Amortizing Rate-1 OT and Applications to PIR and PSI

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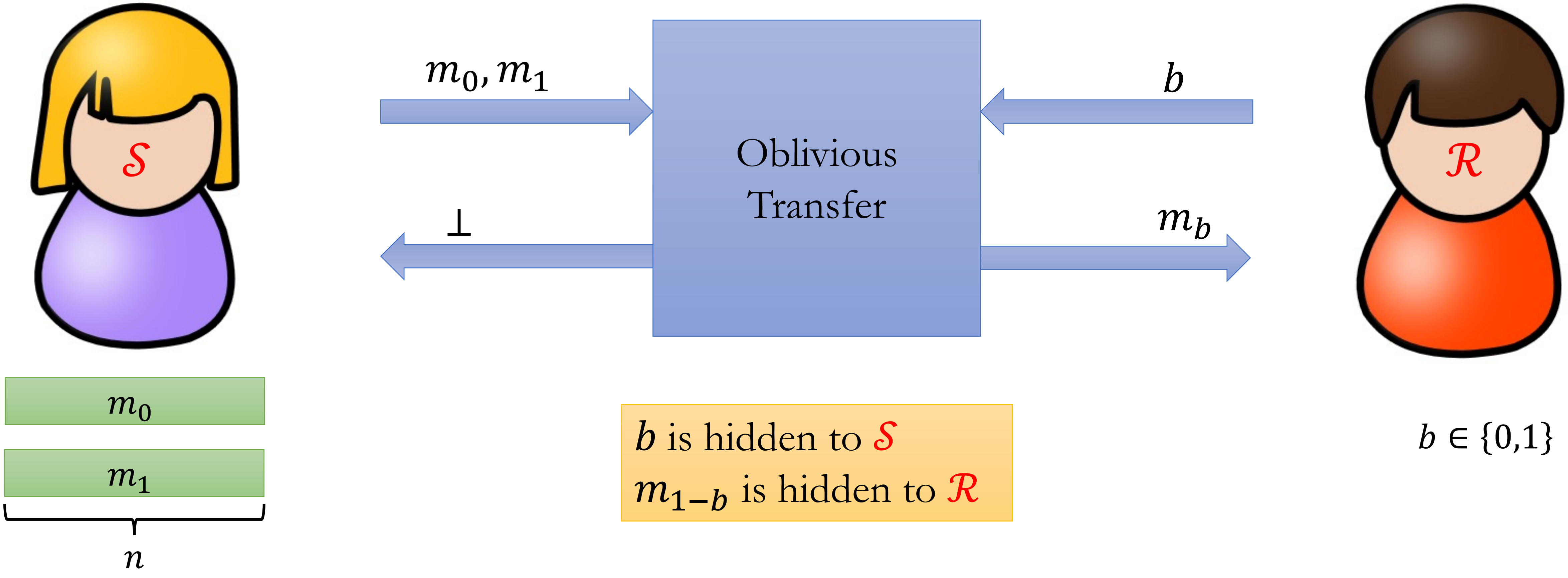
Oblivious Transfer (OT) [Rabin81, EGL82, BCR86, Kilian88]



$\mathcal{S}$

m_0, m_1

$\perp$

Oblivious Transfer

b

m_b

$\mathcal{R}$

m_0

m_1

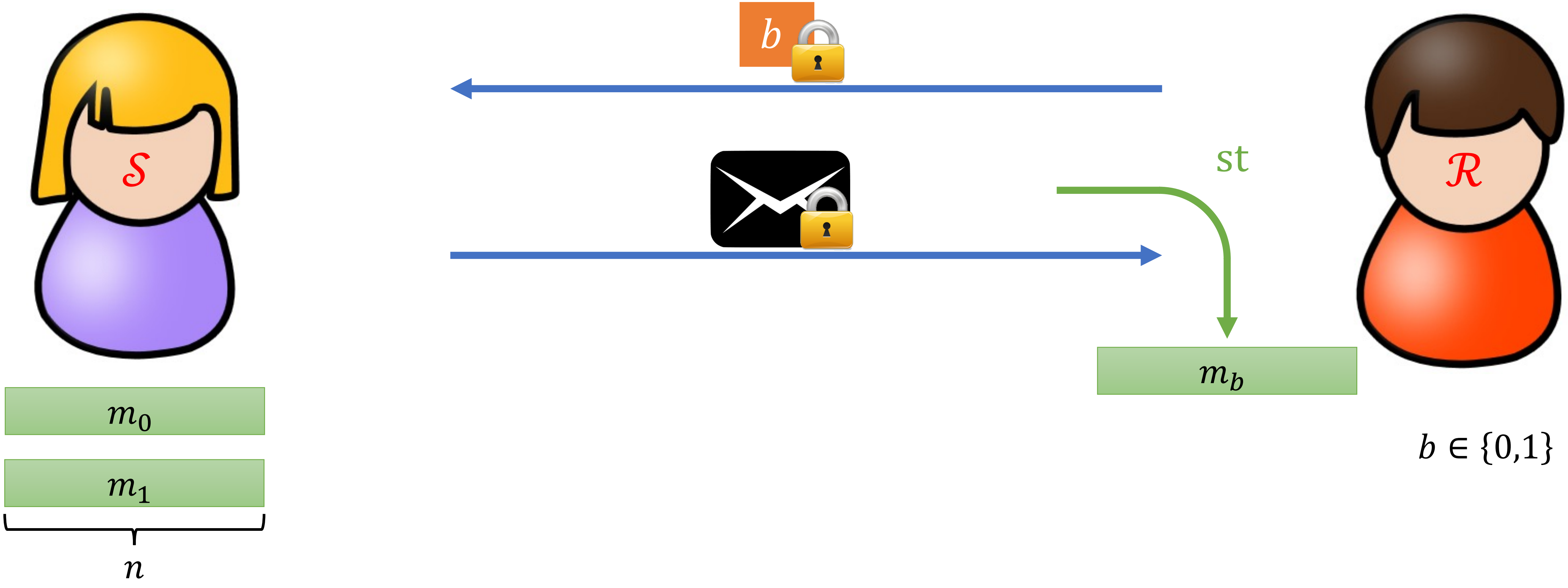
n

b is hidden to $\mathcal{S}$

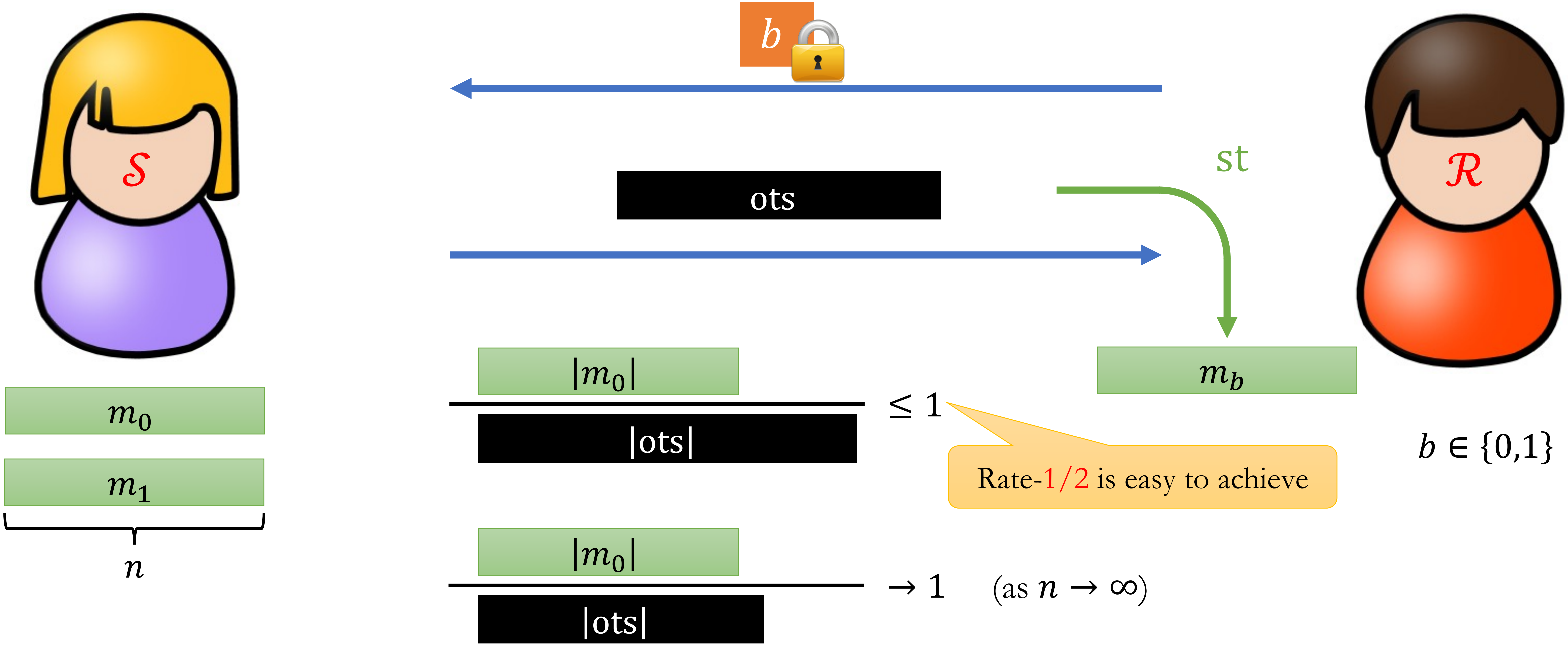
m_{1-b} is hidden to $\mathcal{R}$

$b \in \{0,1\}$

Two-Message OT [AIR01, NP01, PVW08, HK12, DGHMW20]



Rate-1 OT [IP07, DGIMMO19, GHO20]



