Message-recovery Laser Fault Injection Attack on the *Classic McEliece* Cryptosystem

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Introduction

Most public key cryptosystems base their security on the hardness of number theoretic problems. In 1997, Peter Shor showed that quantum algorithms can solve these problems in **polynomial time**[1].

In 2016, NIST started a process[2] for cryptography standards that are **quantum resistant**.

One of the four finalists of Round 3 in the Key Encapsulation Mechanism category (announced July 22, 2020) is *Classic McEliece*[3], based on error-correcting codes.

[1] P. W. Shor. "Polynomial-Time Algorithms for Prime Factorization and Discrete Logarithms on a Quantum Computer". In: *SIAMJournal on Computing* 26.5 (1997), pp. 1484–1509.

[2] https://csrc.nist.gov/Projects/post-quantum-cryptography/

[3] M. R. Albrecht et al. *Classic McEliece*. Tech. rep. https://csrc.nist.gov/projects/post-quantum-cryptography/round-3-submissions. National Institute of Standards and Technology, 2020.

McEliece and Niederreiter PKE schemes

Classic McEliece is based on the Niederreiter[4] cryptosystem:

- KeyGen$(n, k, t) = (\text{pk}, \text{sk})$
  - $\boldsymbol{H}$-parity-check of $\mathcal{C}$[5]
  - Sample $\boldsymbol{S} \in \mathrm{GL}_{n-k}(\mathbb{F}_2)$
  - $\boldsymbol{H}_{\text{pub}} = \boldsymbol{SHP}$
  - $\text{pk} = (\boldsymbol{H}_{\text{pub}}, t)$
  - $\text{sk} = (\boldsymbol{S}, \boldsymbol{H}, \boldsymbol{P})$

- Encrypt$(\boldsymbol{m}, \text{pk}) = \boldsymbol{s}$
  - Encode $\boldsymbol{m} \to \boldsymbol{e}$ ($\boldsymbol{e}$ is a vector of Hamming weight t: $\mathrm{wt}(\boldsymbol{e}) = t$)
  - $\boldsymbol{s} = \boldsymbol{H}_{\text{pub}}\boldsymbol{e}$

[4] H. Niederreiter. "Knapsack-type cryptosystems and algebraic coding theory". In: *Problems of Control and Information Theory* 15.2 (1986), pp. 159–166.

[5] $\mathcal{C}$ is a $[n, k]$ linear code that admits an efficient decoding algorithm able to correct up to t errors

Syndrome decoding problem

Syndrome decoding

Definition (Binary-SDP)

Input: a matrix $\boldsymbol{H} \in \mathcal{M}_{n-k,n}(\mathbb{F}_2)$, a vector $\boldsymbol{s} \in \mathbb{F}_2^{n-k}$ and $t \in \mathbb{N}^+$

Output: a vector $\boldsymbol{x} \in \mathbb{F}_2^n$, with $\mathrm{wt}(\boldsymbol{x}) \leq t$, such that $\boldsymbol{H}\boldsymbol{x} = \boldsymbol{s}$.

Known to be an NP-hard problem[6].

[6] E. R. Berlekamp, R. J. McEliece, and H. C. A. van Tilborg. "On the inherent intractability of certain coding problems (Corresp.)". In: *IEEE Transactions on Information Theory* 24.3 (1978), pp. 384–386.

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Definition (ILP problem)

Let $n, m \in \mathbb{N}^+, \boldsymbol{b} \in \mathbb{N}^n, \boldsymbol{c} \in \mathbb{N}^m$ and $\boldsymbol{A} \in \mathcal{M}_{m,n}(\mathbb{N})$.
The ILP problem is defined as the optimization problem

$$\min\{\boldsymbol{b}^T x | \boldsymbol{A}\boldsymbol{x} = \boldsymbol{c}, \boldsymbol{x} \in \mathbb{N}^n, \boldsymbol{x} \geq 0\}.$$

[6] E. R. Berlekamp, R. J. McEliece, and H. C. A. van Tilborg. "On the inherent intractability of certain coding problems (Corresp.)". In: *IEEE Transactions on Information Theory* 24.3 (1978), pp. 384–386.

$\mathbb{N}$-SDP

Definition ($\mathbb{N}$-SDP)

Input: a matrix $\boldsymbol{H} \in \mathcal{M}_{n-k,n}(\mathbb{N})$ with $h_{i,j} \in \{0,1\}$ for all i, j,
a vector $\boldsymbol{s} \in \mathbb{N}^{n-k}$ and $t \in \mathbb{N}^*$ with $t \neq 0$.

