Efficient Range Proofs

with Transparent Setup from Bounded Integer Commitments

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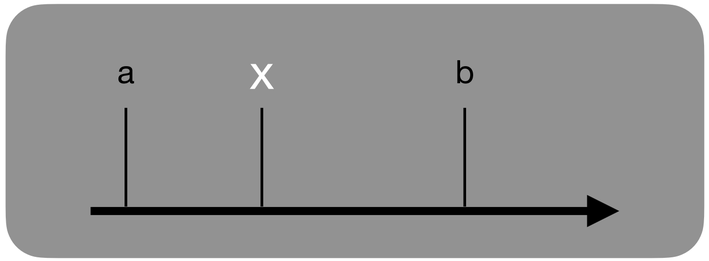
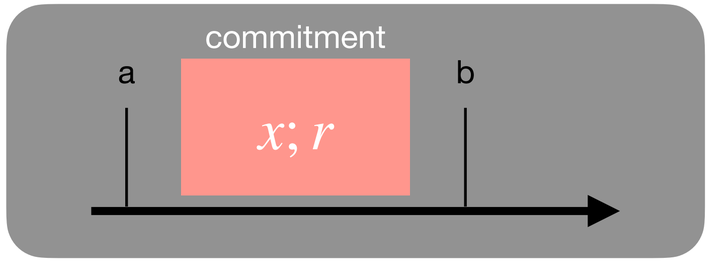
Setting



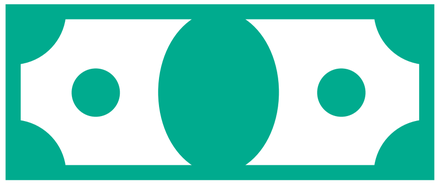


prover

“I know that the committed value **x**
is in range **[a,b]**”

verifier

Applications



Anonymous Transactions

“I have enough money”

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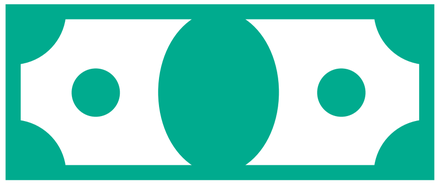

Anonymous Transactions

“I have enough money”

Anonymous Credentials

“I am old enough”

“My ticket in still valid”

Approach

[Bou00], [Lip03], [Gro05], [CPP17]

$$(x-a)(b-x) = \sum_{i=1}^{4} x_i^2$$

- Shows $x \in [a, b]$
- Requires integer commitments
  - trusted setup
  - large parameters

$x \in \mathbb{N}; r$

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- Requires integer commitments
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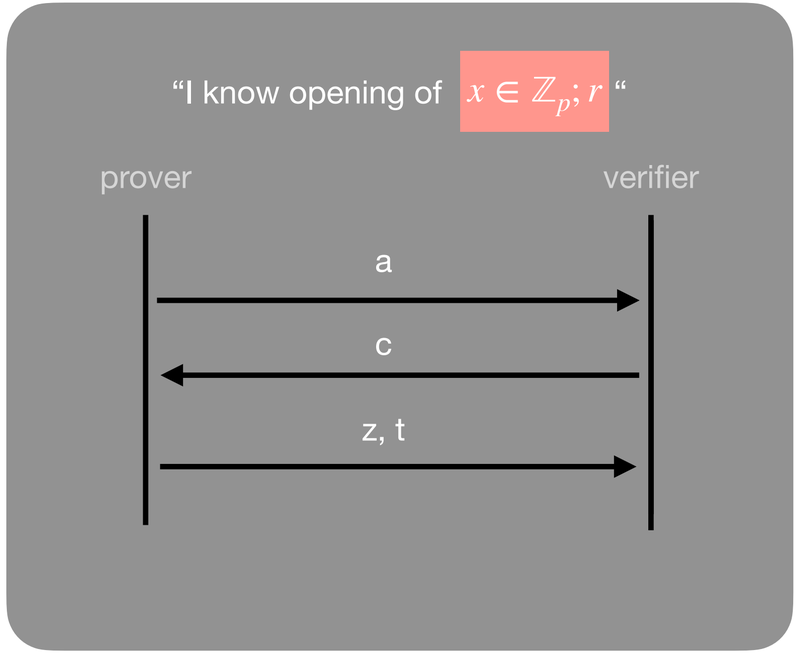
- **Idea:** show decomposition over $\mathbb{Z}_p$
  - need that x is short (no overflow)

