Decentralized Multi-Authority ABE for DNEs from LWE By Pratish Datta, Joint work with Ilan Komargodski, Brent Waters

NTT Research, Hebrew University, UT Austin

EUROCRYPT 2021, October 17-21, 2021

Attribute-Based Encryption [SW05,GPSW06,...]

A secret key allows one to decrypt all ciphertext that satisfy

f(U) =true



Attribute-Based Encryption [SW05,GPSW06,...]

A secret key allows one to decrypt all ciphertext that satisfy

f(U) = true

Key-Policy ABE

f is assoc. w. secret key

is assoc. w. ciphertext



Ciphertext-Policy ABE

U is assoc. w. secret key

f is assoc. w. ciphertext

Key-Policy ABE



Key-Policy ABE

$SK \leftarrow KeyGen(MSK, f)$ $CT \leftarrow Enc(PK, msg, U)$



Key-Policy ABE

 $SK \leftarrow KeyGen(MSK, f)$

 $CT \leftarrow Enc(PK, msg, U)$



Key-Policy ABE

SK \leftarrow KeyGen(MSK, f) CT \leftarrow Enc(PK, msg, U)



Key-Policy ABE

 $SK \leftarrow KeyGen(MSK, f)$ $CT \leftarrow Enc(PK, msg, U)$



Key-Policy ABE



Key-Policy ABE

Bilinear-group-based constructions

Sahai-Waters '05 Goyal-Pandey-Sahai-Waters '06 Ostrovsky-Sahai-Waters '07

- - -



Key-Policy ABE

Bilinear-group-based constructions

Sahai-Waters '05 Goyal-Pandey-Sahai-Waters '06 Solve S

- - -



Key-Policy ABE

Bilinear-group-based constructions

Sahai-Waters '05 Goyal-Pandey-Sahai-Waters '06 Ostrovsky-Sahai-Waters '07

Lattice-based constructions Gorbunov-Vaikuntanathan-Wee '13 Boneh-Gentry-... '14

. . .



Key-Policy ABE

Bilinear-group-based constructions

Sahai-Waters '05 Goyal-Pandey-Sahai-Waters '06 Ostrovsky-Sahai-Waters '07

Lattice-based constructions Gorbunov-Vaikuntanathan-Wee '13 Boneh-Gentry-... '14

. . .



Key-Policy ABE

Bilinear-group-based constructions

Sahai-Waters '05 Goyal-Pandey-Sahai-Waters '06 Ostrovsky-Sahai-Waters '07

Lattice-based constructions Gorbunov-Vaikuntanathan-Wee '13 Boneh-Gentry-... '14

. . .



lacksquare



- \bullet
- In reality, multiple authorities are in charge of different attributes



- In reality, multiple authorities are in charge of different attributes
 - DMV for "holds a driving license"



- In reality, multiple authorities are in charge of different attributes
 - DMV for "holds a driving license"
 - University for "holds a Ph.D"



- In reality, multiple authorities are in charge of different attributes
 - DMV for "holds a driving license"
 - University for "holds a Ph.D"
 - Military for "veteran"



- In reality, multiple authorities are in charge of different attributes
 - DMV for "holds a driving license"
 - University for "holds a Ph.D"
 - Military for "veteran"
- "Multi-Authority" ABE



In ABE, one central authority who verifies attributes and issues secret keys

Chase '07, ..., Lewko-Waters '11, Okamoto-Takashima '13, Rouselakis-Waters '15



- Anyone can become an authority
 - No coordination except global PublicParams
- Different authorities control different attributes
- No bound on # of authorities
- Each authority can issue secret keys to users possessing attributes under their control
 - without any interaction with other authorities





CT1 = Enc("Hi", PhD AND DrivingLicense)

CT2 = Enc("Bye", PhD AND Veteran)







No one can decrypt CT1 (collusion resistance)

Can decrypt CT2

CT1 = Enc("Hi", PhD AND DrivingLicense)

CT2 = Enc("Bye", PhD AND Veteran)





The GID Model

Chase '07

• How to uniquely identify a user?

