The Round Complexity of Quantum Zero-Knowledge

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TCC 2021

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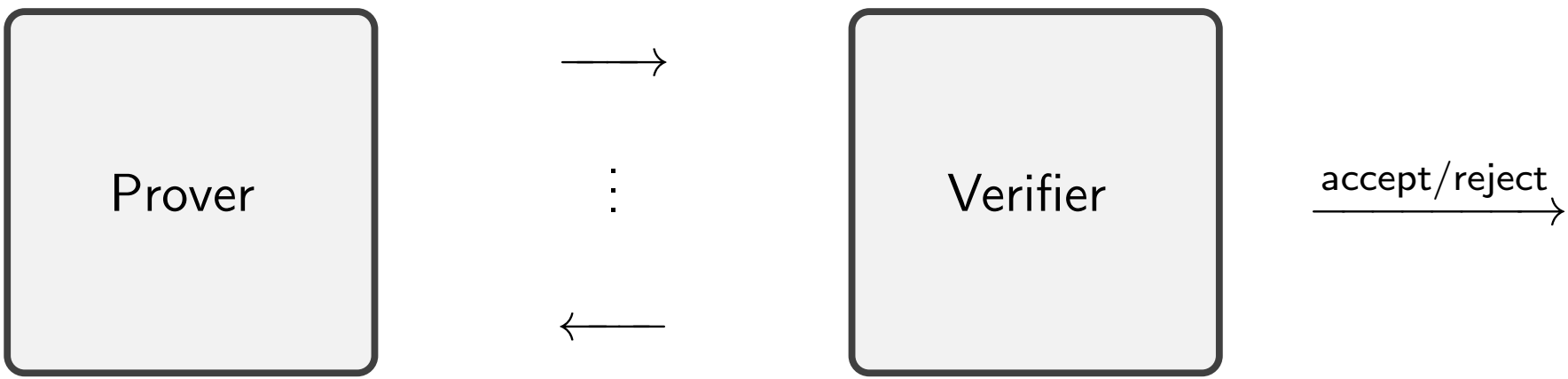
ZK Protocol [GMR89]

Interactive Protocol which allows a Prover to prove the veracity of a statement while revealing nothing beyond that.



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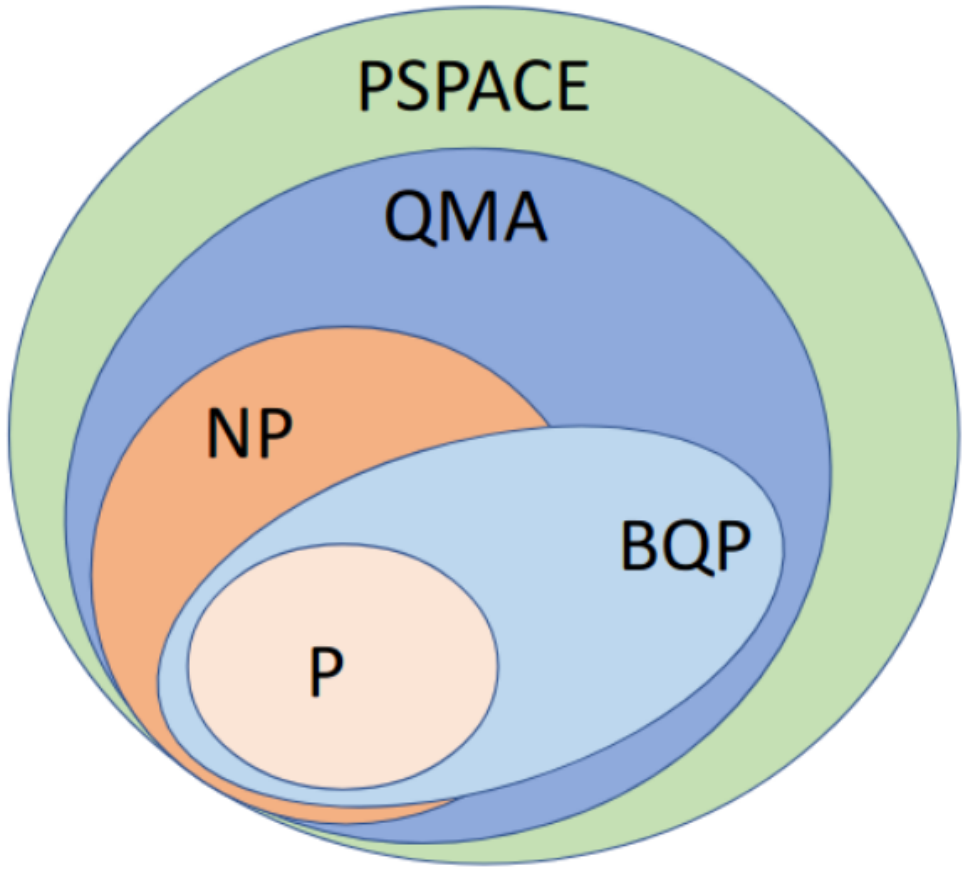
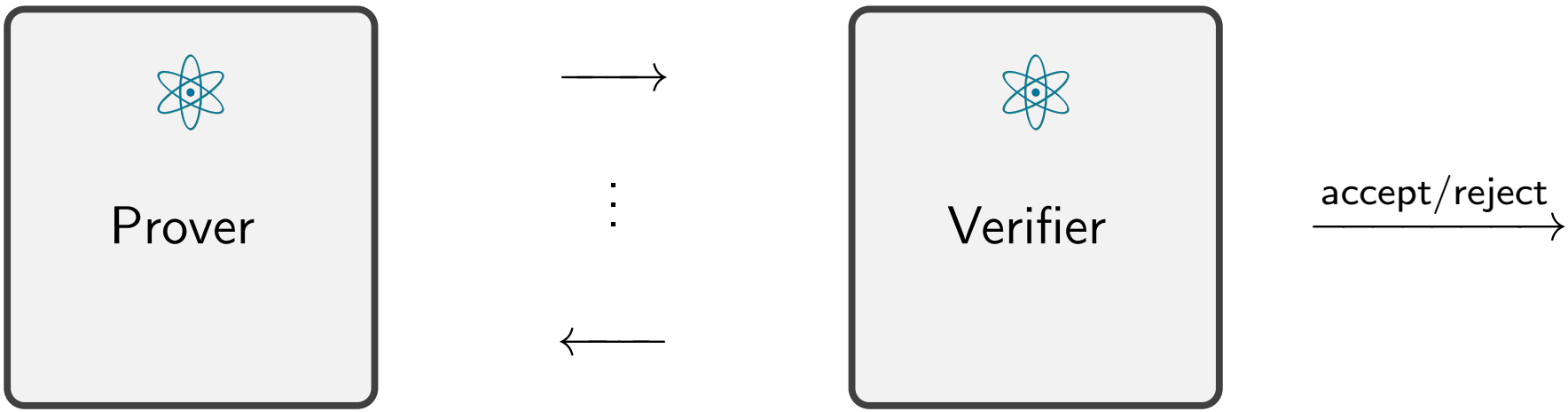
- Any NP statement can be proven in as few as four rounds of interaction [GMW86, GK96].
- 3-round statistical ZK argument for NP (not post-quantum secure) [BP19].

Quantum Complexity



Quantum ZK Protocol [BJSW16]

Allows a Prover to prove the veracity of a statement in QMA while revealing nothing beyond that.



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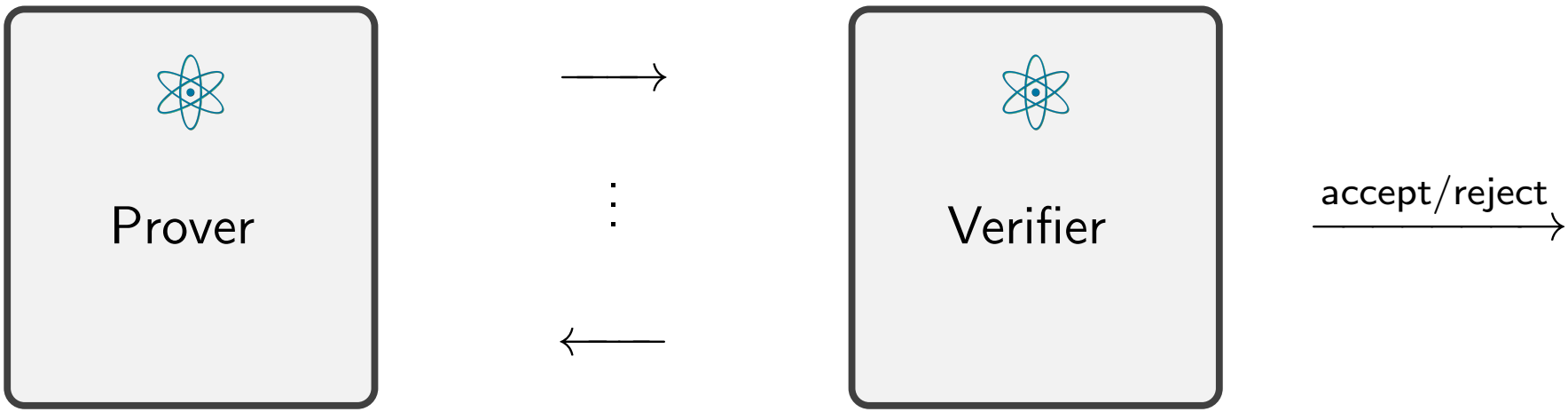

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Best known result in terms of round complexity is in constant rounds [BS20].

Results

2-round statistical WI argument for QMA

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4-round statistical ZK argument for QMA (and NP)

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2-round ZK argument for QMA in the Timing Model

Assuming the quantum quasi-polynomial hardness of LWE, and a non-parallelizing function (resp. a post-quantum time-lock puzzle), there exists a 2-round computational (resp. statistical) ZK argument for QMA in the timing model.