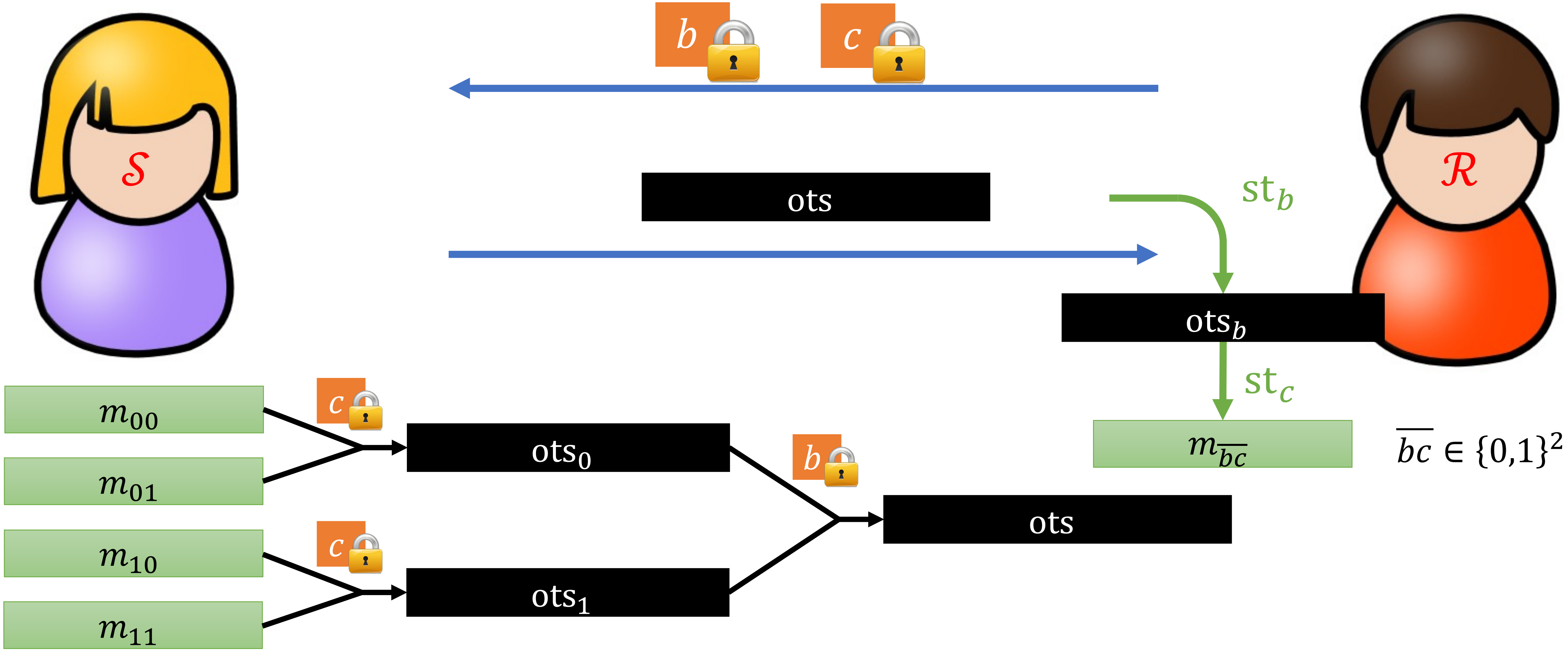
$\frac{|m_0|}{|\text{ots}|} \leq 1$

Rate-1/2 is easy to achieve

$\frac{|m_0|}{|\text{ots}|} \to 1$ (as $n \to \infty$)

Why two-message? Why rate-1?

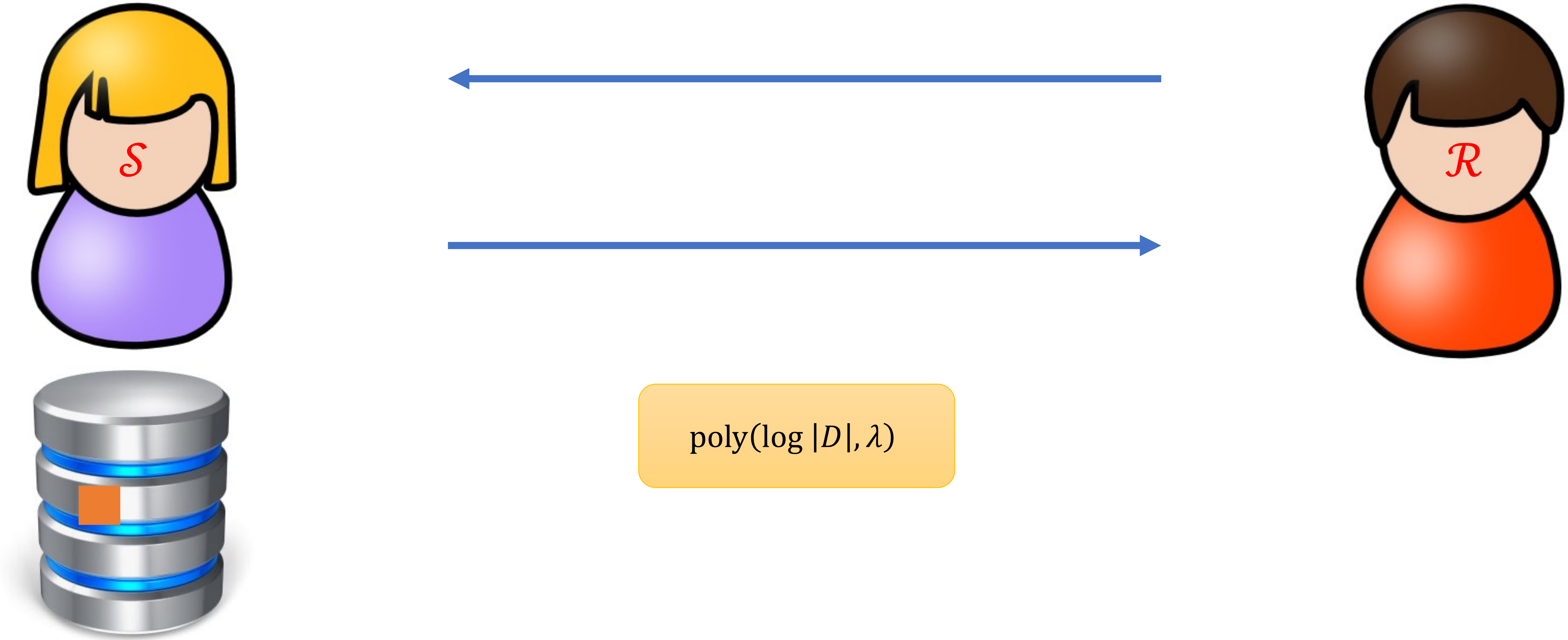
Example: 1-out-of-4 OT



Why two-message? Why rate-1?

Nested OT with low communication

Applications of Rate-1 OT



$\mathrm{poly}(\log |D|, \lambda)$

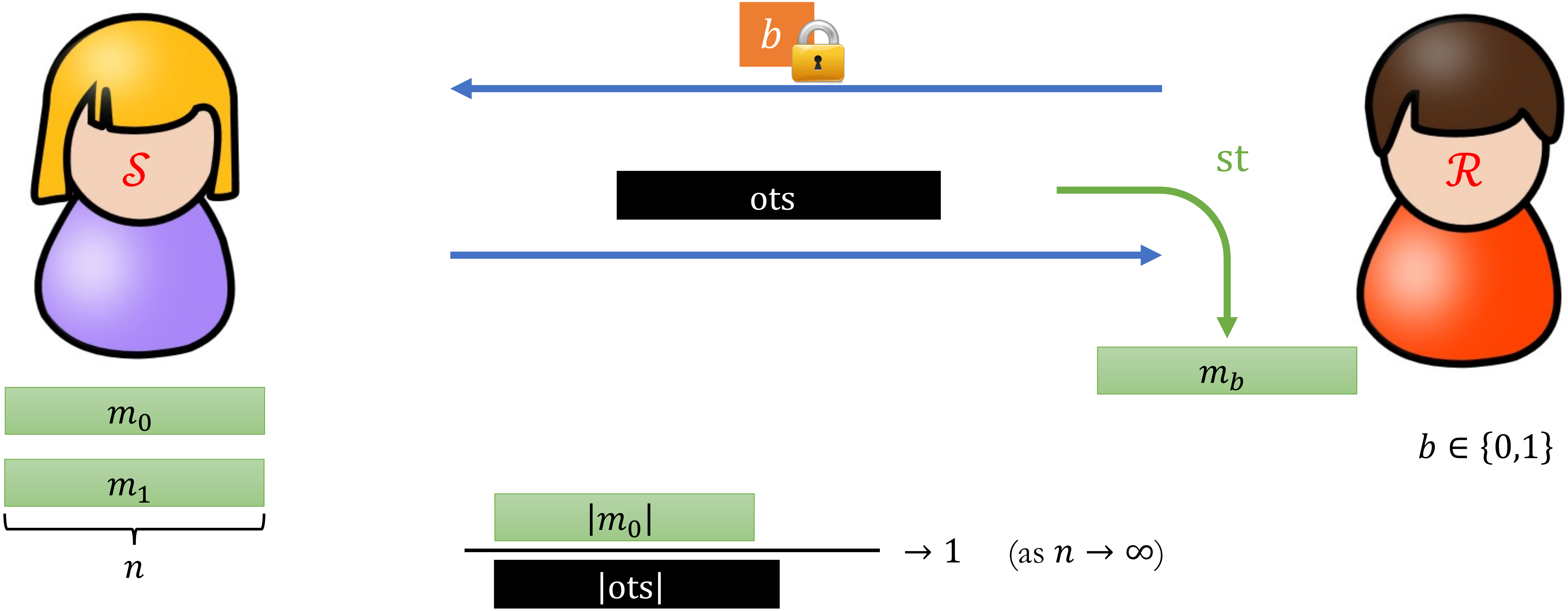
Applications of Rate-1 OT

- Semi-compact homomorphic encryption for branching programs [IP07]
  - Single-server private information retrieval **(PIR)** [KO97] with poly-logarithmic communication
  - Unbalanced private set intersection **(PSI)** with poly-logarithmic communication in the size of the larger set
  - Secure inference on decision trees with communication linear in the tree depth
- Lossy trapdoor functions [PW08, HO12] with optimal rate [DGIMMO19]

Can we achieve Rate-1 OT?

