

# Fine-Grained Secure Attribute-Based Encryption

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# Standard cryptography

Honest party



polynomial-time

Adversary



polynomial-time

Assumption:

- Basic ones (e.g., one-way function)
- More advanced ones (e.g., factoring, discrete logarithm, DDH, LWE)
- Exotic ones (e.g., generic groups, algebraic groups)



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# Fine-grained cryptography

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An honest party uses less  
resources than the  
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The resources of an adversary can be a-prior bounded



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- Based only on mild assumption



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Existing fine-grained primitives:  
NIKE [Mer78], OWF [BC20], PKE [DVV16], verifiable computation [CG18], HPS [EWT19], trapdoor one-way functions [EWT21]

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(signature is not among them)

- Based only on mild assumption





# Our results

Fine-grained secure ABE



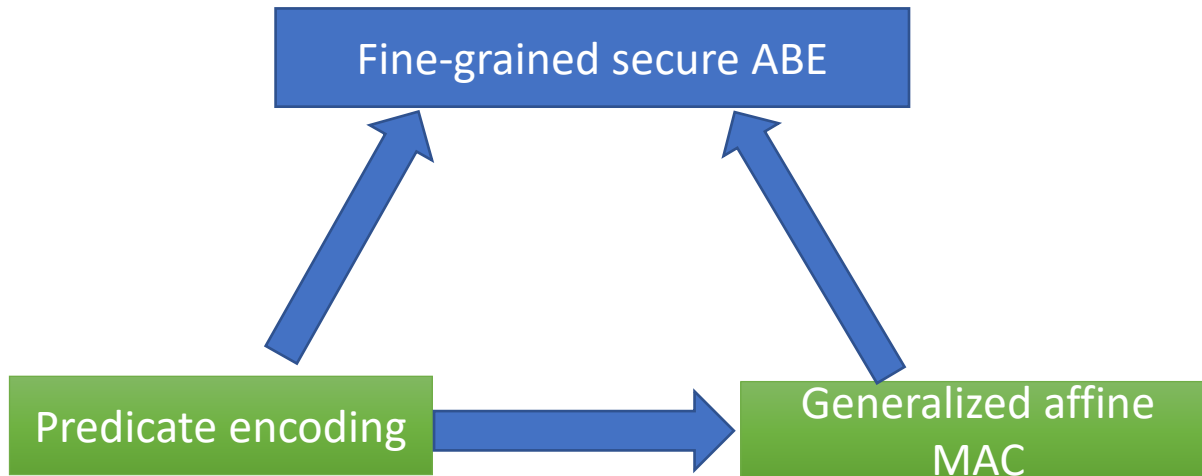
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Fine-grained secure ABE

IBE from affine MAC [BKP14]  
+  
ABE from predicate encodings [CGW15]



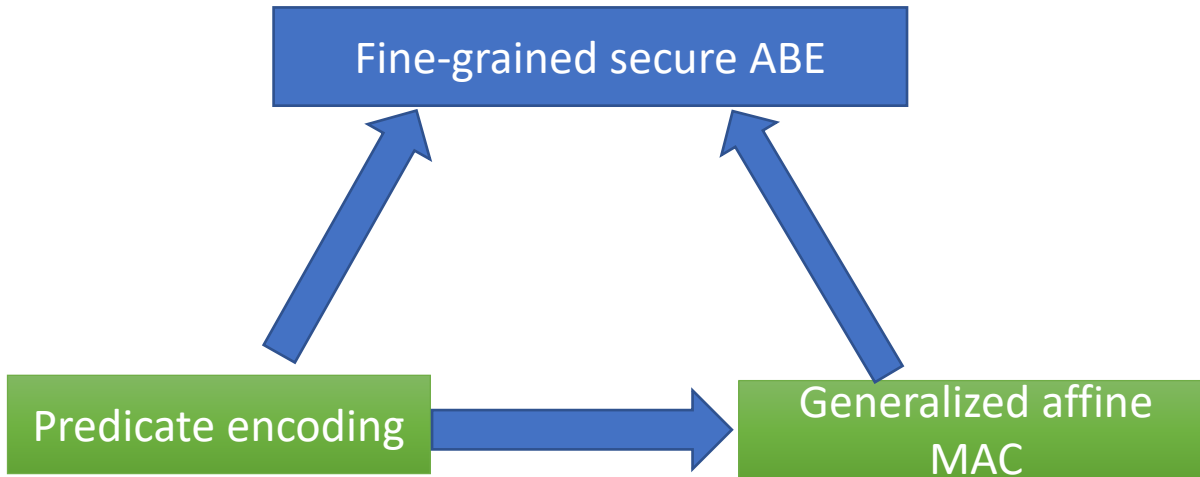
# Our results



IBE from affine MAC [BKP14]  
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# Our results



A dual system in the fine-grained-setting

ABE from predicate encodings [CGW15]



# Our results

By suitably instantiating the underlying **predicate encoding**, we obtain:

1. IBE scheme (which in turn implies a signature scheme)
2. ABEs for
  - a. inner-product encryption
  - b. non-zero inner-product
  - c. encryption spatial encryption
  - d. doubly spatial encryption
  - e. boolean span programs
  - f. arithmetic span programs
3. Broadcast encryption
4. fuzzy IBE

in the **fine-grained setting**.



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(same as the bounded-circuit setting in previous works on fine-grained cryptography [DVV16,CG18,EWT19])



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Circuits with constant depth, polynomial size, and unbounded fan-in using AND, OR, NOT, and PARITY gates.



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Circuits with logarithmic depth, polynomial size, and fan-in 2 gates.



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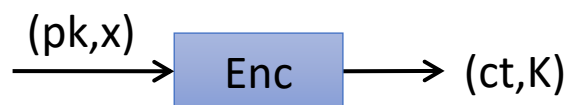
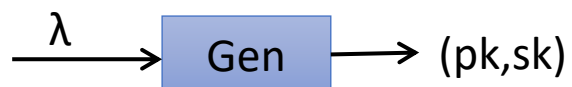
Log space turing machine with parity acceptance.



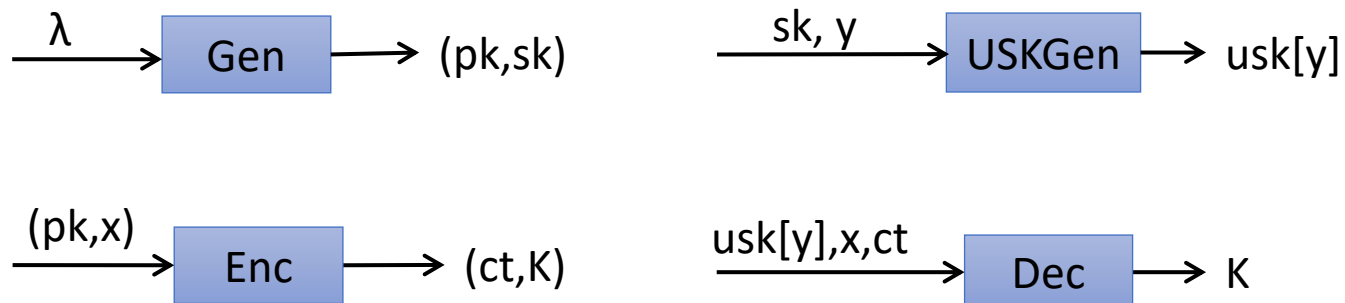
# Attribute-based key encapsulation (ABKEM)