• How to uniquely identify a user?





Distinguishable

- How to uniquely identify a user?
- Associate a unique global verifiable identifier (GID)





- How to uniquely identify a user?
- Associate a unique global verifiable identifier (GID)



• The global identity of a user remains fixed for the entire lifetime of the system

- How to uniquely identify a user?
- Associate a unique global verifiable identifier (GID)
- The global identity of a user remains fixed for the entire lifetime of the system
- Users have no freedom to choose their global identities





- How to uniquely identify a user?
- Associate a unique global verifiable identifier (GID)
- The global identity of a user remains fixed for the entire lifetime of the system
- Users have no freedom to choose their global identities









(Assume one attribute per authority)

 $GP \leftarrow GlobalSetup(1^{\lambda})$

(Assume one attribute per authority)

 $\mathsf{PK}_u, \mathsf{MSK}_u \leftarrow \mathsf{AuthSetup}(\mathsf{GP}, u)$

 $GP \leftarrow GlobalSetup(1^{\lambda})$

$SK_{GID,u} \leftarrow KeyGen(MSK_u, GID)$

(Assume one attribute per authority)

 $\mathsf{PK}_u, \mathsf{MSK}_u \leftarrow \mathsf{AuthSetup}(\mathsf{GP}, u)$ $\mathsf{CT} \leftarrow \mathsf{Enc}(\{\mathsf{Pk}\}$

$GP \leftarrow GlobalSetup(1^{\lambda})$

$SK_{GID,u} \leftarrow KeyGen(MSK_u, GID)$

$CT \leftarrow Enc(\{PK_u\}_{u \in f}, msg, f)$

(Assume one attribute per authority)

 $PK_{u}, MSK_{u} \leftarrow AuthSetup(GP, u)$

 $GP \leftarrow GlobalSetup(1^{\lambda})$

$SK_{GID.u} \leftarrow KeyGen(MSK_u, GID)$

- $CT \leftarrow Enc(\{PK_u\}_{u \in f}, msg, f)$
- $msg' \leftarrow Dec(\{SK_{GID,u}\}_{u \in U}, CT)$ (All with same GID)

(Assume one attribute per authority)

 $\mathsf{PK}_{u}, \mathsf{MSK}_{u} \leftarrow \mathsf{AuthSetup}(\mathsf{GP}, u)$

Correctness: If f(U) = true then msg' = msg

 $GP \leftarrow GlobalSetup(1^{\lambda})$

$SK_{GID,\mu} \leftarrow KeyGen(MSK_{\mu}, GID)$

- $CT \leftarrow Enc(\{PK_u\}_{u \in f}, msg, f)$
- $msg' \leftarrow Dec(\{SK_{GID,\mu}\}_{\mu \in U}, CT)$ (All with same GID)

Security: If f(U) = false then msg is "hidden" (Collusion among users and some corrupt authorities is allowed)

MA-ABE (Main) Known Constructions & Our Main Result

Schemes	Supported Policy Class	Assumption	Attribute per Authority	Security
Lewko-Waters '11	NC1	Subgroup decision (composite order)	Bounded	Adaptive
Okamoto- Takashima '13	NC1	DLIN (prime order)	Bounded	Adaptive
Rouselakis- Waters '15	NC1	q-type (prime order)	Unbounded	Static

(All schemes are in GID model and are proven secure in the random oracle model)

MA-ABE (Main) Known Constructions & Our Main Result

Schemes	Supported Policy Class	Assumption	Attribute per Authority	Security
Lewko-Waters '11	NC1	Subgroup decision (composite order)	Bounded	Adaptive
Okamoto- Takashima '13	NC1	DLIN (prime order)	Bounded	Adaptive
Rouselakis- Waters '15	NC1	q-type (prime order)	Unbounded	Static
This Work	DNF	LWE	Bounded	Static

(All schemes are in GID model and are proven secure in the random oracle model)

Our Main Theorem

There exist an MA-ABE scheme in the GID model for access policies captured by DNF formulas that is statically secure against an arbitrary collusion of parties in the random oracle model and assuming the LWE assumption.