Output: a vector $\boldsymbol{x} \in \mathbb{N}^n$ with $x_i \in \{0,1\}$ and $\mathrm{wt}(\boldsymbol{x}) \leq t$, such that $\boldsymbol{Hx} = \boldsymbol{s}$.

Theorem

Let us suppose that there exists a unique vector $\boldsymbol{x}^* \in \{0,1\}^n$ with $\mathrm{wt}(\boldsymbol{x}^*) = t$, solution to the $\mathbb{N}$-SDP. Then $\boldsymbol{x}^*$ is the optimum solution of an ILP problem.

The $\mathbb{N}$-SDP can be efficiently solved by a linear programming solver.

ILP solver for $\mathbb{N}$-SDP

Input: $\boldsymbol{H}, \boldsymbol{s}, t$

Output: $\boldsymbol{x}$ solution to $\mathbb{N}$-SDP or ERROR

1: Set $\boldsymbol{b} = (1, \ldots, 1)^T$ ▷ Hamming weight constraint
2: Solve $\min\{\boldsymbol{b}^T\boldsymbol{x} \mid \boldsymbol{Hx} = \boldsymbol{s}, 0 \preceq \boldsymbol{x} \preceq 1, \boldsymbol{x} \in \mathbb{R}^n\}$ ▷ Using the LP solver
3: Round the solution $\boldsymbol{x}^*$ to $\boldsymbol{x}^* \in \{0, 1\}^n$
4: **if** $\boldsymbol{Hx}^* = \boldsymbol{s}$ and $\mathrm{wt}(\boldsymbol{x}) \leq t$ **then**
5:     **return** $\boldsymbol{x}^*$
6: **else**
7:     **return** ERROR
8: **end if**

$\boldsymbol{H}$ and t are parameters of the cryptosystem.

Next step

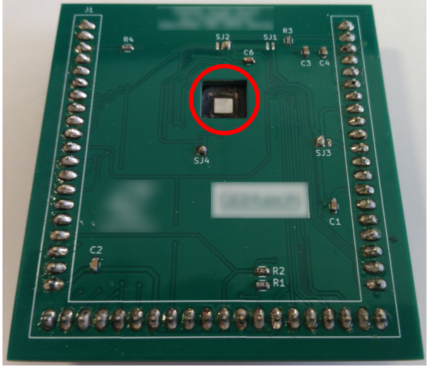
To place ourselves in the $\mathbb{N}$-SDP framework, we must now obtain $\boldsymbol{s} \in \mathbb{N}^{n-k}$.

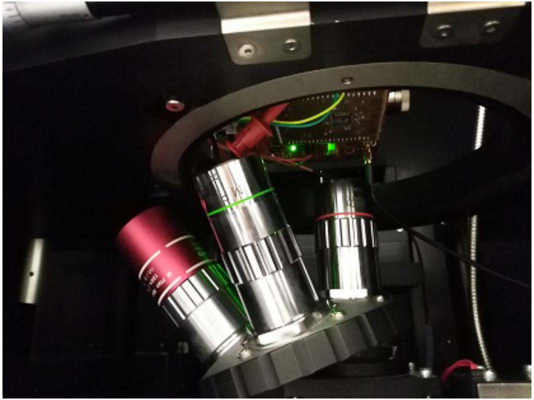
Laser fault injection

Laser fault injection

Laser fault injection was proposed in 2002 by Skorobogatov[7].

Principle: shine an **infrared** laser on the backside of an integrated circuit.





[7] S. P. Skorobogatov and R. J. Anderson. "Optical Fault Induction Attacks". In: *International Workshop on Cryptographic Hardware and Embedded Systems*. Vol. 2523. Redwood Shores, CA, USA: Springer, 2002, pp. 2–12.

Laser fault injection in Flash memory

In a recent line of work, laser fault injection is done in **Flash memory**[8,9,10].

Fault model:

- **mono bit** precision,
- **bit-set** only ($0 \rightarrow 1$),
- **transient** : only **fetched** data is affected, stored data remains **unchanged**.

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[8] D. S. V. Kumar et al. "An In-Depth and Black-Box Characterization of the Effects of Laser Pulses on ATmega328P". In: *International Conference on Smart Card Research and Advanced Applications*. Vol. 11389. Montpellier, France, Nov. 2018, pp. 156–170.