- Damgård-Jurik Cryptosystem [DJ01] from DCR
- Trapdoor Hash Functions [DGIMMO19] from DDH/QR/LWE/DCR

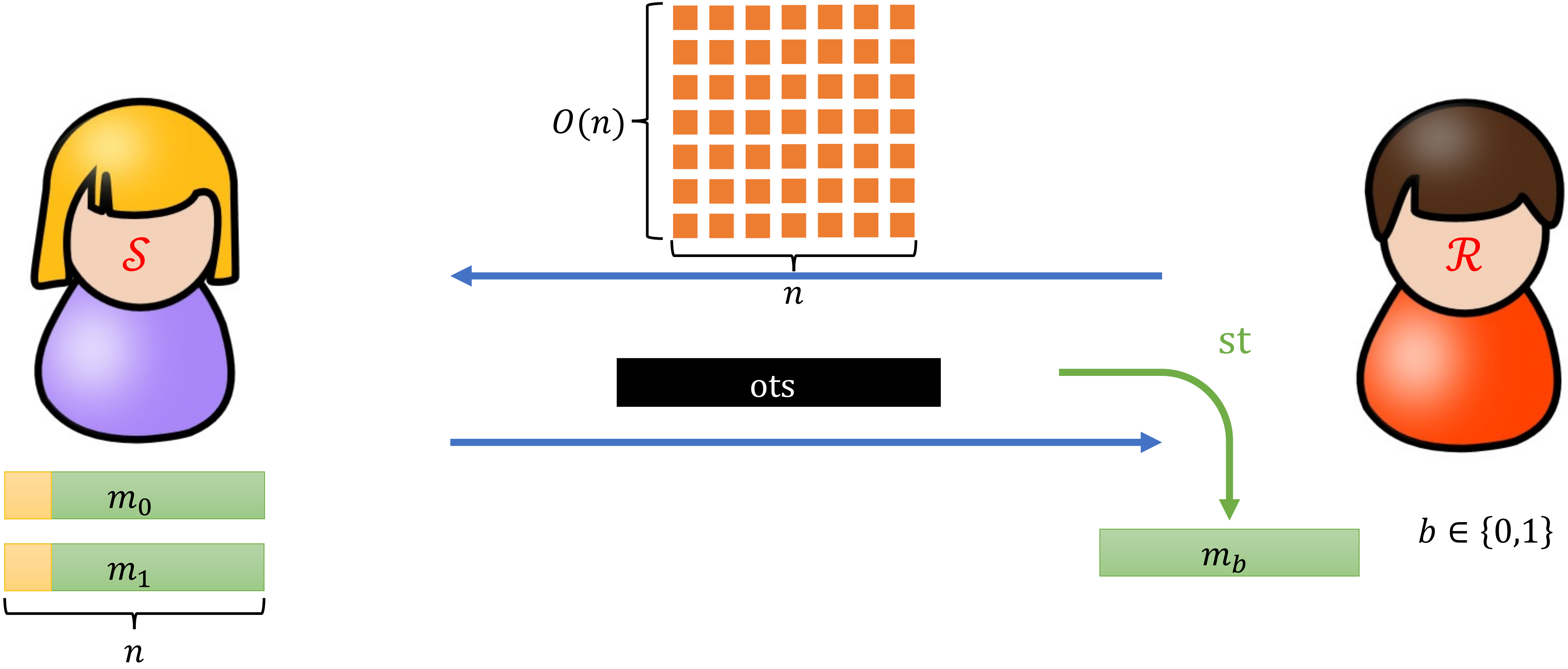
Rate-1 OT [DJ01, DGIMMO19, GHO20]



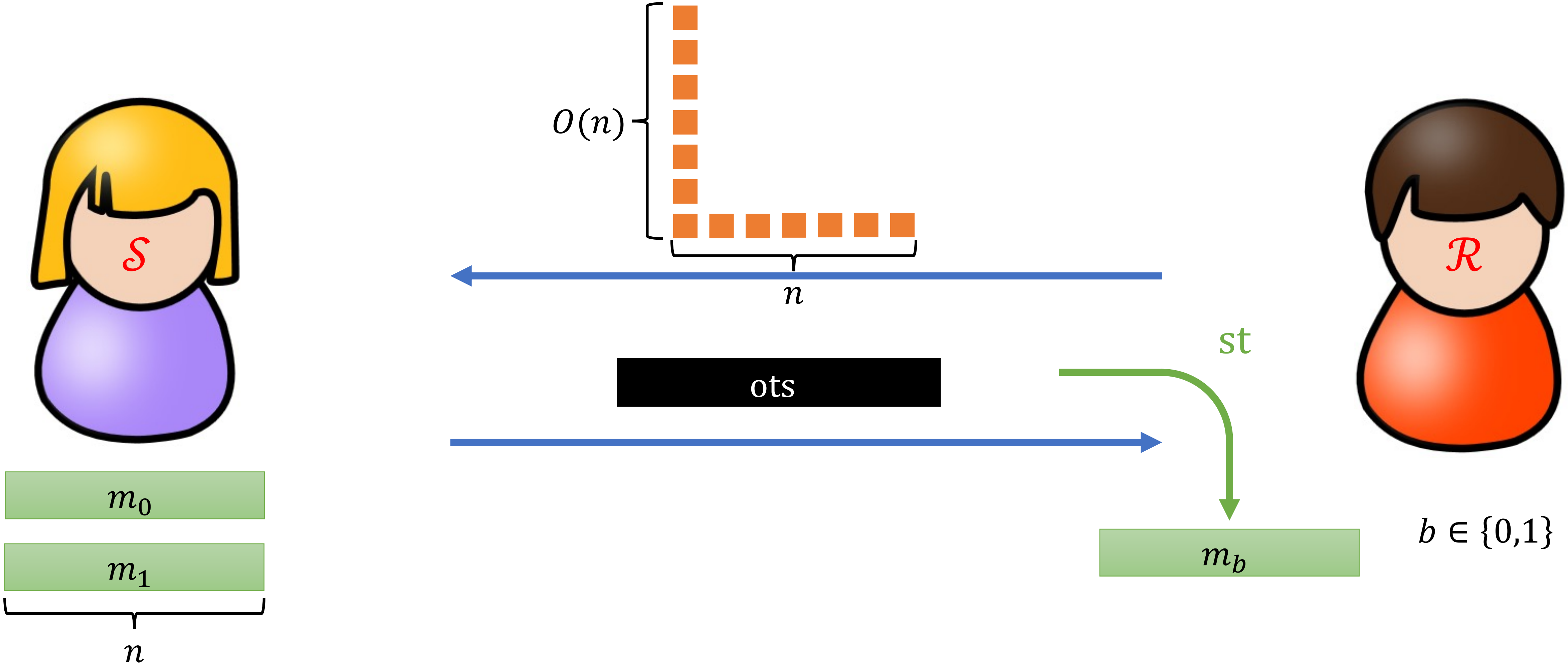
$$\frac{|m_0|}{|\text{ots}|} \to 1 \quad (\text{as } n \to \infty)$$

Receiver Communication?

Rate-1 OT from DDH [DGIMMO19]



Rate-1 OT from Power DDH [GHO20]