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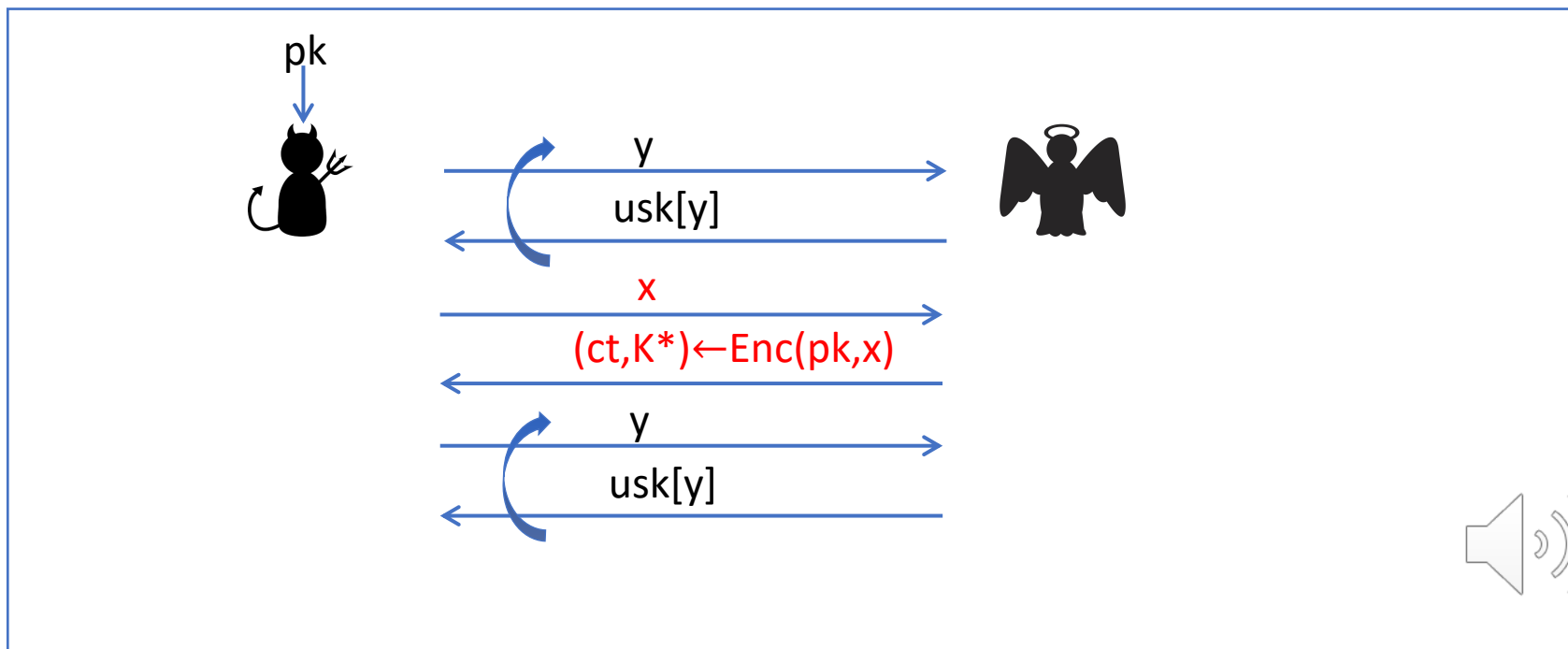
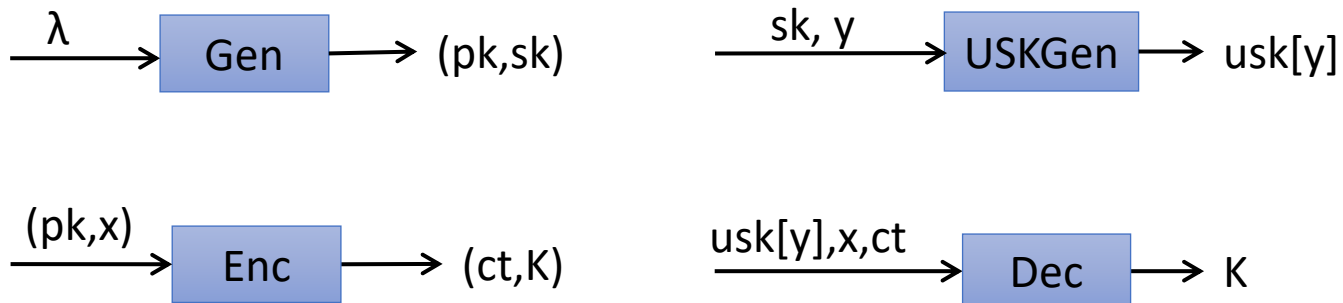
# Attribute-based key encapsulation (ABKEM)



Correctness:  $K$  can be correctly recovered by Dec if  $p(x,y)=1$

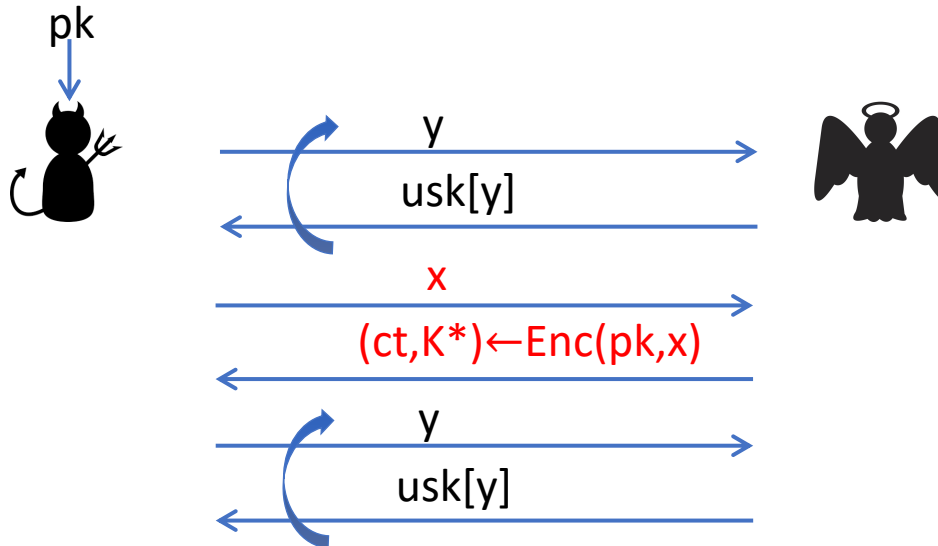
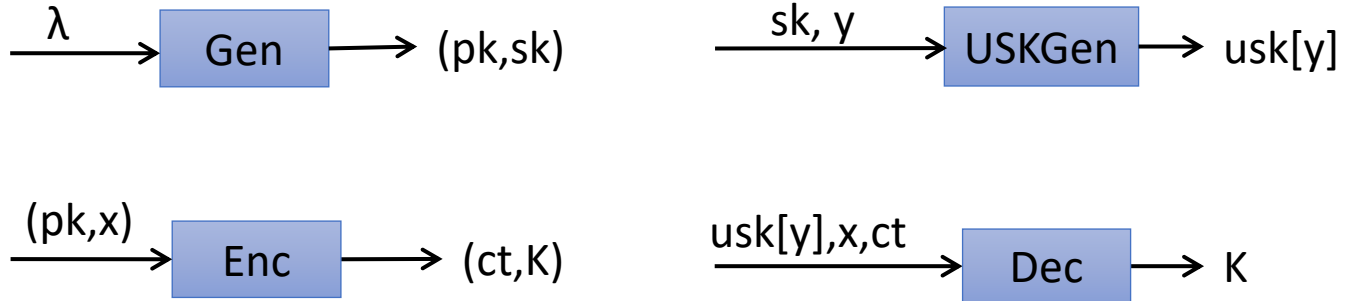


