Password Hashing and Preprocessing

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Passwords

- Still one of the most prevalent forms of authentication
- Use H(pw) in place of pw
- Use H(pw) as a secret key (e.g., to encrypt data)

The Setting

- Attackers attempt to “crack” multiple passwords
- May well use preprocessing (e.g. Rainbow Tables) to speed things up
- Typically the hash function is assumed to be secure (= RO)
- Attackers exploit the low entropy of human-generated passwords

Conventional Wisdom

$\mathsf{sa}_i, \mathsf{H}(\mathsf{sa}_i, \mathsf{pw}_i)$

"Salting Defeats Preprocessing"

Distinct (known) salts $\Longrightarrow$ separate preprocessing effort

Unpredictable salts $\Longrightarrow$ defeat preprocessing effort

Related Works

- Long line of works on preprocessing: [GT00,Unr07,DTT10,DGK17,CDGS18,CDG18,...]
- Bellare, Ristenpart, Tessaro [BRT12]: Highlight the need for multi-instance security in password-based crypto.

Two Settings

Preprocessing

- $\sigma \leftarrow A_0(\mathsf{H})$
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Multi-Instance

- $(\mathsf{pw}'_1, \ldots, \mathsf{pw}'_m) \leftarrow A^{\mathsf{H}}\big(\mathsf{H}(\mathsf{pw}_1), \ldots, \mathsf{H}(\mathsf{pw}_m)\big)$
- Return $(\mathsf{pw}_1, \ldots, \mathsf{pw}_m) = (\mathsf{pw}'_1, \ldots, \mathsf{pw}'_m)$

The Multi-Instance Perspective

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- Effort to compromise *a* single instance may be within reach
- Want: effort to compromise m instances to scale with m

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- Effort to compromise *a* single instance may be within reach
- Want: effort to compromise m instances to scale with m
- Not without salting:

$$\mathbf{Adv}^{\mathrm{rec}}_{[N]^m,\perp,\mathsf{RO}}(T) \geq \left(\frac{T}{N}\right)^m$$

The Multi-Instance Perspective

- $(\mathsf{pw}'_1, \ldots, \mathsf{pw}'_m) \leftarrow A^{\mathsf{H}}\big(\mathsf{H}(\mathsf{sa}_1, \mathsf{pw}_1), \ldots, \mathsf{H}(\mathsf{sa}_n, \mathsf{pw}_m)\big)$
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- With sufficiently large salts:

$$\mathbf{Adv}^{\mathrm{rec}}_{[N]^m,[K],\mathsf{RO}}(T) \geq \left(\frac{T}{m \cdot N}\right)^m$$

Our Goal

Understand the security of password hashing
in the presence of multiple instances and preprocessing

The Setting

Preprocessing

- $\sigma \leftarrow A_1(\mathsf{H})$
- Run G with $A_1^{\mathsf{H}}(\sigma)$

Multi-Instance

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Combine!