$\mathcal{S}$

$\mathcal{R}$

$O(n)$

n

ots

st

m_0

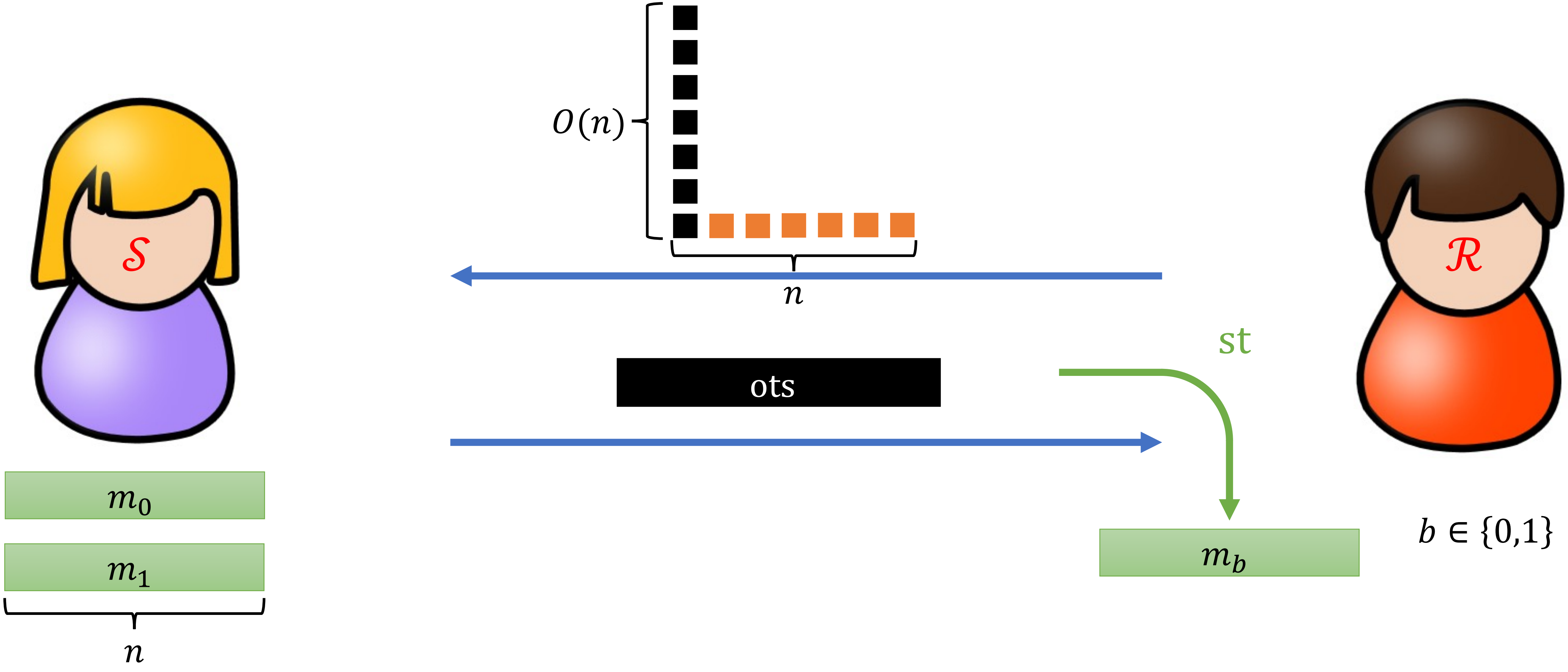
m_1

n

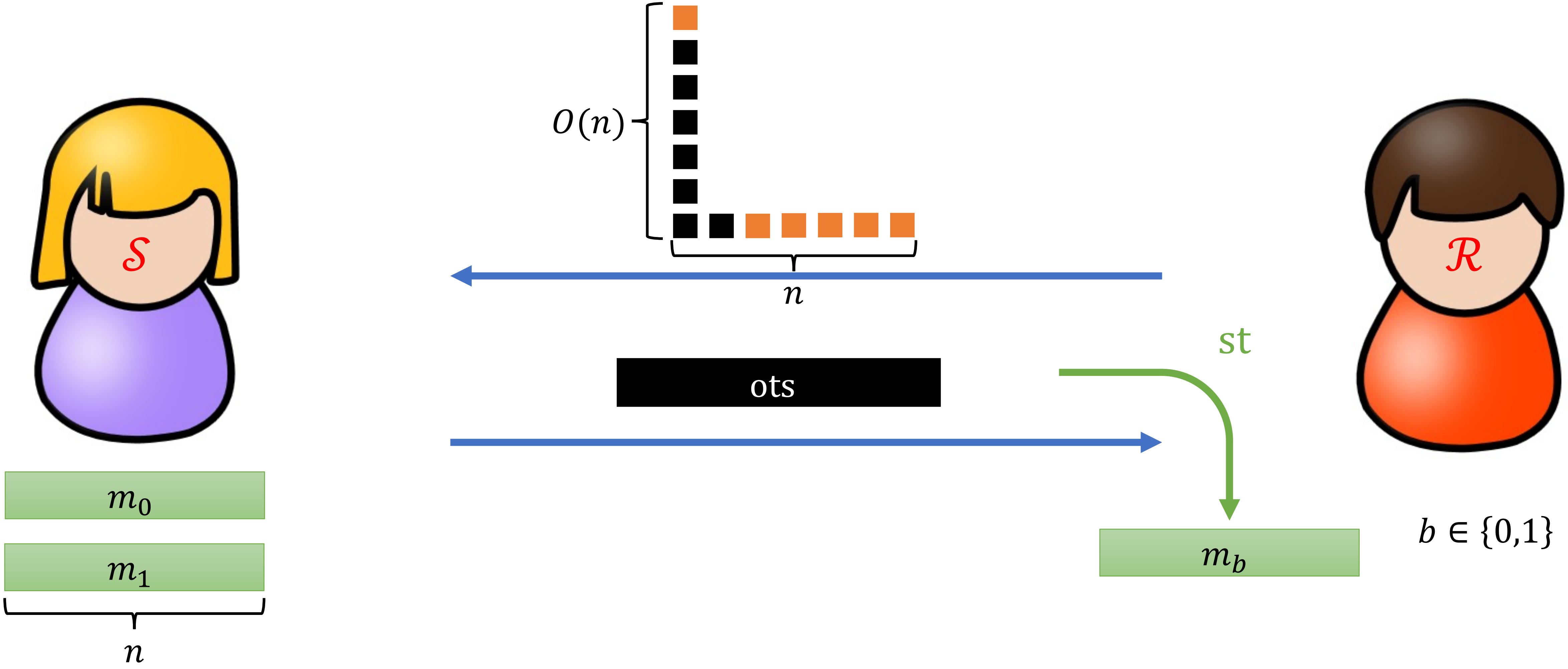
m_b

$b \in \{0,1\}$

Rate-1 OT from Power DDH [GHO20]



Rate-1 OT from Power DDH [GHO20]



$\mathcal{S}$

$\mathcal{R}$

$O(n)$

n

ots

st

m_0

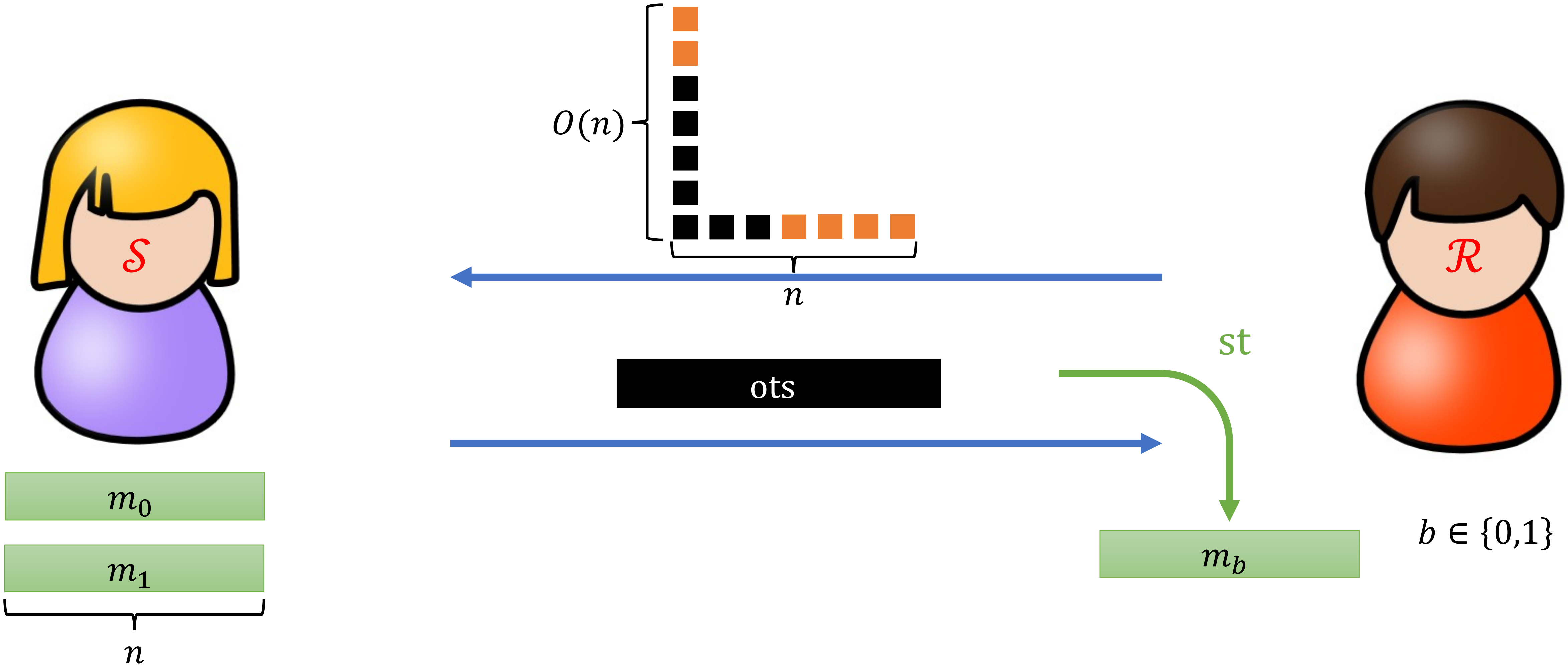
m_1

n

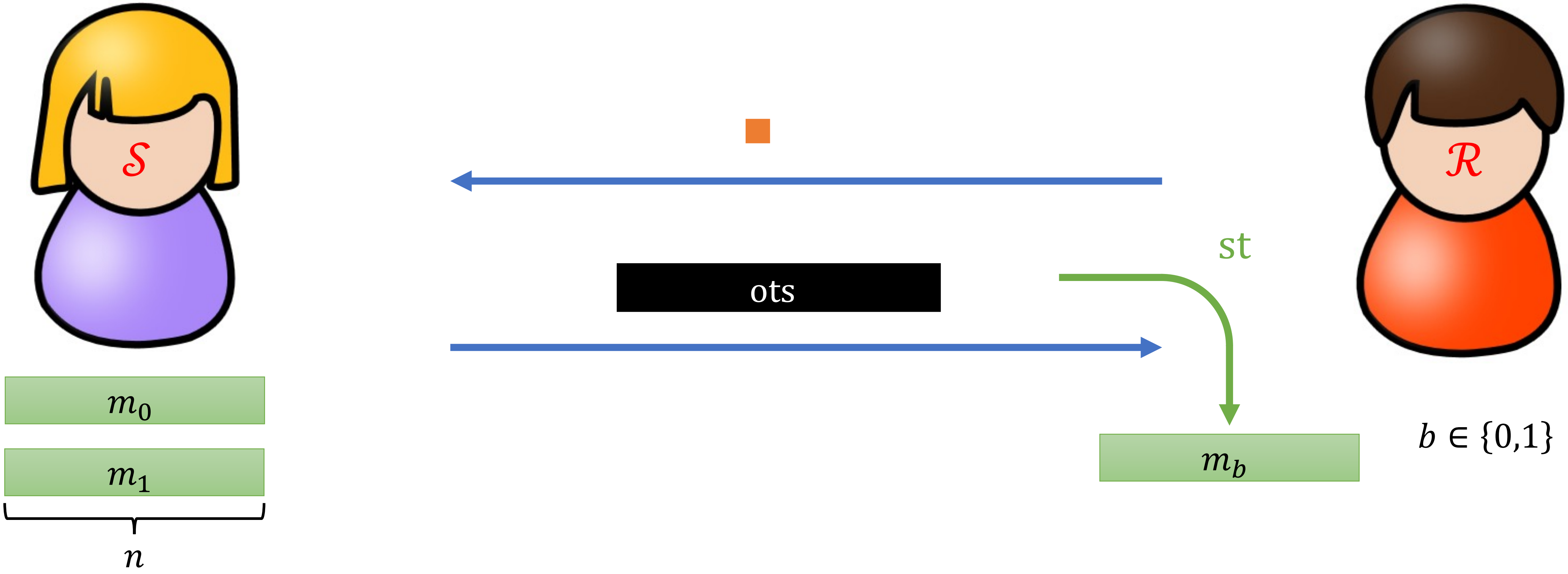
m_b

$b \in \{0,1\}$

Rate-1 OT from Power DDH [GHO20]



Further reduce receiver communication?