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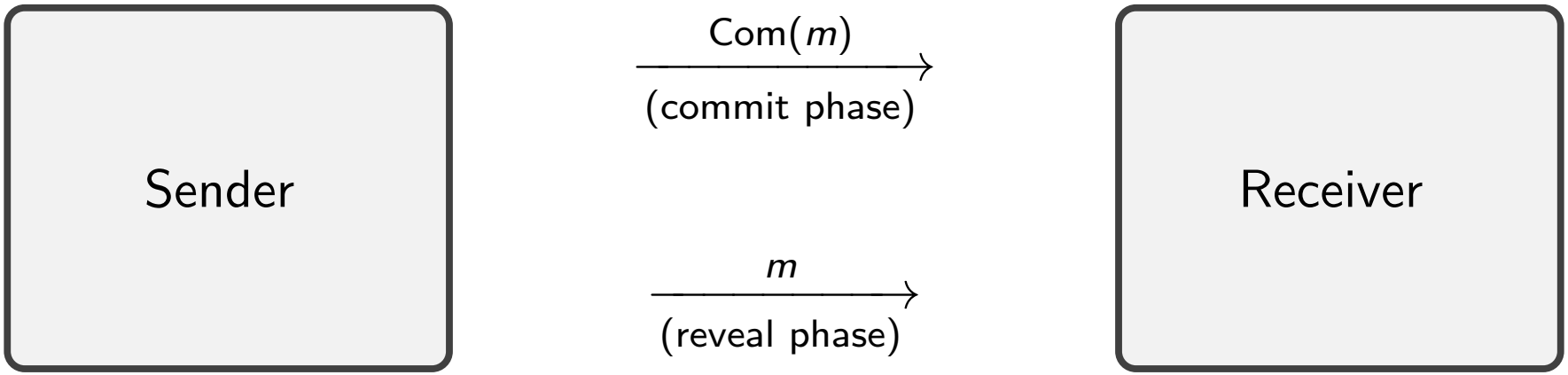
Commitment Scheme



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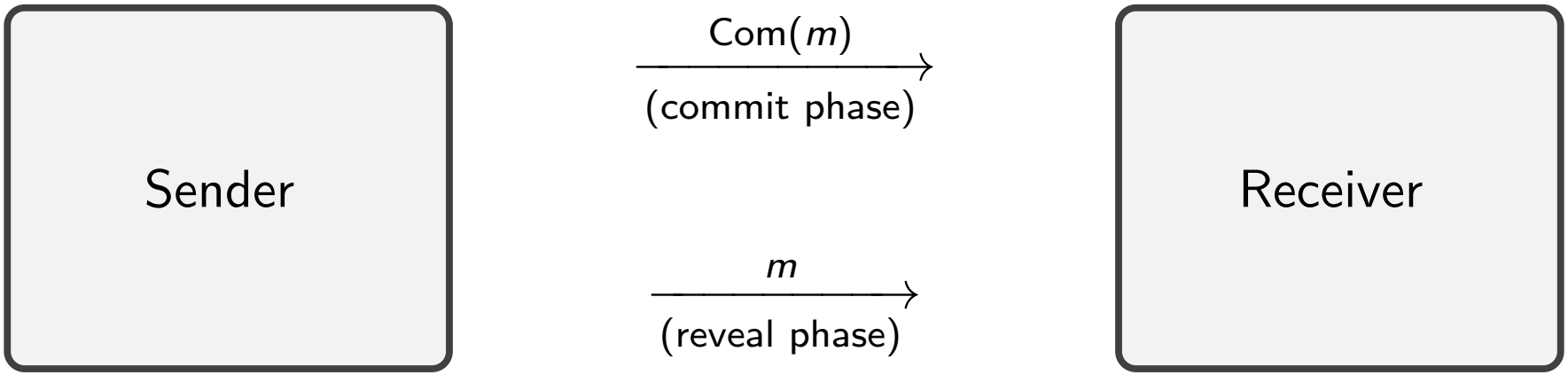

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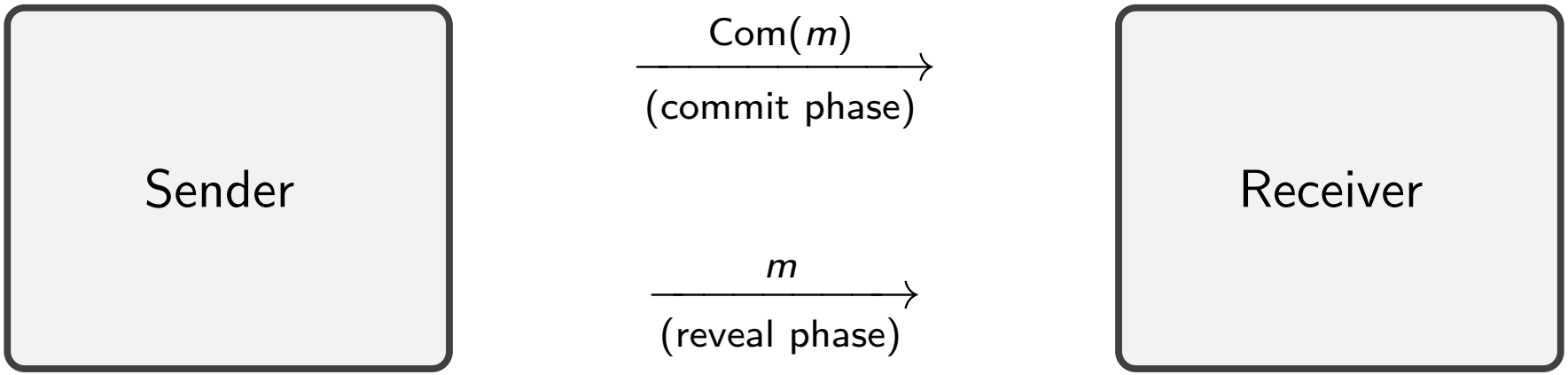

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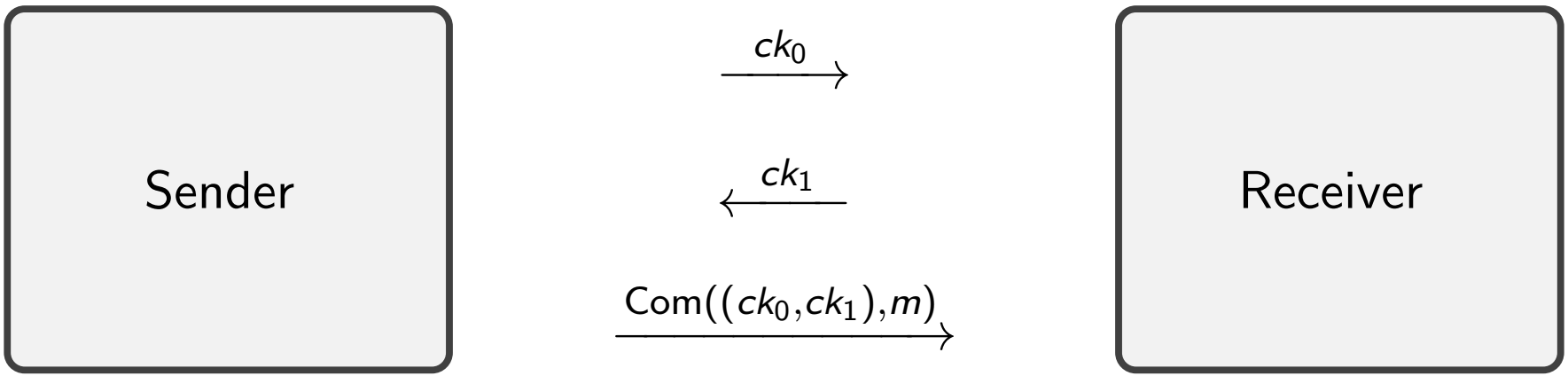
$$\mathrm{Com}(m_0) \approx \mathrm{Com}(m_1)$$

- Binding: The commitment $\mathrm{Com}(m)$ can only be opened to the message m.

Sometimes Binding Statistically Hiding Commitment

An SBSH commitment is a a special type of commitment [LVW20].



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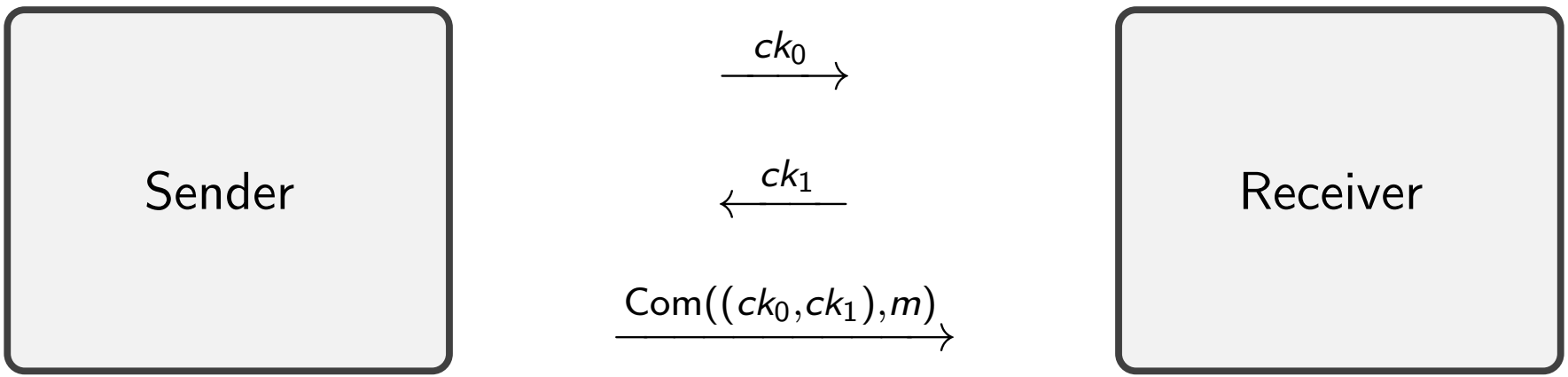

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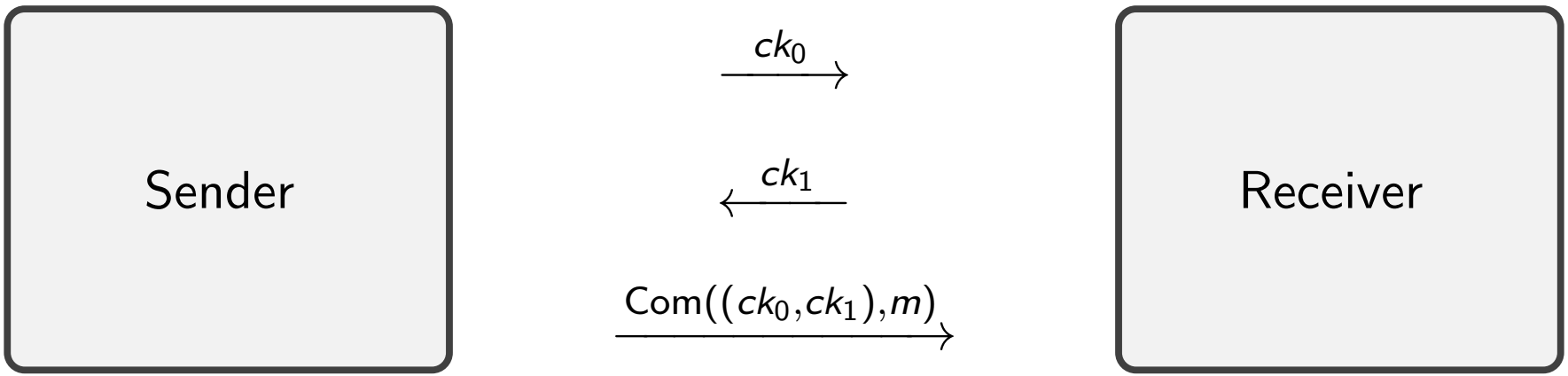