We Consider

No Salts, Distinct Salts, & Random Salts

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Historical, Amplification

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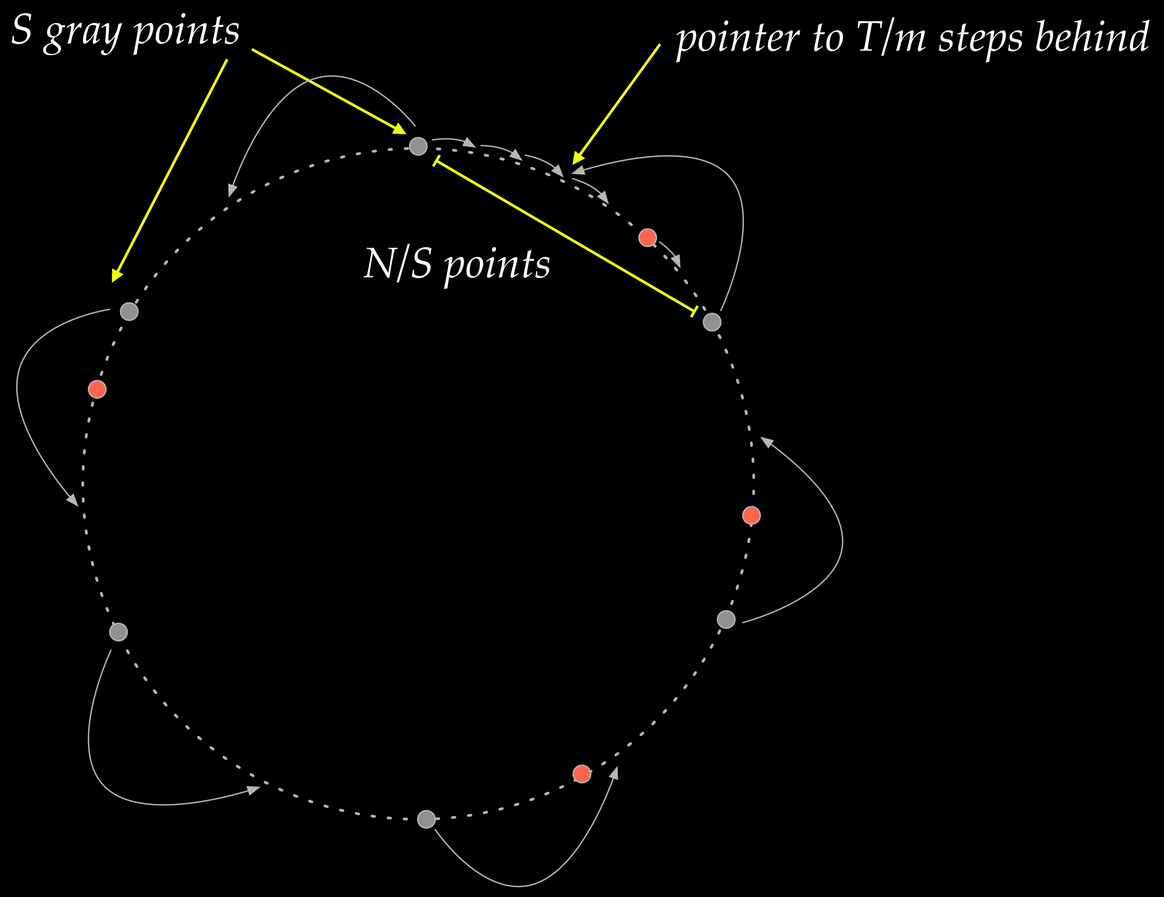
Practice

Multi-Instance Hellman



$$\Pr\left[\textbf{All reds within } T/m \textbf{ of a gray}\right] \approx \left(\frac{T/m}{N/S}\right)^m = \left(\frac{ST}{mN}\right)^m$$

Password Unguessability [BRT12]

Game $\textsc{Guess}_{\mathcal{P}}^{\mathcal{A}}$:	Proc. $\textsc{Test}(i, pw)$:	Proc. $\textsc{Cor}(i)$:
$(\mathbf{pw}, z) \twoheadleftarrow \mathcal{P}$	$\mathsf{win}_i \leftarrow (pw = \mathbf{pw}[i])$	$\mathsf{win}_i \leftarrow \mathsf{true}$
$y \twoheadleftarrow \mathcal{A}^{\textsc{Test},\textsc{Cor}}(z)$	return win_i	return $\mathbf{pw}[i]$
return $\bigwedge_{i=1}^{m} \mathsf{win}_i$		

$$\mathbf{Adv}_{\mathcal{P}}^{\text{guess}}(T, c) := \max_{\mathcal{A}} \Pr\left[\textsc{Guess}_{\mathcal{P}}^{\mathcal{A}}\right]$$

The Basic Measure

Game $\textsc{Guess}^{\mathcal{A}}_{\mathcal{P}}$:	Proc. $\textsc{Test}(i, pw)$:	Proc. $\textsc{Cor}(i)$:
$(\mathbf{pw}, z) \twoheadleftarrow \mathcal{P}$	$\mathsf{win}_i \leftarrow (pw = \mathbf{pw}[i])$	$\mathsf{win}_i \leftarrow \mathsf{true}$
$y \twoheadleftarrow \mathcal{A}^{\textsc{Test},\textsc{Cor}}(z)$	return win_i	return $\mathbf{pw}[i]$
return $\bigwedge_{i=1}^{m} \mathsf{win}_i$		

$$\mathbf{Adv}^{\mathrm{guess}}_{\mathcal{P}}(m, 0) = 2^{-\tilde{\mathbf{H}}_{\infty}(\mathcal{P}|\mathcal{Z})}$$