[9] B. Colombier et al. "Laser-induced Single-bit Faults in Flash Memory: Instructions Corruption on a 32-bit Microcontroller". In: *IEEE International Symposium on Hardware Oriented Security and Trust*. McLean, VA, USA, May 2019, pp. 1–10.

[10] K. Garb and J. Obermaier. "Temporary Laser Fault Injection into Flash Memory: Calibration, Enhanced Attacks, and Countermeasures". In: *International Symposium on On-Line Testing and Robust System Design*. Napoli, Italy: IEEE, July 2020, pp. 1–7.

Capabilities

It is possible to **corrupt the instructions**[11] which are fetched from Flash memory before being executed by the microcontroller.

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0

EORS instruction: R_d = R_m ^ R_n

0	1	0	0	0	0	0	0	0	1	R_m	R_{dn}

ADCS instruction: R_d = R_m + R_n

0	1	0	0	0	0	0	1	0	1	R_m	R_{dn}

We can turn addition in $\mathbb{F}_2$ into addition in $\mathbb{N}$.

[11] The opcodes for the instructions are available in the ARMv7-M Architecture Reference Manual `https://developer.arm.com/documentation/ddi0403/ee/`

Target function

We target the **matrix-vector multiplication** for **syndrome computation**: $\boldsymbol{s} = \boldsymbol{H}_{\text{pub}}\boldsymbol{e}$.

```
function MAT_VEC_MULT(matrix, error_vector)
    for row ← 0 to (n − k − 1) do
        syndrome[row] = 0
    for row ← 0 to (n − k − 1) do
        for col ← 0 to (n − 1) do
            syndrome[row] ^= matrix[row][col] & error_vector[col]
    return syndrome
```

Outcome

After performing $(n-k) \times n$ faults during **one** encryption, we get $\boldsymbol{s}^* \in \mathbb{N}$.

Experimental results

Experiments

Scipy LP solver[12] and cryptographic parameters of the *Classic McEliece* submission.

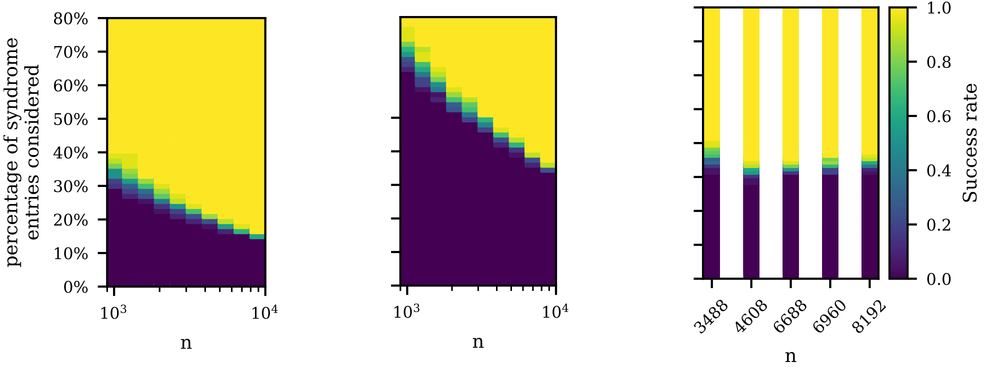
n	3488	4608	6688	6960	8192
k	2720	3360	5024	5413	6528
t	64	96	128	119	128
Equivalent bit-level security	128	196	256	256	256

- $\boldsymbol{H}_{\text{pub}}$ has $n-k$ rows and n columns, parity check matrix of a **random linear code**,
- $\boldsymbol{e}$ is of size n and of Hamming weight t.

[12] `https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.linprog.html`

Required number of faulty syndrome entries

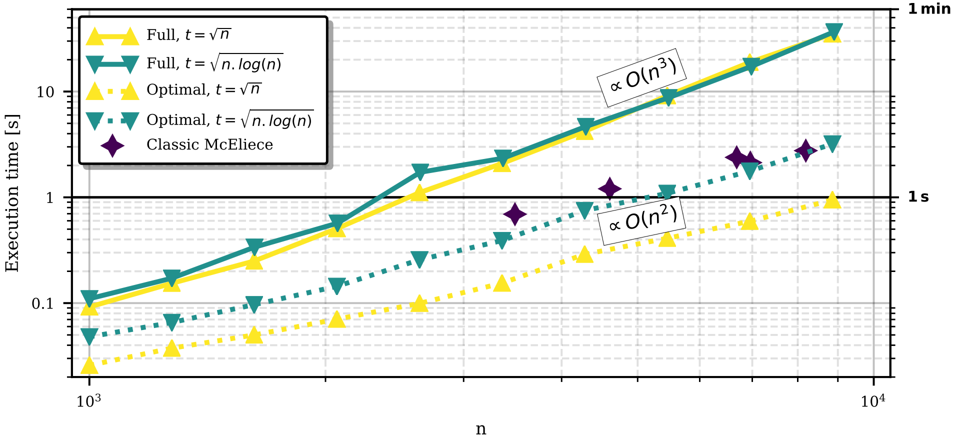
A **fraction** of the faulty syndrome entries is enough to solve the problem.