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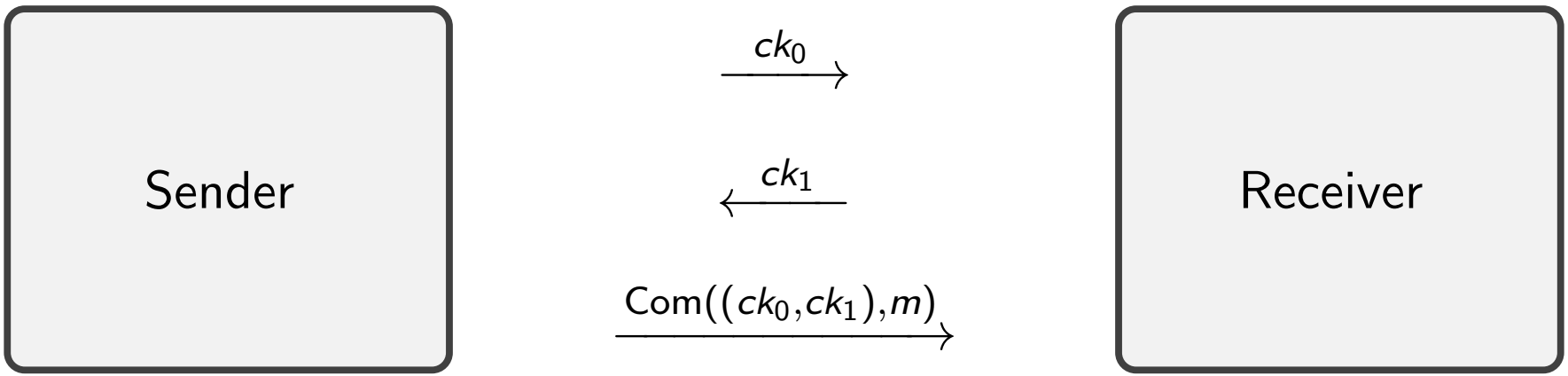

- Statistically Hiding: $\mathsf{Com}(m_0) \approx_S \mathsf{Com}(m_1)$
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- Straight line Extraction of m when binding.

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- Weaker notion of ZK.

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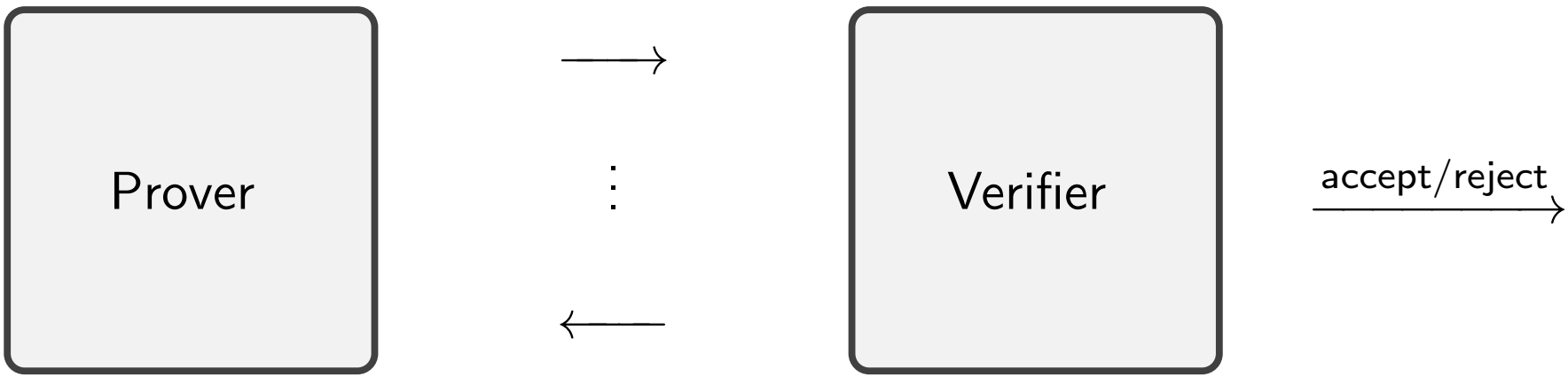

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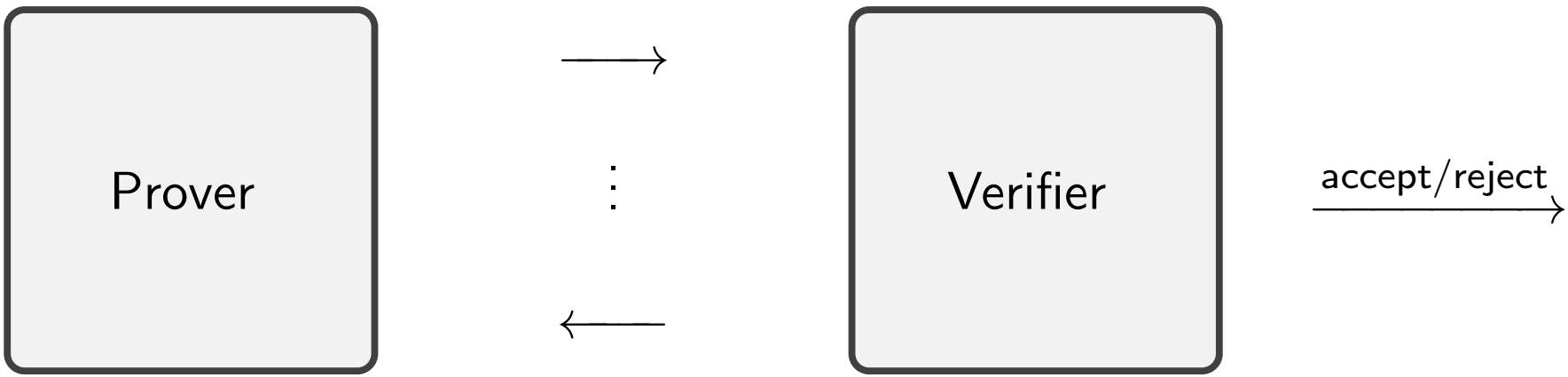

- The verifier cannot tell the difference between valid witnesses used by the prover.

Sigma Protocol for QMA

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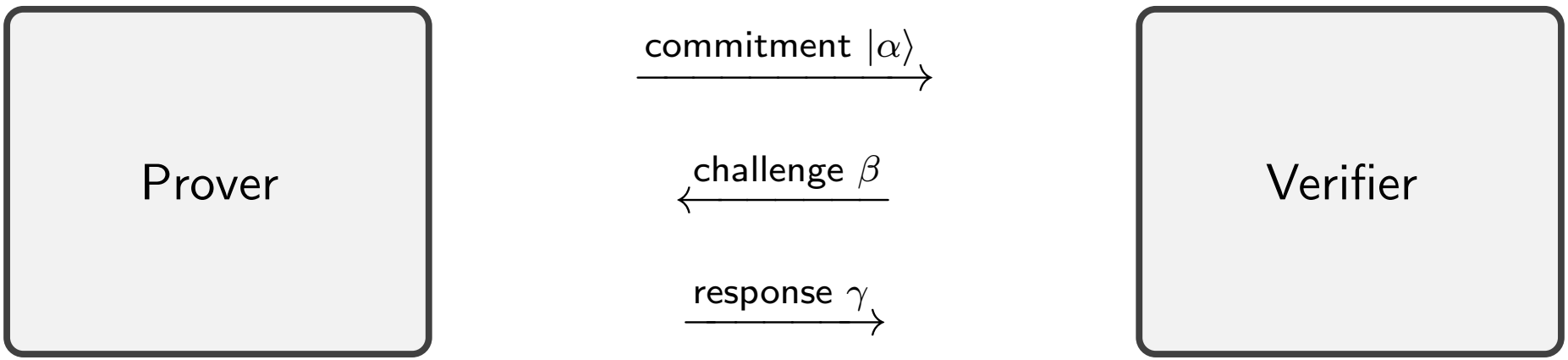

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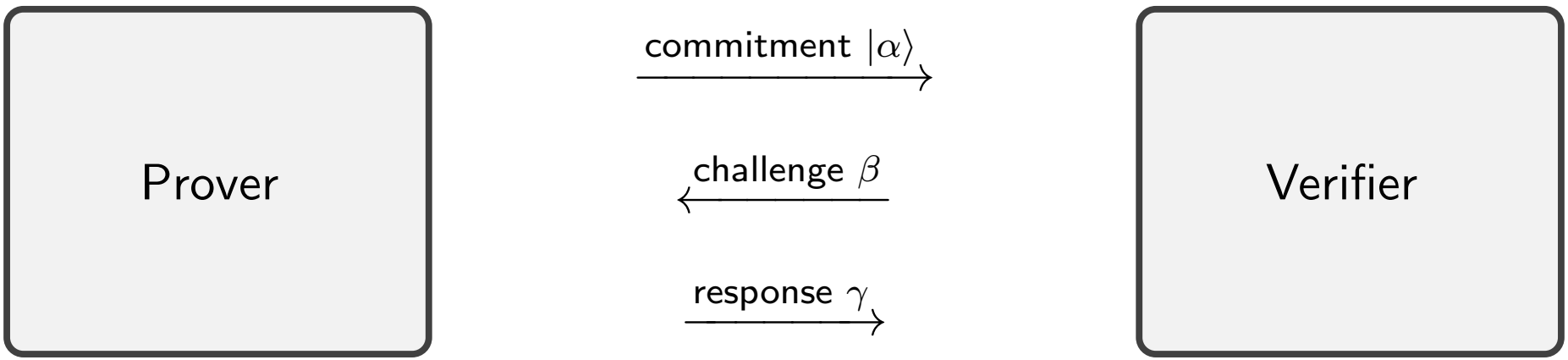

- Quantum Σ-Protocol where the computation of β and γ is completely classical [BG20].

