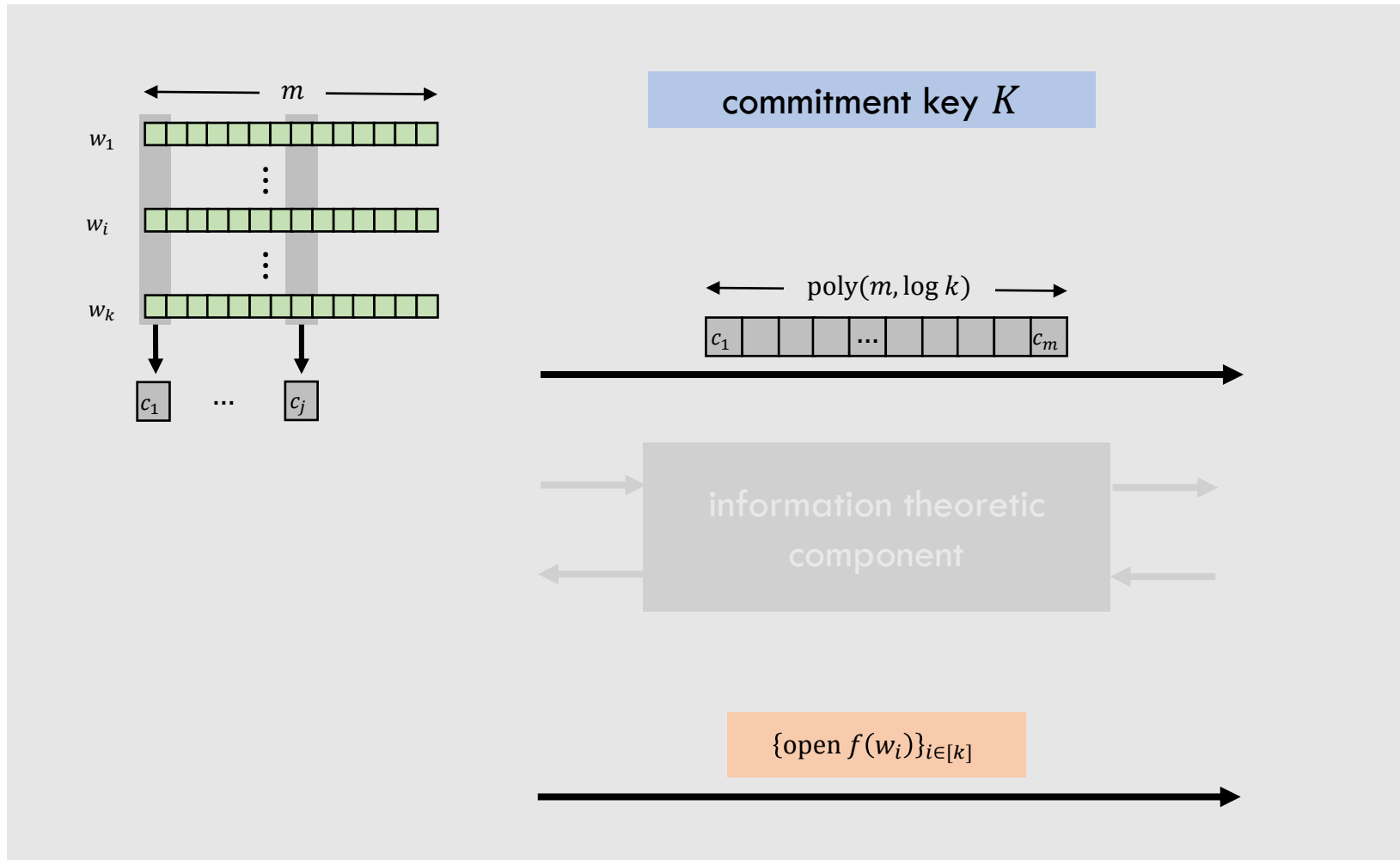
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

f determined by the information theoretic component.

Dual Mode Batch Argument

Protocol Template

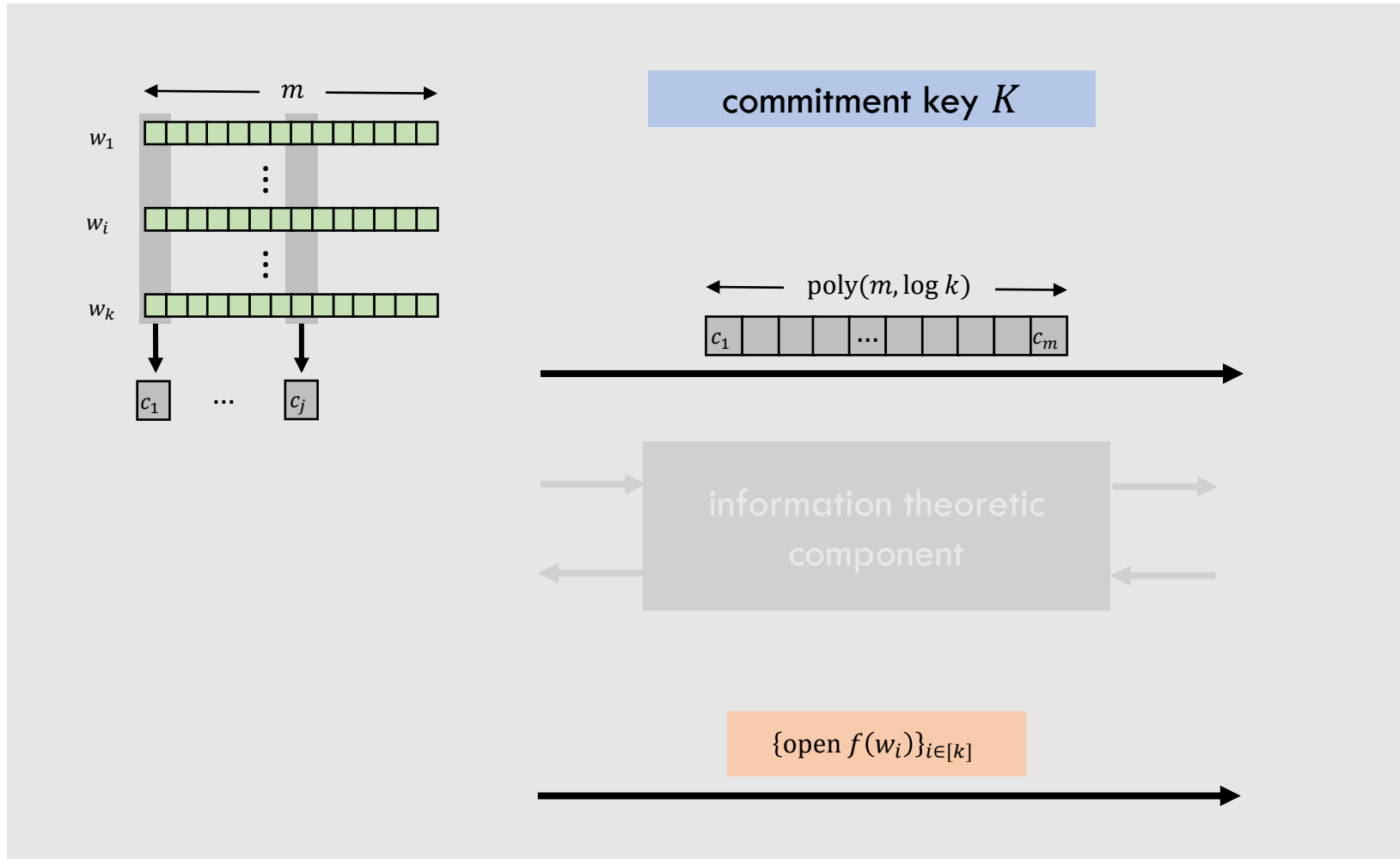


$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Dual Mode Batch Argument

Protocol Template



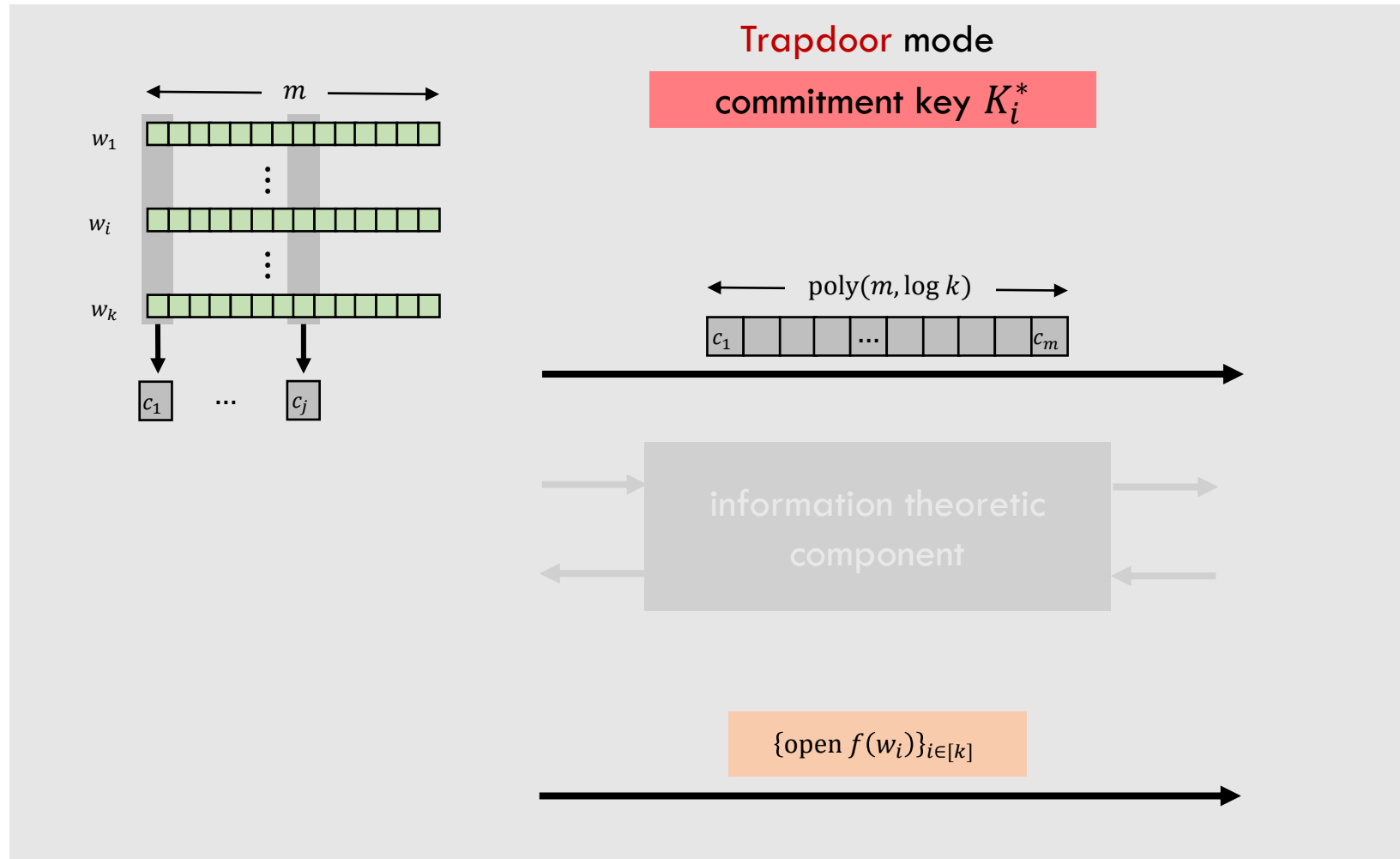
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Somewhere Statistically
Binding (SSB) Commitment
Scheme

Dual Mode Batch Argument

Protocol Template



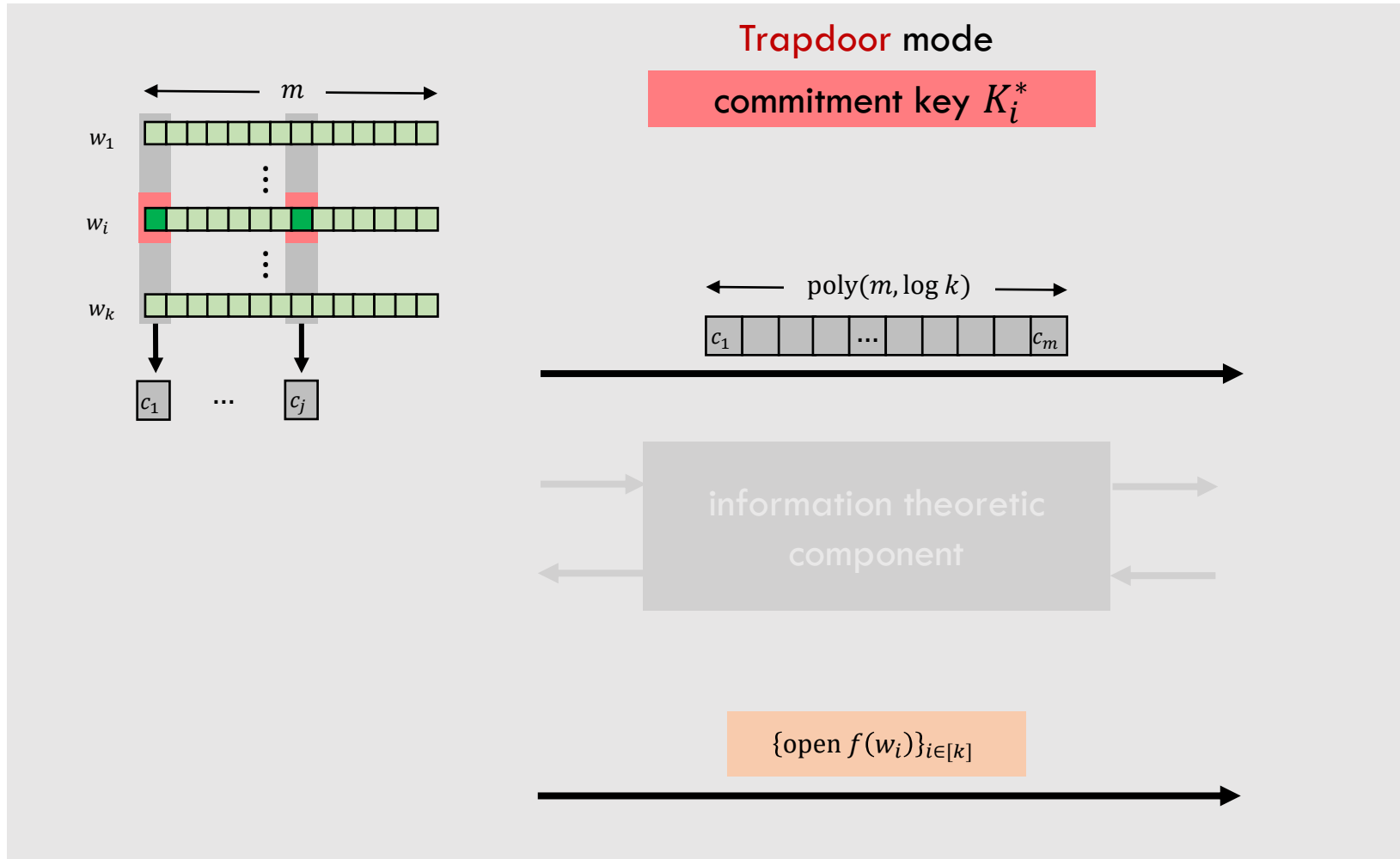
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Somewhere Statistically
Binding (SSB) Commitment
Scheme