Our Main Theorem

in the random oracle model and assuming the LWE assumption.



There exist an MA-ABE scheme in the GID model for access policies captured by DNF formulas that is statically secure against an arbitrary collusion of parties

Our Main Theorem

in the random oracle model and assuming the LWE assumption.



There exist an MA-ABE scheme in the GID model for access policies captured by DNF formulas that is statically secure against an arbitrary collusion of parties



Challenges



 $\mathsf{SK} = (\mathsf{SK}_1, \mathsf{SK}_2, \mathsf{SK}_3)$









Collusion resistance is obtained by using fresh randomness for every SK specific to the user

The components of SK (and SK[']) are compatible within themselves, but incompatible across sets



Collusion resistance is obtained by using fresh randomness for every SK specific to the user

The components of SK (and SK[']) are compatible within themselves, but incompatible across sets

MA challenge 1

The randomness used to tie together different key components for a specific user is obtained from H(GID)

Randomness is essentially public

Existing LWE-based CP-ABE schemes fail to satisfy public randomness



Collusion resistance is obtained by using fresh randomness for every SK specific to the user The components of SK (and SK[']) are compatible within themselves, but incompatible across sets

MA challenge 1

The randomness used to tie together different key components for a specific user is obtained from H(GID)

Randomness is essentially public

and a second second

Existing LWE-based CP-ABE schemes fail to satisfy public randomness

MA challenge 2

Should support arbitrary authorities joining on the fly

Keys should be "piecewise". Master PK and user SK should consist of components specific to attributes

Existing LWE-based CP-ABE schemes fail to satisfy modular keys

a way to see the way to a second the second to be a second to be a second to be a second to be a second to be a



(Non-Monotone) Linear Secret Sharing Schemes (LSSS)

[Shamir,KW93...]

A secret sharing scheme where sharing & reconstruction are linear functions



(Non-Monotone) Linear Secret Sharing Schemes (LSSS)

[Shamir,KW93...]

- Equivalent to span programs (M, ρ)

A secret sharing scheme where sharing & reconstruction are linear functions



(Non-Monotone) Linear Secret Sharing Schemes (LSSS)

[Shamir,KW93...]

- Equivalent to span programs (M, ρ)



A secret sharing scheme where sharing & reconstruction are linear functions

			S
,2	• • •	$v_{1,s-1}$	$v_{1,s}$
2,2	• • •	$v_{2,s-1}$	$V_{2,s}$
8,2	• • •	$v_{3,s-1}$	<i>v</i> _{3,s}
I,2	• • •	$v_{4,s-1}$	$V_{4,s}$
2,2	• • •	$v_{\ell,s-1}$	$\mathcal{V}_{\mathcal{C},S}$



- Small reconstruction coefficients: Reconstruction of the secret can be done by small coefficients, i.e., coming from {0,1}.
- Linear independence for unauthorized rows: Any unauthorized set of rows of the share generating matrix are linearly independent.

- Small reconstruction coefficients: Reconstruction of the secret can be done by small coefficients, i.e., coming from {0,1}.
- Linear independence for unauthorized rows: Any unauthorized set of rows of the share generating matrix are linearly independent.



Theorem: There exists such a non-monotone LSSS* for LOGSPACE (implicit in GVW13)

- Small reconstruction coefficients: Reconstruction of the secret can be done by small coefficients, i.e., coming from {0,1}.
- Linear independence for unauthorized rows: Any unauthorized set of rows of the share generating matrix are linearly independent.



Theorem: There exists such a non-monotone LSSS* for LOGSPACE (implicit in GVW13)

A different construction for NC1 in the paper

Width of LSSS \approx policy size



- Small reconstruction coefficients: Reconstruction of the secret can be done by small coefficients, i.e., coming from {0,1}.
- Linear independence for unauthorized rows: Any unauthorized set of rows of the share generating matrix are linearly independent.