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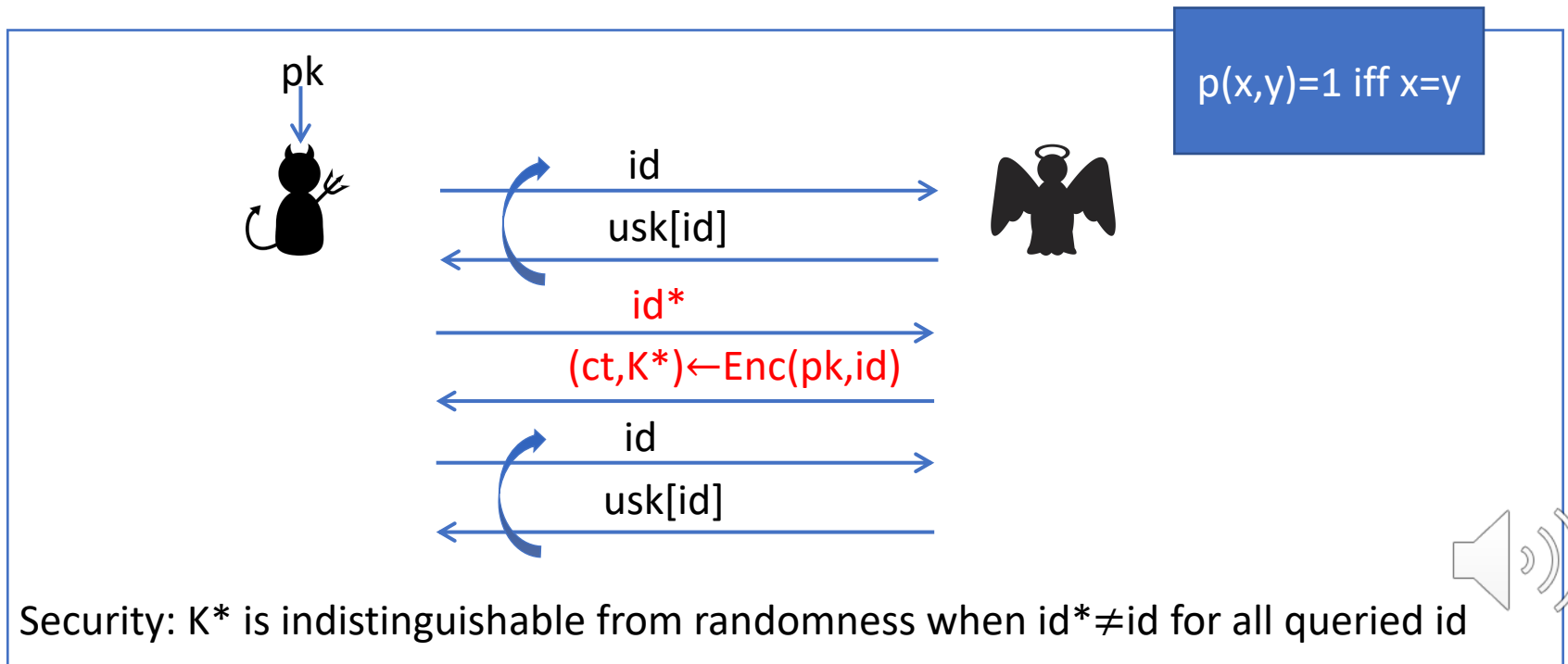
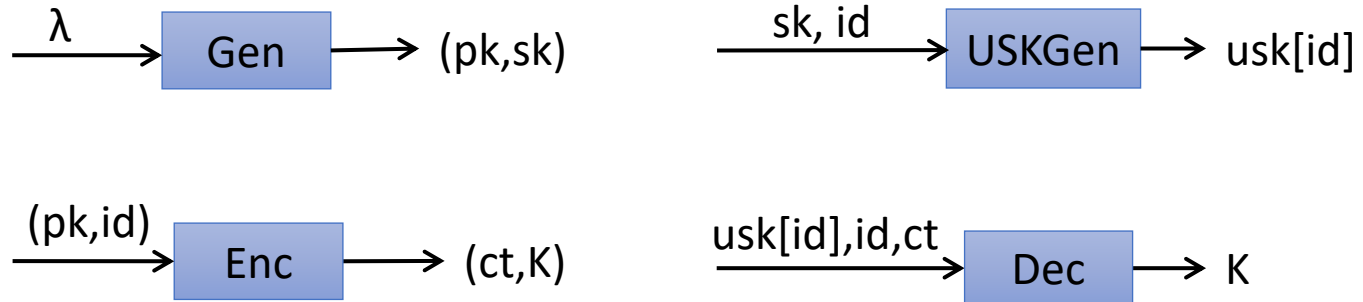
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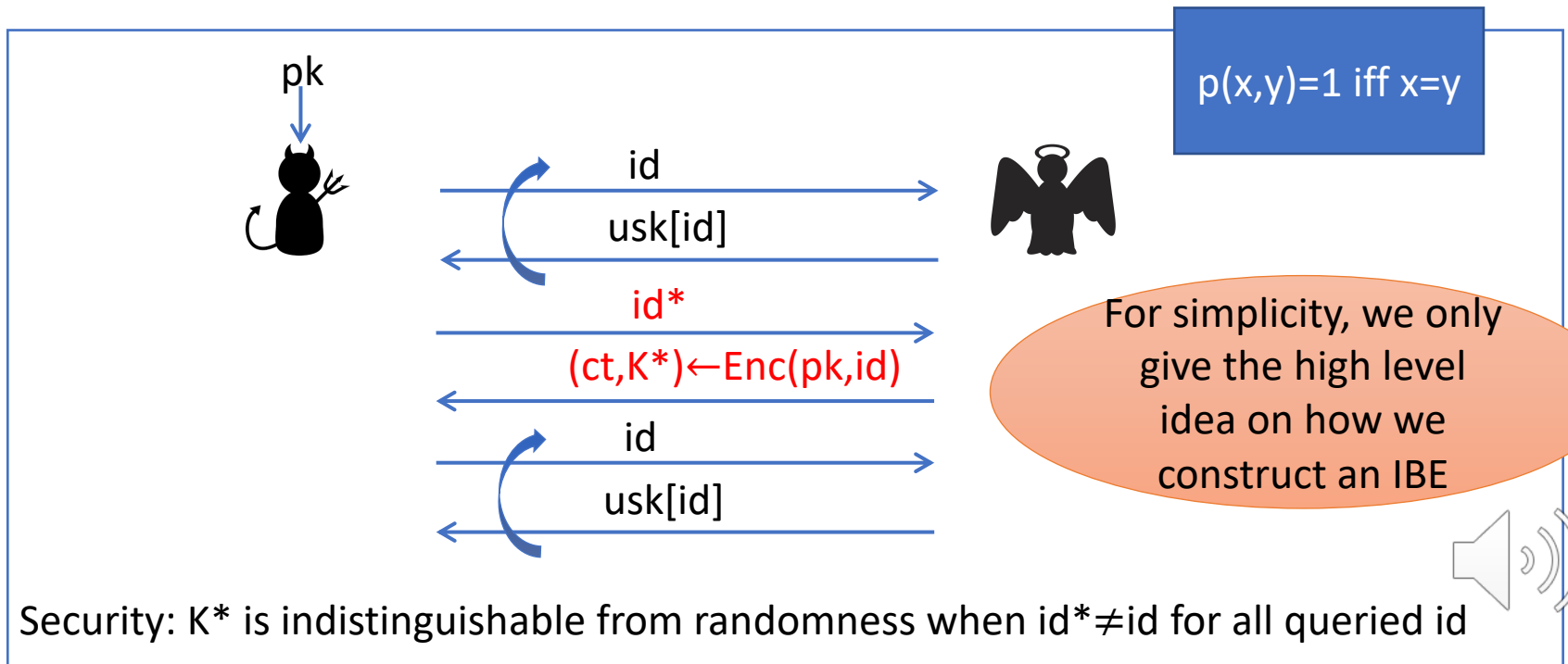
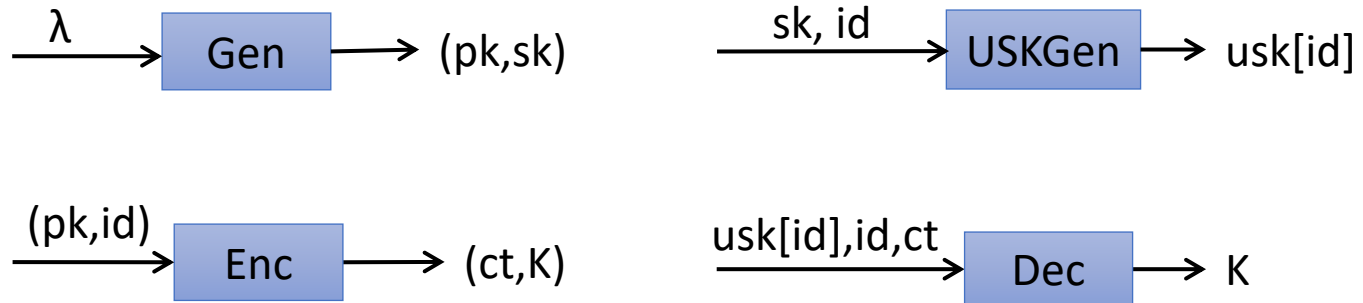
Security:  $K^*$  is indistinguishable from randomness when  $p(x, y) \neq 1$  for all queried  $y$



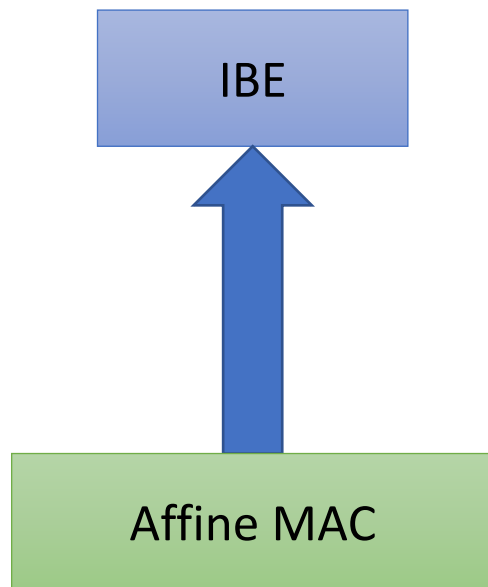
# Identity-based key encapsulation (IBKEM)



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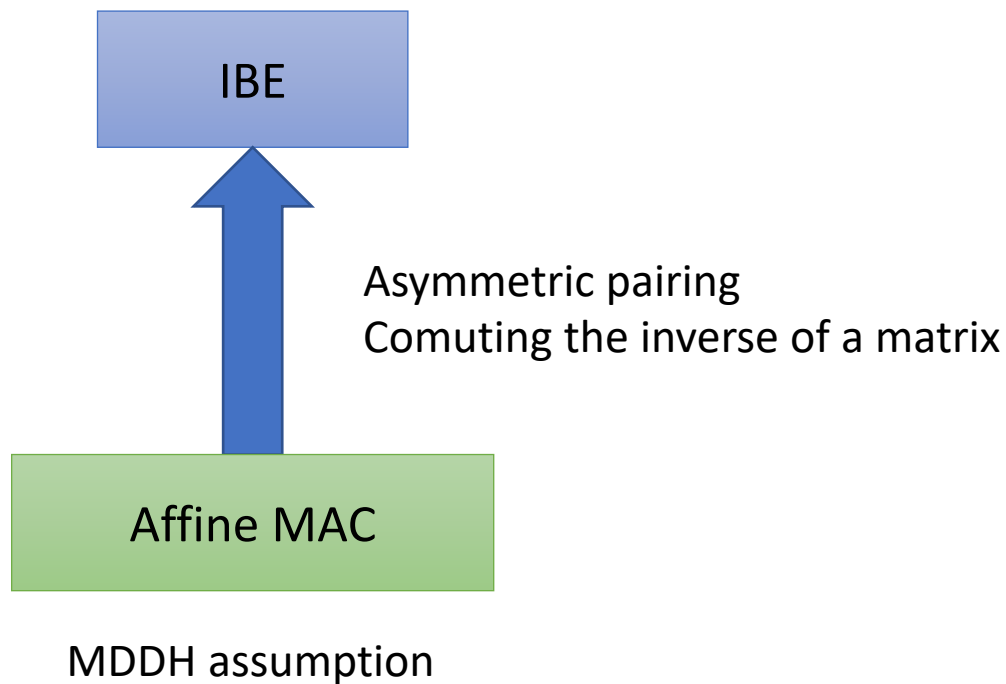
# The BKP framework