Dual Mode Batch Argument

Protocol Template



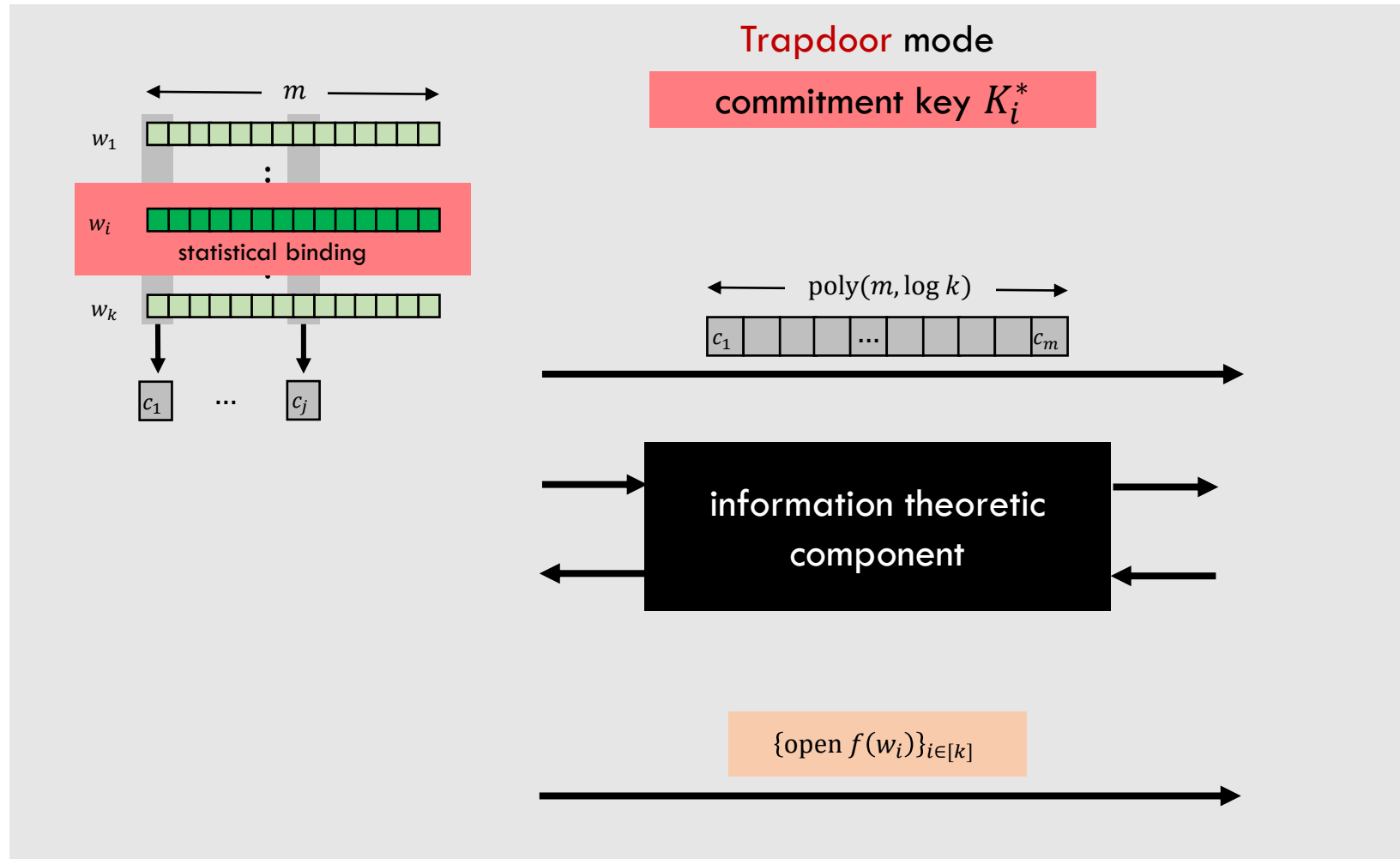
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Somewhere Statistically
Binding (SSB) Commitment
Scheme

Dual Mode Batch Argument

Protocol Template



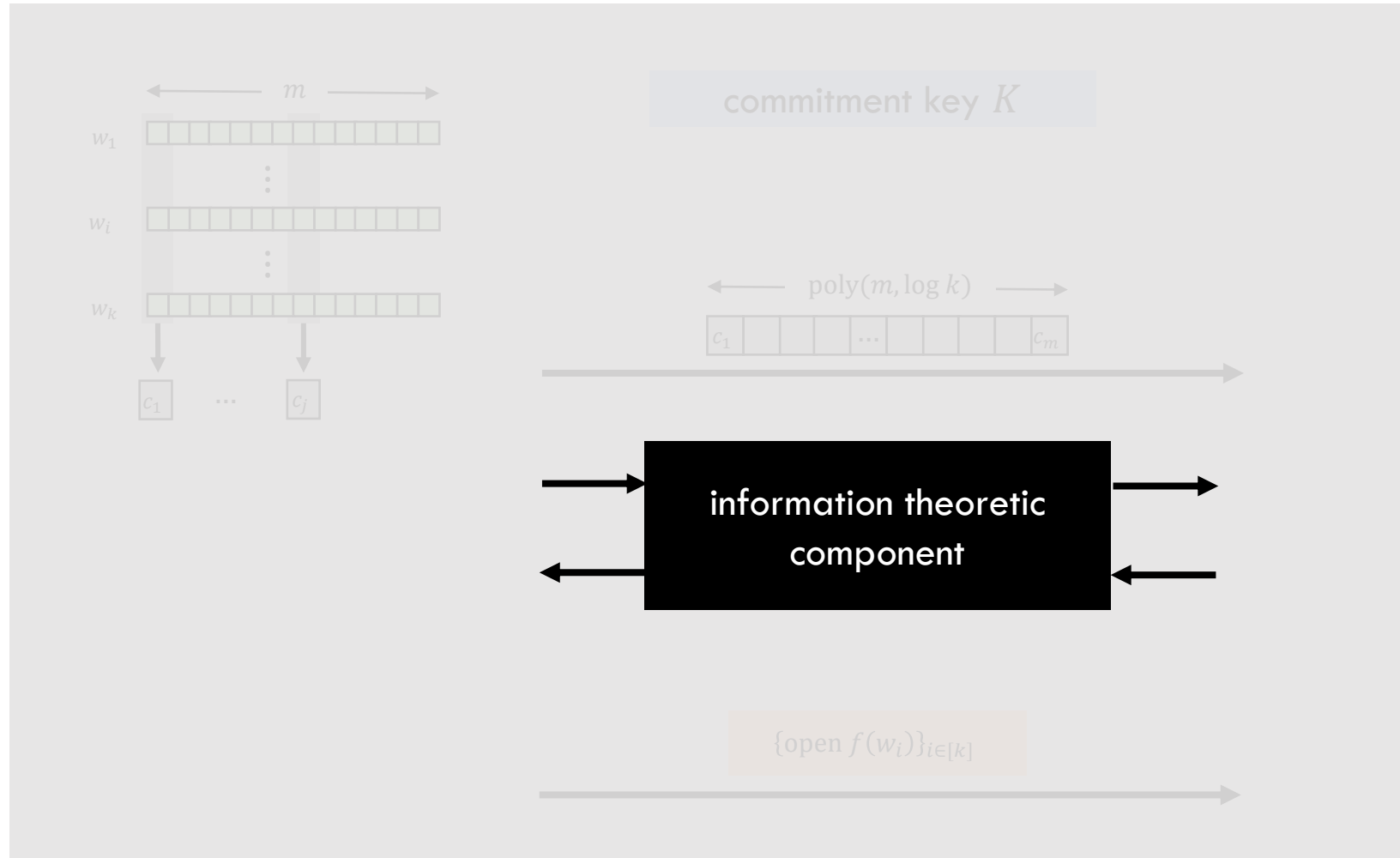
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

Somewhere Statistically
Binding (SSB) Commitment
Scheme

Dual Mode Batch Argument

Protocol Template



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

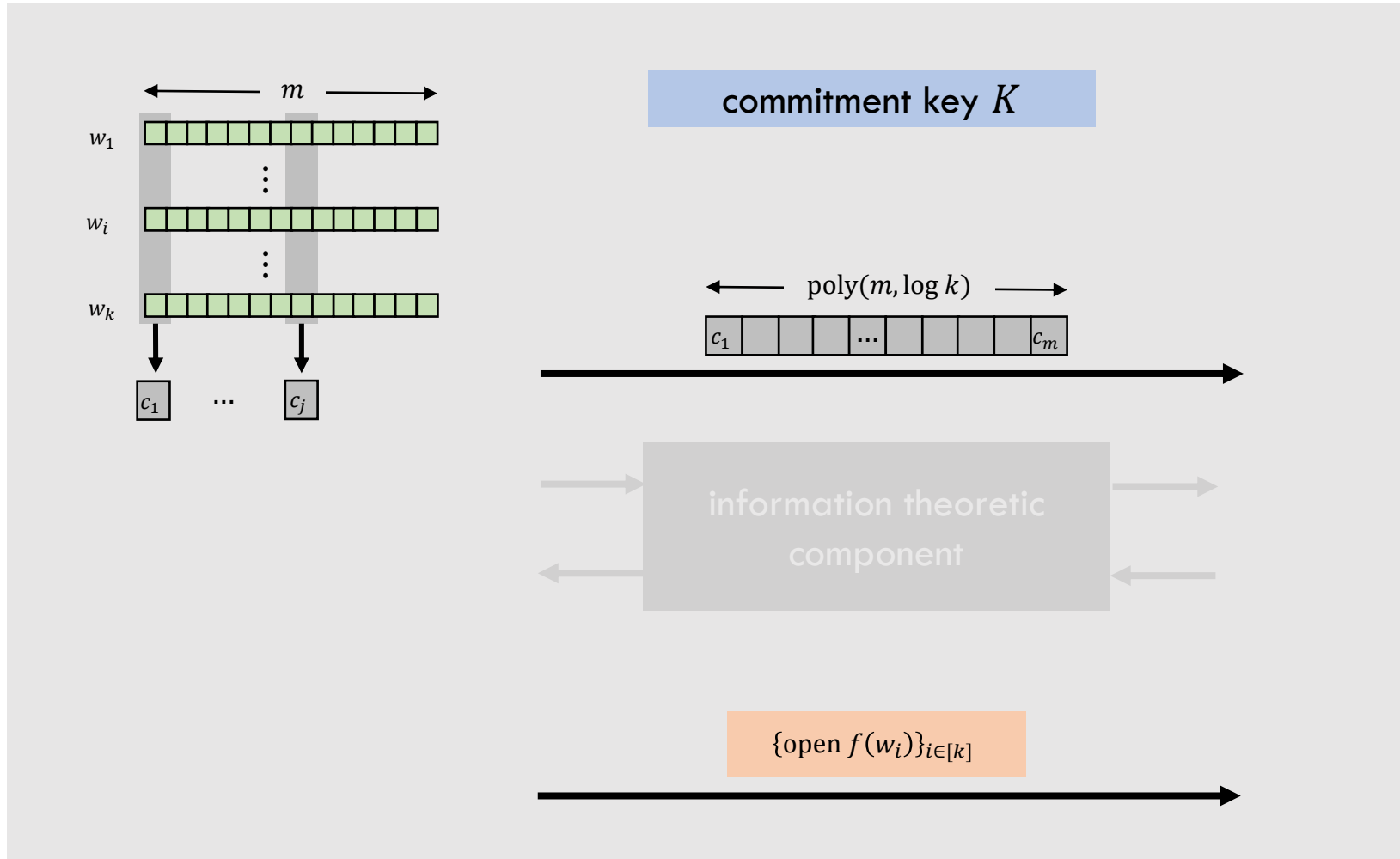
Somewhere Statistically Binding (SSB) Commitment Scheme

Needs to be Fiat-Shamir friendly.