Agrawal et al. '20, Lewko-Waters '11

Theorem: There exists such a monotone LSSS* for DNFs Theorem: There exists such a non-monotone LSSS* for LOGSPACE (implicit in GVW13) A different

A different construction for NC1 in the paper

Width of LSSS \approx policy size



- Small reconstruction coefficients: Reconstruction of the secret can be done by small coefficients, i.e., coming from {0,1}.
- Linear independence for unauthorized rows: Any unauthorized set of rows of the share generating matrix are linearly independent.

Agrawal et al. '20, Lewko-Waters '11

Theorem: There exists such a monotone LSSS* for DNFs

Open: such a monotone LSSS for NC1?

Theorem: There exists such a non-monotone LSSS* for LOGSPACE (implicit in GVW13) A different construction for NC1 in the paper

Width of LSSS \approx policy size



The Recipe

Non-monotone LSSS* with linear

ind. property for C

CP-ABE

for \mathscr{C}









In CP-ABE, central authority enforces no user gets the key for an attribute and its negation. In MA-ABE, an adversary colluding with corrupt authority knows both.



Setup $(1^{\lambda}, \mathbb{U})$:

For each attribute $u \in U$, sample



•
$$\mathbf{H}_{u} \leftarrow \mathbb{Z}_{q}^{n \times m}$$

Sample $\mathbf{y} \leftarrow \mathbb{Z}_q^n$

Output:

 $\mathsf{PK} = (\mathbf{y}, \{\mathbf{A}_{u}\}, \{\mathbf{H}_{u}\}), \quad \mathsf{MSK} = \{\mathsf{PK}, \mathbf{A}_{u}^{-1}\}$





KeyGen(MSK, U): // U is a set of attributes

Sample $\hat{\mathbf{t}} \leftarrow \text{noise}^{m-1}$ and set $\mathbf{t} = (1, \hat{\mathbf{t}})$ For each attribute u, sample $\mathbf{k}_{u} \leftarrow \mathbf{A}_{u}^{-1}(\mathbf{H}_{u} \cdot \mathbf{t}^{\top})$ Output:

$SK_U = (\{\mathbf{k}_u\}, \mathbf{t})$

$PK = (y, \{A_u\}, \{H_u\})$ $MSK = \{A_u^{-1}\}$

KeyGen(MSK, U): // U is a set of attributes

Sample $\hat{\mathbf{t}} \leftarrow \text{noise}^{m-1}$ and set $\mathbf{t} = (1, \hat{\mathbf{t}})$ For each attribute u, sample $\mathbf{k}_{u} \leftarrow \mathbf{A}_{u}^{-1}(\mathbf{H}_{u} \cdot \mathbf{t}^{\top})$ Output:

$$\mathsf{SK}_U = (\{\mathbf{k}_u\}, \mathbf{t})$$

t is the (public) randomness that ties together different key components

$PK = (y, \{A_u\}, \{H_u\})$ $MSK = \{A_u^{-1}\}$

KeyGen(MSK, U): // U is a set of attributes

Sample $\hat{\mathbf{t}} \leftarrow \text{noise}^{m-1}$ and set $\mathbf{t} = (1, \hat{\mathbf{t}})$ For each attribute u, sample $\mathbf{k}_{u} \leftarrow \mathbf{A}_{u}^{-1}(\mathbf{H}_{u} \cdot \mathbf{t}^{\top})$ Output:

$$\mathsf{SK}_U = (\{\mathbf{k}_u\}, \mathbf{t})$$

Public nature of randomness is important for MA-ABE

t is the (public) randomness that ties together different key components

$PK = (y, \{A_u\}, \{H_u\})$ $MSK = \{A_u^{-1}\}$

 $\mathbf{c}_i = \mathbf{s}\mathbf{A}_{\rho(i)} + \text{noise}_i$ $\hat{\mathbf{c}}_{i} = M_{i} \cdot \begin{bmatrix} \mathbf{s} \mathbf{y}^{\top} & 0 & \dots & 0 \\ & \mathbf{v}_{2} & & \\ & \mathbf{v}_{3} & & \\ & \dots & & \\ & \mathbf{v}_{i} & & \end{bmatrix} - \mathbf{s} \mathbf{H}_{\rho(i)} + \hat{\text{noise}}_{i}$