$x \in \mathbb{N}; r$

$x \in \mathbb{Z}_p; r$

Proof of Opening



- Homomorphic commitment over $\mathbb{Z}_p$

Zero-Knowledge:
reveals no information about witness

Soundness:
witness can be extracted from transcripts

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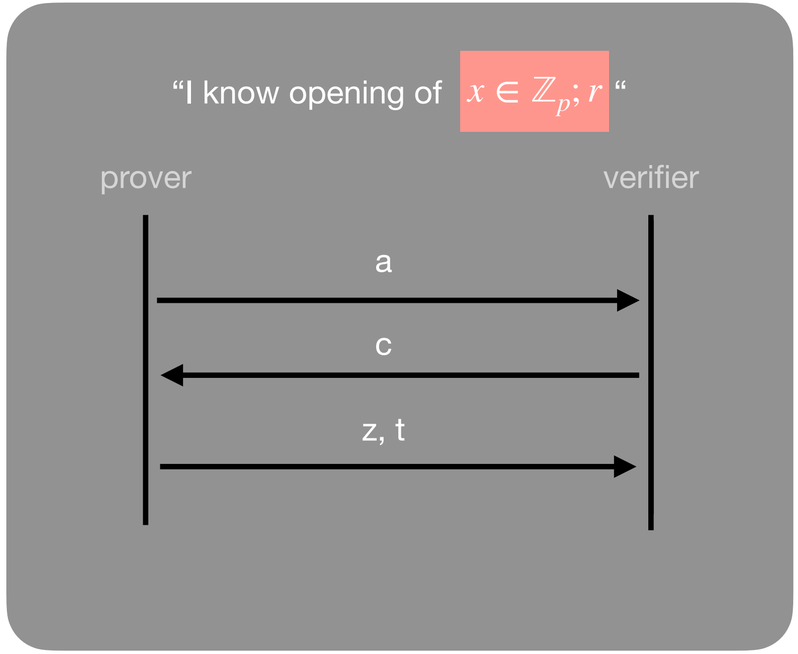

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  - $x = \frac{z - z'}{c - c'} \mod p$
  - can ensure that $z - z'$ and $c - c'$ are short

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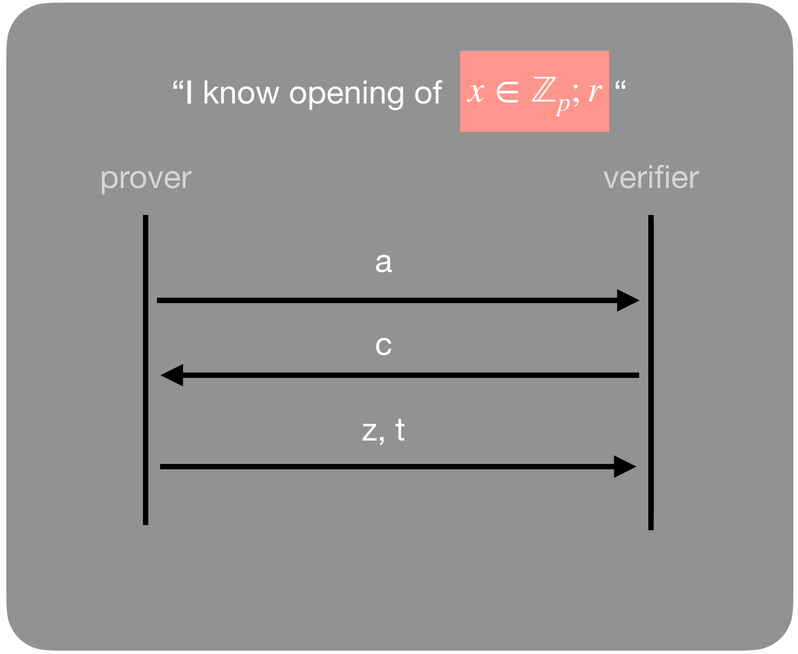