The Basic Measure

Game $\textsc{Guess}^{\mathcal{A}}_{\mathcal{P}}$:	Proc. $\textsc{Test}(i, pw)$:	Proc. $\textsc{Cor}(i)$:
$(\mathbf{pw}, z) \twoheadleftarrow \mathcal{P}$ $y \twoheadleftarrow \mathcal{A}^{\textsc{Test},\textsc{Cor}}(z)$ return $\bigwedge_{i=1}^{m} \mathsf{win}_i$	$\mathsf{win}_i \leftarrow (pw = \mathbf{pw}[i])$ return win_i	$\mathsf{win}_i \leftarrow \mathsf{true}$ return $\mathbf{pw}[i]$

$$\mathbf{Adv}^{\text{guess}}_{\mathcal{P}}(m - c, c)$$

A Simple Argument

Theorem 2 (Unguessability). *For any m-sampler $\mathcal{P}$ and any $T, c \in \mathbb{N}$,*

$$\mathbf{Adv}^{\text{guess}}_{\mathcal{P}}(T, c) \leq \binom{T}{m-c} \cdot \mathbf{Adv}^{\text{guess}}_{\mathcal{P}}(m-c, c) \ .$$

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Proof:

- Guess the (m — c) out of T queries to TEST that will return T. Relay these to own TEST.
- Answer the rest of TEST queries with F.
- Relay all COR queries to own COR.

Resolves Open Question in [BRT12]

DISCUSSION. We note that although the bound in Theorem 3.2 depends only on q_t, q_c, we are not claiming that it holds for any (q_t, q_c)-guessing adversary, but only for a $(\mathbf{q}, q_c)$-guessing adversary with $q_t = \mathbf{q}[1] + \cdots + \mathbf{q}[m]$. The same is true for the better bound we just quoted for the $q_c = 0$ case. An example from [27] shows that our bound of $(q_t/m2^{\mu})^m$ for the $q_c = 0$ case in fact does not hold for arbitrary (q_t, q_c)-guessing adversaries. For $q_t = 11$, $m = 2$ and $\mathcal{P}$ which picks random, 3-bit passwords, they present an attack with advantage higher than $(q_t/m2^{\mu})^m$. Obtaining a (good) bound for (q_t, q_c)-guessing adversaries is an interesting open question.

- BRT considered *a priori* bounds T[i] for *each* index i.
- We remove this restriction:

  Adversary can adapt T[i] as it makes progress

So far...

We looked at
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Next: hashed salted passwords

Unrecoverability in AI-RO

Game $\text{AI-REC}^{\mathcal{A}_0,\mathcal{A}_1}_{\mathcal{P},\ell,\mathsf{Gen},\mathsf{KD}}$:

$\mathsf{H} \twoheadleftarrow \mathrm{Fun}(D, R)$
$\sigma \twoheadleftarrow \mathcal{A}_0(\mathsf{H})$
$(\mathbf{pw}, z) \twoheadleftarrow \mathcal{P}$
for $(i, j) \in [m] \times [\ell]$ do
  $\mathbf{sa}[i, j] \twoheadleftarrow \mathsf{Gen}(i, j)$
  $\mathbf{k}[i, j] \leftarrow \mathsf{KD}^{\mathsf{H}}(\mathbf{pw}[i], \mathbf{sa}[i, j])$
$\mathbf{pw}' \twoheadleftarrow \mathcal{A}_1^{\mathsf{H},\text{COR}}(\mathbf{sa}, \mathbf{k}, \sigma, z)$
return $(\mathbf{pw}' = \mathbf{pw})$

Proc. $\text{COR}(i)$:

return $\mathbf{pw}[i]$

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return $(\mathbf{pw}' = \mathbf{pw})$

Proc. $\text{COR}(i)$:
return $\mathbf{pw}[i]$

Game $\text{BF-REC}^{\mathcal{A}_0,\mathcal{A}_1}_{\mathcal{P},\ell,\mathsf{Gen},\mathsf{KD}}$:

$\mathsf{H} \twoheadleftarrow \mathrm{Fun}(D,R)$
$(\sigma, L) \twoheadleftarrow \mathcal{A}_0()$
$(\mathbf{pw}, z) \twoheadleftarrow \mathcal{P}$
for $(i,j) \in [m] \times [\ell]$ do
  $\mathbf{sa}[i,j] \twoheadleftarrow \mathsf{Gen}(i,j)$
  $\mathbf{k}[i,j] \leftarrow \mathsf{KD}^{\mathsf{H}[L]}(\mathbf{pw}[i], \mathbf{sa}[i,j])$
$\mathbf{pw}' \twoheadleftarrow \mathcal{A}_1^{\mathsf{H}[L],\text{COR}}(\mathbf{sa}, \mathbf{k}, \sigma, z)$
return $(\mathbf{pw}' = \mathbf{pw})$

Proc. $\text{COR}(i)$:
return $\mathbf{pw}[i]$

Unrecoverability in AI-RO

[CDGS18]

Game $\text{AI-REC}^{\mathcal{A}_0,\mathcal{A}_1}_{\mathcal{P},\ell,\mathsf{Gen},\mathsf{KD}}$:

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$\sigma \twoheadleftarrow \mathcal{A}_0(\mathsf{H})$
$(\mathbf{pw}, z) \twoheadleftarrow \mathcal{P}$
for $(i,j) \in [m] \times [\ell]$ do
  $\mathbf{sa}[i,j] \twoheadleftarrow \mathsf{Gen}(i,j)$
  $\mathbf{k}[i,j] \leftarrow \mathsf{KD}^{\mathsf{H}}(\mathbf{pw}[i], \mathbf{sa}[i,j])$
$\mathbf{pw}' \twoheadleftarrow \mathcal{A}_1^{\mathsf{H},\text{COR}}(\mathbf{sa}, \mathbf{k}, \sigma, z)$
return $(\mathbf{pw}' = \mathbf{pw})$

Proc. $\text{COR}(i)$:
return $\mathbf{pw}[i]$

Game $\text{BF-REC}^{\mathcal{A}_0,\mathcal{A}_1}_{\mathcal{P},\ell,\mathsf{Gen},\mathsf{KD}}$:

$\mathsf{H} \twoheadleftarrow \text{Fun}(D,R)$
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for $(i,j) \in [m] \times [\ell]$ do
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  $\mathbf{k}[i,j] \leftarrow \mathsf{KD}^{\mathsf{H}[L]}(\mathbf{pw}[i], \mathbf{sa}[i,j])$
$\mathbf{pw}' \twoheadleftarrow \mathcal{A}_1^{\mathsf{H}[L],\text{COR}}(\mathbf{sa}, \mathbf{k}, \sigma, z)$
return $(\mathbf{pw}' = \mathbf{pw})$

Proc. $\text{COR}(i)$:
return $\mathbf{pw}[i]$

BF-Rec Bound (Uniform Salts)

Theorem 7. *Let* $\mathsf{KD}^{\mathsf{H}}(pw, sa) := \mathsf{H}(pw|sa)$ *for random oracle* H. *Then for any* m*-sampler* $\mathcal{P}$*, any salt generator* $\mathsf{Gen} := [K]$ *that outputs uniform salts in a set of size* K*, and any* $\ell, P, T, c \in \mathbb{N}$*,*

$$\mathbf{Adv}^{\text{bf-rec}}_{\mathcal{P},\ell,[K],\mathsf{H}}(P,T,c) \leq \left(\binom{T}{m-c} + \frac{m\ell}{K}\binom{T+P}{m-c}\right) \cdot \mathbf{Adv}^{\text{guess}}_{\mathcal{P}}(m-c,c) + \frac{m^2\ell^2}{K} \ .$$

Other Bounds (See Paper)

Theorem 6. *Let* $\mathsf{KD}^{\mathsf{H}}(pw, sa) := \mathsf{H}(pw|sa)$ *for random oracle* H. *Then for any* m*-sampler* $\mathcal{P}$*, any salt generator* Gen*, and any* $\ell, P, T, c \in \mathbb{N}$*,*

$$\mathbf{Adv}^{\text{bf-rec}}_{\mathcal{P},\ell,\mathsf{Gen},\mathsf{H}}(P, T, c) \leq \mathbf{Adv}^{\text{sa-guess}}_{\mathcal{P},\ell,\mathsf{Gen}}(T + P, c) \ .$$

Theorem 4. *For any* m*-sampler* $\mathcal{P}$ *and any* $\ell, T \in \mathbb{N}$*,*

$$\mathbf{Adv}^{\text{sa-guess}}_{\mathcal{P},\ell,\perp}(T, 0) \leq T^m \cdot \mathbf{Adv}^{\text{guess}}_{\mathcal{P}}(m, 0) \ .$$

Our Main Bounds

	No salts	Known distinct salts	Uniform salts
$S = 0$	$\left(\frac{6T}{N}\right)^m$	$\left(\frac{6\mathrm{e}T}{mN}\right)^m$	$\left(1 + \frac{m\ell}{K}\right) \cdot \left(\frac{6\mathrm{e}T}{mN}\right)^m$
"Large" $S \geq 3m$	$\left(\frac{6ST}{mN}\right)^m$	$\left(\frac{6\mathrm{e}ST}{m^2N}\right)^m$	$\left(\frac{2\mathrm{e}T}{mN}\right)^m + \frac{m\ell}{K} \cdot \left(\frac{6\mathrm{e}ST}{m^2N}\right)^m$

Composable AI-KDF Security (Following BRT)

Game AI-KDF-REAL$^{\mathcal{D}_0,\mathcal{D}_1}_{\mathcal{P},\ell,\mathsf{Gen},\mathsf{KD}}$:

$\mathsf{H} \twoheadleftarrow \mathrm{Fun}(D,R)$
$\sigma \twoheadleftarrow \mathcal{D}_0(\mathsf{H})$
$(\mathbf{pw}, z) \twoheadleftarrow \mathcal{P}$
for $(i,j) \in [m] \times [\ell]$ do
  $\mathbf{sa}[i,j] \twoheadleftarrow \mathsf{Gen}(i,j)$
  $\mathbf{k}[i,j] \leftarrow \mathsf{KD}^{\mathsf{H}}(\mathbf{pw}[i], \mathbf{sa}[i,j])$
$b' \twoheadleftarrow \mathcal{D}_1^{\text{PRIM}}(\mathbf{pw}, \mathbf{sa}, \mathbf{k}, z, \sigma)$
return b'

Proc. PRIM(w):

return $\mathsf{H}(w)$

Game BF/AI-KDF-IDEAL$^{\mathcal{D}_1}_{\mathcal{P},\ell,\mathsf{Gen},\mathcal{S}_0,\mathcal{S}_1}$:

$(\sigma, st) \twoheadleftarrow \mathcal{S}_0()$
$(\mathbf{pw}, z) \twoheadleftarrow \mathcal{P}$
for $(i,j) \in [m] \times [\ell]$ do
  $\mathbf{sa}[i,j] \twoheadleftarrow \mathsf{Gen}(i,j)$
  $\mathbf{k}[i,j] \twoheadleftarrow \{0,1\}^k$
$b' \twoheadleftarrow \mathcal{D}_1^{\text{PRIM}}(\mathbf{pw}, \mathbf{sa}, \mathbf{k}, z, \sigma)$
return b'

Proc. PRIM(w):

$(y, st) \twoheadleftarrow \mathcal{S}_1^{\text{TEST}}(w; st)$
return y

Proc. TEST(pw, sa):

$S \leftarrow \{i \in [m] : \exists j \in [\ell] \text{ st. } (\mathbf{pw}[i], \mathbf{sa}[i,j]) = (pw, sa)\}$
return $\mathbf{k}[S]$

Theorem 13. AI-KDF security composes in the presence of *aux info* and multiple instances:

$$\mathbf{Adv}^{\text{ai-multi}}_{\mathrm{G},\mathcal{P},\mathsf{Gen},\mathsf{KD}}(\mathcal{A}_0,\mathcal{A}_1) \le 2 \cdot \mathbf{Adv}^{\text{ai-kdf}}_{\mathcal{P},\ell,\mathsf{Gen},\mathsf{KD},\mathcal{S}_0,\mathcal{S}_1}(\mathcal{D}_0,\mathcal{D}_1) + \\ + 2 \cdot \mathbf{Adv}^{\text{sa-guess}}_{\mathcal{P},\ell,\mathsf{Gen}}(T,c) + m \cdot \mathbf{Adv}^{\text{single}}_{\mathrm{G}}(\mathcal{B}) \ .$$

AI-KDF Security of Iteration

$$\mathsf{KD}_r^{\mathsf{H}}(pw, sa) := \underbrace{\mathsf{H} \circ \cdots \circ \mathsf{H} \circ \mathsf{H}}_{r}(pw|sa)$$

BF-KDM Security:

$$\mathbf{Adv}^{\text{bf-kdf}}_{\mathcal{P},\ell,\mathsf{Gen},\mathsf{KD}_r,\mathcal{S}_0,\mathcal{S}_1}(\mathcal{D}_0, \mathcal{D}_1) \leq \frac{(r+1)m\ell T}{N} + 3 \cdot \left(\frac{rm\ell(rm\ell + P)}{N} + \frac{m\ell P}{K} + \frac{m^2\ell^2}{K} \right)$$

(Next: use [CDGS18] and optimize P to get an AI-KDF bound.)

Take-Away Message

	No salts	Known distinct salts	Uniform salts
$S = 0$	$\left(\frac{6T}{N}\right)^m$	$\left(\frac{6\mathrm{e}T}{mN}\right)^m$	$\left(1 + \frac{m\ell}{K}\right) \cdot \left(\frac{6\mathrm{e}T}{mN}\right)^m$
"Large" $S \geq 3m$	$\left(\frac{6ST}{mN}\right)^m$	$\left(\frac{6\mathrm{e}ST}{m^2N}\right)^m$	$\left(\frac{2\mathrm{e}T}{mN}\right)^m + \frac{m\ell}{K} \cdot \left(\frac{6\mathrm{e}ST}{m^2N}\right)^m$

Plus: Composable AI-KDF security of iterated hashing.

Thanks.