Sigma Protocol for QMA

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- The Sigma-Protocol [BG20] works for computational ZK and statistical soundness.
- Extend to statistical ZK and computational soundness:
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  - parallel repetition of [BS20] to get negligible soundness.

Necessary tools

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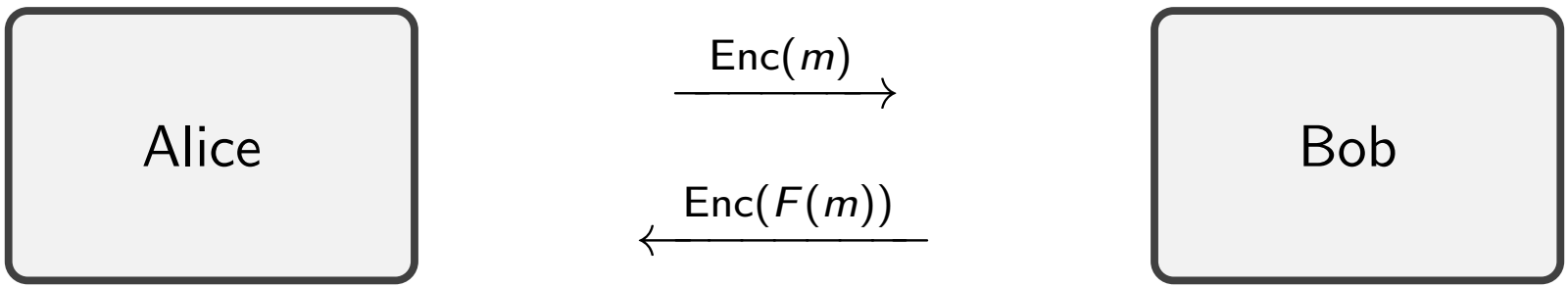

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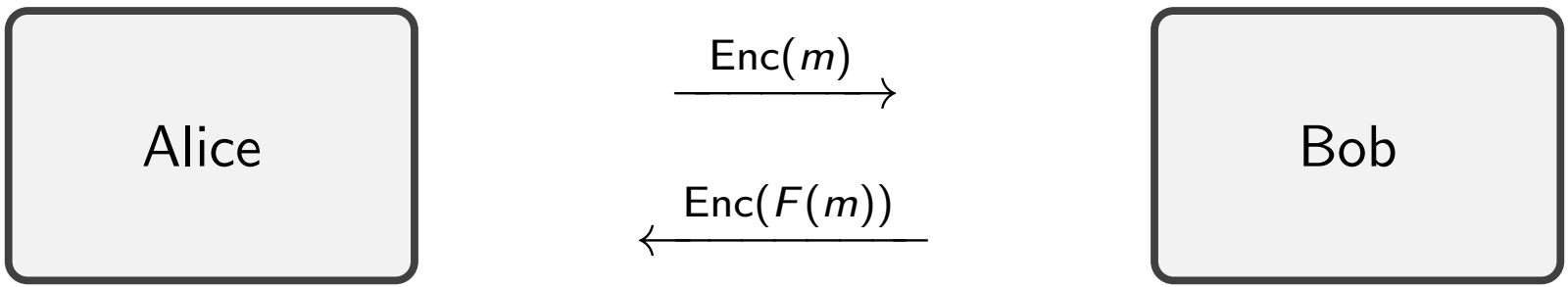
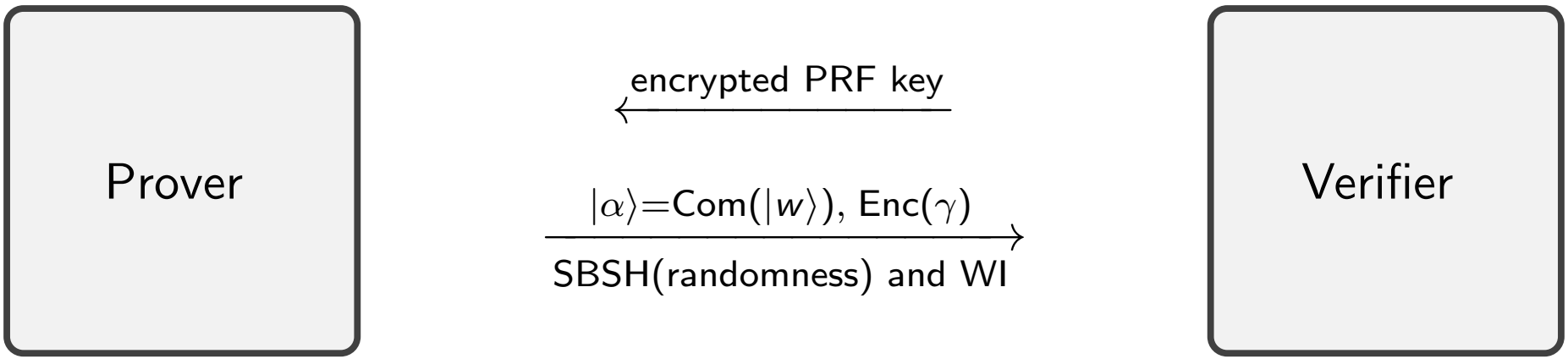


- Non-Interactive WI argument for NP [BFJ+20, GJJM20].

2-Round WI Argument for QMA



recomputes β and decrypts Enc(γ).

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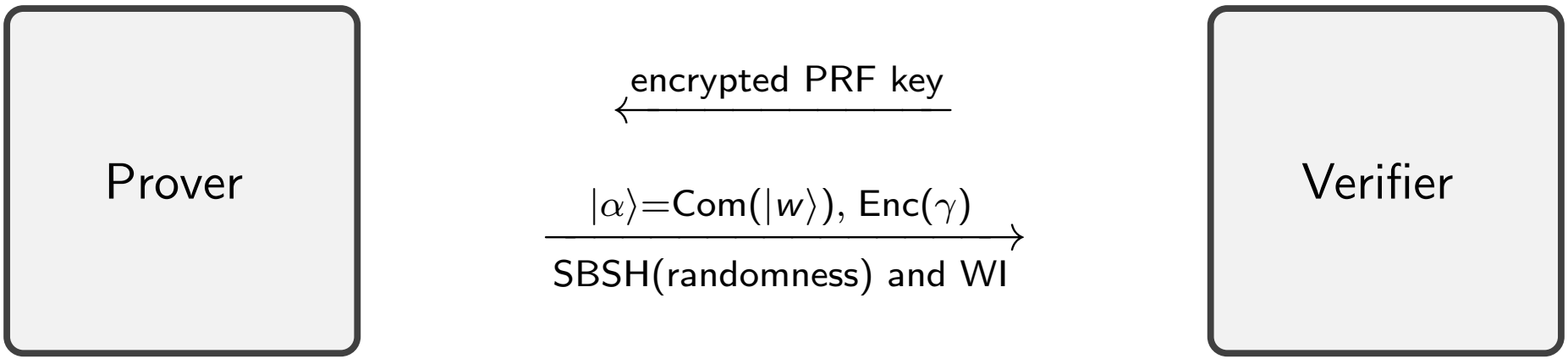

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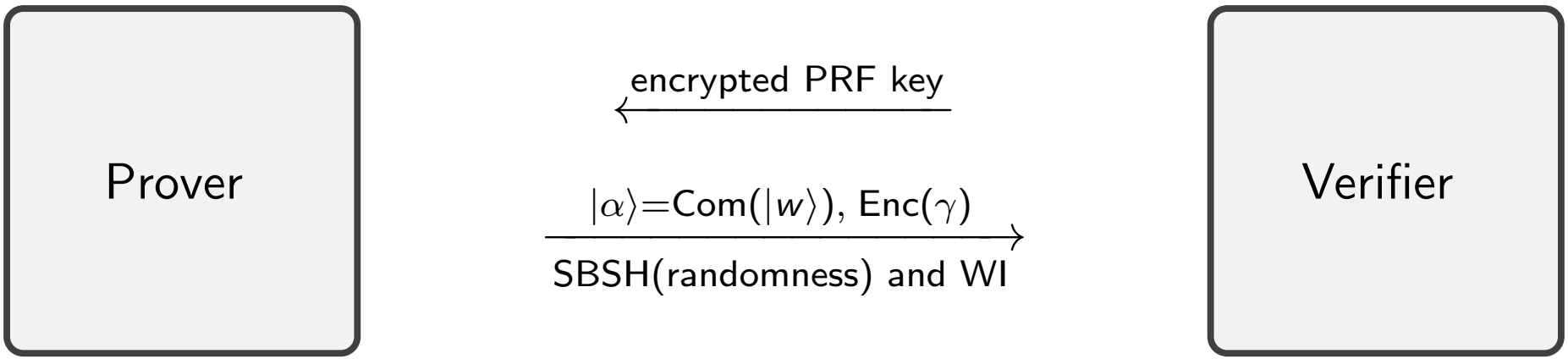

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- Assume quasi-polynomial hardness of LWE.

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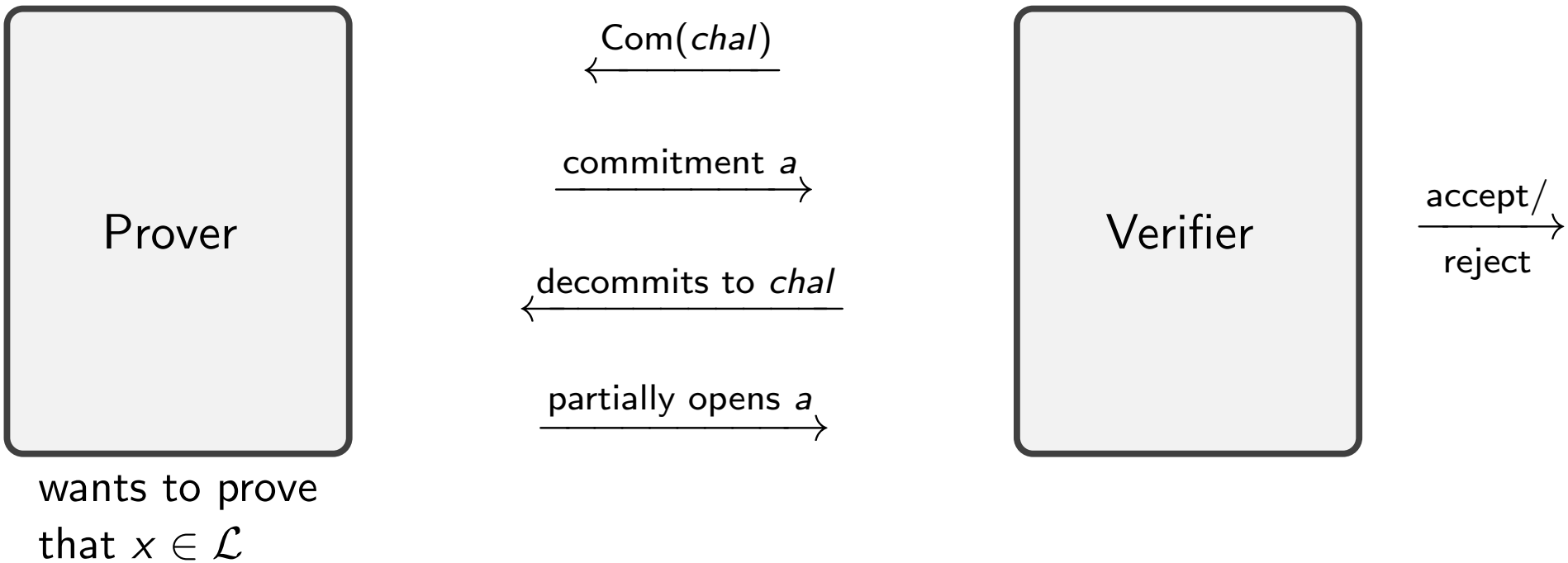