$\mathcal{S}$

$\mathcal{R}$

🟧

ots

st

m_0

m_1

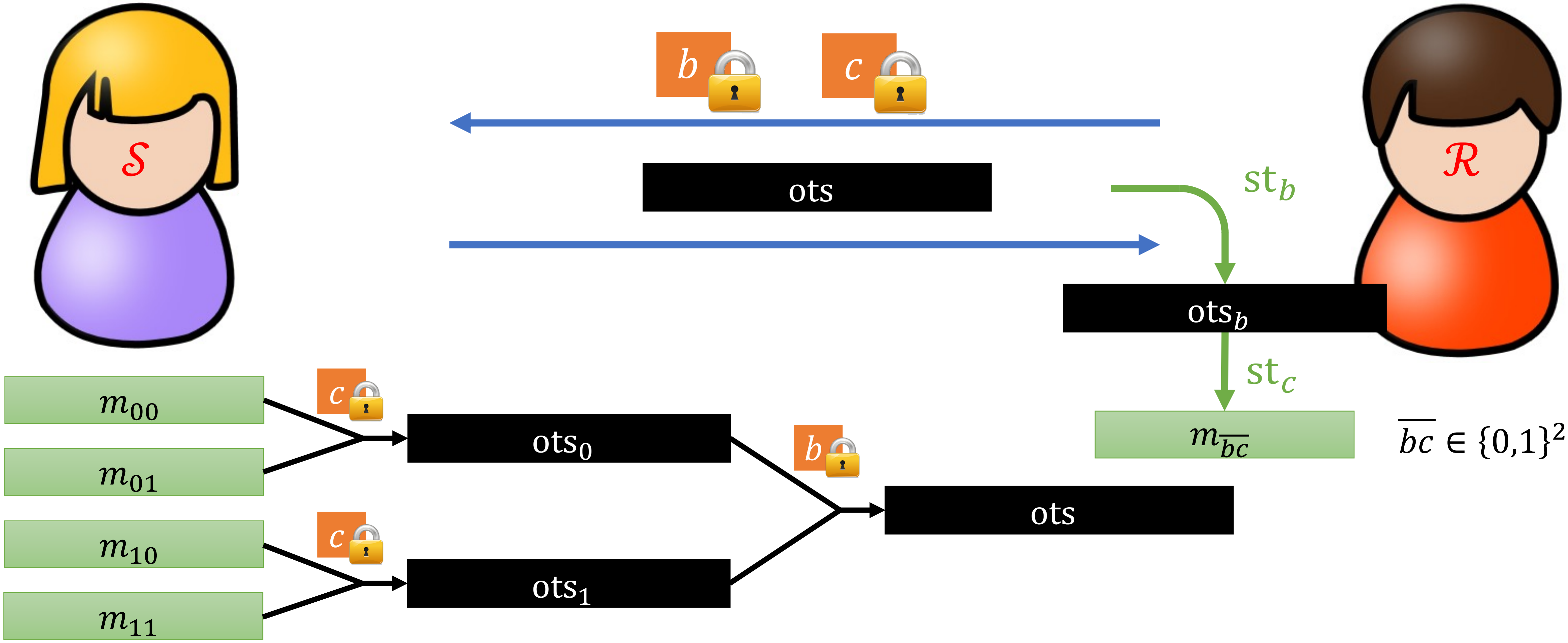
n

m_b

$b \in \{0,1\}$

Why do we care?