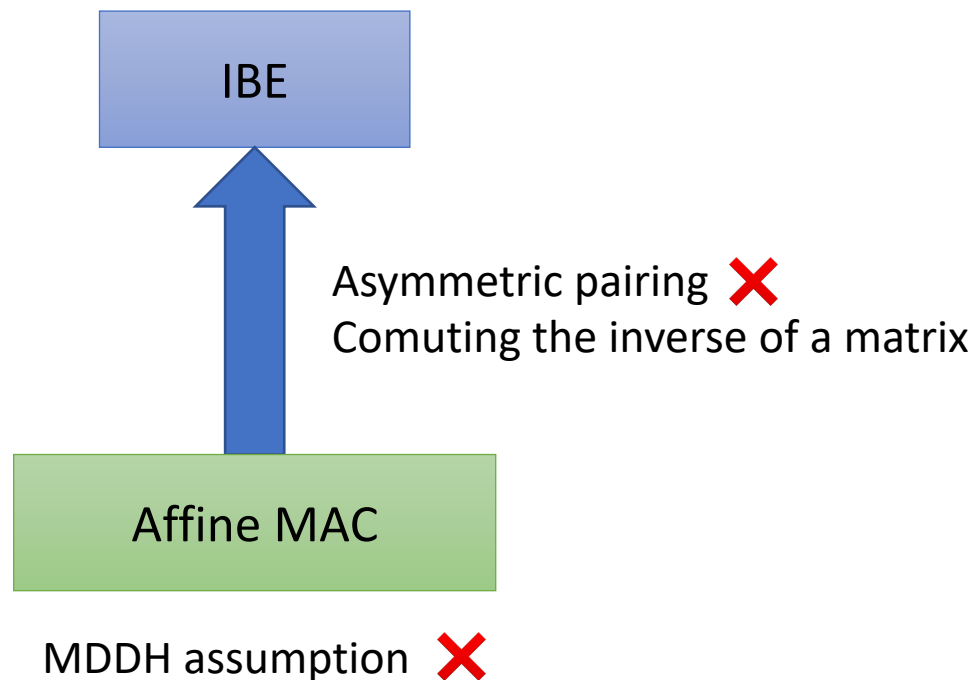
MDDH assumption



# The BKP framework



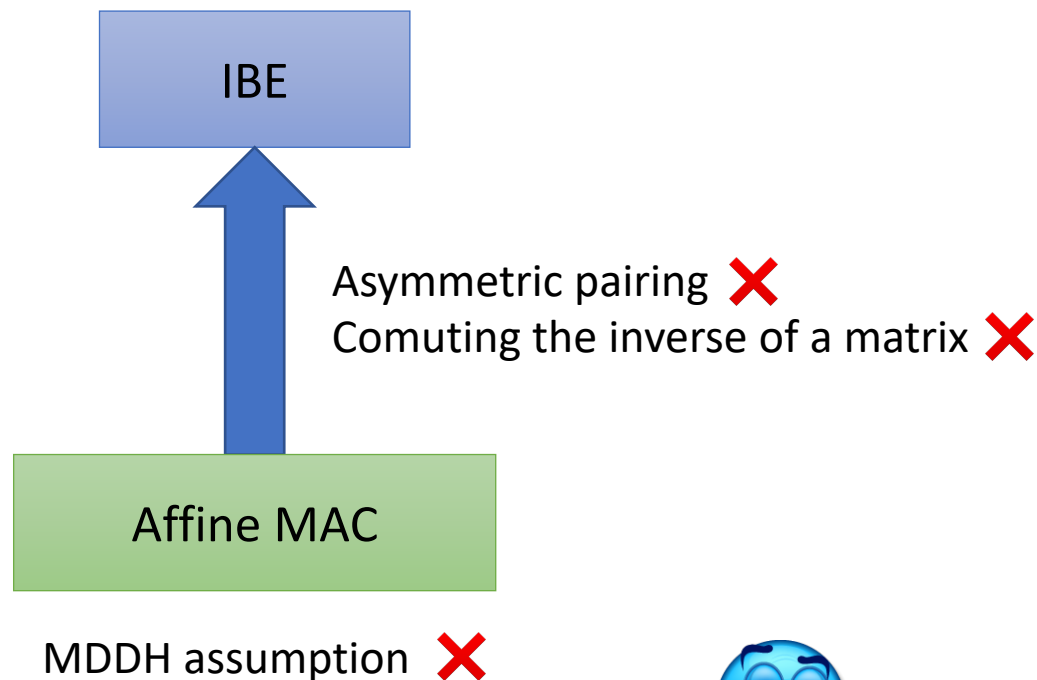
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We have no pairing and  
MDDH assumption in  $NC^1$



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We cannot compute the  
inverse of of a matrix in  
 $NC^1$



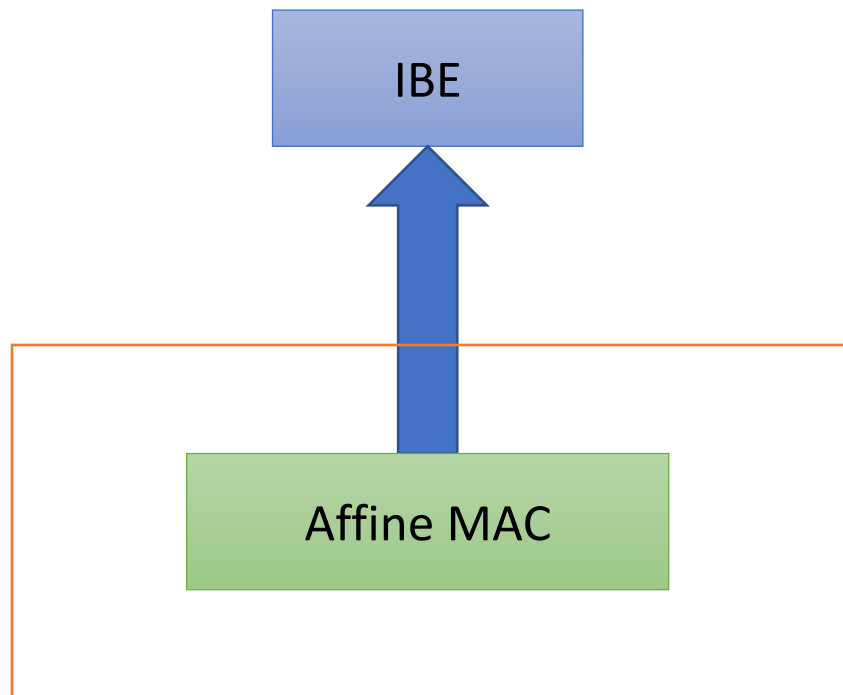








# Affine MAC



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$\text{Gen}_{\text{MAC}}(\lambda)$ :

- $B^T \leftarrow \text{ZeroSamp}(\lambda)$
- $x_i \leftarrow \{0,1\}^\lambda$  for  $i=0, \dots, n$
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$sk_{\text{MAC}} = (B, x_0, \dots, x_n, x')$

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Affine equation of  $x_i^T t$  and  $x'$   
with coefficients derived  
from the message