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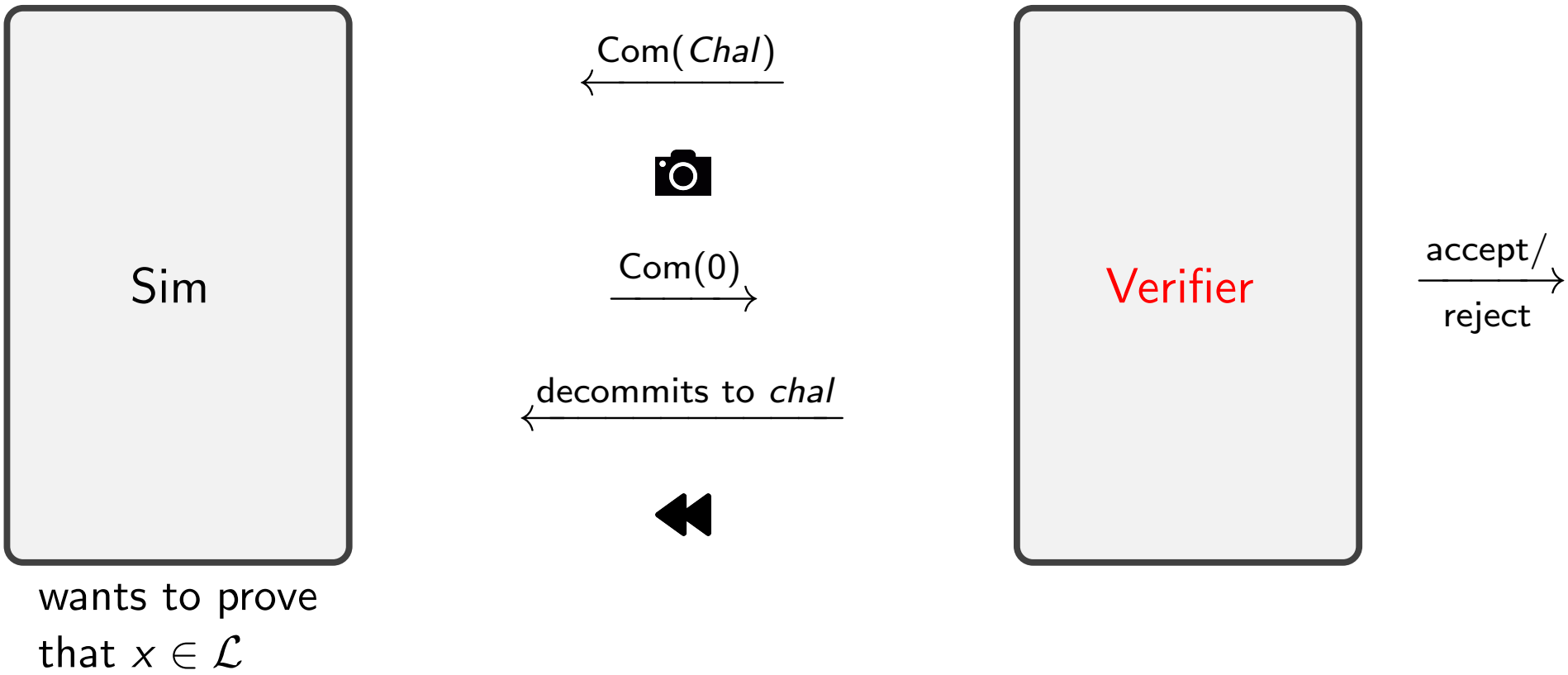
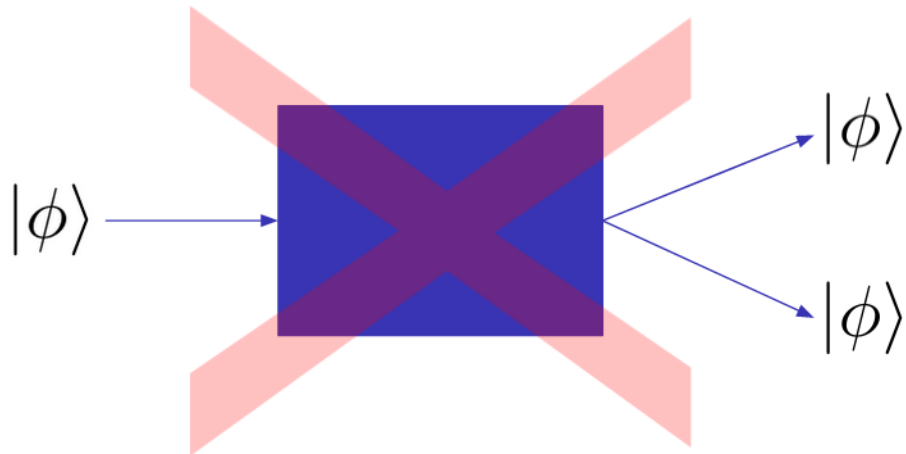


No-Cloning Theorem

No quantum procedure transforms $|\phi\rangle \rightarrow |\phi\rangle\,|\phi\rangle$, for all $|\phi\rangle$.



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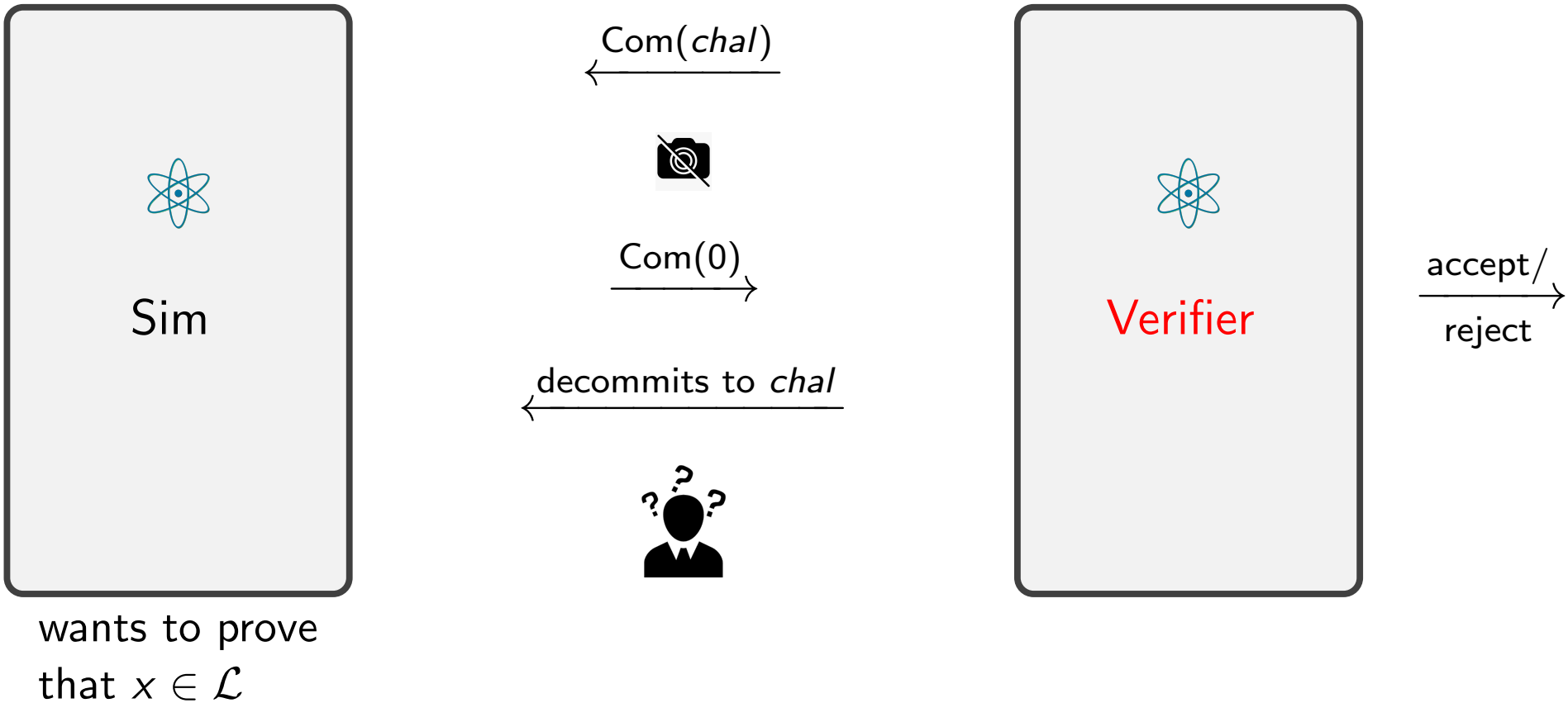
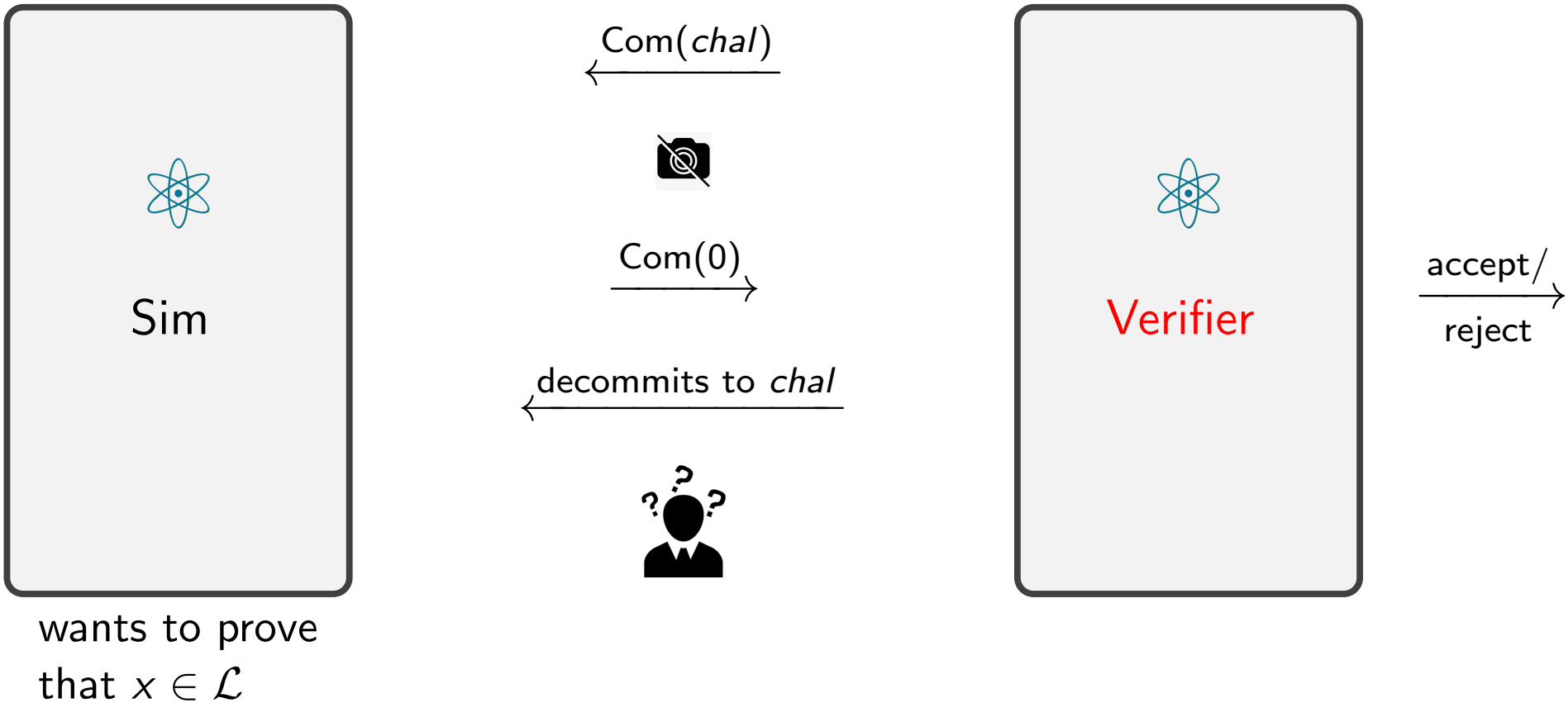