Based on LWE/sub-exp DDH

Dual Mode Batch Argument

Protocol Template



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

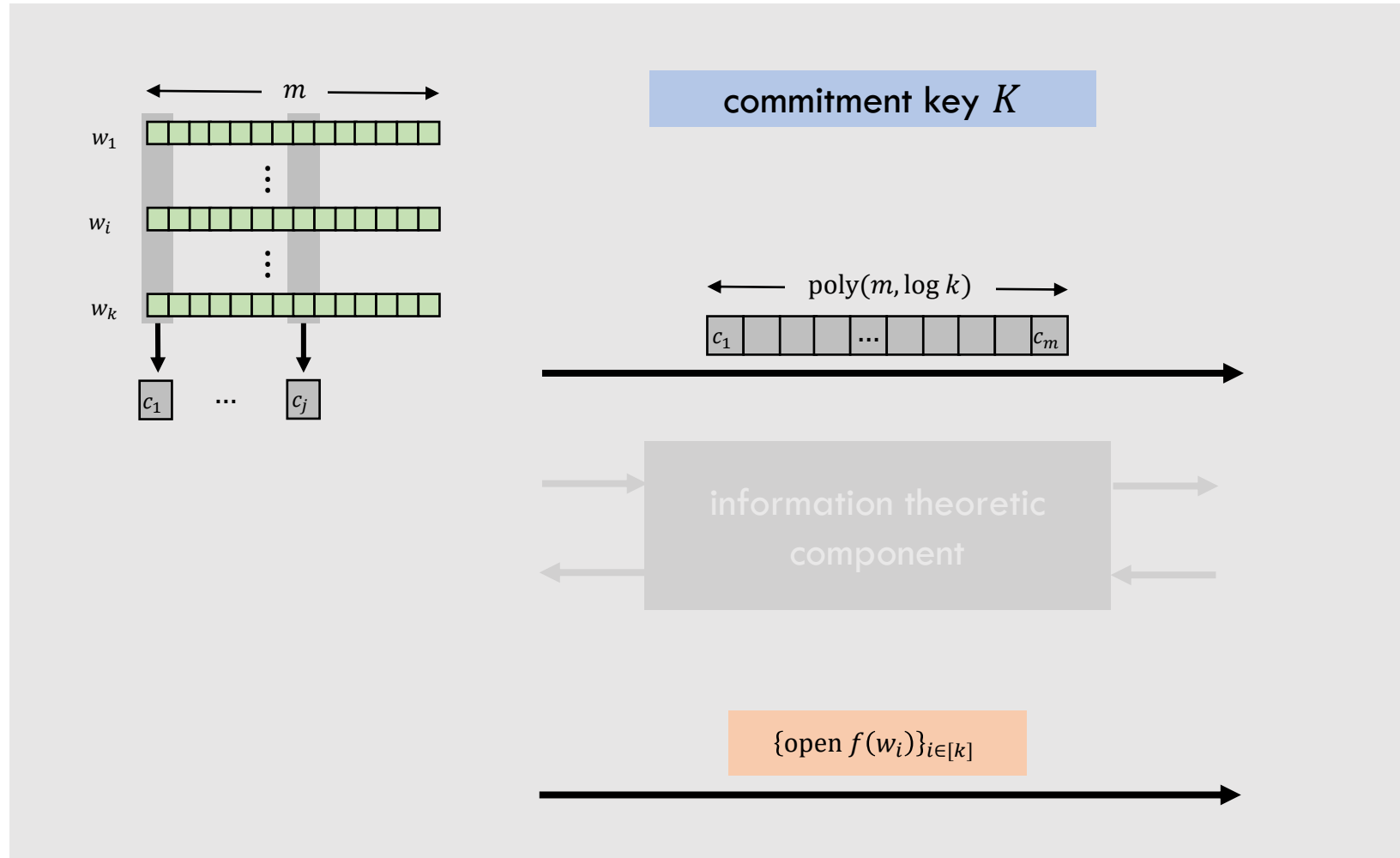
Somewhere Statistically Binding (SSB) Commitment Scheme

Needs to be Fiat-Shamir friendly.

Based on LWE/sub-exp DDH

Dual Mode Batch Argument

Protocol Template



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

We **construct** SSB with appropriate opening to f (with additional properties) based on QR

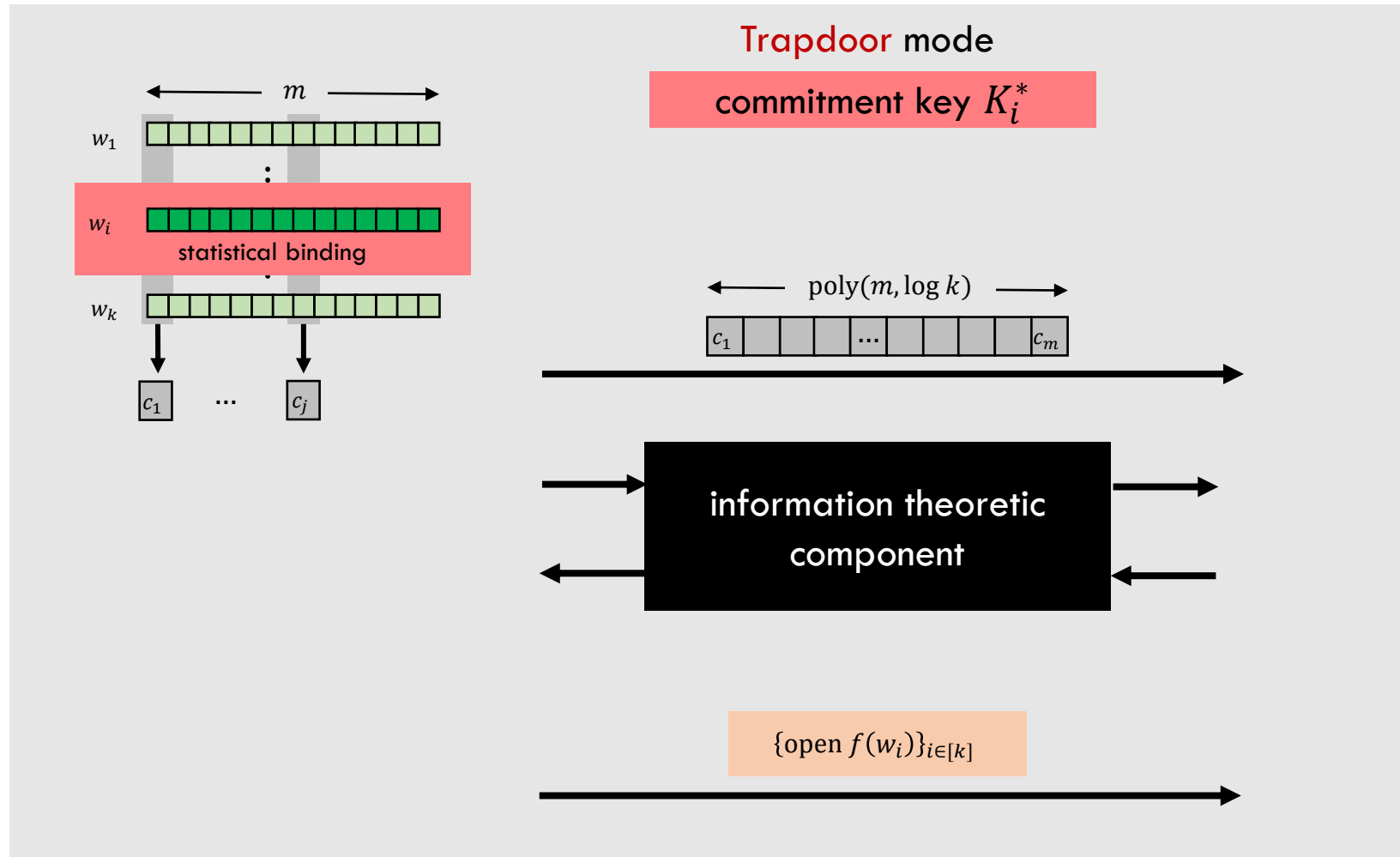
Needs to be **Fiat-Shamir** friendly.

Based on **LWE**/sub-exp **DDH**

(Some) Technical Details

Dual Mode Batch Argument

Protocol Template



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

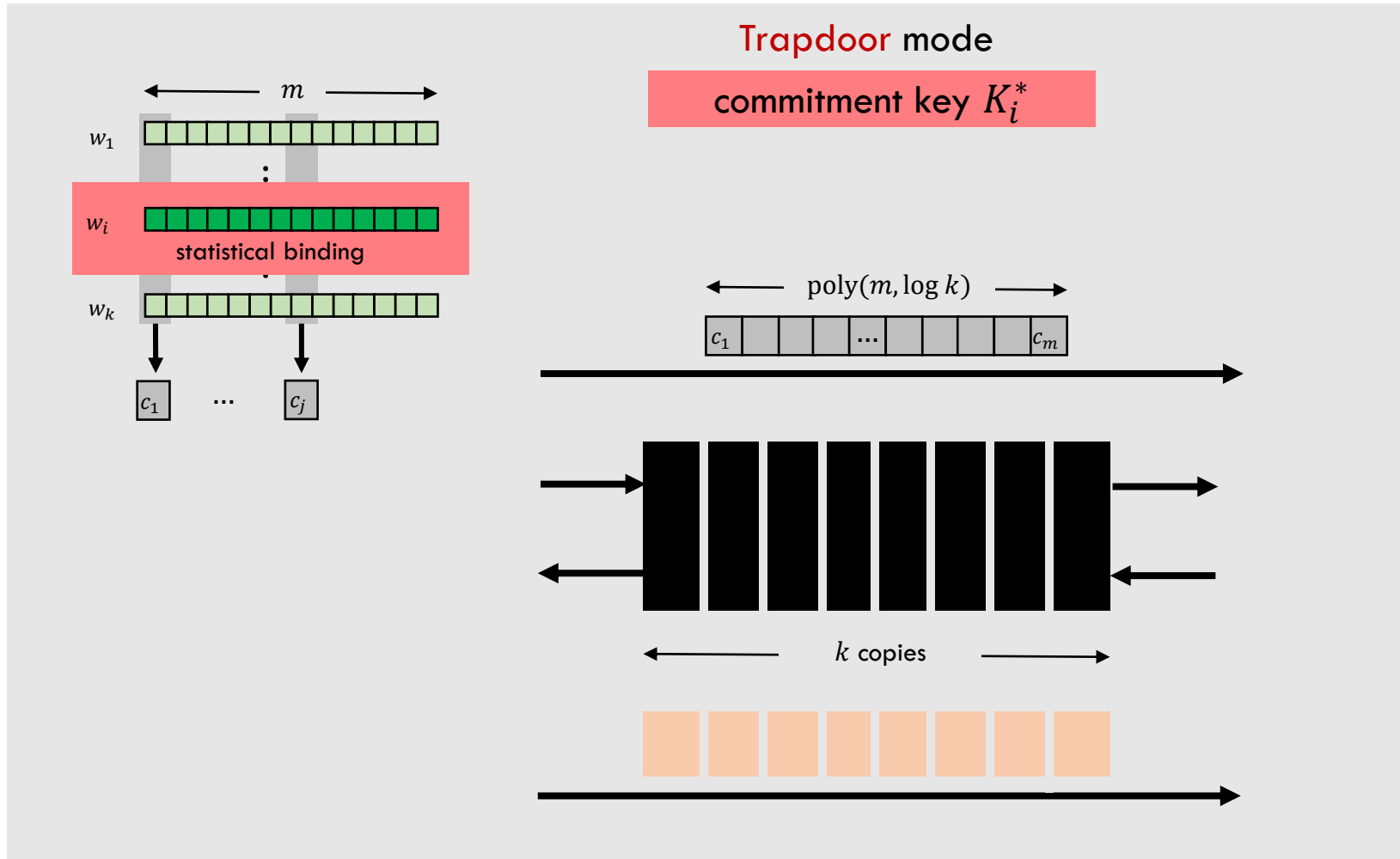
We **construct** SSB with appropriate opening to f (with additional properties) based on QR

Needs to be **Fiat-Shamir friendly**.

Based on **LWE/sub-exp DDH**

Dual Mode Batch Argument

Protocol Template



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

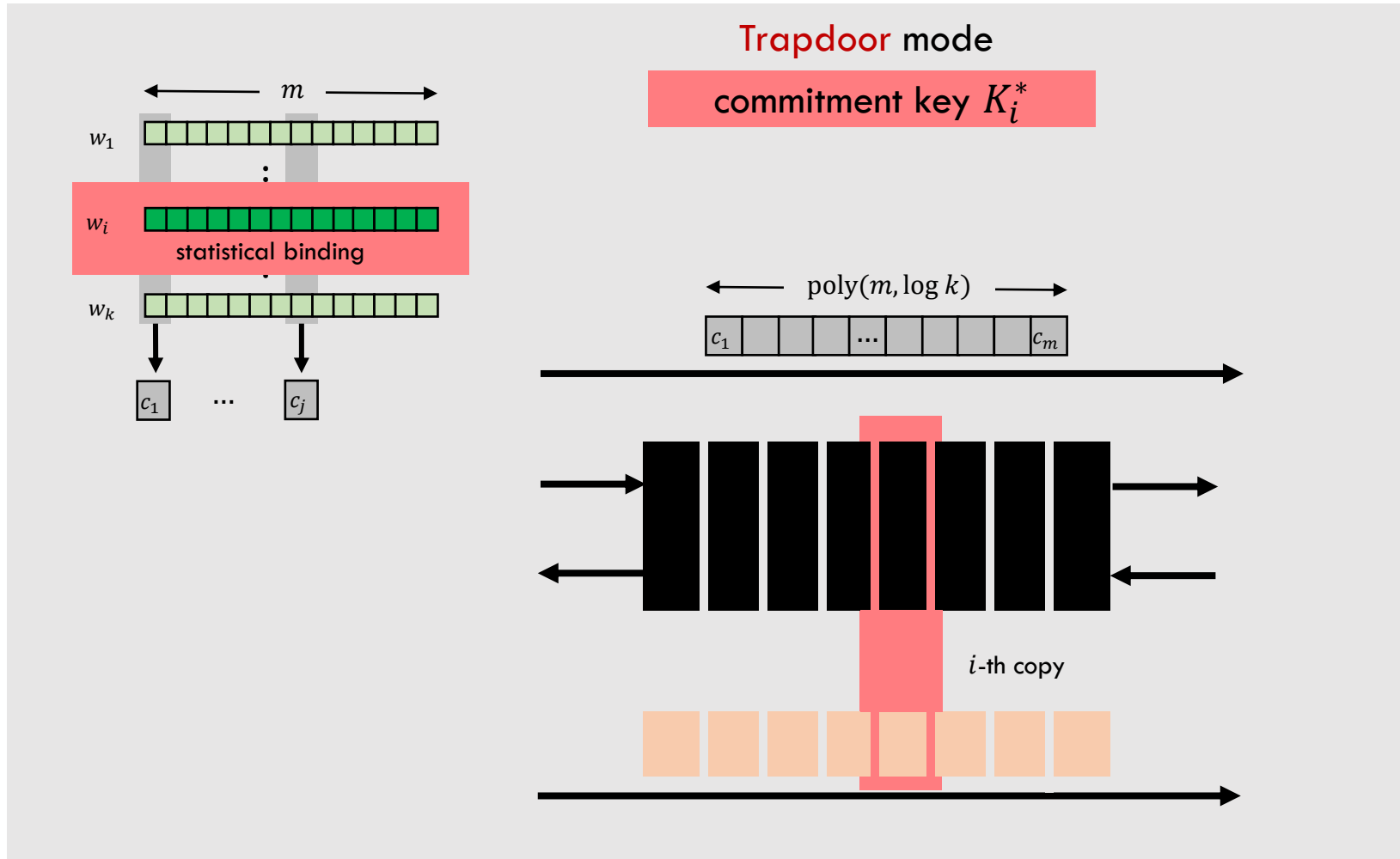
We construct SSB with appropriate opening to f (with additional properties) based on QR

Needs to be Fiat-Shamir friendly.

