Fast verification of masking schemes in characteristic two

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Eurocrypt 2021
2021–10–19

Masking schemes for finite field multiplication

Probing security model

Computationally checking security in $\mathbb{F}_2$

Results

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The context

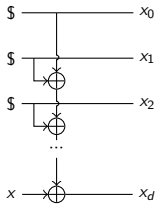
Context: Crypto implementations on observable devices

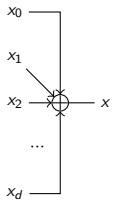
Objective: secure finite-field multiplication with leakage

- Implement $(a, b) \mapsto c = a \times b$, a, b, $c \in \mathbb{K}$
  - Used in non-linear ops in symmetric crypto (e.g. S-boxes)
  - Inputs/output usually secret!
- Problem: computations leak information

Basic idea: masking

- Split a, b, c into *shares* (i.e. use a secret-sharing scheme)
  - Typically simple and additive:
    $x = \sum_{i=0}^{d} x_i$, $x_{0,\ldots,d-1} \xleftarrow{\$} \mathbb{K}$, $x_d = x - \sum_{i=0}^{d-1} x_i$

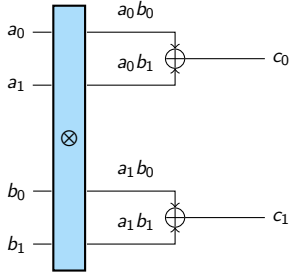




- Compute the operation over the shared operands; obtain a shared result
- Ensure that no information can (easily) be gained on a, b or c

First attempt

- We want to compute $c = \sum_k c_k = \sum_i a_i \times \sum_j b_j = \sum_{i,j} a_i b_j$
- So maybe define $c_k = \sum_{j=0}^{d} a_k b_j$?



- Problem: any single c_k reveals information about b
- This strategy is intuitively not secure.

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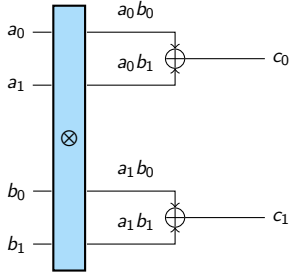
- So maybe define $c_k = \sum_{j=0}^{d} a_k b_j = a_k \times b$?



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Quick defs.

Gadget

A gadget for a function f is a (randomized) circuit C on (additively) shared inputs/outputs $\boldsymbol{x}_i$, $\boldsymbol{y}_j$ s.t. for every set of coins $\mathcal{R}$, $(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_m) \leftarrow\!\shortmid C(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n; \mathcal{R})$ satisfies:

$$\left(\sum_{j=1}^{v} \boldsymbol{y}_{1,j}, \ldots, \sum_{j=1}^{v} \boldsymbol{y}_{m,j}\right) = f\left(\sum_{j=1}^{u} \boldsymbol{x}_{1,j}, \ldots, \sum_{j=1}^{u} \boldsymbol{x}_{m,j}\right)$$

Probe

A probe on C maps a wire to the value it takes in a run of the gadget

d-privacy

Introduced by Ishai, Sahai & Wagner (2003)

- If every set of $t \leq d$ probes is independent from the unmasked values on which the gadget is evaluated $\rightsquigarrow$ d-privacy

ISW (2003) designed a d-private masking scheme with quadratic complexity in d

- $2d(d+1)$ sums
- $(d+1)^2$ products
- $d(d+1)/2$ additional random masks

Secure multiplication gadgets

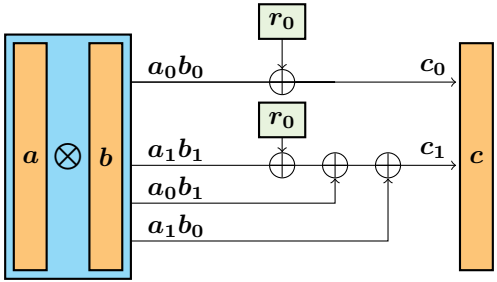

Figure: Multiplication gadget at order $d = 1$ from ISW, 2003.