Sim
wants to prove
that $x \in \mathcal{L}$
Com(*chal*)
Com(0)
decommits to *chal*
Verifier
accept/
reject

Quantum ZK



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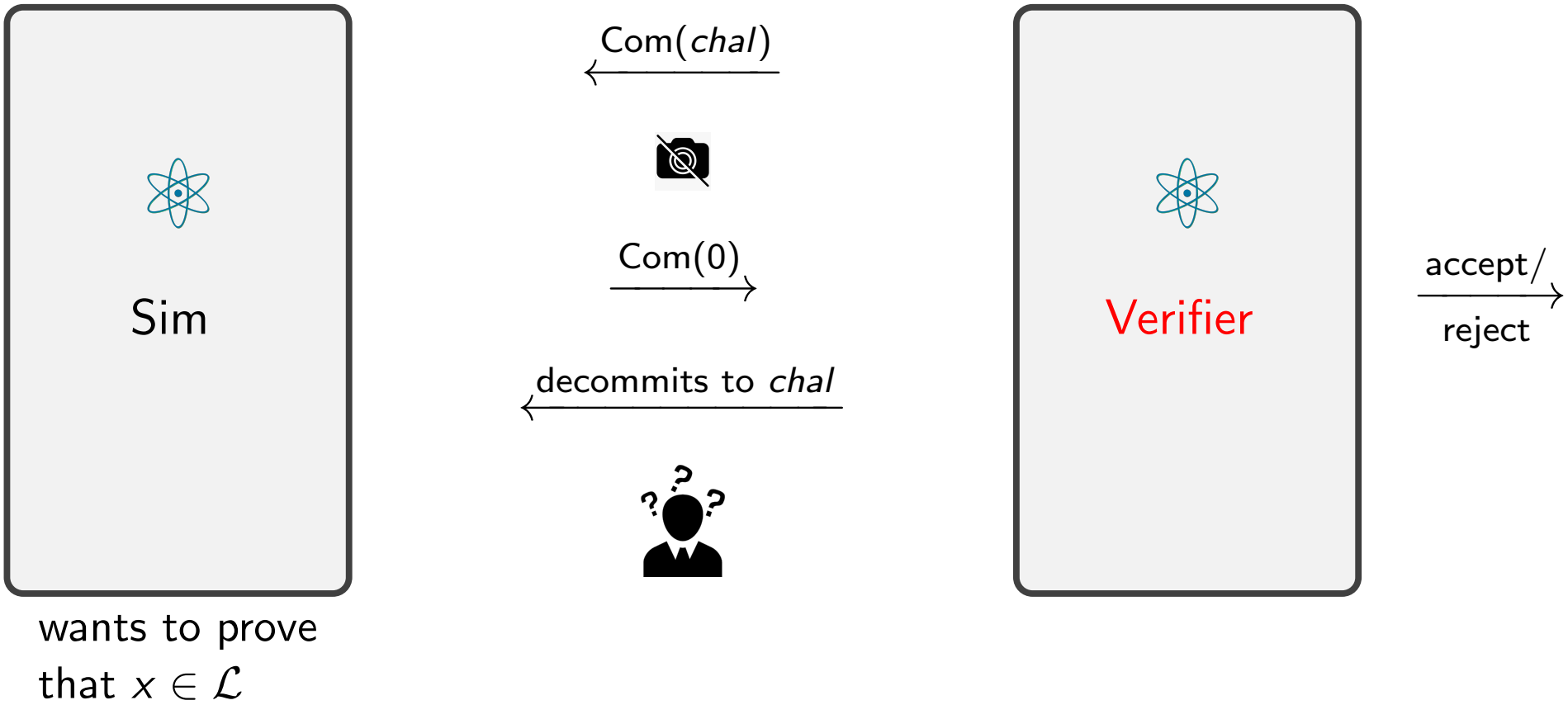
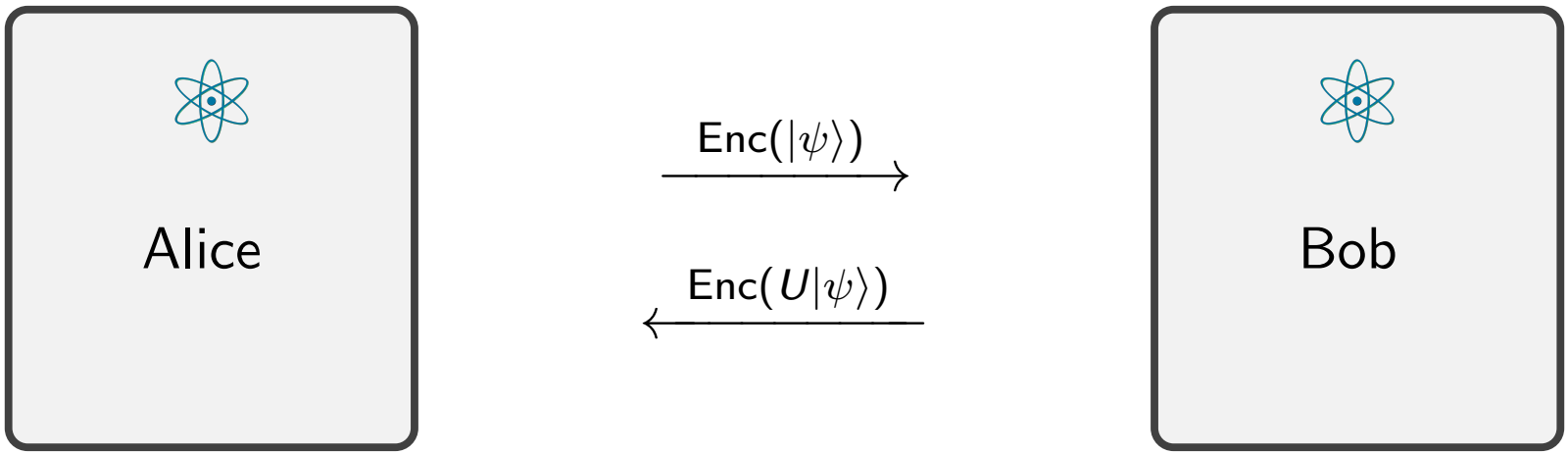


- Simulation techniques that don't use rewinding still rely on cloning.
- Linear Extraction Technique in [AL20, BS20].