Example: 1-out-of-4 OT



$\mathcal{S}$

b

c

ots

$\mathcal{R}$

st_b

ots_b

st_c

$m_{\overline{bc}}$

$\overline{bc} \in \{0,1\}^2$

m_{00}

m_{01}

m_{10}

m_{11}

c

ots_0

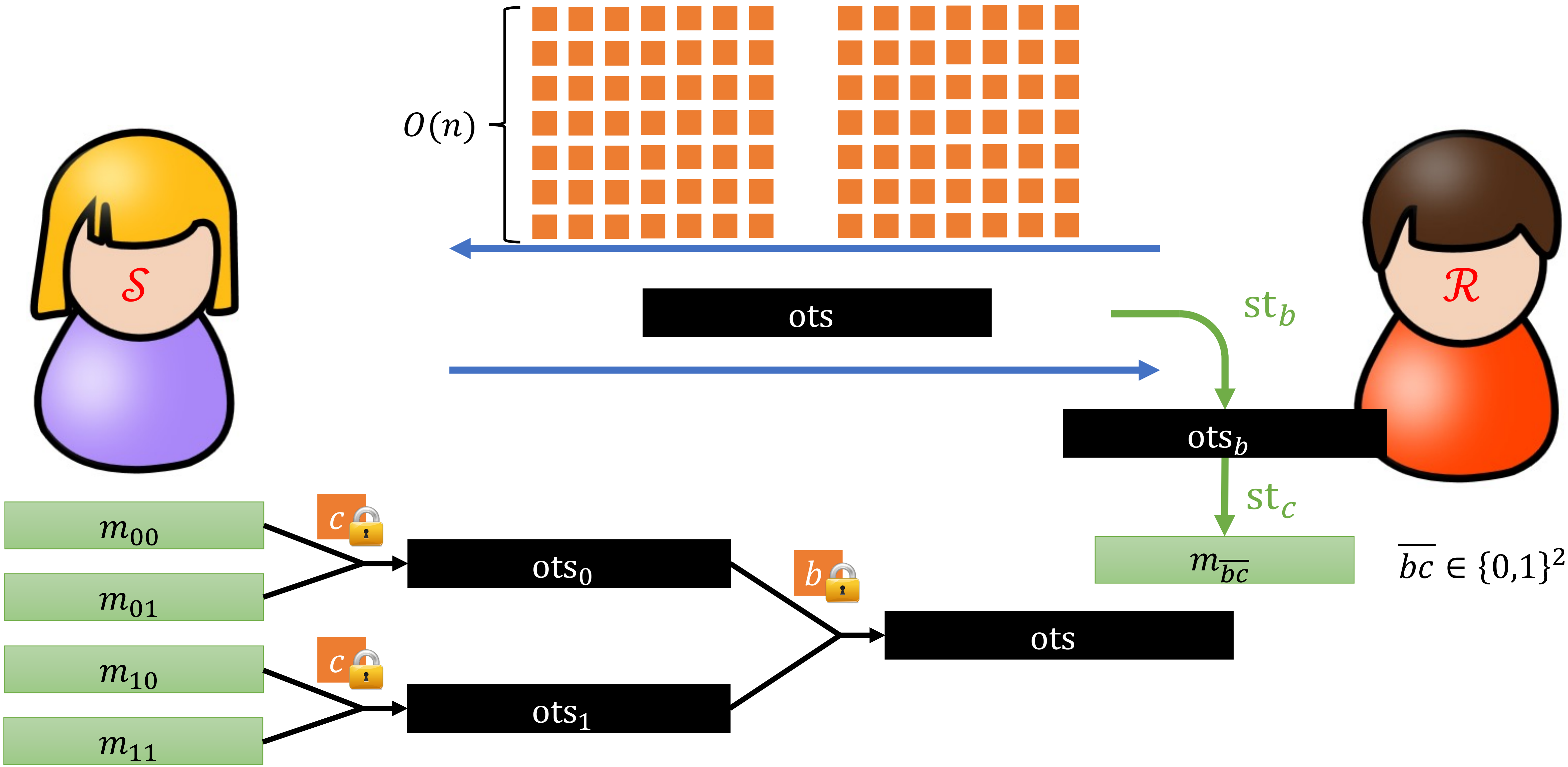
c

ots_1

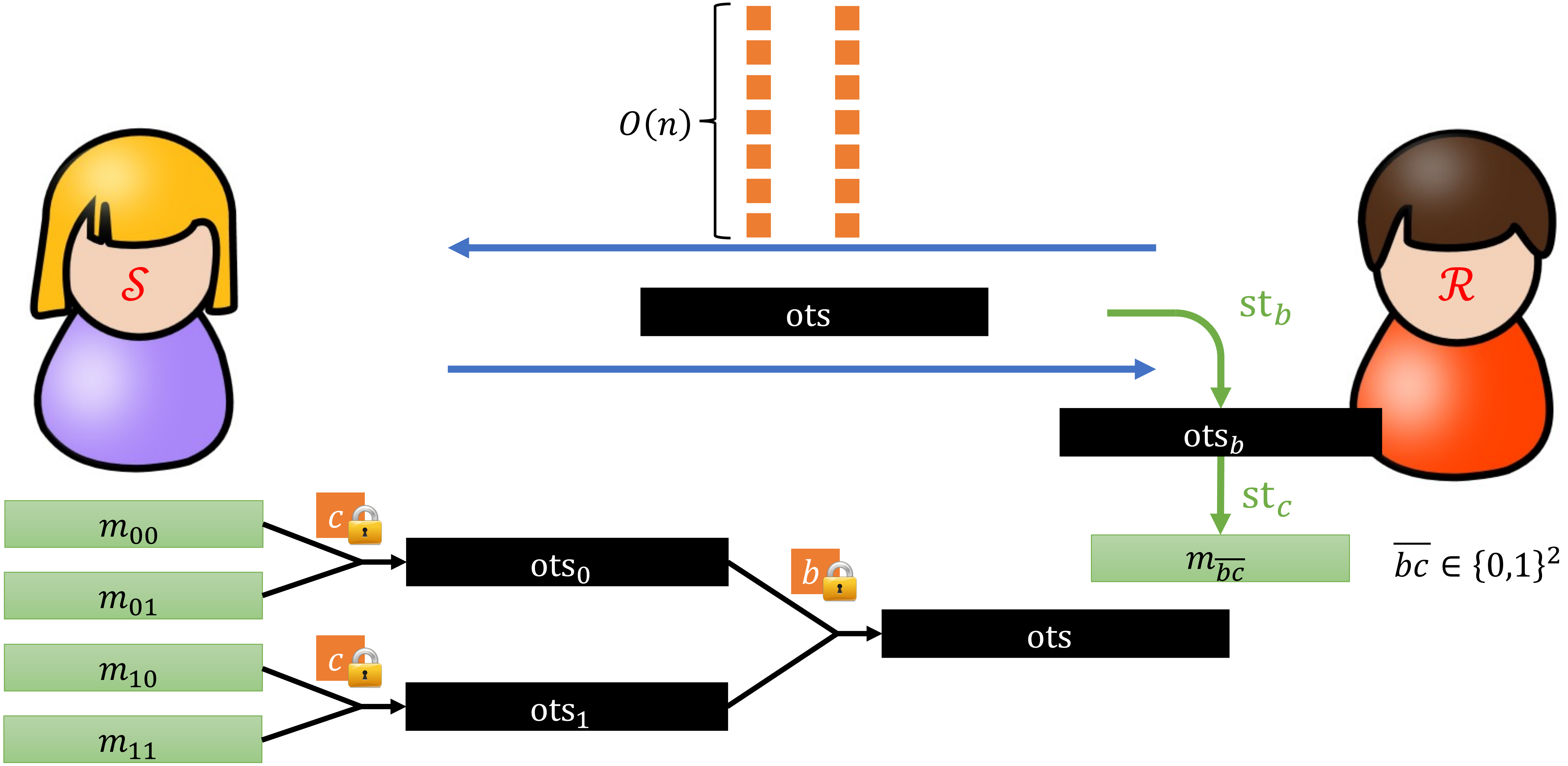
b

ots

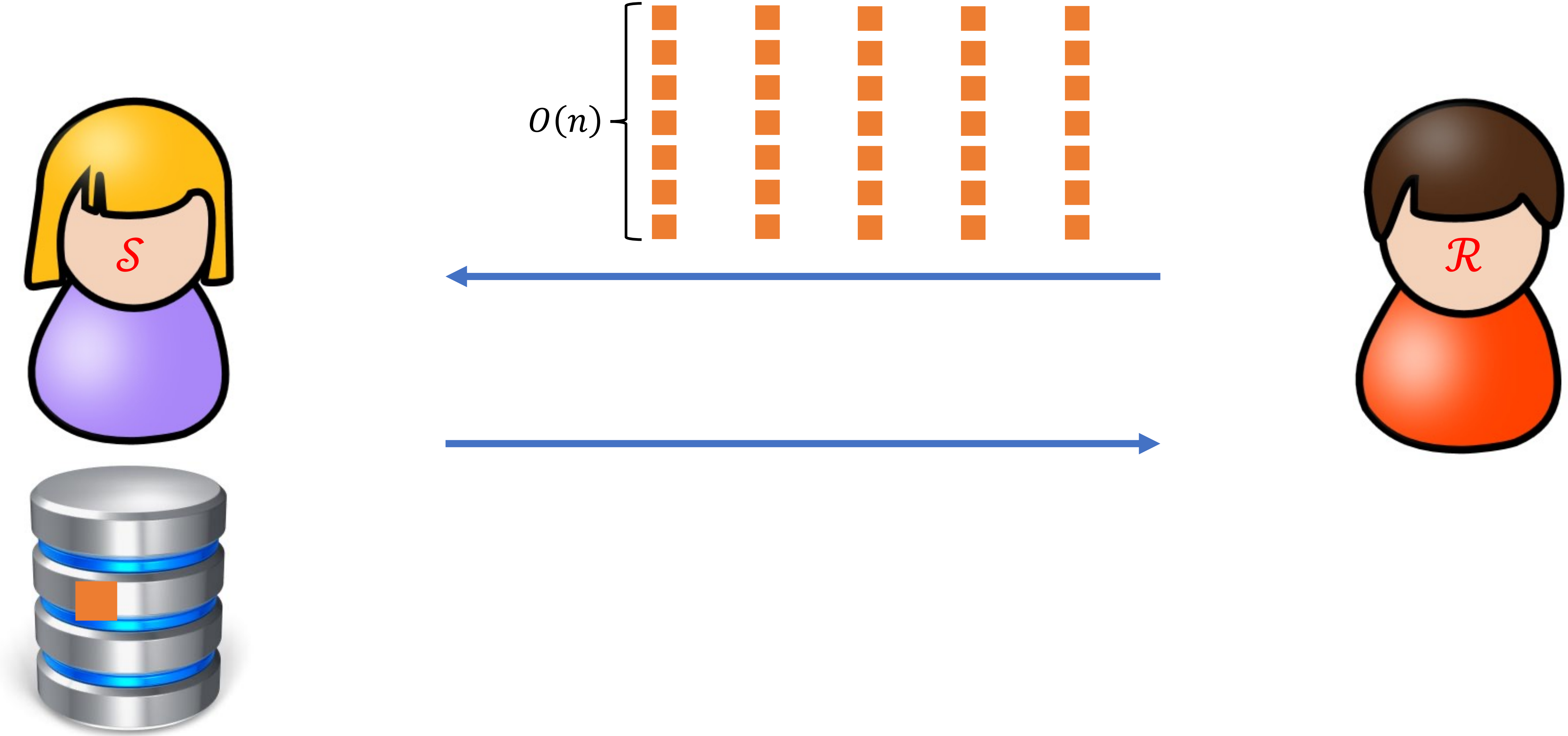
1-out-of-4 OT from DDH [DGIMMO19]



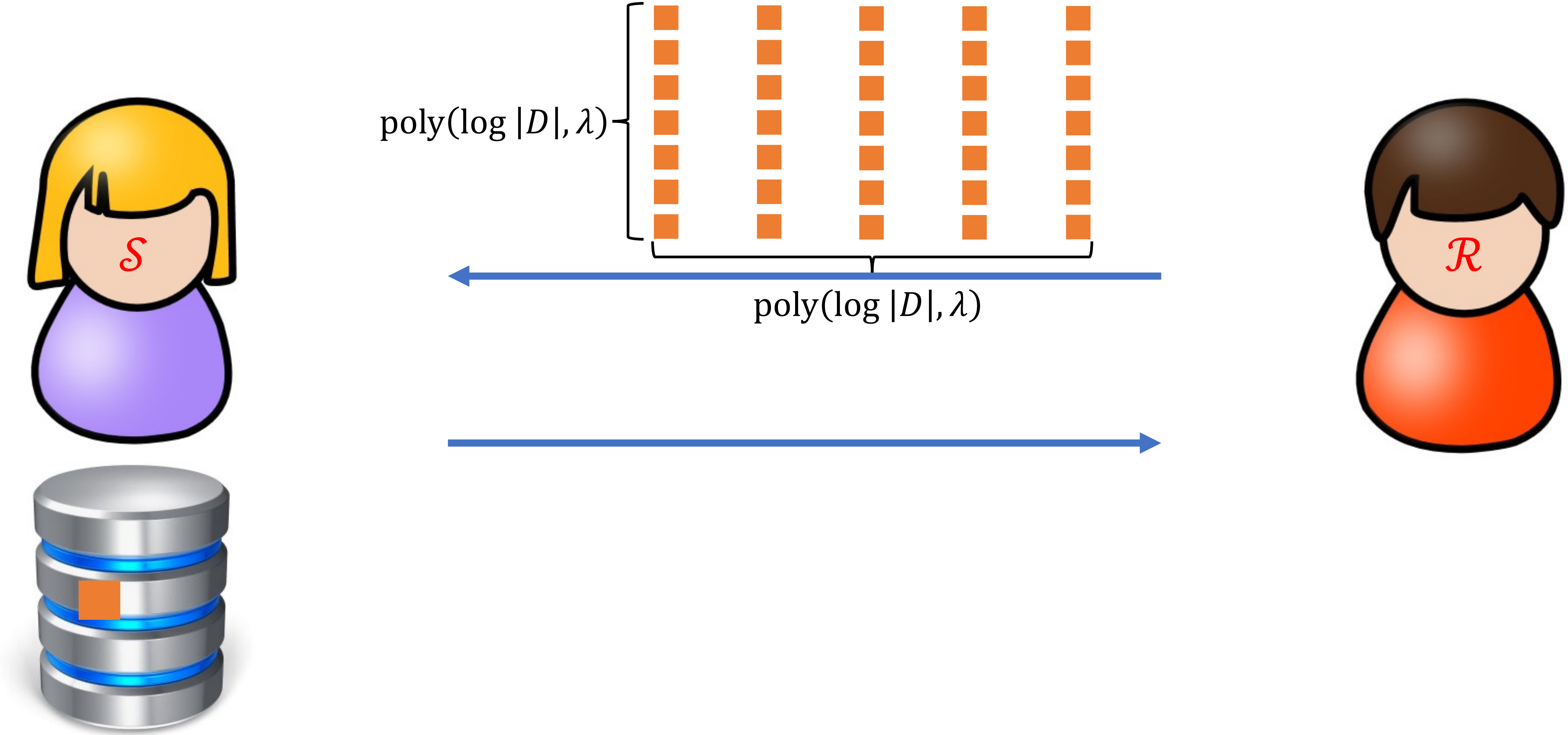
1-out-of-4 OT from Power DDH [DGIMMO19]