Based on LWE/sub-exp DDH

Dual Mode Batch Argument

Protocol Template



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

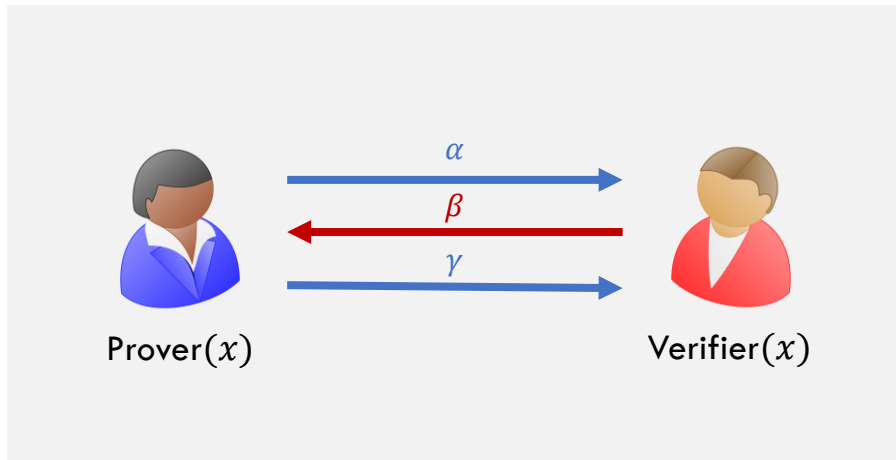
$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

We **construct** SSB with appropriate opening to f (with additional properties) based on QR

Needs to be **Fiat-Shamir** friendly.

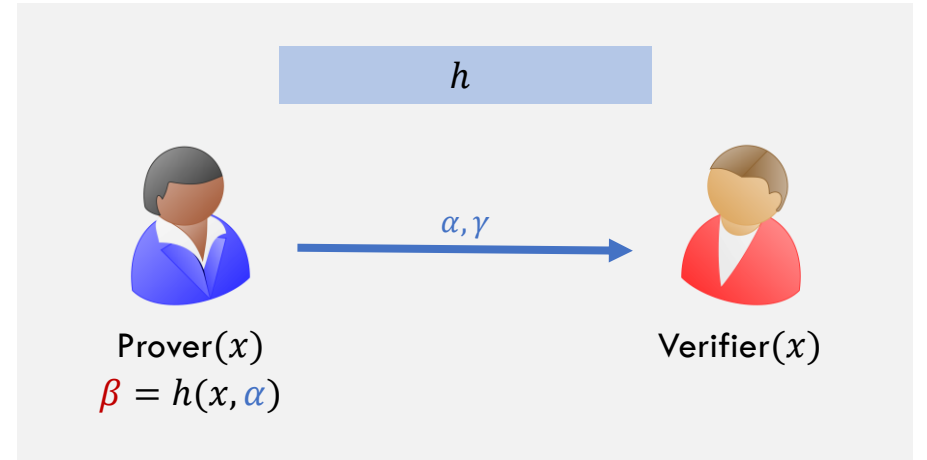
Based on **LWE**/sub-exp **DDH**

Fiat-Shamir (FS) Methodology

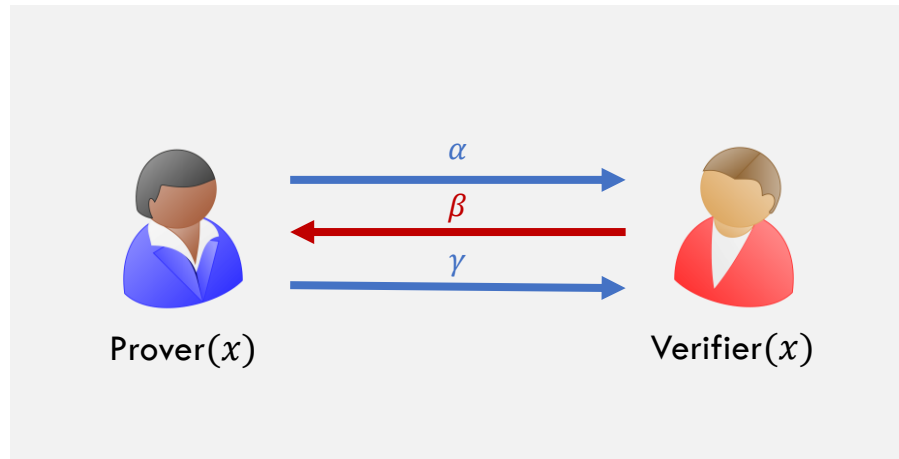


β is a random string

[Fiat-Shamir'86]

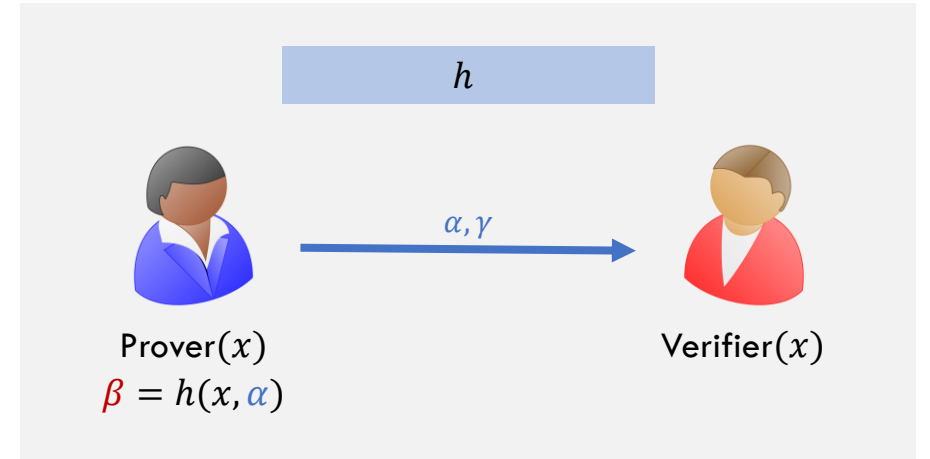


Fiat-Shamir (FS) Methodology



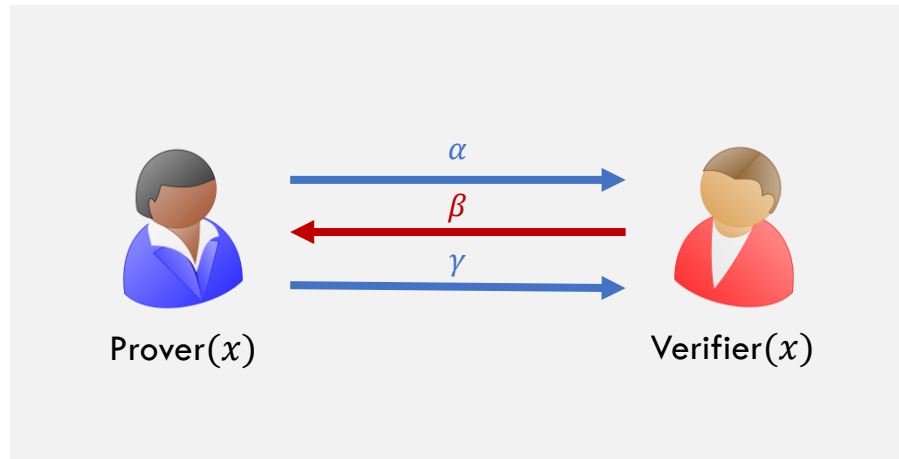
β is a random string

[Fiat-Shamir'86]



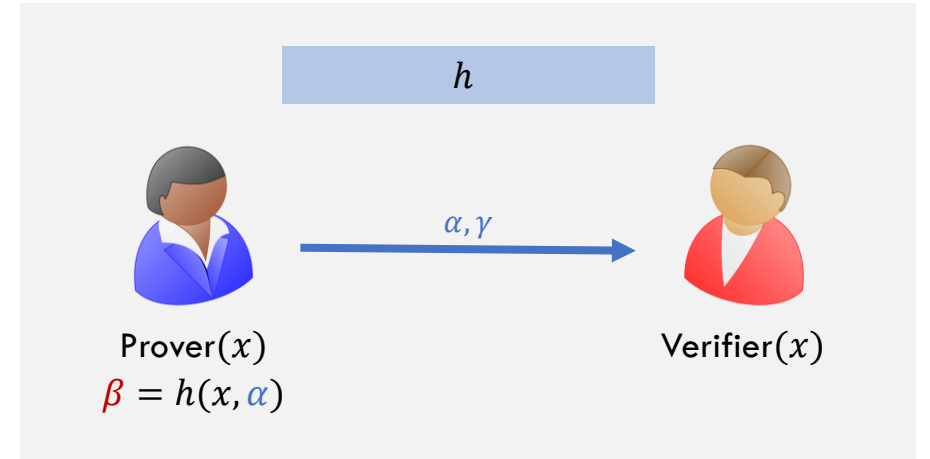
$$\forall x \notin \mathcal{L}$$
$$\text{BAD}_{x,\alpha} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$$

Fiat-Shamir (FS) Methodology




β is a random string

→
[Fiat-Shamir'86]

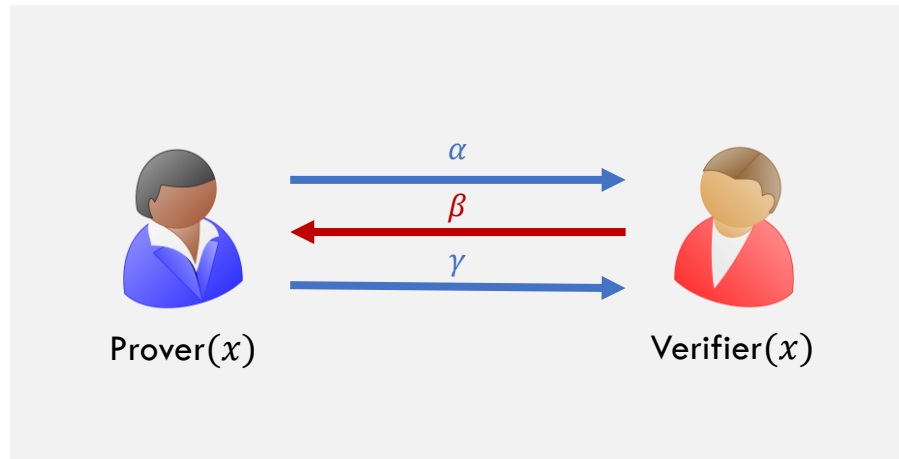


$\forall x \notin \mathcal{L}$
 $BAD_{x,\alpha} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$

If $x \notin \mathcal{L}$, no PPT  can find α such that

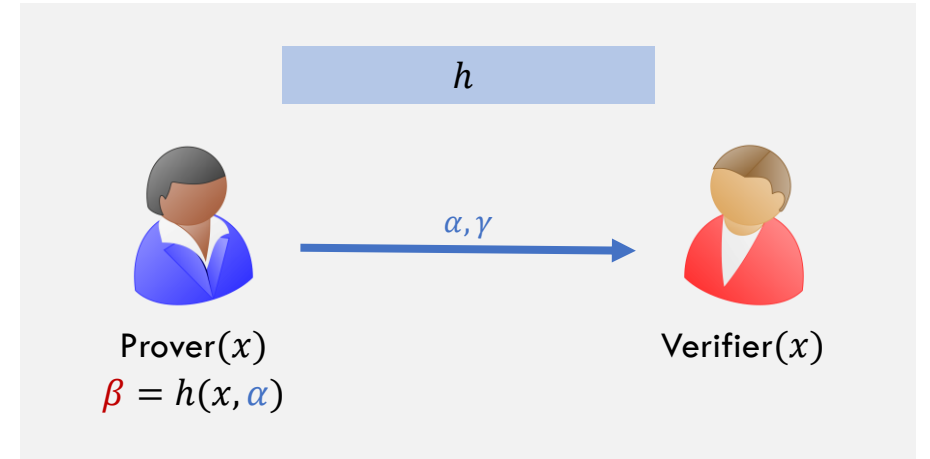
$h(x, \alpha) \in BAD_{x,\alpha}$

Fiat-Shamir (FS) Methodology



β is a random string

→
[Fiat-Shamir'86]