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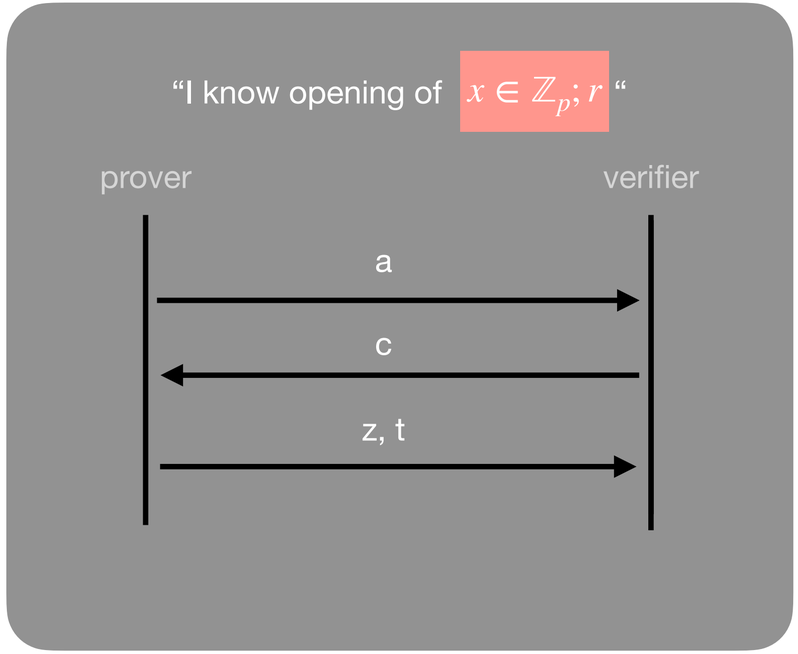

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- **Problem:** extracted x not short
- **Idea:** map $x = \frac{z - z'}{c - c'} \in \mathbb{Z}_p$ to $x = \left\lfloor \frac{z - z'}{c - c'} \right\rfloor \in \mathbb{N}$

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- Relax commitment scheme:
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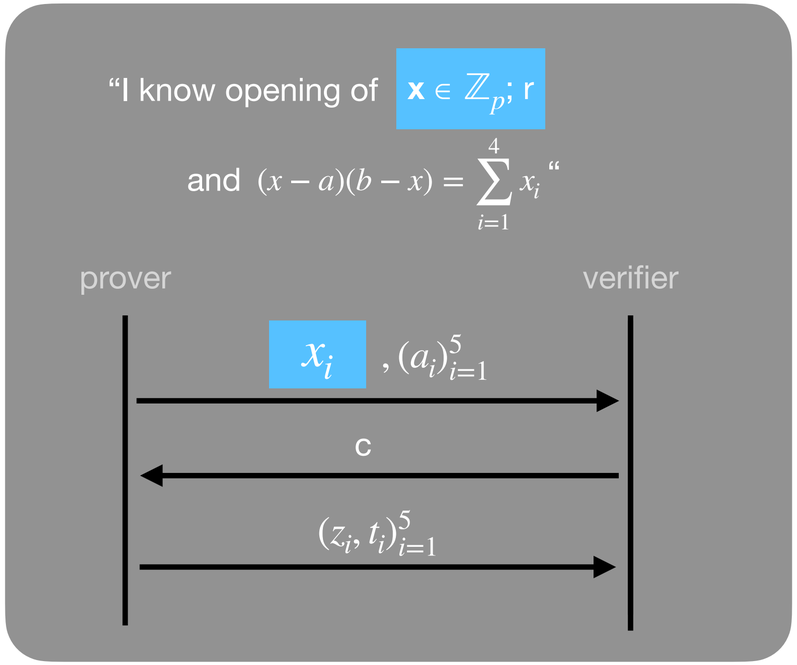

Evaluation

- Simple Scheme
- Good Efficiency
- Transparent Setup

- Large groups required
- Weaker homomorphic properties

Thank You

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