Applications from Power DDH [GHO20]



Applications from Power DDH [GHO20]



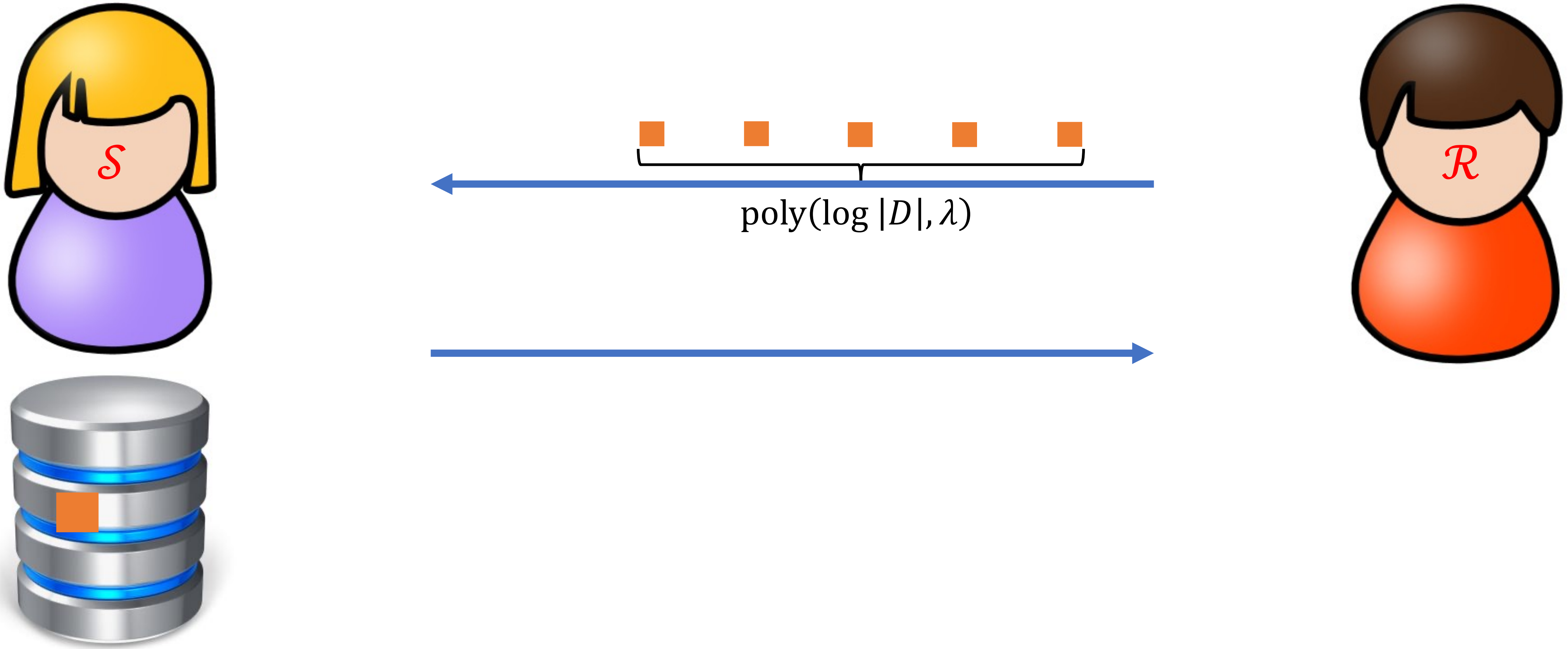
$\mathcal{S}$

$\mathcal{R}$

$\text{poly}(\log |D|, \lambda)$

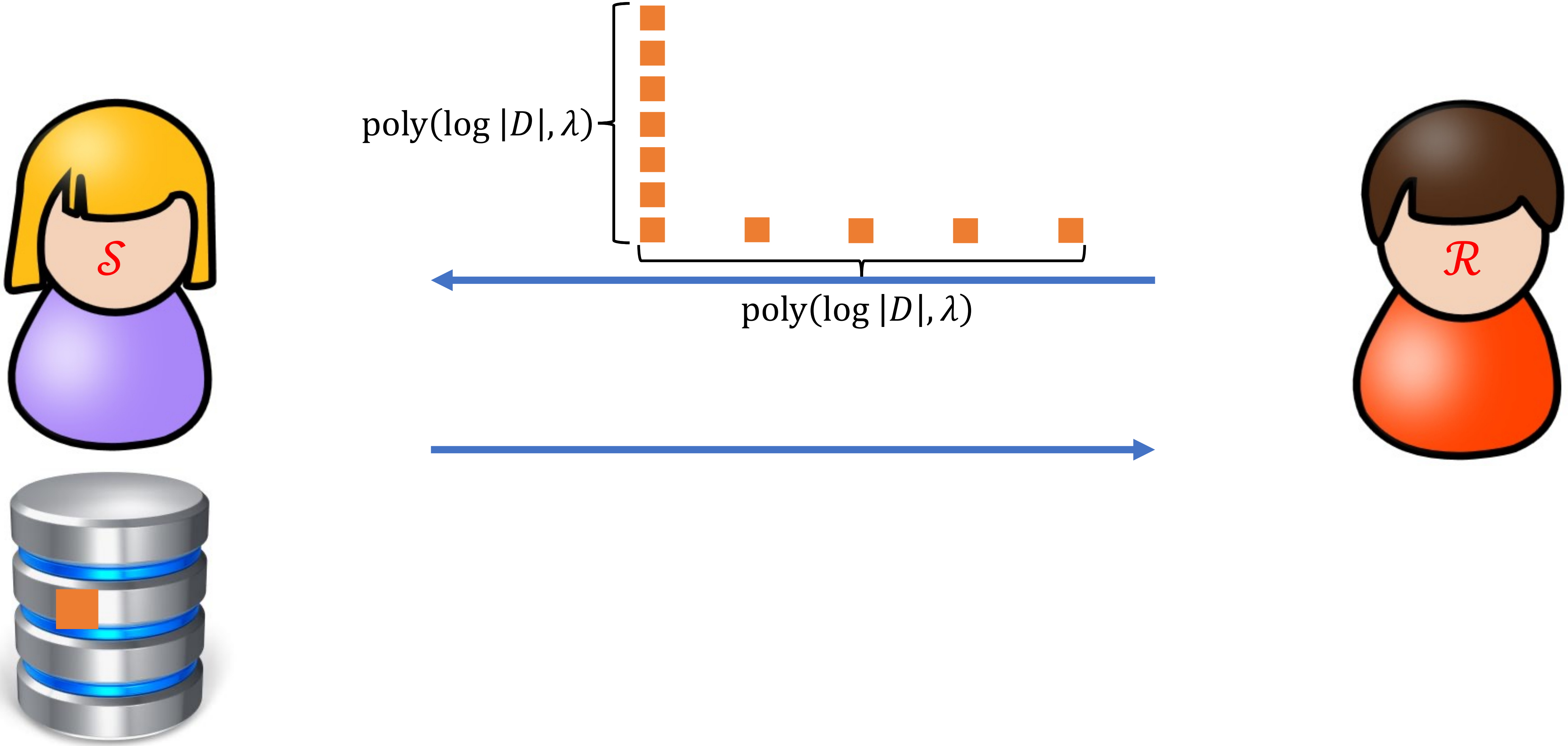
$\text{poly}(\log |D|, \lambda)$

Reduce receiver communication?