# Affine MAC (security)

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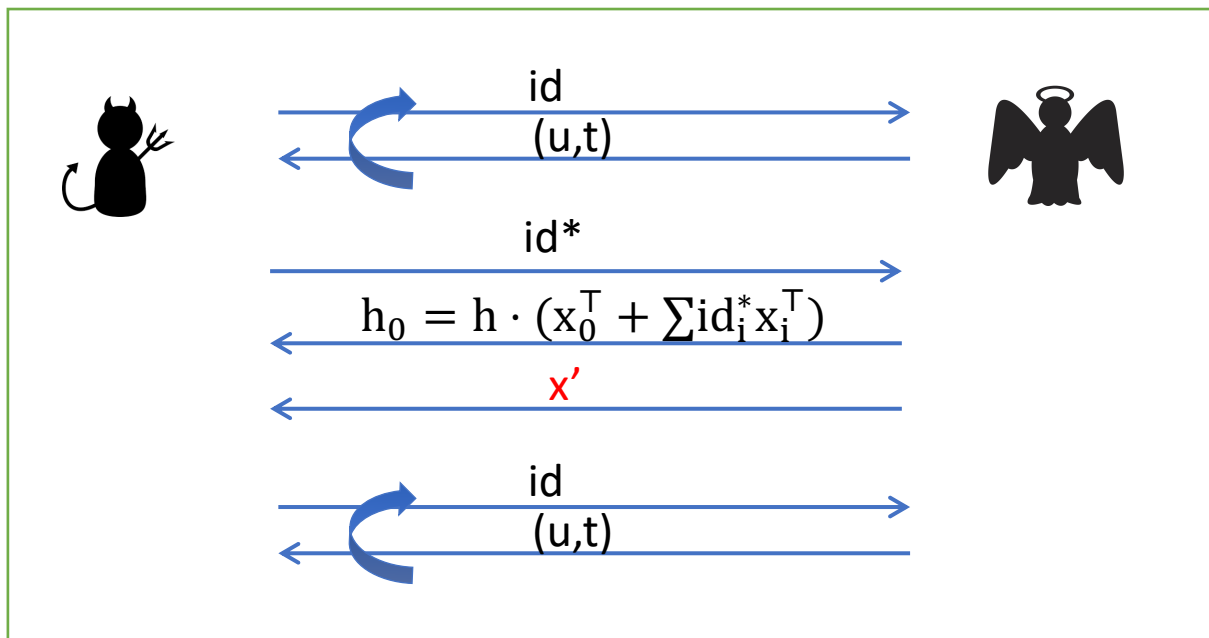
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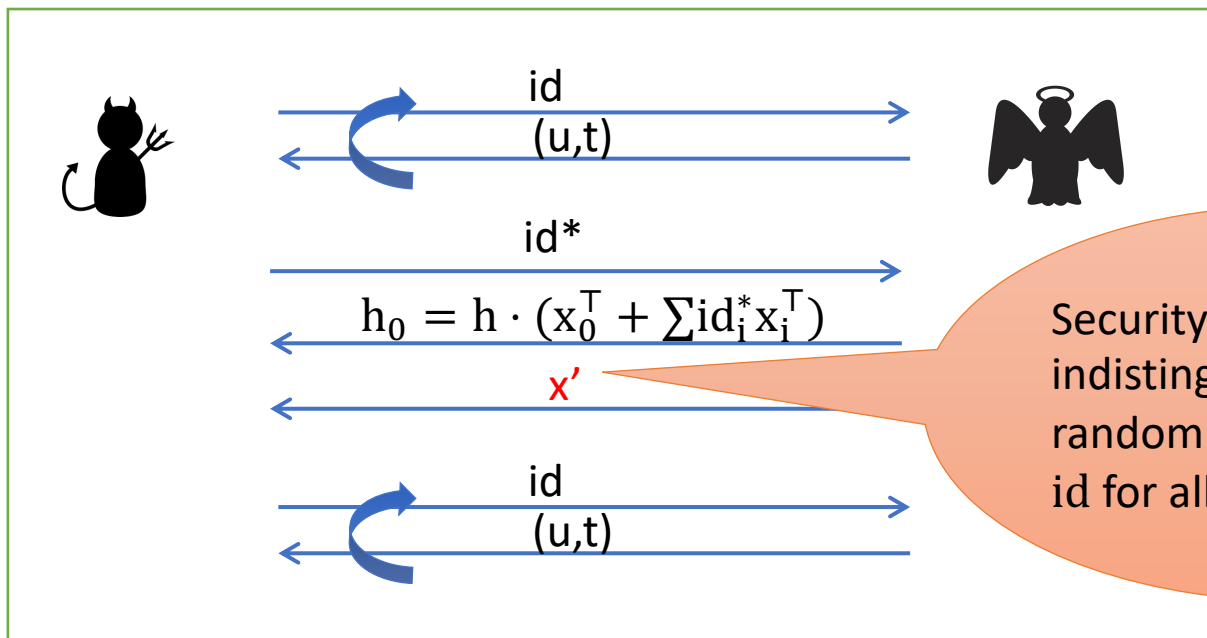
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Security holds if  $x'$  is indistinguishable with a random bit when  $\text{id}^* \neq \text{id}$  for all queried  $\text{id}$



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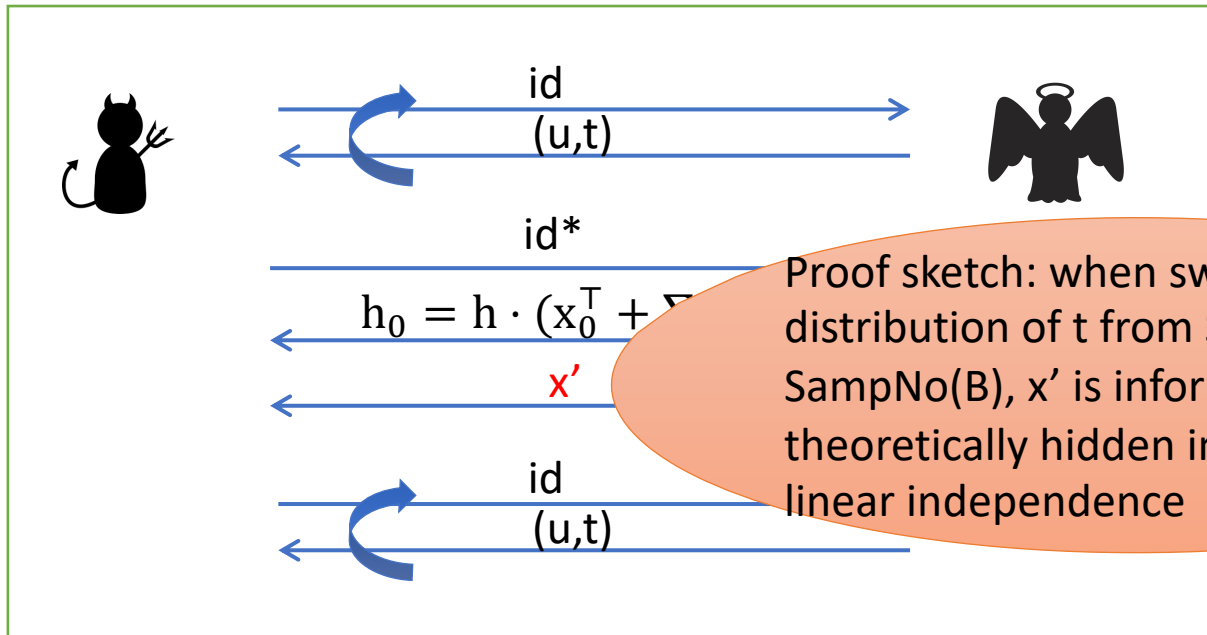
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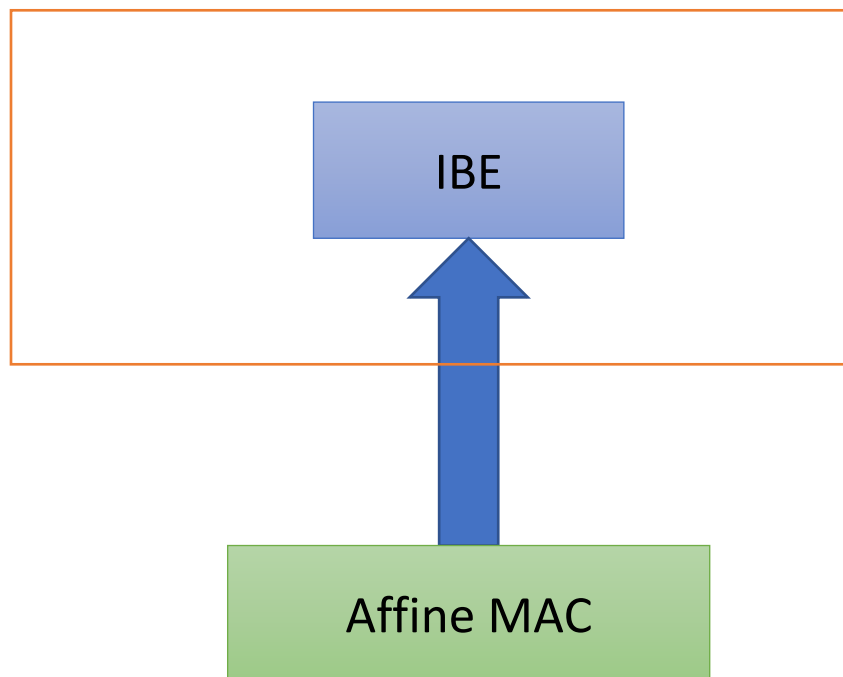
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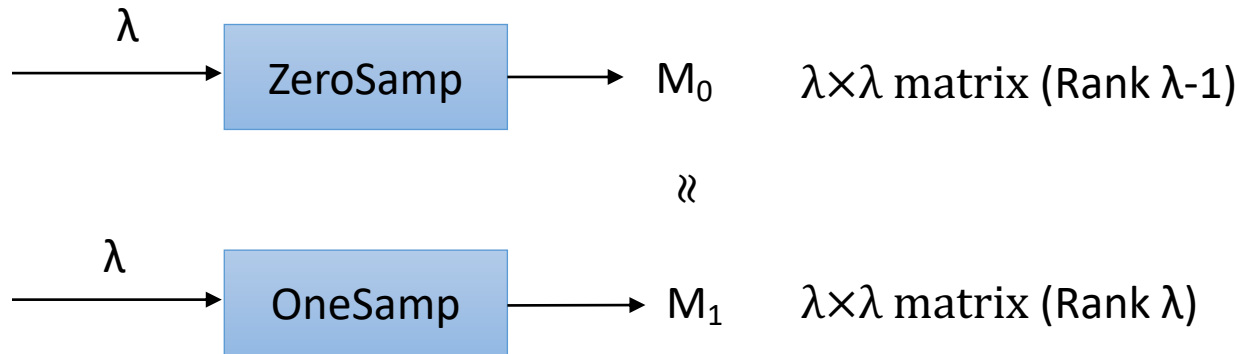
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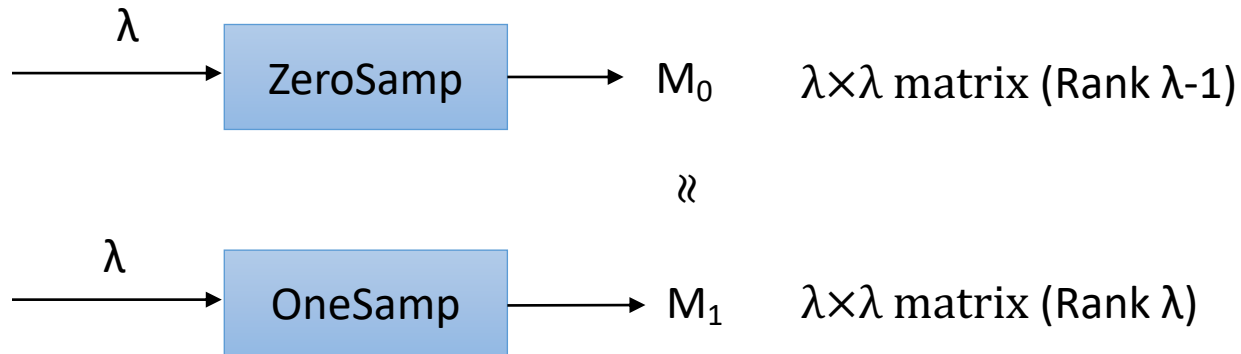
# Construction of IBKEM