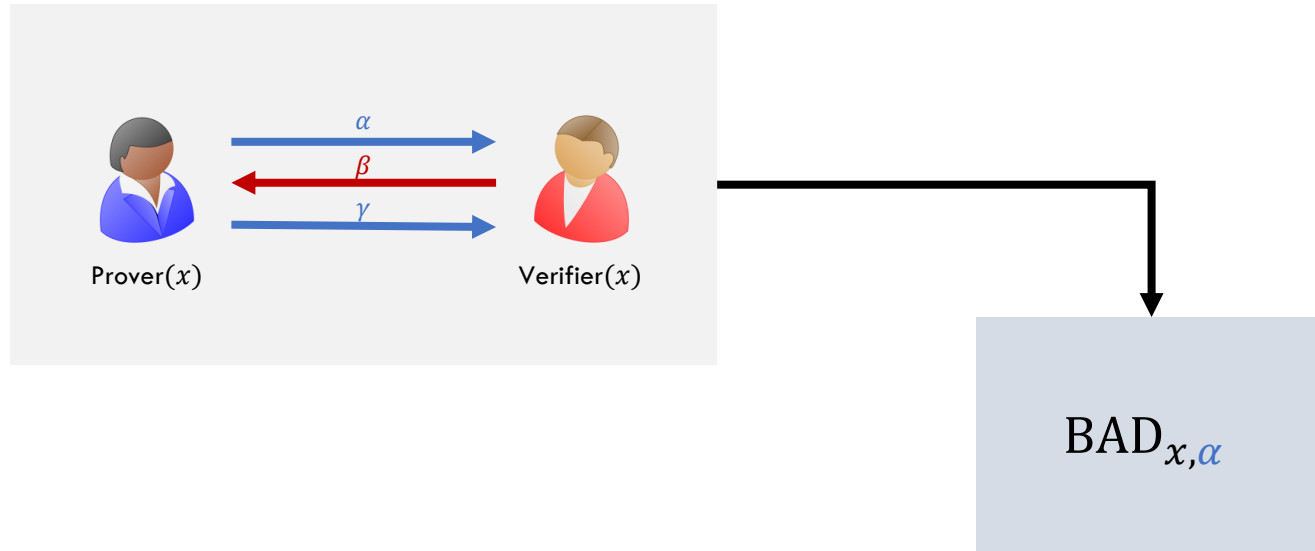
$$\forall x \notin \mathcal{L}$$
$$\text{BAD}_{x,\alpha} = \{\beta \mid \exists \gamma \text{ s.t. Verifier accepts } (\alpha, \beta, \gamma)\}$$

If $x \notin \mathcal{L}$, no PPT  can find α such that

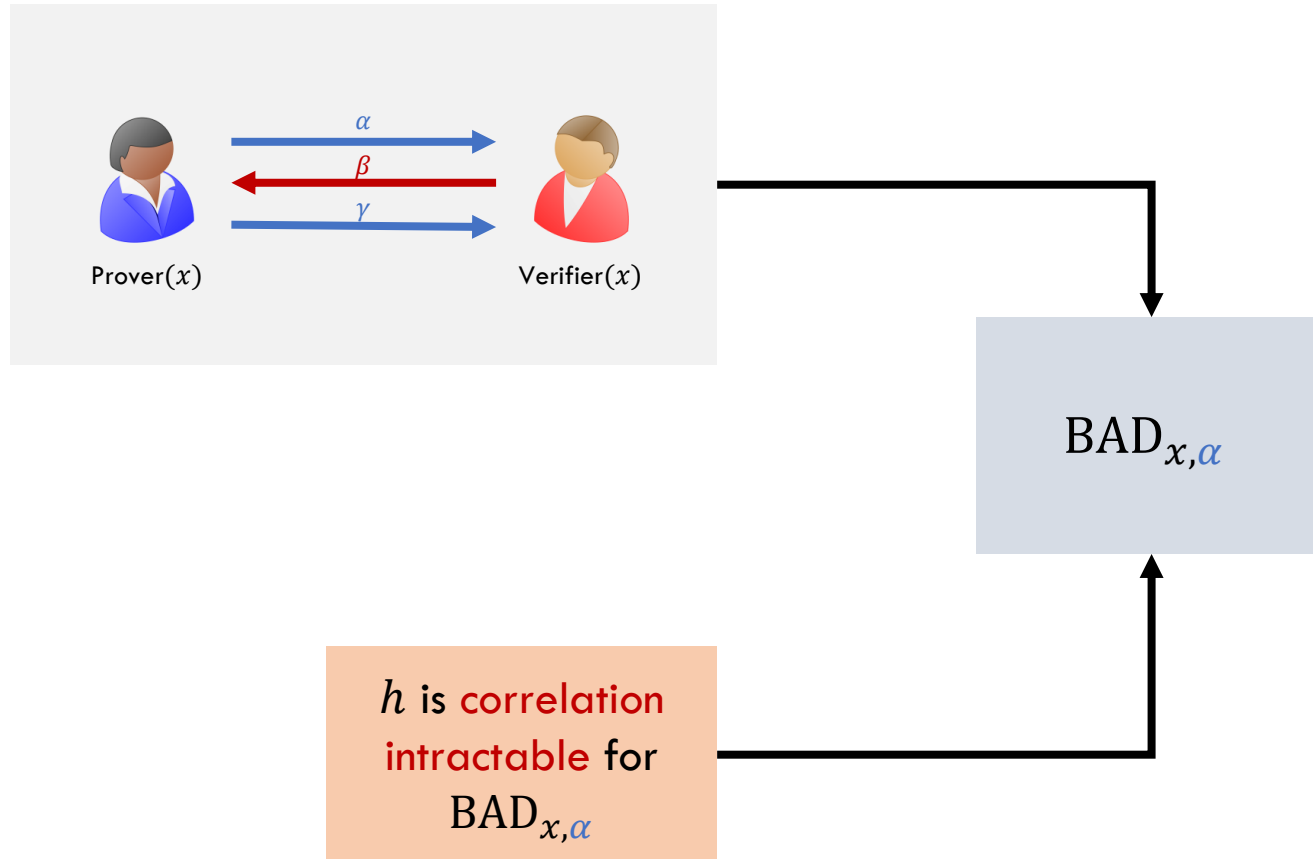
$$h(x, \alpha) \in \text{BAD}_{x,\alpha}$$