Secure multiplication gadgets

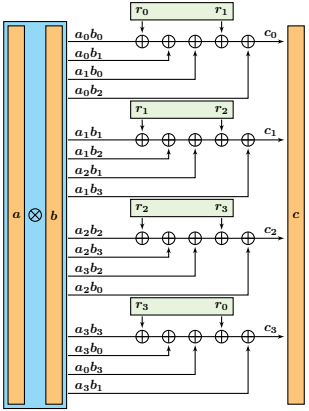

Figure: Multiplication gadget at order $d = 3$ from Barthe et al., 2017.

A composable security model based on simulatability

- d-privacy is not composable.
- Barthe et al. (2016) introduced composable models called *(strong) non-interference* based on simulatability

Simulatability

A given set of probes $\mathcal{P} := \{p_1, p_2, \ldots\}$ on C is t-simulatable if for fixed input sharings $(\boldsymbol{x}_1, \boldsymbol{x}_2, \ldots)$, all the distributions induced on $\mathcal{P}$ by $\mathcal{R}$ can be simulated with the knowledge of $\leq t$ $\boldsymbol{x}_{1,i}$s; $\leq t$ $\boldsymbol{x}_{2,i}$s; etc.

d-(Strong) Non-Interference

d-Non-Interference (d-NI)

C is d-NI iff. any set of at most d probes is d-simulatable

d-Strong Non-Interference (d-SNI)

C is d-SNI iff. any set of at most $d_1 + d_2 \leq d$ probes is d_1-simulatable, where d_2 probes are on the output wires only

These security models are useful because:

- d-SNI $\Longrightarrow$ d-NI $\Longrightarrow$ d-private;
- under some independence hypotheses, SNI$\circ$NI = SNI.

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How to check a masking scheme?

To efficiently check if a masking scheme is d-NI, in absence of a generic proof, we want:

- An easy-to-check condition for simulatability of subsets of probes for a scheme defined over $\mathbb{F}_2$
- An efficient enumeration of all subsets of probes
- (Not shown here) A set of probes to check as small as possible
- (Not shown here) Extension to check d-SNI
- (Not shown here) Extension to check in the more hardware-oriented robust probing model

A linear condition on bilinear probes

Condition 3.2 (Belaïd et al., 2017)

A set of bilinear probes $\mathcal{P} = \{p_1, \ldots, p_\ell\}$ on a gadget C for a function $f : \mathbb{K}^2 \to \mathbb{K}$ satisfies Cond. 3.2 iff. $\exists\, \boldsymbol{\lambda} \in \mathbb{K}^\ell$, $\boldsymbol{M} \in \mathbb{K}^{(d+1)\times(d+1)}$, $\boldsymbol{\mu}$, $\boldsymbol{\nu} \in \mathbb{K}^{d+1}$, and $\tau \in \mathbb{K}$ s.t. $\sum_{i=1}^{\ell} \boldsymbol{\lambda}_i p_i = \boldsymbol{a}^t \boldsymbol{M} \boldsymbol{b} + \boldsymbol{a}^t \boldsymbol{\mu} + \boldsymbol{b}^t \boldsymbol{\nu} + \tau$ and all the rows of the block matrix $\begin{pmatrix} \boldsymbol{M} & \boldsymbol{\mu} \end{pmatrix}$ or all the columns of the block matrix $\begin{pmatrix} \boldsymbol{M} \\ \boldsymbol{\nu}^t \end{pmatrix}$ are non-zero

- No $\boldsymbol{r}$-dependency: cannot be simulated with a uniform distribution
- No zero rows/columns: full functional dependence on the $d+1$ shares of $\boldsymbol{a}$ or $\boldsymbol{b}$