# Two facts on ZeroSamp and OneSamp [EWT19]



# Two facts on ZeroSamp and OneSamp [EWT19]



The distribution of  $M_0^T + N$  is identical to that of  $M_1^T$

$$N = \begin{pmatrix} 0 & \cdots & & 0 \\ \vdots & 0 & \cdots & 0 \\ 0 & & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \end{pmatrix}$$





# Construction of IBKEM

Gen( $\lambda$ ):

- $A^T \leftarrow \text{ZeroSamp}(\lambda)$ ,  $sk_{MAC} = (B, x_0, \dots, x_n, x') \leftarrow \text{Gen}_{MAC}$
  - For  $i=0, \dots, n$ ,  $Y_i \leftarrow \{0,1\}^{(\lambda-1) \times \lambda}$ ,  $Z_i = (Y_i^T || x_i)A$
  - $y' \leftarrow \{0,1\}^{(\lambda-1)}$ ,  $z' = (y'^T || x')A$
- $pk = (A, (Z_i)_i, z')$ ,  $sk = (sk_{MAC}, (Y_i)_i, y')$



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Committing  $sk_{MAC}$



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USKGen( $sk_{MAC}$ , id):

- $(t, u) \leftarrow \text{Tag}(sk_{MAC}, id)$ ,  $v = t^T (Y_0^T + \sum id_i Y_i^T) + y'^T$
- $usk[id] = (t, u, v)$

Affine equation of  $Y_i t$  and  $y'$   
(a proof that the tag was correctly computed)





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Enc( $pk$ , id):

- $r \leftarrow \{0\} \times \{0,1\}^{\lambda-1}$ ,  $c_0 = Ar$ ,  $c_1 = (Z_0 + \sum id_i Z_i)r$
- $ct = (c_0, c_1)$ ,  $K = z'r$

Affine equation  
of  $Z_i r$



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Dec( $\text{usk}[\text{id}]$ , ct):  
 $K = (v | u)c_0 - t^T c_1$

Pairing is not necessary now  
since the computations are not  
in groups



# Construction of IBKEM

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Crucial step in the security game: to construct a reduction breaking the security of the affine MAC

USKGen( $sk_{MAC}$ , id):

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Core of the proof:

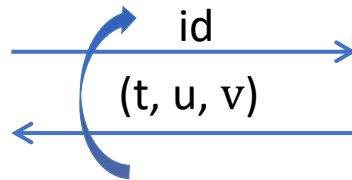
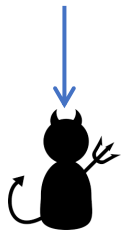
A new technique to extract the forgery of the affine MAC from the adversary.

=> switching the distribution of A twice and changing the distribution of r during the switching procedure

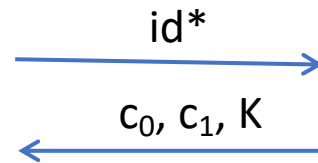


# Proof sketch (Game 0)

$A^T \leftarrow \text{ZeroSamp}(\lambda), (Z_i = (Y_i^T || x_i)A)_i, z' = (y^T || x')A$



$(t, u) \leftarrow \text{Tag}(\text{sk}_{\text{MAC}}, \text{id})$   
 $v = t^T (Y_0^T + \sum \text{id}_i Y_i^T) + y'^T$

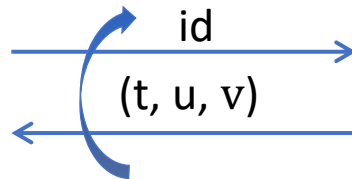
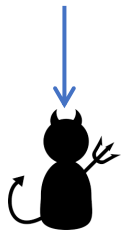


$r \leftarrow \{0\} \times \{0, 1\}^{\lambda-1}$   
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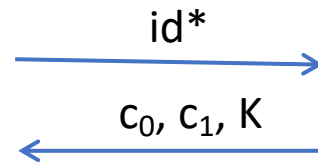
# Proof sketch (Game 1)

$A^\top \leftarrow \text{ZeroSamp}(\lambda)$ ,  $(Z_i = (Y_i^\top || x_i)A)_i$ ,  $z' = (y^\top || x')A$



$(t, u) \leftarrow \text{Tag}(\text{sk}_{\text{MAC}}, \text{id})$

$v = t^\top (Y_0^\top + \sum \text{id}_i Y_i^\top) + y'^\top$



$r \leftarrow \{0\} \times \{0, 1\}^{\lambda-1}$

$c_0 = (A + N)r$

$c_1 = (Y_0^\top || x_0)(A + N)r + \sum \text{id}_i (A + N)r$

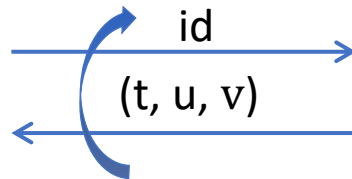
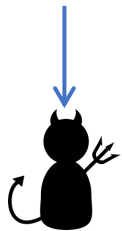
$K = (y'^\top || x')(A + N)r$

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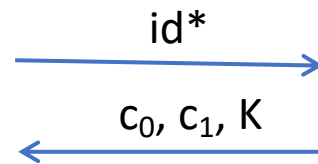
# Proof sketch (Game 1)

$A^T \leftarrow \text{ZeroSamp}(\lambda), (Z_i = (Y_i^T || x_i)A)_i, z' = (y^T || x')A$



$(t, u) \leftarrow \text{Tag}(\text{sk}_{\text{MAC}}, \text{id})$

$v = t^T (Y_0^T + \sum \text{id}_j Y_j^T) + \dots$



$r \leftarrow \{0\} \times \{0, 1\}^{\lambda-1}$

$c_0 = (A + N)r$

$c_1 = (Y_0^T || x_0)(A + N)r + \sum \text{id}_i (A + N)r$

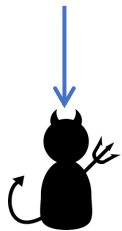
$K = (y'^T || x')(A + N)r$

The distribution of  $c_1$  does not change since  $Nr=0$



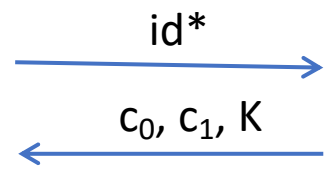
# Proof sketch (Game 2)

$$A^T \leftarrow \text{OneSamp}(\lambda), (Z_i = (Y_i^T | |x_i)A)_i, z' = (y^T | |x')A$$



Switch the distribution of  $A^T$  to  $\text{OneSamp}(c, id)$

$$v = t^T (Y_0^T + \sum id_i Y_i^T) + y'^T$$



$$r \leftarrow \{0\} \times \{0,1\}^{\lambda-1}$$

$$c_0 = (A+N)r$$

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$$c_1 = (Y_0^T | x_0)(A + N)r + \sum id_i (A + N)r$$

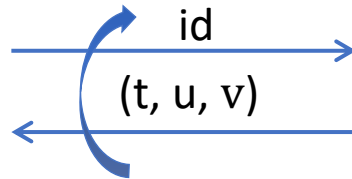
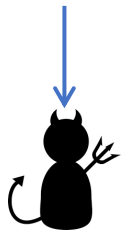
$$K = (y'^T | x')(A + N)r$$