h is **correlation intractable (CI)** for $\text{BAD}_{x,\alpha}$

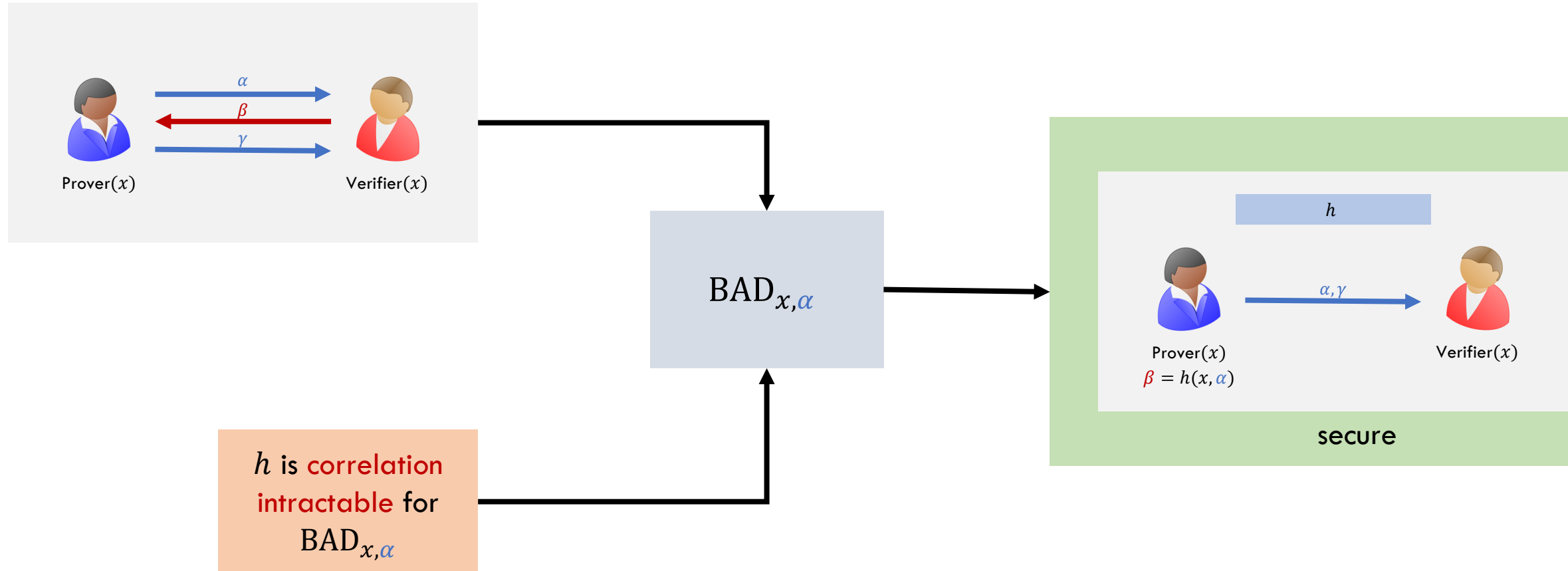
Instantiating the FS Transform



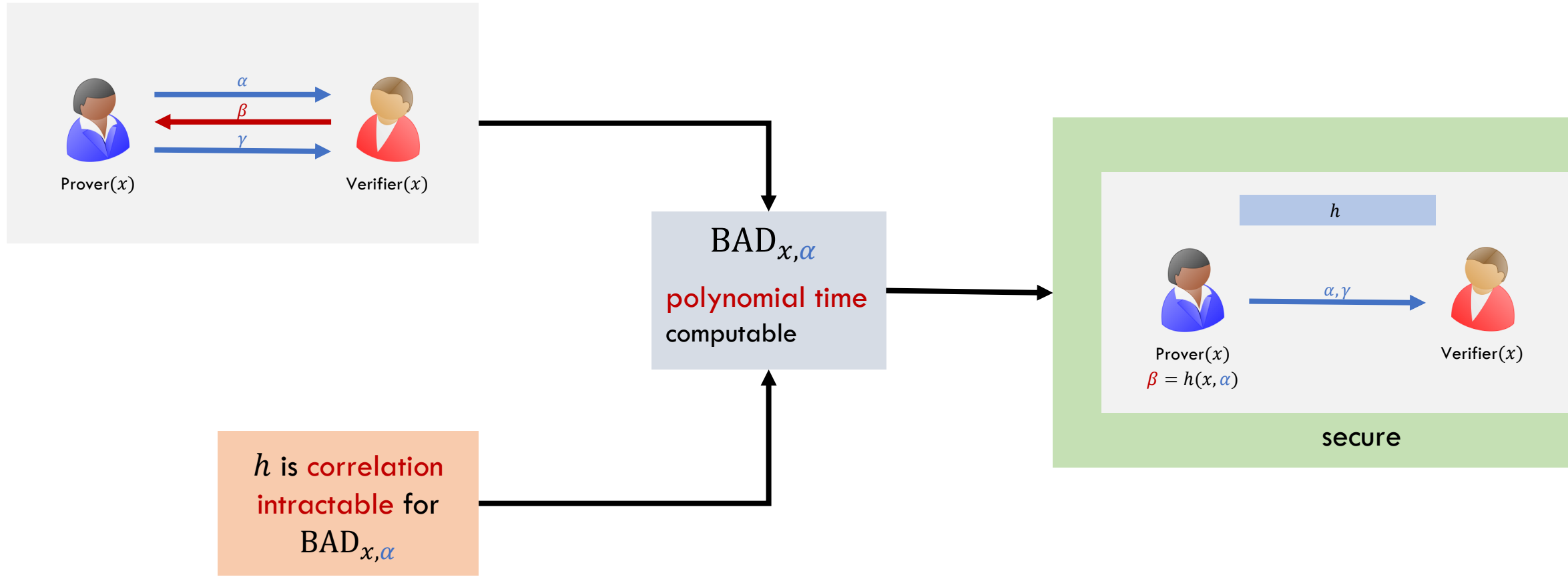
Instantiating the FS Transform



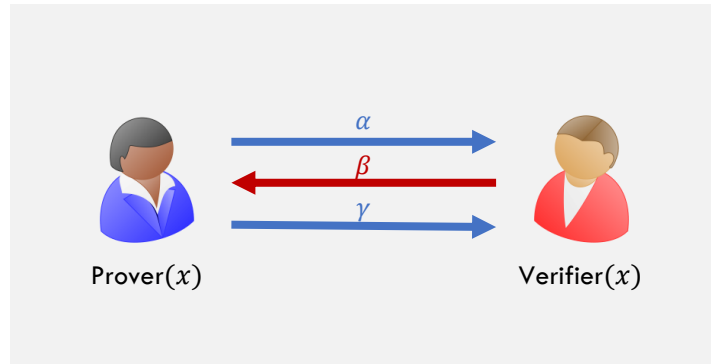
Instantiating the FS Transform



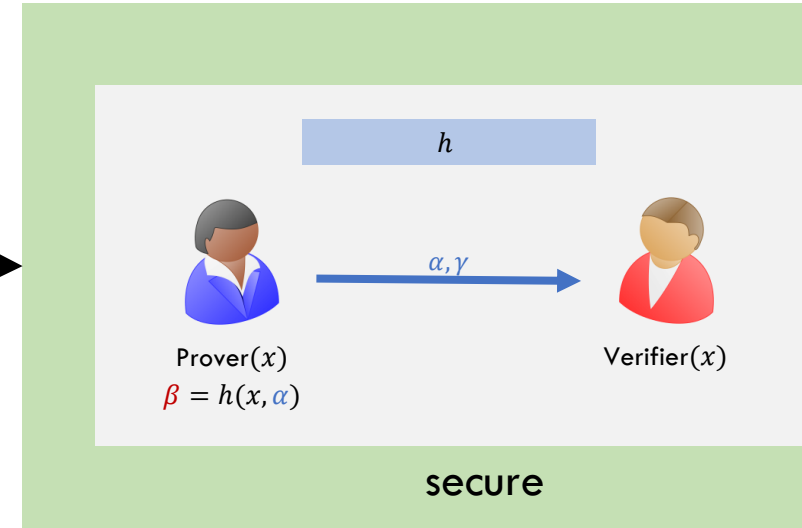
Instantiating the FS Transform



Instantiating the FS Transform



$BAD_{x,\alpha}$
polynomial time
computable

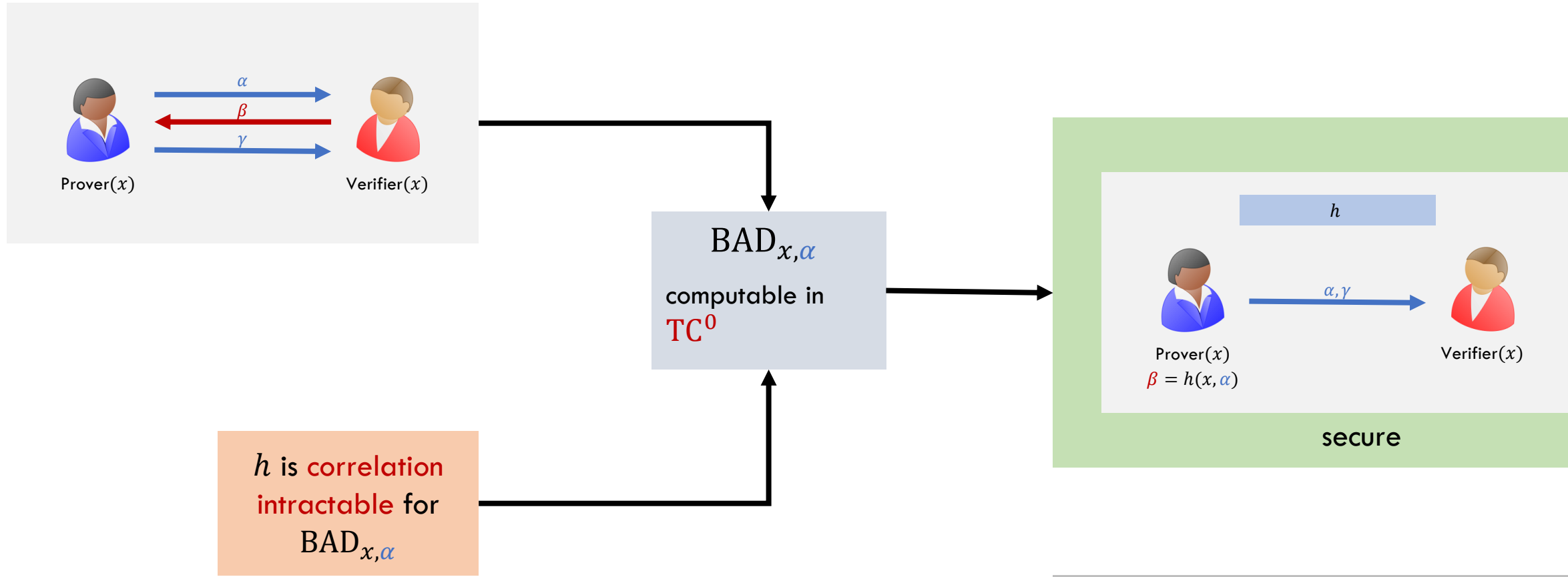


[Peikert-Shiehian'19]

LWE

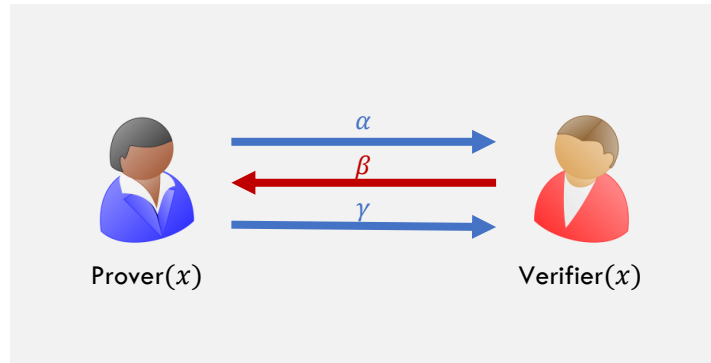
h is correlation
intractable for
 $BAD_{x,\alpha}$

Instantiating the FS Transform

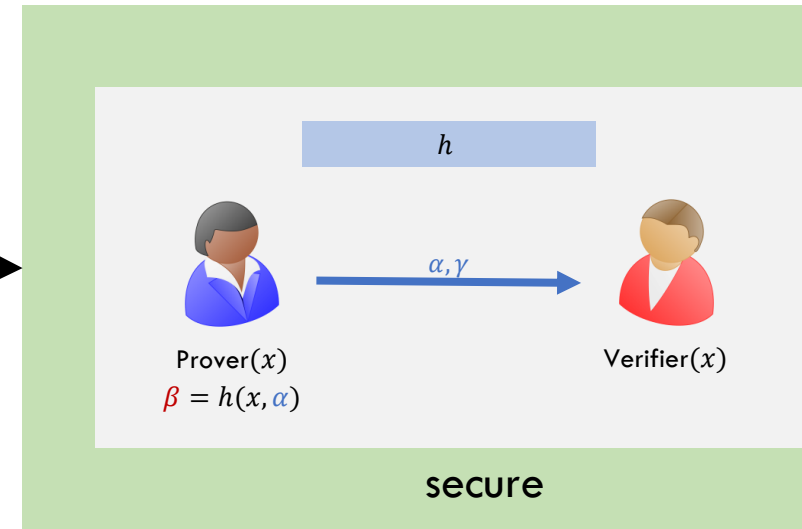


TC^0 - Constant depth polynomial-size threshold circuits

Instantiating the FS Transform



$BAD_{x,\alpha}$
computable in
 TC^0



[Jain-Jin'21]

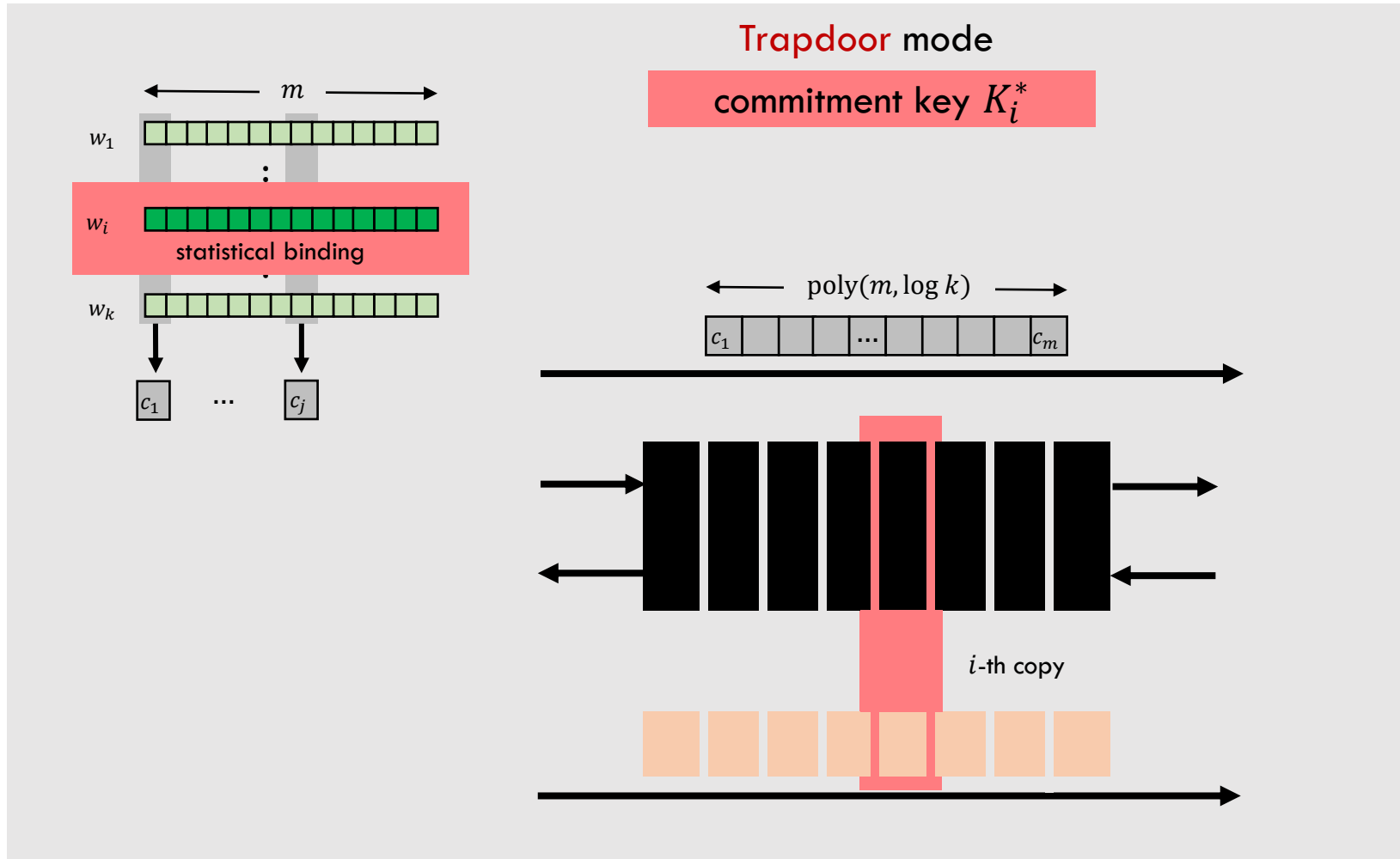
sub-exp
DDH

h is correlation
intractable for
 $BAD_{x,\alpha}$

TC^0 - Constant depth polynomial-size
threshold circuits

Dual Mode Batch Argument

Protocol Template



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

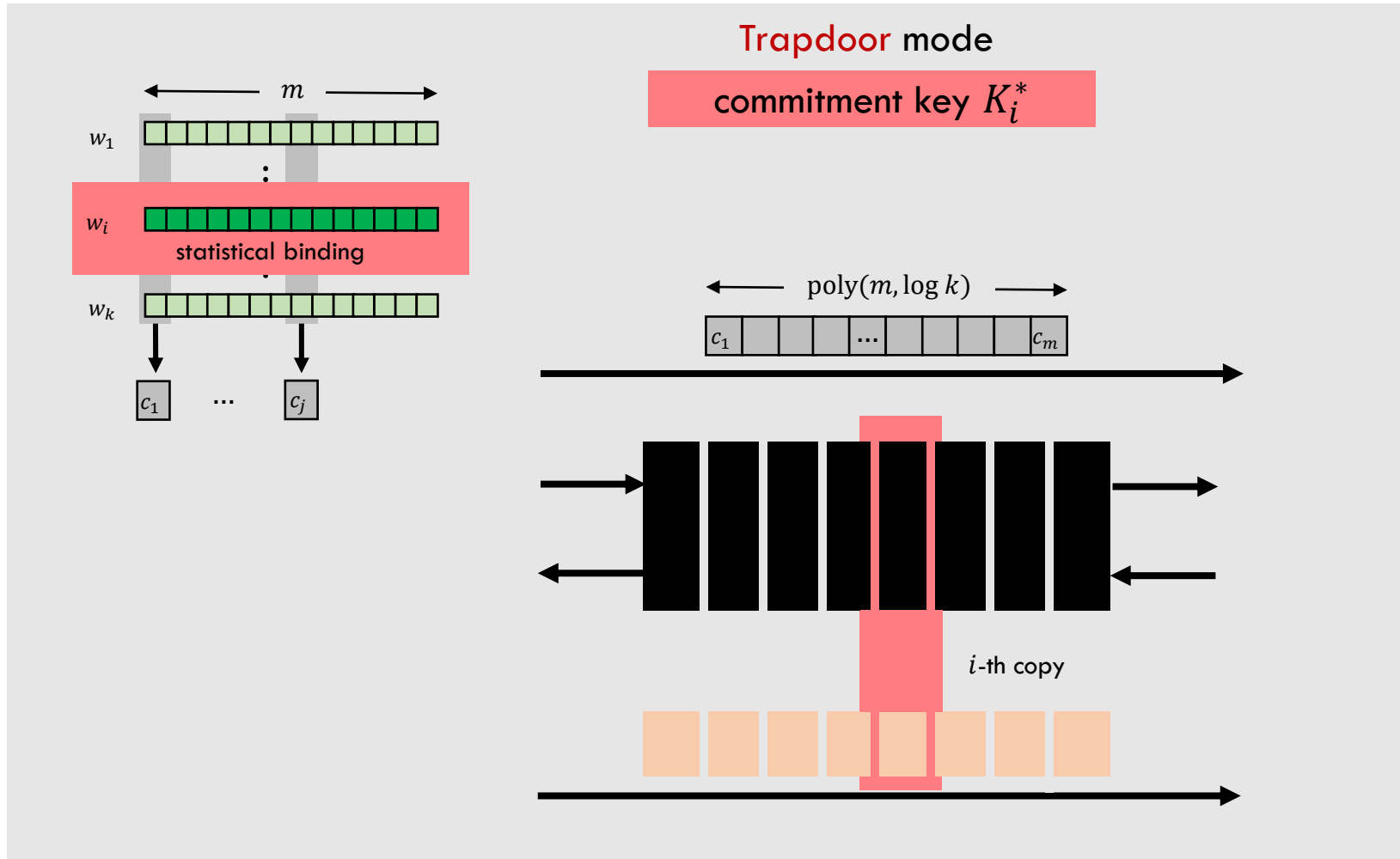
We construct SSB with appropriate opening to f (with additional properties) based on QR

Needs to be Fiat-Shamir friendly.