Theorem for simulatability

The previous condition is useful to analyse the security of a gadget

Theorem (Belaïd et al., 2017)

If $\mathcal{P}$ satisfies Cond. 3.2, then it is not d-simulatable
If $\mathcal{P}$ is not d-simulatable *and* $\#\mathbb{K} > d + 1$, then it satisfies Cond 3.2

Corollary(Belaïd et al., 2017)

If $\#\mathbb{K} > d + 1$ and no set of $\leq d$ probes on C satisfies Cond. 3.2, then it is d-NI

A **new** linear condition on bilinear probes

Condition 12

A set of bilinear probes $\mathcal{P} = \{p_1, \ldots, p_\ell\}$ on a gadget C for a function $f : \mathbb{K}^2 \to \mathbb{K}$ satisfies Cond. 12 iff. $\exists \boldsymbol{\lambda} \in \mathbb{K}^\ell$, $\boldsymbol{M} \in \mathbb{K}^{(d+1)\times(d+1)}$, $\boldsymbol{\mu}$, $\boldsymbol{\nu} \in \mathbb{K}^{d+1}$, and $\tau \in \mathbb{K}$ s.t. $\sum_{i=1}^{\ell} \boldsymbol{\lambda}_i p_i = \boldsymbol{a}^t \boldsymbol{M} \boldsymbol{b} + \boldsymbol{a}^t \boldsymbol{\mu} + \boldsymbol{b}^t \boldsymbol{\nu} + \tau$ and *the block matrix* $\begin{pmatrix} \boldsymbol{M} & \boldsymbol{\mu} \end{pmatrix}$ *has* **at least** $\ell + 1$ *non-zero rows or the block matrix* $\begin{pmatrix} \boldsymbol{M} \\ \boldsymbol{\nu}^t \end{pmatrix}$ *has* **at least** $\ell + 1$ *non-zero columns.*

- No $\boldsymbol{r}$-dependency: cannot be simulated with a uniform distribution
- At least $\ell + 1$ non-zero rows/columns: functional dependence on more shares of $\boldsymbol{a}$ or $\boldsymbol{b}$ than the number of probes

New theorem for simulatability

The previous condition is useful to analyse the security of a gadget

Theorem

If $\mathcal{P}$ of ℓ probes satisfies Condition 12, then it is not ℓ-simulatable.
If $\mathcal{P}$ is not d-simulatable, then there exists $\mathcal{P}' \subseteq \mathcal{P}$ that satisfies Condition 12.

Corollary

If no set of $\leq d$ probes on C satisfies Condition 12, then it is d-NI

Efficient software implementation

Check Condition 12 for all subset of $\leq d$ probes.
To do it efficiently:

- Rewrite the condition in terms of weight of indicator matrices;
- Use vectorization to compute the matrices & weights;
- Use combination Gray codes for efficient enumeration;

$\rightsquigarrow$ Peak performance (@2.60 GHz) of $\approx 2^{27.5}$ checks/s
$\rightsquigarrow$ Easy parallelization

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Overview of our contributions (model and tool)

- New condition for d-(S)NI security over small fields
- New algorithm to check d-(S)NI over $\mathbb{F}_2$
- An efficient implementation
- Improved verification performance by 3 orders of magnitude
  - e.g. 8-SNI verif.: 13 days (4 threads, maskVerif, Barthe et. al. 2019) vs < 10 minutes (1 thread, our software)

Overview of our contributions (applications)

- NI/SNI verif. of concrete masking schemes up to order $d = 11$ (previously verified only to order 7)
- Disproving conjecture of Barthe et. al. on the security of generic transformation of NI gadgets into SNI gadgets
- Slightly better SNI masking schemes for high-order
  - e.g. at order $d = 7$, 17% less additional random masks (r_i)

References

- Full version: https://eprint.iacr.org/2019/1165
  (with additional examples and figures)
- Implementation: https://github.com/NicsTr/binary_masking