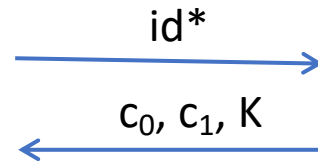


# Proof sketch (Game 3)

$A^T \leftarrow \text{OneSamp}(\lambda)$ ,  $(Z_i = (Y_i^T || x_i)A)_i$ ,  $z' = (y^T || x')A$



$(t, u) \leftarrow \text{Tag}(\text{sk}_{\text{MAC}}, \text{id})$   
 $v = t^T (Y_0^T + \sum \text{id}_i Y_i^T) + y'^T$



$r \leftarrow \{1\} \times \{0, 1\}^{\lambda-1}$

$c_0 = (A + N)r$

$c_1 = (Y_0^T || x_0)(A + N)r + \sum \text{id}_i (A + N)r$

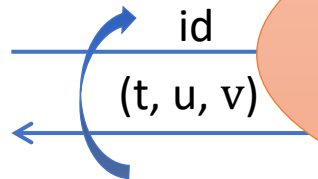
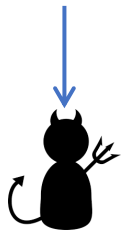
$K = (y'^T || x')(A + N)r$

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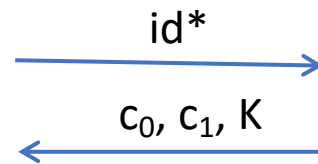


# Proof sketch (Game 3)

$A^\top \leftarrow \text{OneSamp}(\lambda)$ ,  $(Z_i = (Y_i^\top || x_i)A)_i$ ,  $z' = (y^\top || x')A$



This does not affect the distribution of  $(A+N)r$ , since  $(A+N)^\top \in \text{ZeroSamp}(\lambda)$



$$r \leftarrow \{1\} \times \{0,1\}^{\lambda-1}$$

$$c_0 = (A+N)r$$

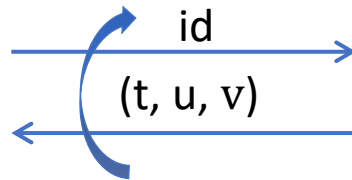
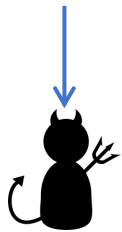
$$c_1 = (Y_0^\top || x_0)(A+N)r + \sum \text{id}_i (A+N)r$$

$$K = (y'^\top || x')(A+N)r$$



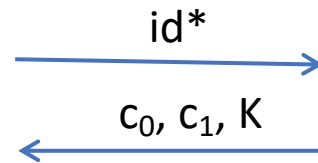
# Proof sketch (Game 3)

$A^T \leftarrow \text{OneSamp}(\lambda)$ ,  $(Z_i = (Y_i^T || x_i)A)_i$ ,  $z' = (y^T || x')A$



$(t, u) \leftarrow \text{Tag}(sk_{\text{MAC}}, id)$

$v = t^T (Y_0^T + \sum id_i Y_i^T) + y'^T$



$r \leftarrow \{1\} \times \{0, 1\}^{\lambda-1}$

$c_0 = (A + N)r$

$c_1 = (Y_0^T || x_0)(A + N)r + \sum id_i (A + N)r$

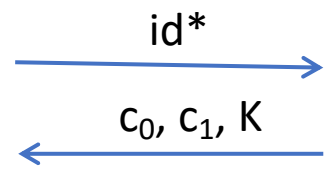
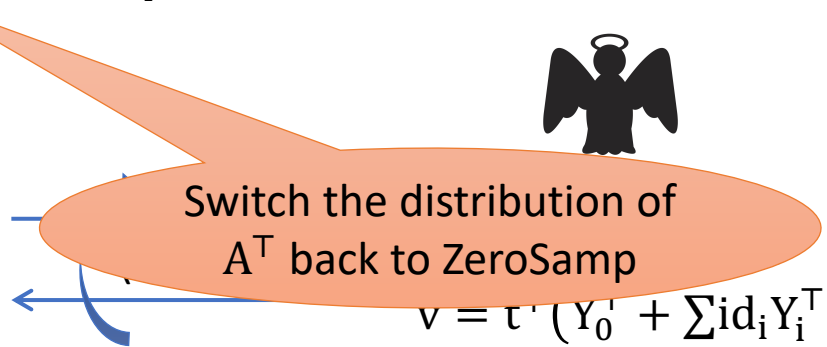
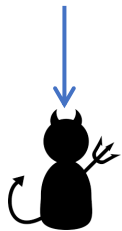
$K = (y'^T || x')(A + N)r$

Also, notice that  $(Y_0^T || x_0)Nr = x_0$  and  $(y'^T || x')Nr = x'$  now



# Proof sketch (Game 4)

$$A^T \leftarrow \text{ZeroSamp}(\lambda), (Z_i = (Y_i^T || x_i)A)_i, z' = (y^T || x')A$$



$$r \leftarrow \{1\} \times \{0,1\}^{\lambda-1}$$

$$c_0 = (A+N)r$$

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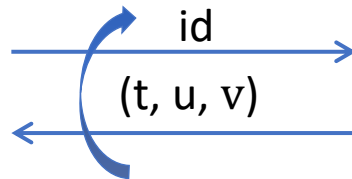
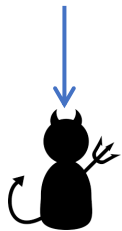
$$c_1 = (Y_0^T || x_0)(A + N)r + \sum id_i (A + N)r$$

$$K = (y'^T || x')(A + N)r$$



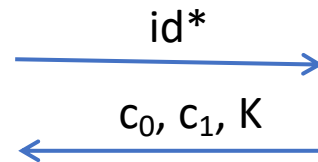
# Proof sketch (Game 5)

$A^T \leftarrow \text{ZeroSamp}(\lambda)$ ,  $(Z_i = (0 \parallel D_i)R_0^T)_i$ ,  $z' = (0 \parallel d')R_0^T$



$(t, u) \leftarrow \text{Tag}(\text{sk}_{\text{MAC}}, \text{id})$

$$v = t^T(D_0 + \sum \text{id}_i D_i) + d' + us^T$$



$r \leftarrow \{1\} \times \{0, 1\}^{\lambda-1}$

$$c_0 = (A + N)r$$

$$c_1 = Z_0 r + x_0 + \sum \text{id}_i (Z_i r + x_i)$$

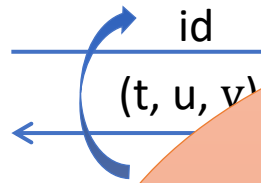
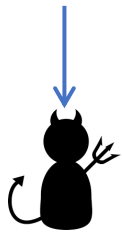
$$K = z' r + x'$$

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# Proof sketch (Game 5)

$A^T \leftarrow \text{ZeroSamp}(\lambda)$ ,  $(Z_i = (0 \parallel D_i) R_0^T)_i$ ,  $z' = (0 \parallel d') R_0^T$

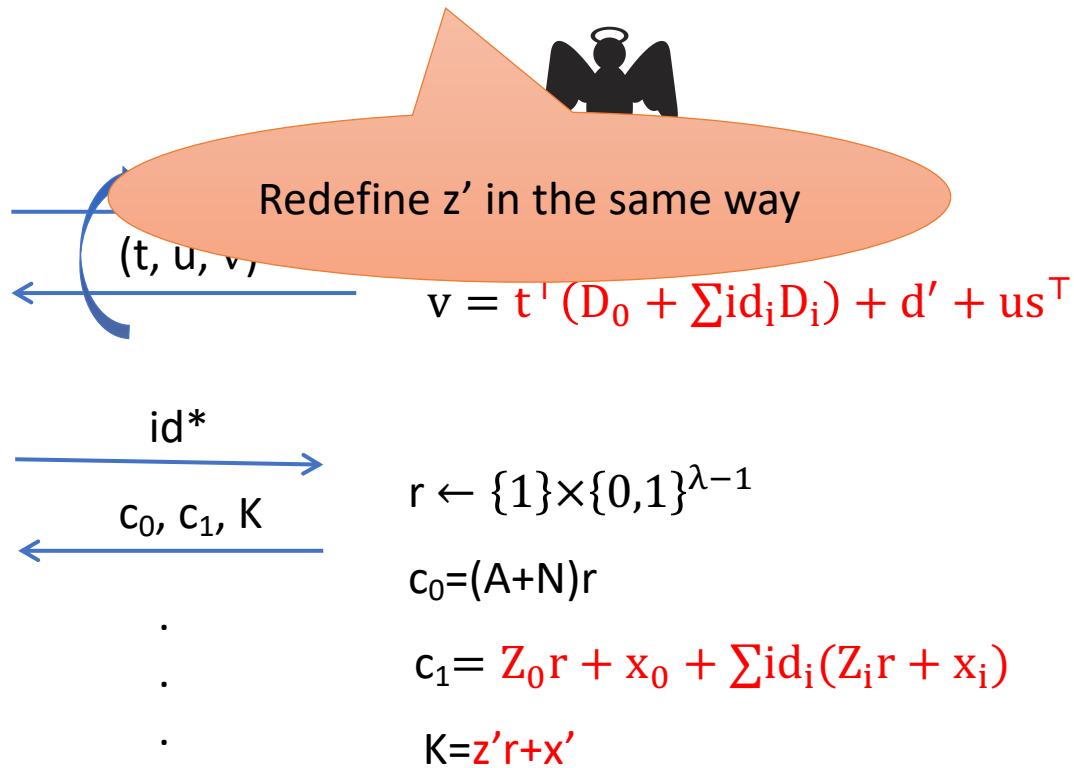
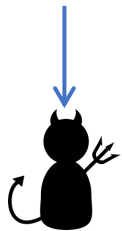