Based on LWE/sub-exp DDH

Dual Mode Batch Argument

Protocol Template



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

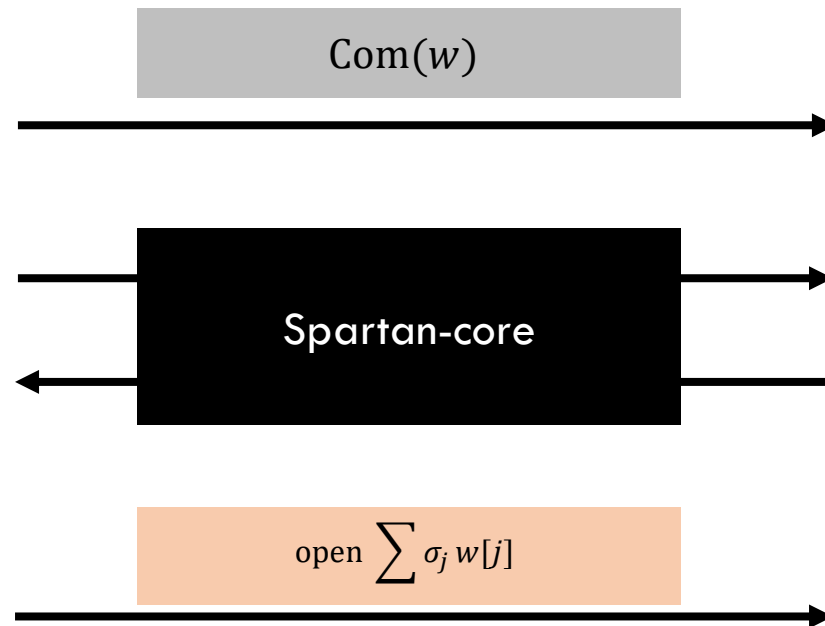
We construct SSB with appropriate opening to f (with additional properties) based on QR

BAD needs to be computable in TC^0 .

Based on LWE/sub-exp DDH

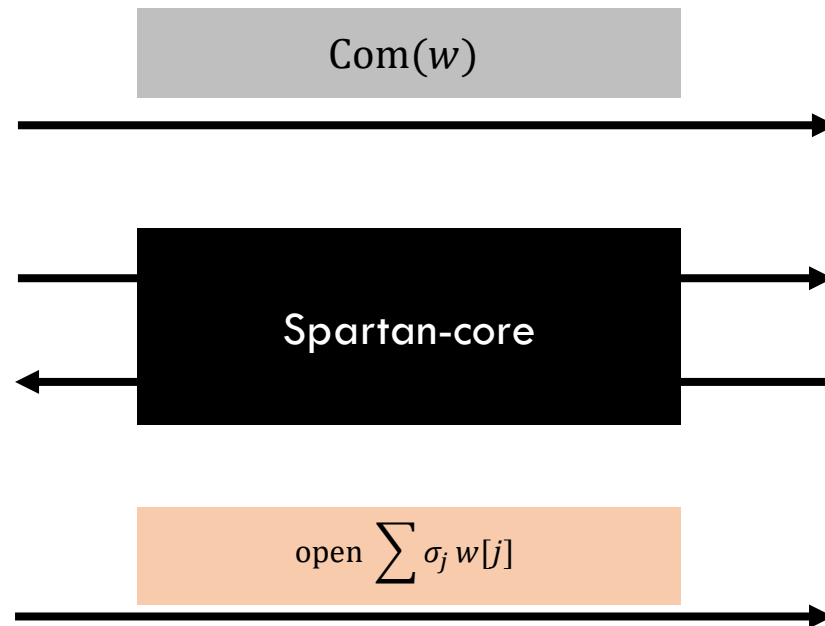
Building Block: Spartan [Setty'20]

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$



Building Block: Spartan [Setty'20]

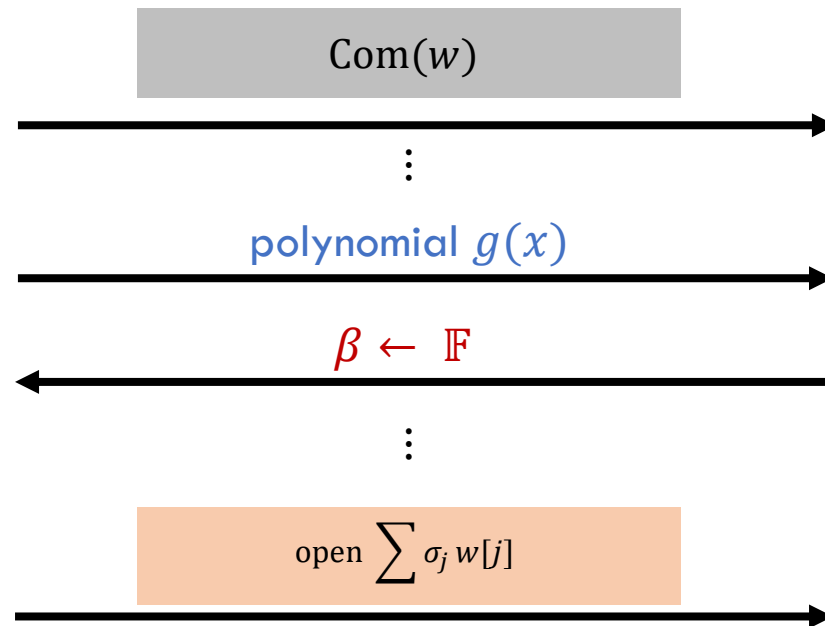
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$



Spartan-core primarily consists of the [Sumcheck protocol](#).

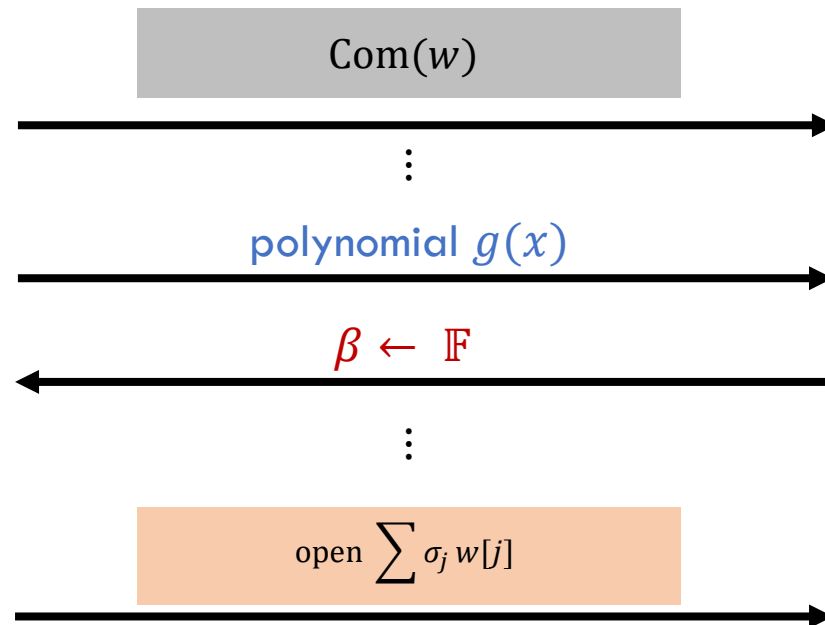
Building Block: Spartan [Setty'20]

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$



Building Block: Spartan [Setty'20]

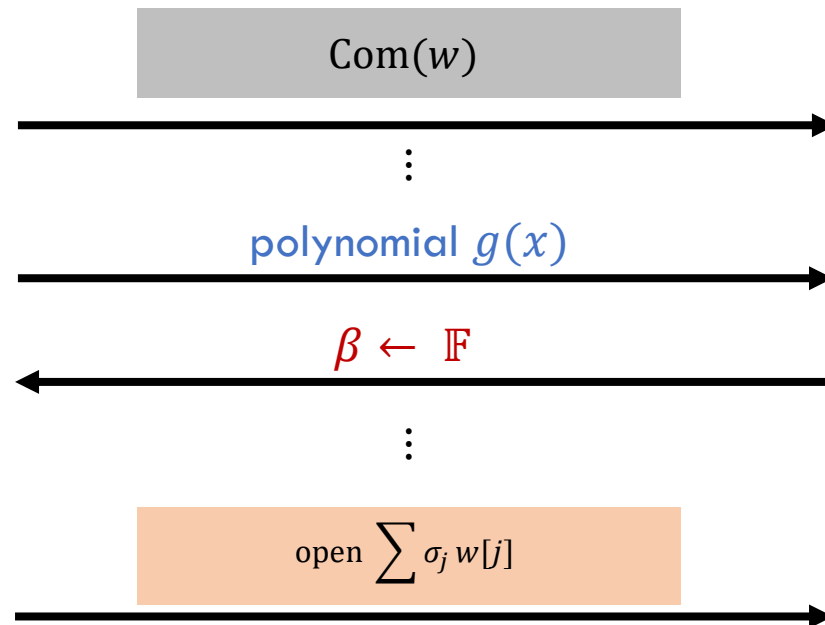
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$



$$\text{BAD} = \{\beta \in \mathbb{F} \mid \beta \text{ is a root of } g(x) - g_w^*(x)\}$$

Building Block: Spartan [Setty'20]

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$



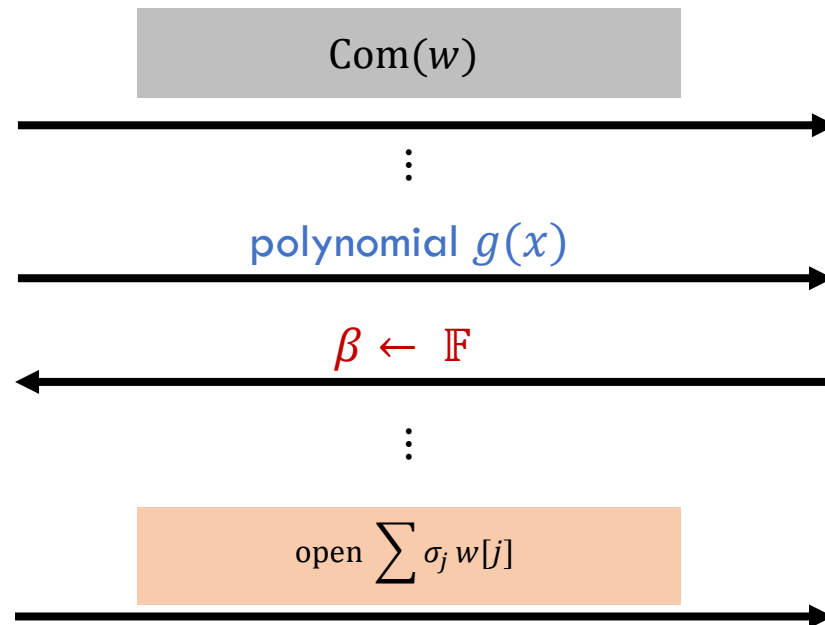
$$\text{BAD} = \{\beta \in \mathbb{F} \mid \beta \text{ is a root of } g(x) - g_w^*(x)\}$$

the "true" polynomial an honest prover would have sent



Building Block: Spartan [Setty'20]

$$\text{SAT} = \{(C, x) \mid \exists w \text{ s.t. } C(x, w) = 1\}$$

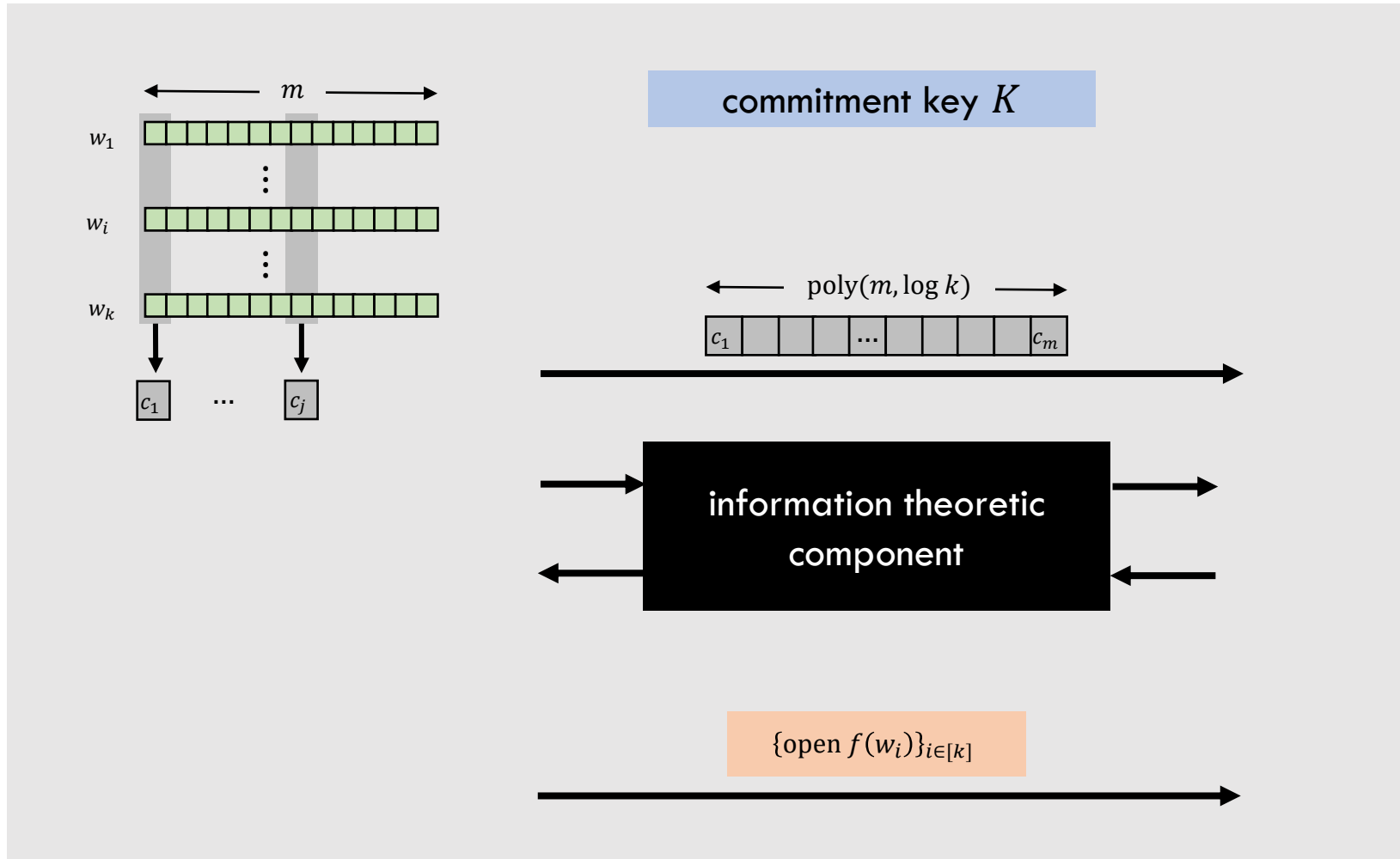


$$\text{BAD} = \{\beta \in \mathbb{F} \mid \beta \text{ is a root of } g(x) - g_w^*(x)\}$$

Show that appropriate field \mathbb{F} , **BAD** can be computed in TC^0 .