Necessary Tools

Quantum FHE Scheme [Mah18, Bra18]

A Quantum FHE scheme [Mah18, Bra18] allows Alice to encrypt some message $|\psi\rangle$ such that later Bob (holding any unitary U) can compute



Necessary Tools

Compute-and-Compare Obfuscation [WZ17, GKW17, GKVW19]

- Compute-and-Compare Program:

$$\mathbf{CC}[f, s, z](x) = \begin{cases} z, & f(x) = s \\ \perp, & \text{else} \end{cases}$$

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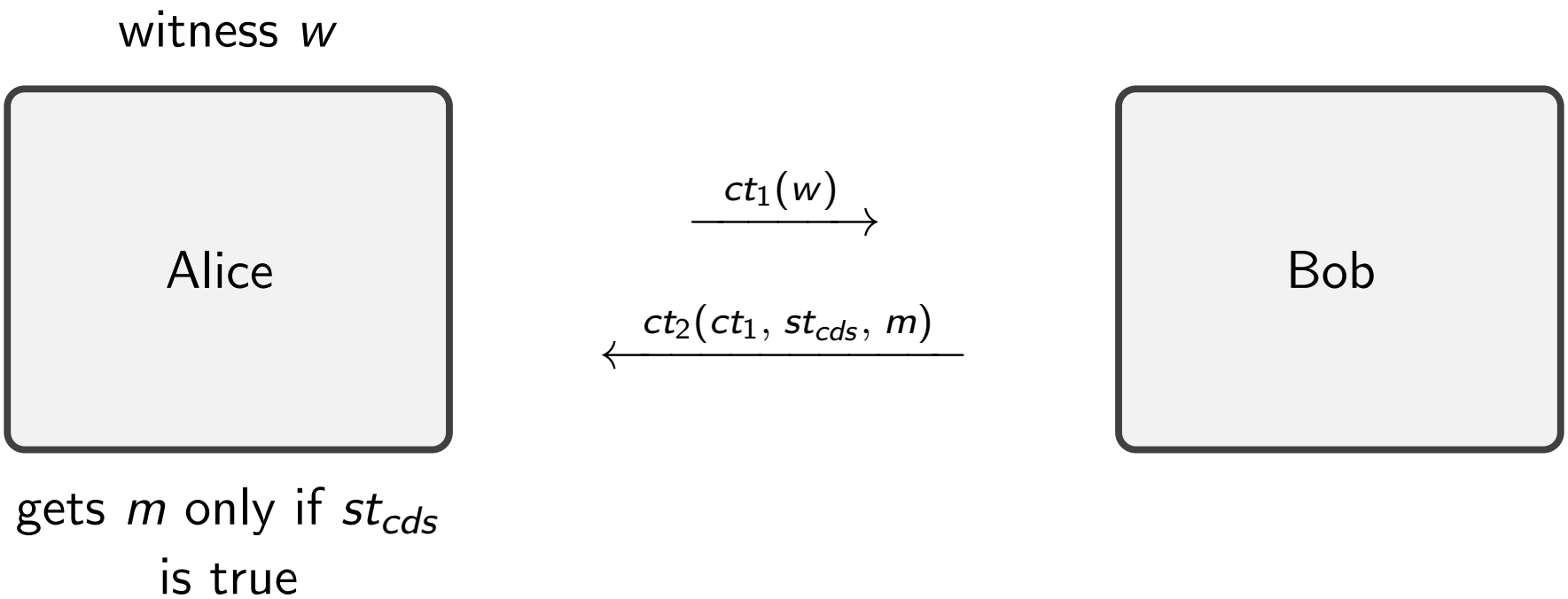
$$\mathbf{CC}[f, s, z](x) = \begin{cases} z, & f(x) = s \\ \perp, & \text{else} \end{cases}$$

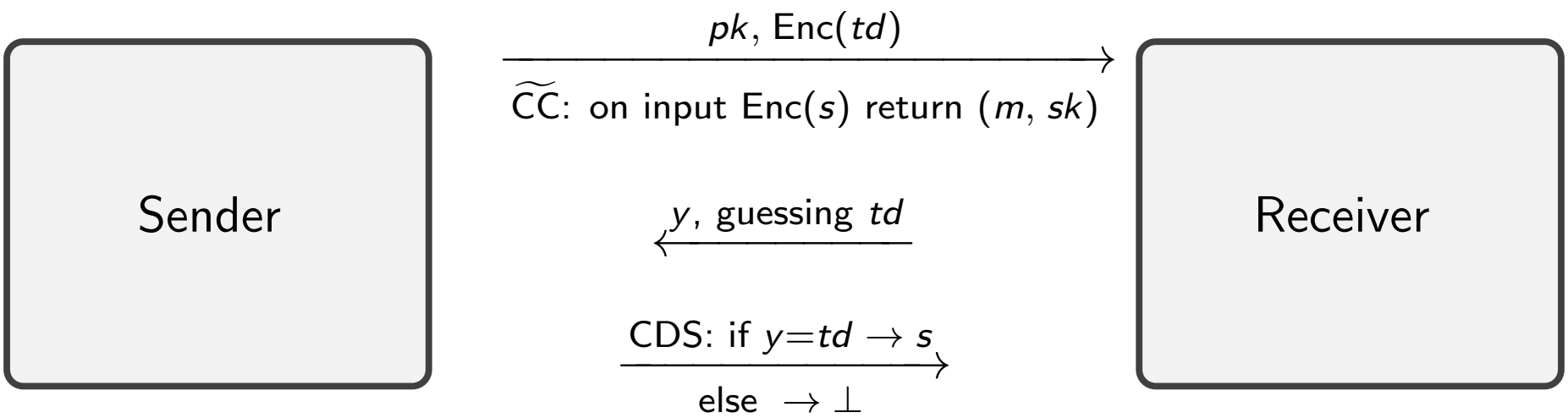
- Compute-and-Compare Obfuscator: Compiles a **CC** program to the obfuscated program $\widetilde{\mathbf{CC}}$, where the implementation is hidden.

Necessary Tools

Conditional Disclosure of Secrets [AIR01]



Homomorphic Trapdoor Technique [AL20, BS20]



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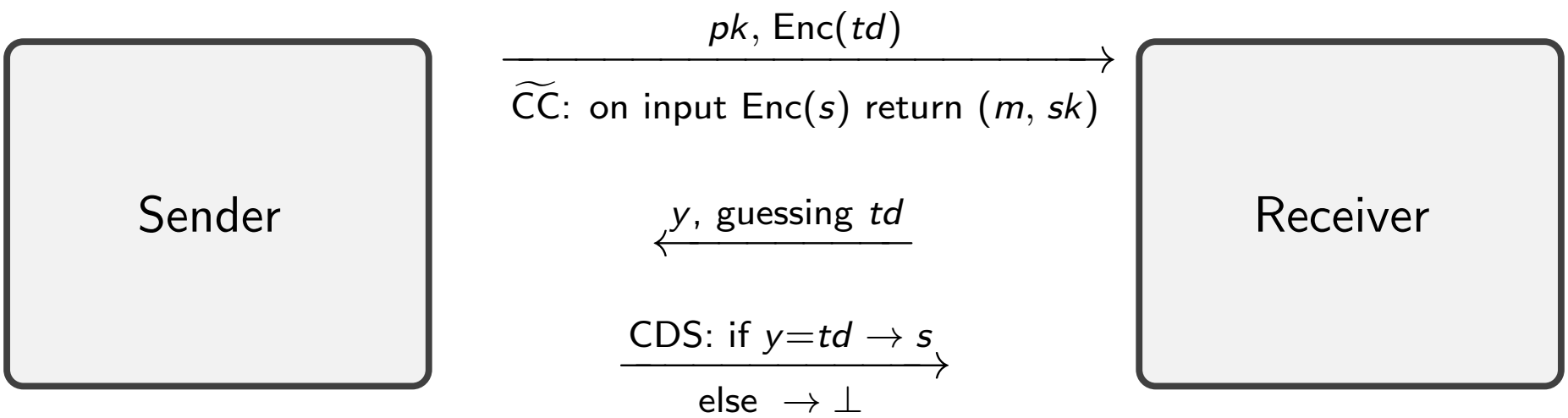

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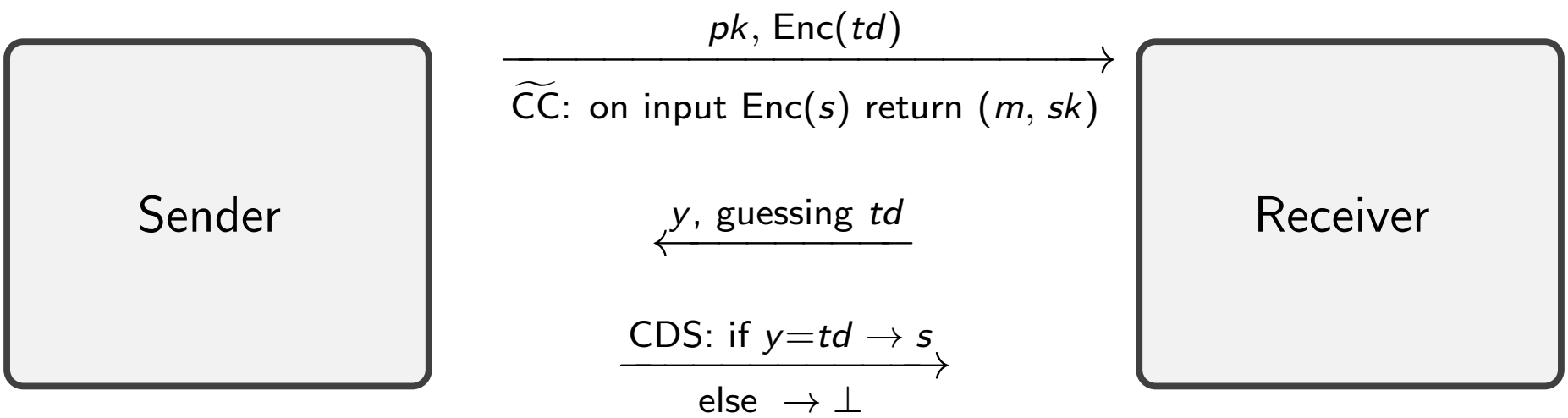

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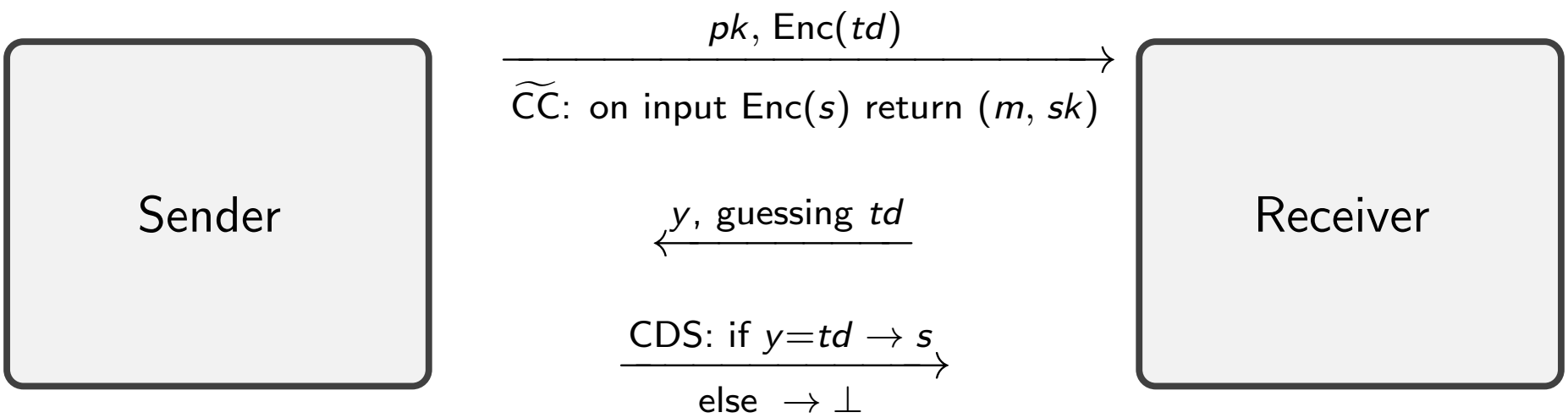

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- $\text{Enc}(td) \xrightarrow{\text{homomorphically}} \text{Enc}(s)$.