Output:

 $CT = (\{\mathbf{c}_i\}_{i \in [\ell]}, \{\hat{\mathbf{c}}_i\}_{i \in [\ell]}, C = \mathsf{MSB}(\mathbf{s}\mathbf{y}^{\mathsf{T}}) \oplus \mathsf{msg})$

 $\begin{array}{l} \textbf{Free CP-ABE Scheme} \\ \textbf{Enc(PK, msg \in \{0,1\}, (M, \rho)):} \\ \text{Sample } s \leftarrow \mathbb{Z}_q^n, \textbf{v}_2, \dots, \textbf{v}_s \leftarrow \mathbb{Z}_q^m \end{array} \end{array} \begin{bmatrix} M_1 \\ M_2 \\ \dots \\ M_\ell \end{bmatrix} \in \mathbb{Z}_q^{\ell \times s} \\ \textbf{K}_U = (\{\textbf{k}_u\}, \textbf{t}) \end{aligned}$



Dec(SK $_U$, CT):

If U does not satisfy (M, ρ) , output \bot Else do the following:

- I Indices of rows of M corresponding to available attributes
- $\{w_i\}_{i \in I} \in \{0,1\}$ Reconstruction coefficients

Compute:

$$K' = \sum_{i \in I} w_i (\mathbf{c}_i \mathbf{k}_{\rho(i)}^\top + \hat{\mathbf{c}}_i \mathbf{t}^\top)$$

Output:

$$msg' = C \oplus MSB(K')$$

 $PK = (y, \{A_{u}\}, \{H_{u}\})$ $MSK = \{A_{u}^{-1}\}$ $SK_{U} = (\{k_{u}\}, t)$ $CT = (\{\mathbf{c}_i\}_{i \in [\ell]}, \{\hat{\mathbf{c}}_i\}_{i \in [\ell]}, \\ C = \mathsf{MSB}(\mathbf{s}\mathbf{y}^{\mathsf{T}})$ \oplus msg)





 $K' = \sum w_i (\mathbf{c}_i \mathbf{k}_{\rho(i)}^{\mathsf{T}} + \hat{\mathbf{c}}_i \mathbf{t}^{\mathsf{T}})$ $msg' = C \oplus MSB(K')$

$\mathbf{c}_{i}\mathbf{k}_{\rho(i)}^{\mathsf{T}} + \hat{\mathbf{c}}_{i}\mathbf{t}^{\mathsf{T}} = \mathbf{s}\mathbf{A}_{\rho(i)}\mathbf{k}_{\rho(i)}^{\mathsf{T}} + M_{i} \cdot \begin{bmatrix} \mathbf{s}\mathbf{y}^{\mathsf{T}} & 0 & \dots & 0 \\ & \mathbf{v}_{2} & & \\ & \mathbf{v}_{3} & & \\ & \ddots & & \\ & \mathbf{v} & & \\ \end{bmatrix} \mathbf{t}^{\mathsf{T}} - \mathbf{s}\mathbf{H}_{\rho(i)}\mathbf{t}^{\mathsf{T}}$ **V**_S





Recall $\mathbf{A}_{\rho(i)}\mathbf{k}_{\rho(i)}^{\top} = \mathbf{H}_{\rho(i)}\mathbf{t}^{\top}$

 $K' = \sum w_i (\mathbf{c}_i \mathbf{k}_{\rho(i)}^\top + \hat{\mathbf{c}}_i \mathbf{t}^\top)$ $msg' = C \oplus MSB(K')$