(a) $t = \sqrt{n}$ (b) $t = \sqrt{n \log(n)}$ (c) *Classic McEliece*

For *Classic McEliece*, **less than 40 %** faulty syndrome entries is enough.

Execution time



On a desktop computer (6 cores at 2.8 GHz and 32 GB of RAM):

- *Classic McEliece* with **128**-bit security: **less than 1 second**,
- *Classic McEliece* with **256**-bit security: **less than 3 seconds**.

Conclusion

We presented a **message-recovery attack**:

Given the public matrix ($\boldsymbol{H}_{\mathrm{pub}}$) and a syndrome ($\boldsymbol{s}$), we aim at recovering the private error vector $\boldsymbol{e}$ such that $\boldsymbol{s} = \boldsymbol{H}_{\mathrm{pub}}\boldsymbol{e}$.

1. Perform laser fault injection during the encryption process to modify the instructions, turning the `XOR` into an `ADD`,
2. Obtain a faulty syndrome $\boldsymbol{s}^* \in \mathbb{N}^n$,
3. Solve the resulting problem using an Integer Linear Programming solver,
4. Recover the private error vector in polynomial time.

Perspectives

- Export the idea to key recovery attacks,
- Better study the attack complexity in the "Full" and "Optimal" cases,
- Extend our attack to any code-based cryptosystem for which the implementation is vulnerable to laser fault injection,
- Inspect other fault injection techniques to corrupt the instructions,
- Examine other types of corruption of instructions (`XOR`->Shift, ...) and try to define new challenging theoretical problems.

Backup slides

Micro-controller implementations of code-based cryptosystems

Existing work done in the 2010's[13,14].

Recent work too, since ARM Cortex-M4 became the de facto standard for this kind of embedded system implementations[15,16]

[13] S. Heyse. "Low-Reiter: Niederreiter Encryption Scheme for Embedded Microcontrollers". In: *International Workshop on Post-Quantum Cryptography*. Ed. by N. Sendrier. Vol. 6061. Lecture Notes in Computer Science. Darmstadt, Germany: Springer, May 2010, pp. 165–181.

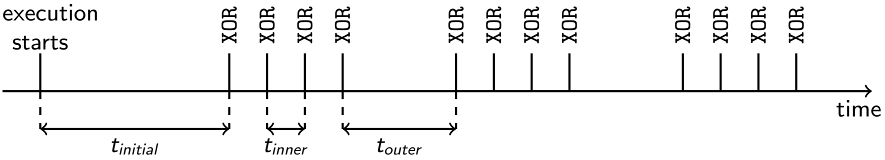
[14] T. Eisenbarth, T. Güneysu, S. Heyse, and C. Paar. "MicroEliece: McEliece for Embedded Devices". In: *International Workshop on Cryptographic Hardware and Embedded Systems*. Ed. by C. Clavier and K. Gaj. Vol. 5747. Lecture Notes in Computer Science. Lausanne, Switzerland: Springer, Sept. 2009, pp. 49–64.

[15] M.-S. Chen and T. Chou. "Classic McEliece on the ARM Cortex-M4". In: *IACR Transactions on Cryptographic Hardware and Embedded Systems* 2021.3 (2021), pp. 125–148.

[16] J. Roth, E. G. Karatsiolis, and J. Krämer. "Classic McEliece Implementation with Low Memory Footprint". In: *International Conference on Smart Card Research and Advanced Application*. Ed. by P.-Y. Liardet and N. Mentens. Vol. 12609. Lecture Notes in Computer Science. Virtual Event: Springer, Nov. 2020, pp. 34–49.

Fault injection timing

Constant-time code makes fault injection much easier.



3 delays to tune:

- $t_{initial}$
- t_{inner}
- t_{outer}