ZK Protocols for QMA

- No Cloning Extraction $\rightarrow$ Constant-Round Computational ZK Argument for QMA [BS20]

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- Statistical WI for QMA $\xrightarrow[\text{Extraction}]{\text{No Cloning}}$ 4-Round Statistical ZK Argument for QMA

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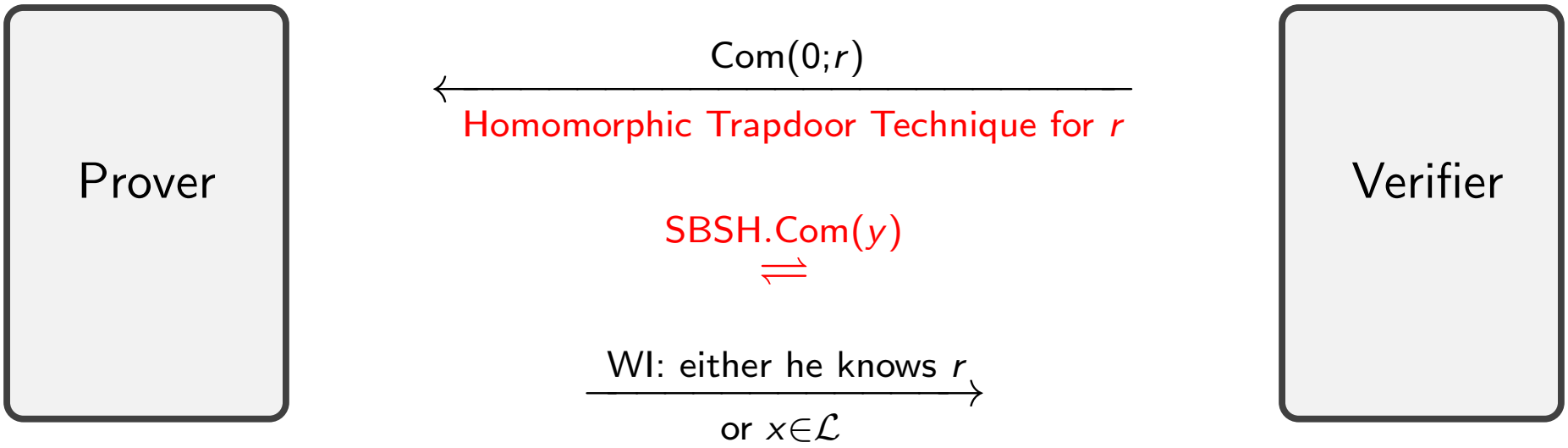
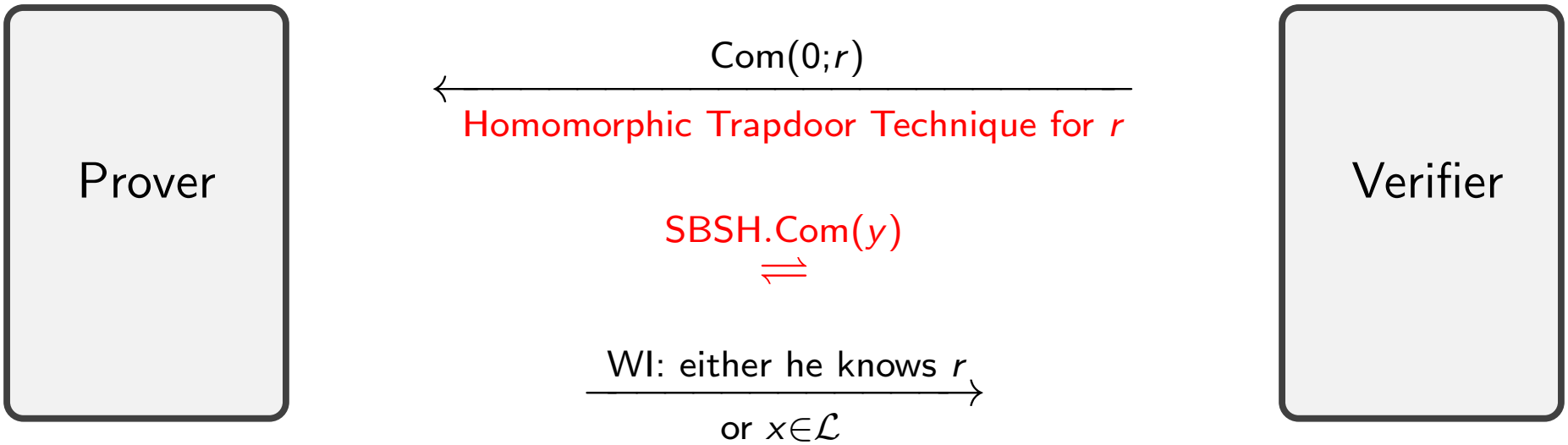
4-round ZK Argument for QMA Construction



4-round ZK Argument for QMA Construction



- Simulator extracts r and proves the other WI clause.

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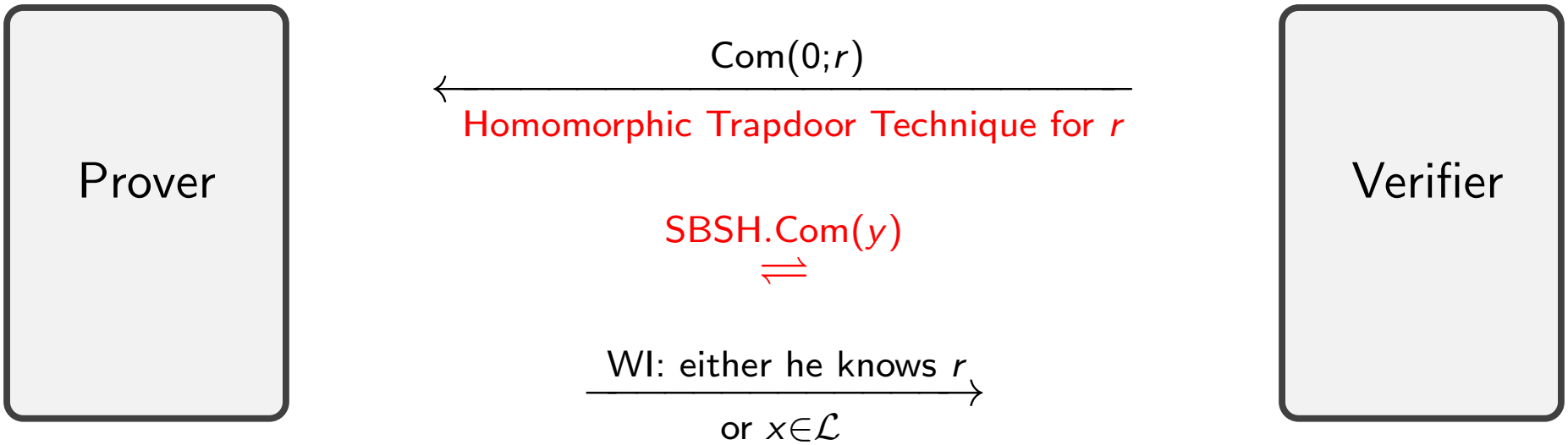

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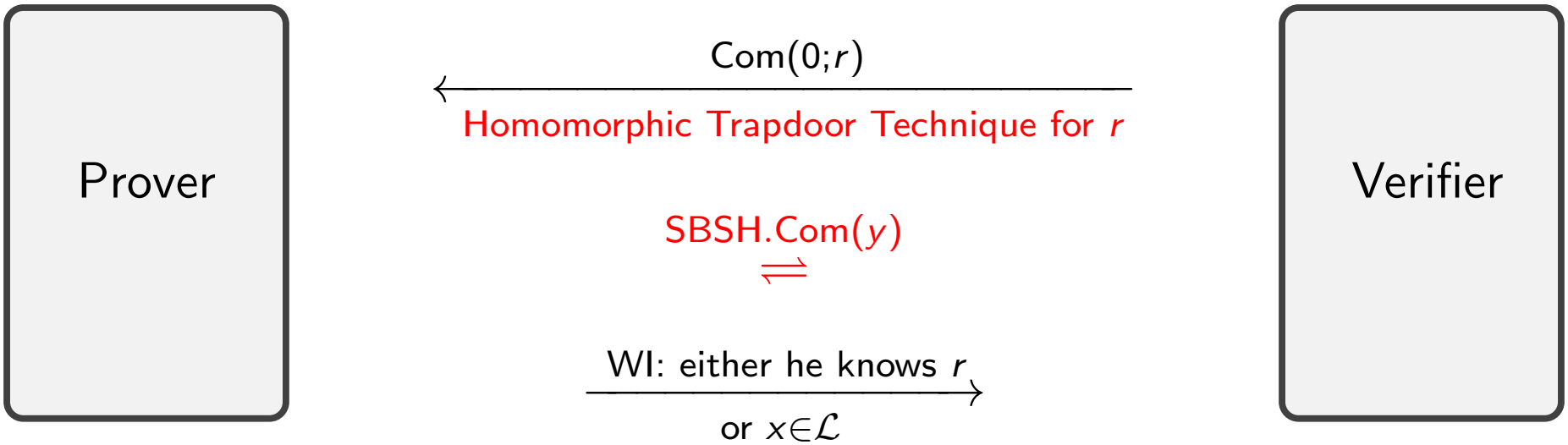

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- Works for non-Malicious Verifiers:
  - Non-Aborting.
  - Explainable (messages can always be explained).

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Aborting

- Our simulator fails in the aborting case, gets stuck with Verifier's encrypted inner state.

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- A combined simulator guesses which one should be used, using Watrous' Quantum Rewinding Lemma [Wat09].

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Non-Explainable

- Add proof from the verifier to the prover that the messages were computed honestly.

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- Verifier's messages are classical, so ZK for NP suffices.

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Non-Explainable

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- Verifier's messages are classical, so ZK for NP suffices.
- To maintain statistical ZK and round complexity we need a 3-round post-quatum ZK proof for NP.

Obstacles

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- **Solved by constructing a 3-round sometimes-extractable statistically receiver private oblivious transfer (SRP-OT).**

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- Simulator (straight line) runs in polynomial time with an exponentially small success probability.
- Security parameters of other primitives are set to account for this exponential loss.

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Conclusion

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