Dual Mode Batch Argument

Protocol Template



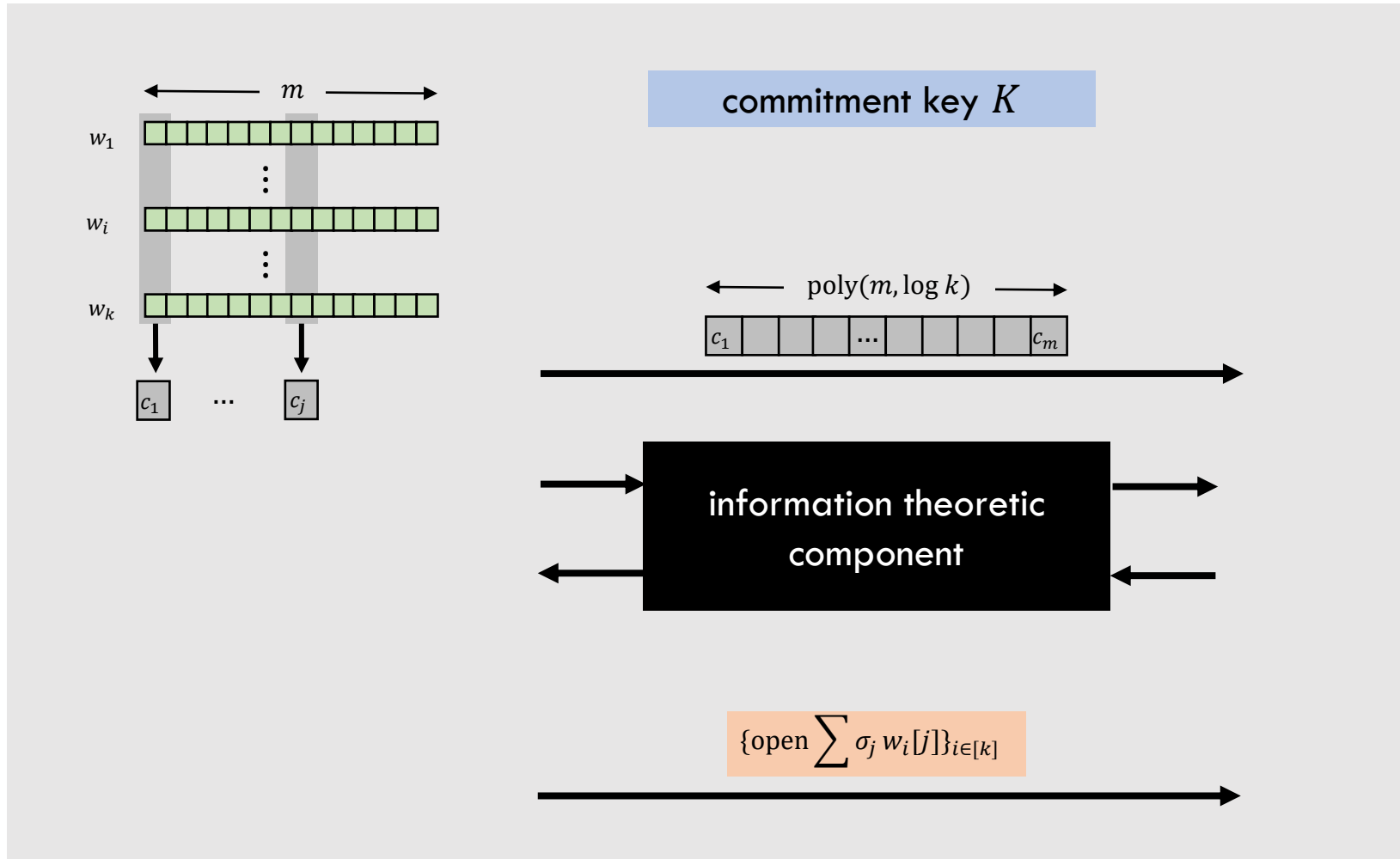
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

We **construct** SSB with appropriate opening to f (with additional properties) based on QR

Dual Mode Batch Argument

Protocol Template



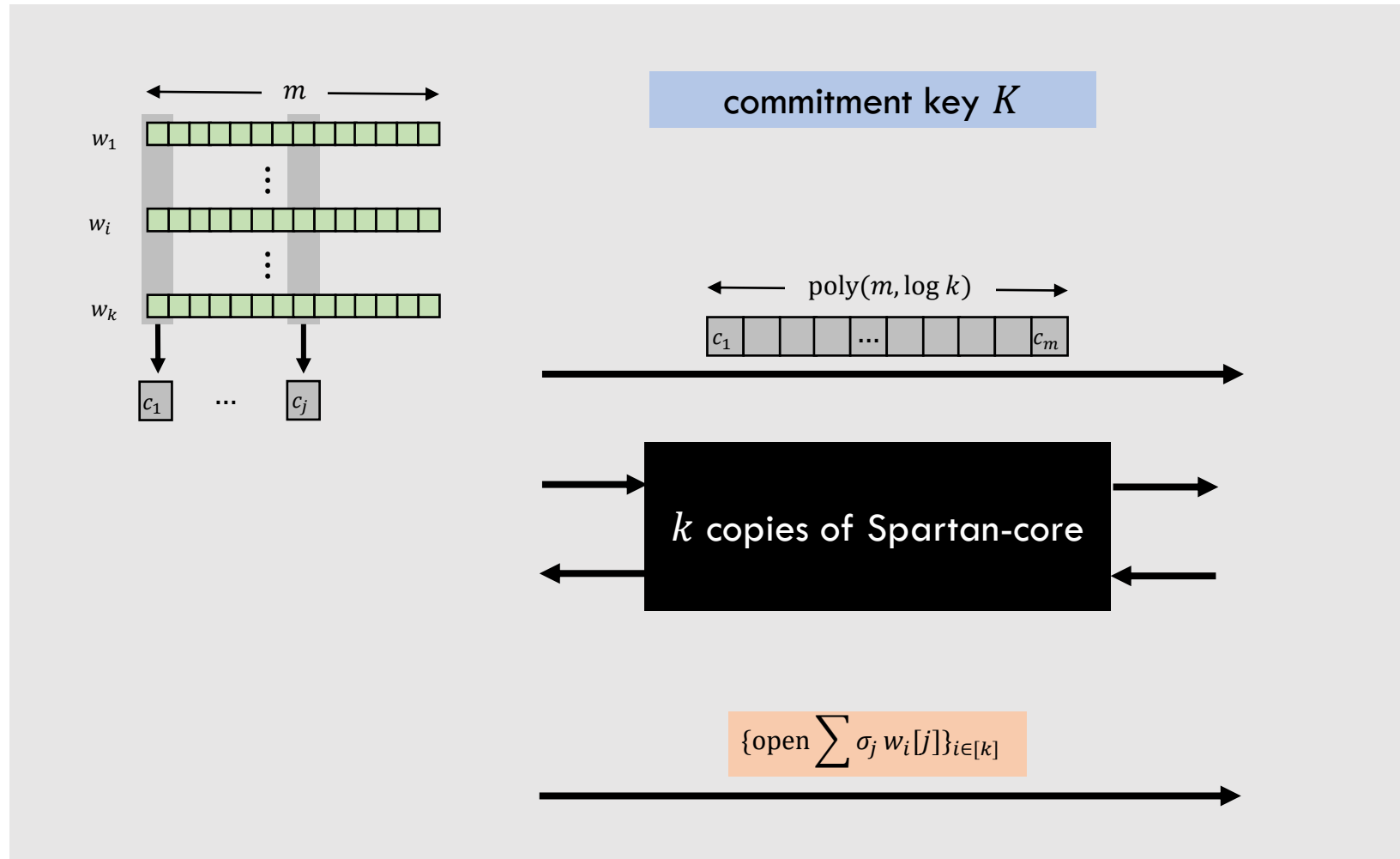
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

We construct SSB with linear homomorphic opening (with additional properties) based on QR

Dual Mode Batch Argument

Protocol Template



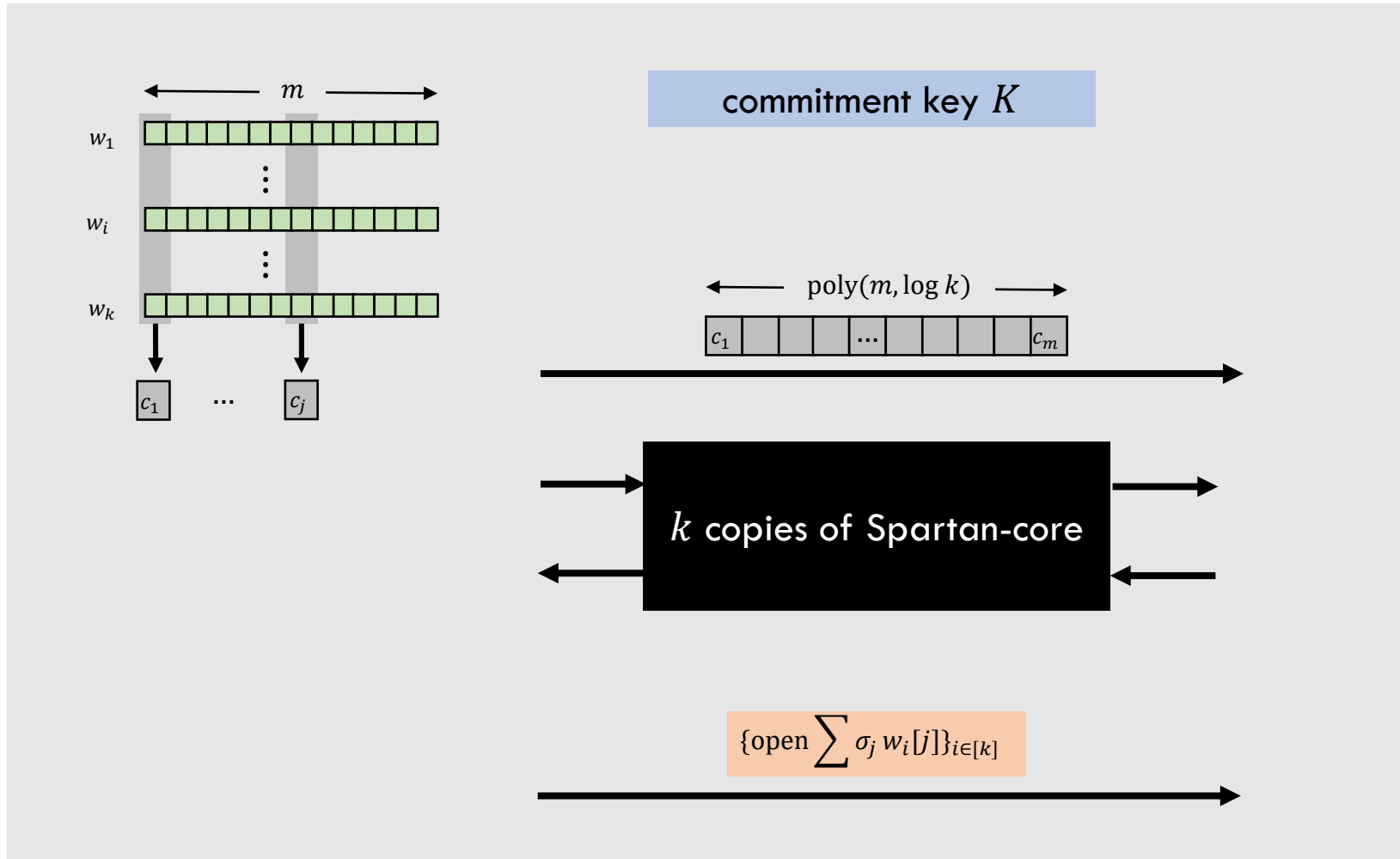
$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

We construct SSB with linear homomorphic opening (with additional properties) based on QR

Dual Mode Batch Argument

Protocol Template



$$\text{SAT} = \{(C, x) \mid \exists w \text{ s. t. } C(x, w) = 1\}$$

$$\forall i \in [k], (C, x_i) \in \text{SAT}$$

We construct SSB with linear homomorphic opening (with additional properties) based on QR

BAD computable in TC^0 .

Concluding Remarks

Theorem

Assuming QR + (LWE/sub-exp DDH) there exists a non-interactive batch argument for NP where

$$|\Pi| = \tilde{O}(|C| + \sqrt{k|C|})$$

Concluding Remarks

Theorem

Assuming QR + (LWE/sub-exp DDH) there exists a non-interactive batch argument for NP where

$$|\Pi| = \tilde{O}(|C| + \sqrt{k|C|})$$

Follow up work [C-Jain-Jin'21 b] (ia.cr/2021/808)

- Batch arguments for NP with improved parameters
- Application of batch arguments to construct delegation scheme for \mathcal{P}

Thank you. Questions?

Arka Rai Choudhuri

achoud@cs.jhu.edu

ia.cr/2021/807