$D_i = Y_i^T + x_i s^T$  and  
reveals no information on  $x_i$ , and  $R_0$  and  
 $s$  are intermediate values used by  
ZeroSamp



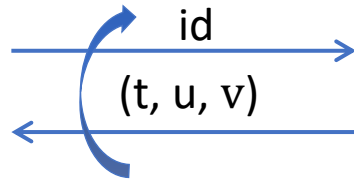
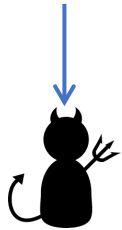
# Proof sketch (Game 5)

$A^T \leftarrow \text{ZeroSamp}(\lambda)$ ,  $(Z_i = (0 \parallel D_i)R_0^T)_i$ ,  $z' = (0 \parallel d')R_0^T$



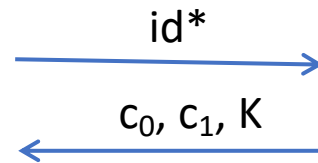
# Proof sketch (Game 5)

$A^T \leftarrow \text{ZeroSamp}(\lambda)$ ,  $(Z_i = (0 \parallel D_i)R_0^T)_i$ ,  $z' = (0 \parallel d')R_0^T$



$(t, u) \leftarrow \text{Tag}(\text{sk}_{\text{MAC}}, \text{id})$

$v = t^T(D_0 + \sum \text{id}_i D_i) + d' + us^T$



$r \leftarrow \{1\} \times \{0, 1\}^{\lambda-1}$

$c_0 = (A + N)r$

$c_1 = Z_0 r + x_0 + \sum \text{id}_i (Z_i r + x_i)$

$K = z' r + x'$

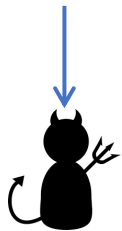
v reveals no information on the secrets except for u





# Proof sketch (Game 5)

$$A^T \leftarrow \text{ZeroSamp}(\lambda), (Z_i = (0 \parallel D_i)R_0^T)_i, z' = (0 \parallel d')R_0^T$$



$c_1$  reveals no information on  $x_i$  except for  $x_0 + \sum id_i x_i$  (exactly part of the token in the security game for the affine MAC)

$c_0, c_1, K$

$$c_0 = (A+N)r$$

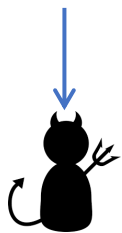
$$c_1 = Z_0 r + x_0 + \sum id_i (Z_i r + x_i)$$

$$K = z' r + x'$$



# Proof sketch (Game 5)

$$A^T \leftarrow \text{ZeroSamp}(\lambda), (Z_i = (0 \parallel D_i)R_0^T)_i, z' = (0 \parallel d')R_0^T$$



$x'$  is indistinguishable from randomness due to the security of affine MAC, i.e.,  $K$  can be switched to randomness

$c_0, c_1, K$

$$c_0 = r$$

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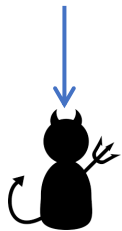
$$c_1 = Z_0 r + x_0 + \sum id_i (Z_i r + x_i)$$

$$K = z' r + x'$$



# Proof sketch (Game 0)

$$A^T \leftarrow \text{ZeroSamp}(\lambda), (Z_i = (Y_i^T || x_i)A)_i, z' = (y^T || x')A$$



id



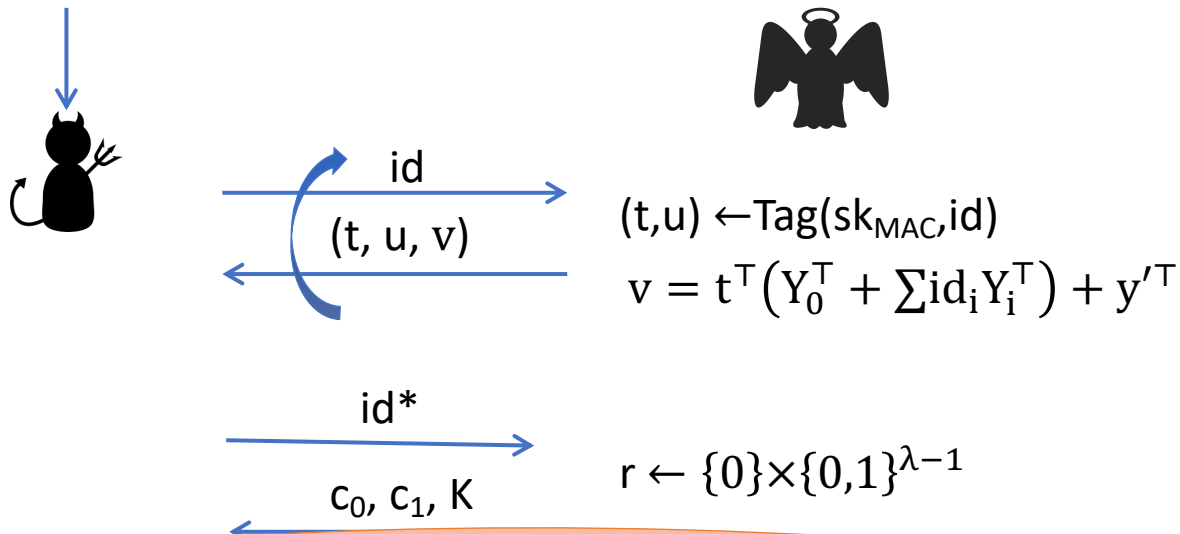
By doing the steps in the reverse order, we can prove that  $K$  is indistinguishable from randomness in the original game.

$$\begin{aligned} & \cdot \\ & \cdot \\ & \cdot \end{aligned} \quad \begin{aligned} & c_1 \dots + \sum id_i Z_i) r \\ & K = z' r \end{aligned}$$



# Proof sketch (Game 0)

$A^T \leftarrow \text{ZeroSamp}(\lambda), (Z_i = (Y_i^T || x_i)A)_i, z' = (y^T || x')A$



In the security proof, all the computations are in NC1.



# Extension to ABKEM

The red parts essentially use encoding for equality and can be generalized as predicate encodings [CGW15] to achieve ABKEM

Gen( $\lambda$ ):

- $A^T \leftarrow \text{ZeroSamp}(\lambda)$ ,  $\text{sk}_{\text{MAC}} = (B, x_0, \dots, x_n, x) \leftarrow \text{Gen}_{\text{MAC}}(\lambda)$
  - For  $i=0, \dots, n$ ,  $Y_i \leftarrow \{0,1\}^{(\lambda-1) \times \lambda}$ ,  $Z_i = (Y_i^T || x_i)A$
  - $y' \leftarrow \{0,1\}^{(\lambda-1)}$ ,  $z' = (y'^T || x')A$
- $\text{pk} = (A, (Z_i)_i, z')$ ,  $\text{sk} = (\text{sk}_{\text{MAC}}, (Y_i)_i, y')$

Tag( $\text{sk}_{\text{MAC}}$ ,  $\text{id} = (\text{id}_i)_{i=1, \dots, n}$ ):

$t \leftarrow \text{SampYes}(B)$

$$u = x_0^T t + \sum \text{id}_i x_i^T t + x'$$

Return  $(t, u)$

USKGen( $\text{sk}_{\text{MAC}}$ ,  $\text{id}$ ):

- $(t, u) \leftarrow \text{Tag}(\text{sk}_{\text{MAC}}, \text{id})$ ,  $v = t^T (Y_0^T + \sum \text{id}_i Y_i^T) + y'^T$
- $\text{usk}[\text{id}] = (t, u, v)$

Enc( $\text{pk}$ ,  $\text{id}$ ):

- $r \leftarrow \{0\} \times \{0,1\}^{\lambda-1}$ ,  $c_0 = Ar$ ,  $c_1 = (Z_0 + \sum \text{id}_i Z_i)r$
- $\text{ct} = (c_0, c_1)$ ,  $K = z'r$

Dec( $\text{usk}[\text{id}]$ ,  $\text{ct}$ ):

$$K = (v | u) c_0 - t^T c_1$$



# Conclusion

- Generic construction of fine-grained ABE secure against  $NC^1$  adversaries under  $NC^1 \neq \bigoplus L/poly$  and computable in  $AC^0[2]$ .



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Instantiations:

1. IBE scheme (which in turn implies a signature scheme)
2. ABEs for
  - inner-product encryption
  - non-zero inner-product
  - encryptionspatial encryption
  - doubly spatial encryption
  - boolean span programs
  - arithmetic span programs
3. Broadcast encryption
4. fuzzy IBE



# Conclusion

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  3. Broadcast encryption
  4. fuzzy IBE
- More application of our techniques : an efficient fine-grained QA-NIZK.