$\mathbf{c}_{i}\mathbf{k}_{\rho(i)}^{\mathsf{T}} + \hat{\mathbf{c}}_{i}\mathbf{t}^{\mathsf{T}} = \mathbf{s}\mathbf{A}_{\rho(i)}\mathbf{k}_{\rho(i)}^{\mathsf{T}} + M_{i} \cdot \begin{bmatrix} \mathbf{s}\mathbf{y}^{\mathsf{T}} & 0 & \dots & 0 \\ & \mathbf{v}_{2} & & \\ & \mathbf{v}_{3} & & \\ & \dots & & \\ & \mathbf{v}_{s} & & \end{bmatrix} \mathbf{t}^{\mathsf{T}} - \mathbf{s}\mathbf{H}_{\rho(i)}\mathbf{t}^{\mathsf{T}}$





$$\mathbf{c}_{i}\mathbf{k}_{\rho(i)}^{\mathsf{T}} + \hat{\mathbf{c}}_{i}\mathbf{t}^{\mathsf{T}} = \mathbf{s}\mathbf{A}_{(i)}\mathbf{k}_{\rho(i)}^{\mathsf{T}} + M_{i} \cdot$$

Recall $\mathbf{A}_{\rho(i)}\mathbf{k}_{\rho(i)}^{\dagger} = \mathbf{H}_{\rho(i)}\mathbf{t}^{\dagger}$

 $K' = \sum w_i (\mathbf{c}_i \mathbf{k}_{\rho(i)}^{\mathsf{T}} + \hat{\mathbf{c}}_i \mathbf{t}^{\mathsf{T}})$ i∈I $msg' = C \oplus MSB(K')$ $\begin{bmatrix} \mathbf{s}\mathbf{y}^{\mathsf{T}} & \mathbf{0} & \dots & \mathbf{0} \\ & \mathbf{v}_2 \\ & \mathbf{v}_3 \\ & \ddots \\ & \mathbf{v}_s \end{bmatrix}$ t⊤ - **S**H





$$\mathbf{c}_{i}\mathbf{k}_{\rho(i)}^{\mathsf{T}} + \hat{\mathbf{c}}_{i}\mathbf{t}^{\mathsf{T}} = \mathbf{s}\mathbf{A}_{(i)}\mathbf{k}_{\rho(i)}^{\mathsf{T}} + M_{i} \cdot$$

Recall $\mathbf{A}_{\rho(i)}\mathbf{k}_{\rho(i)}^{\top} = \mathbf{H}_{\rho(i)}\mathbf{t}^{\top}$

Reconstruction gives sy⁺

 $K' = \sum w_i (\mathbf{c}_i \mathbf{k}_{\rho(i)}^{\mathsf{T}} + \hat{\mathbf{c}}_i \mathbf{t}^{\mathsf{T}})$ i∈I $\begin{bmatrix} \mathbf{s}\mathbf{y}^{\mathsf{T}} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{v}_2 & & & \\ \mathbf{v}_3 & & & \\ \mathbf{v}_3 & & & \\ \mathbf{v}_s & & & \end{bmatrix} \mathbf{t}^{\mathsf{T}} - \mathbf{t}^{\mathsf{T}} \mathbf{v}_s$ $msg' = C \oplus MSB(K')$



Conclusion

- The first MA-ABE for a non-trivial class of access policies from LWE
- A direct LWE-based approach for CP-ABE

ass of access policies from LWE

Conclusion

- The first MA-ABE for a non-trivial class of access policies from LWE
- A direct LWE-based approach for CP-ABE

Open problems:

- More expressive policies than DNFs
- Better security (we only get static)
- Better parameters (even for CP-ABE)
- Unbounded number of attributes per authority (we only get bounded)

Conclusion

- The first MA-ABE for a non-trivial class of access policies from LWE
- A direct LWE-based approach for CP-ABE

Open problems:

- More expressive policies than DNFs
- Better security (we only get static)
- Better parameters (even for CP-ABE)
- Unbounded number of attributes per authority (we only get bounded)