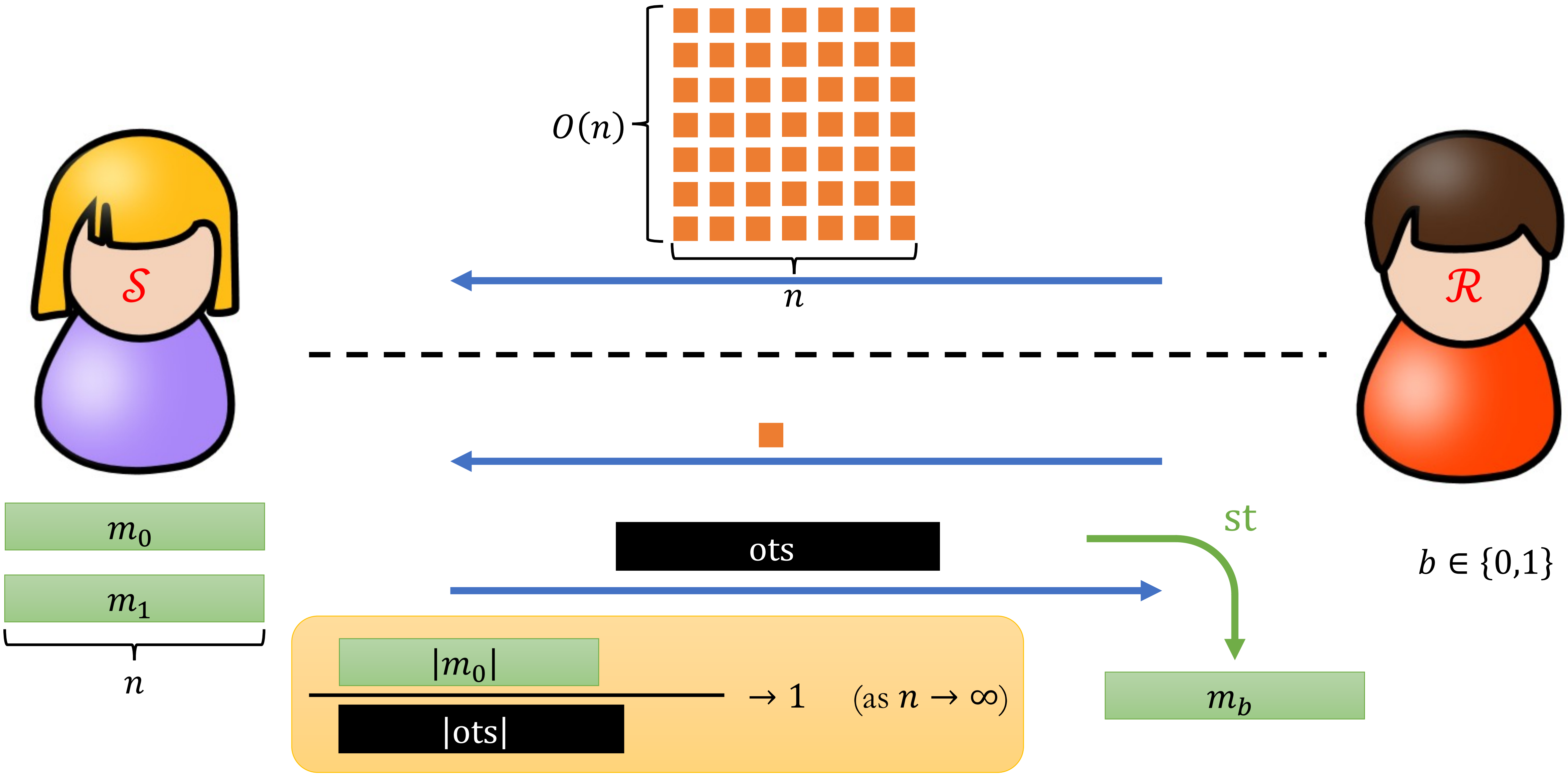


$\text{poly}(\log |D|, \lambda)$

Reduce receiver communication?


$\mathcal{S}$
$\mathcal{R}$
$\text{poly}(\log|D|, \lambda)$
$\text{poly}(\log|D|, \lambda)$

Our Results: Amortized Rate-1 OT



$O(n)$

n

$\mathcal{S}$

$\mathcal{R}$

m_0

m_1

n

ots

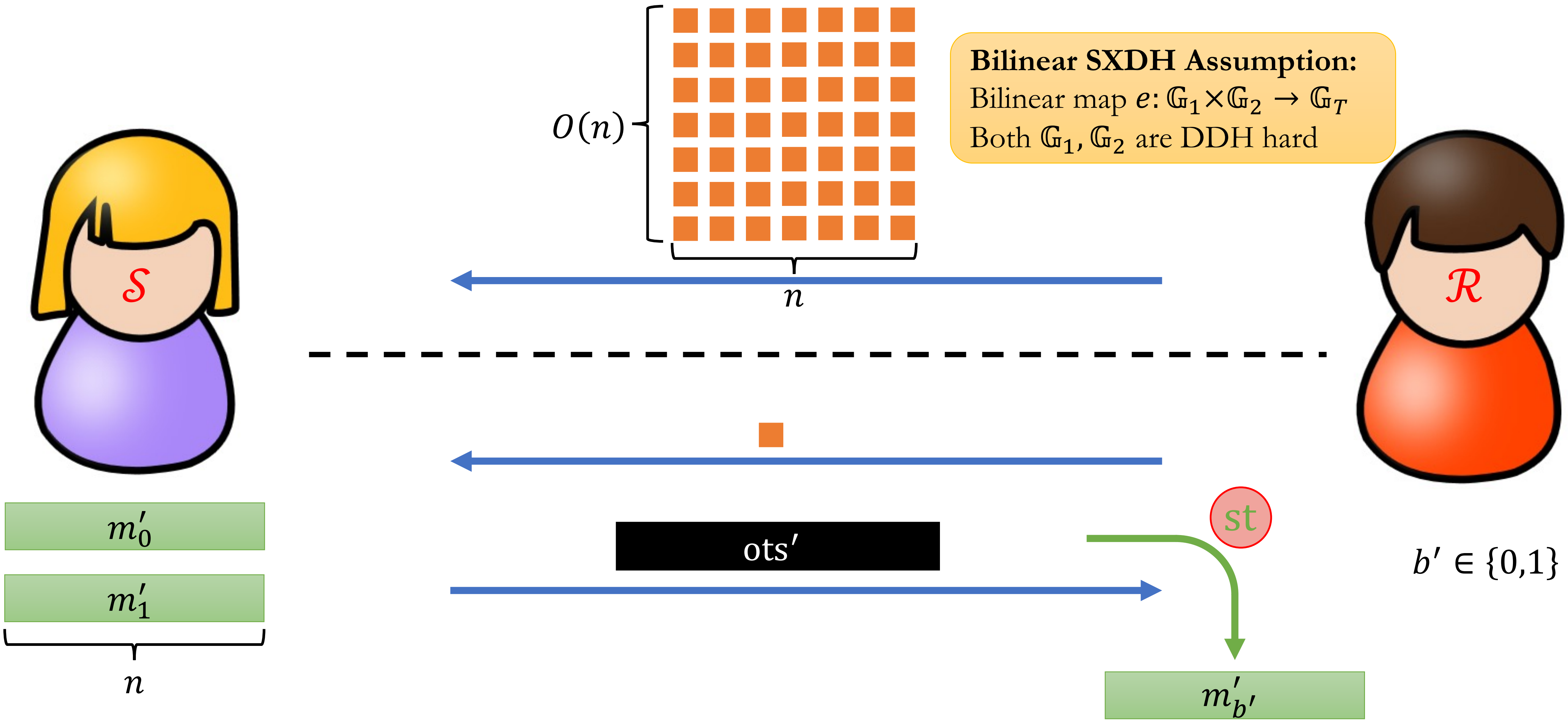
st

$b \in \{0,1\}$

m_b

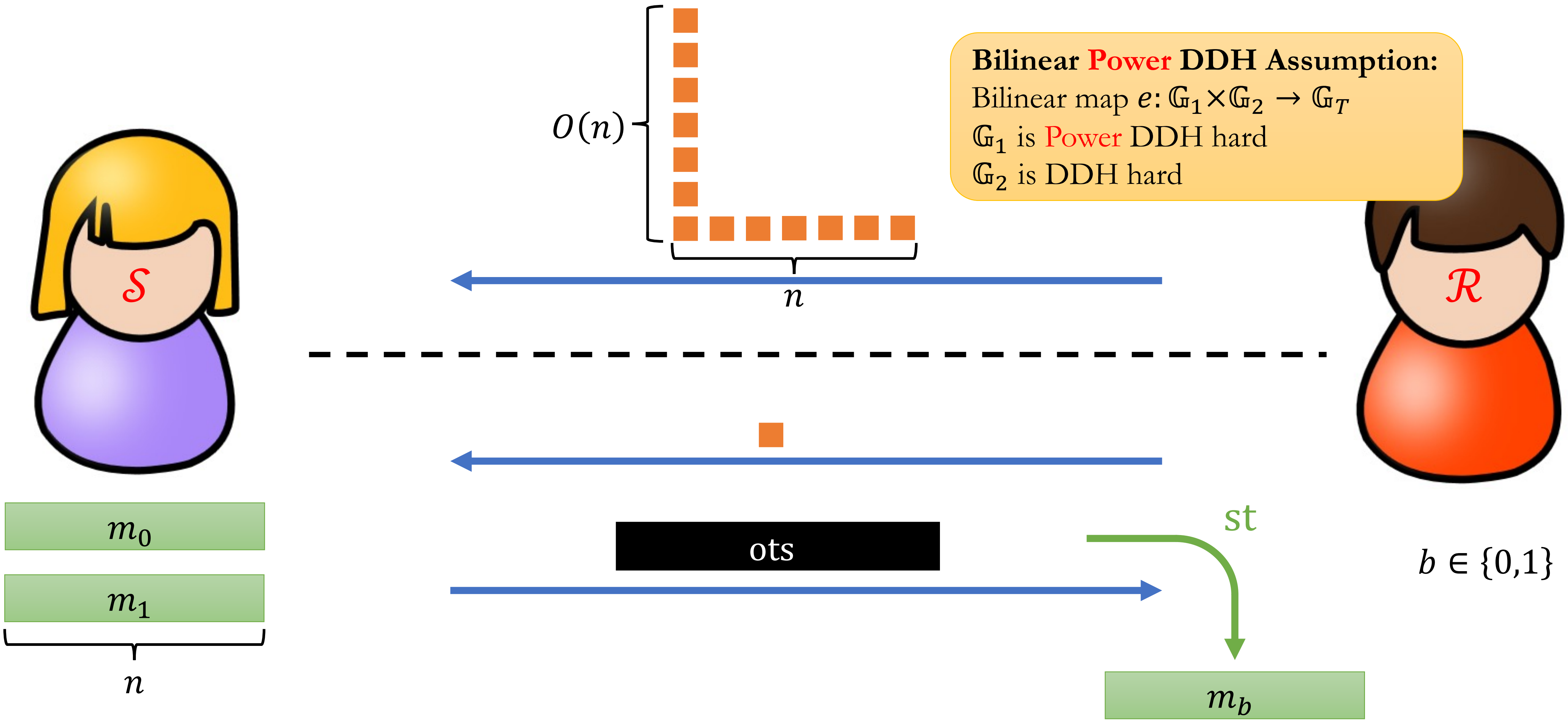
$$\frac{|m_0|}{|\mathrm{ots}|} \to 1 \quad (\text{as } n \to \infty)$$

Our Results: Amortized Rate-1 OT



Bilinear SXDH Assumption:
Bilinear map $e\colon \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$
Both $\mathbb{G}_1, \mathbb{G}_2$ are DDH hard

Our Results: Amortized Rate-1 OT



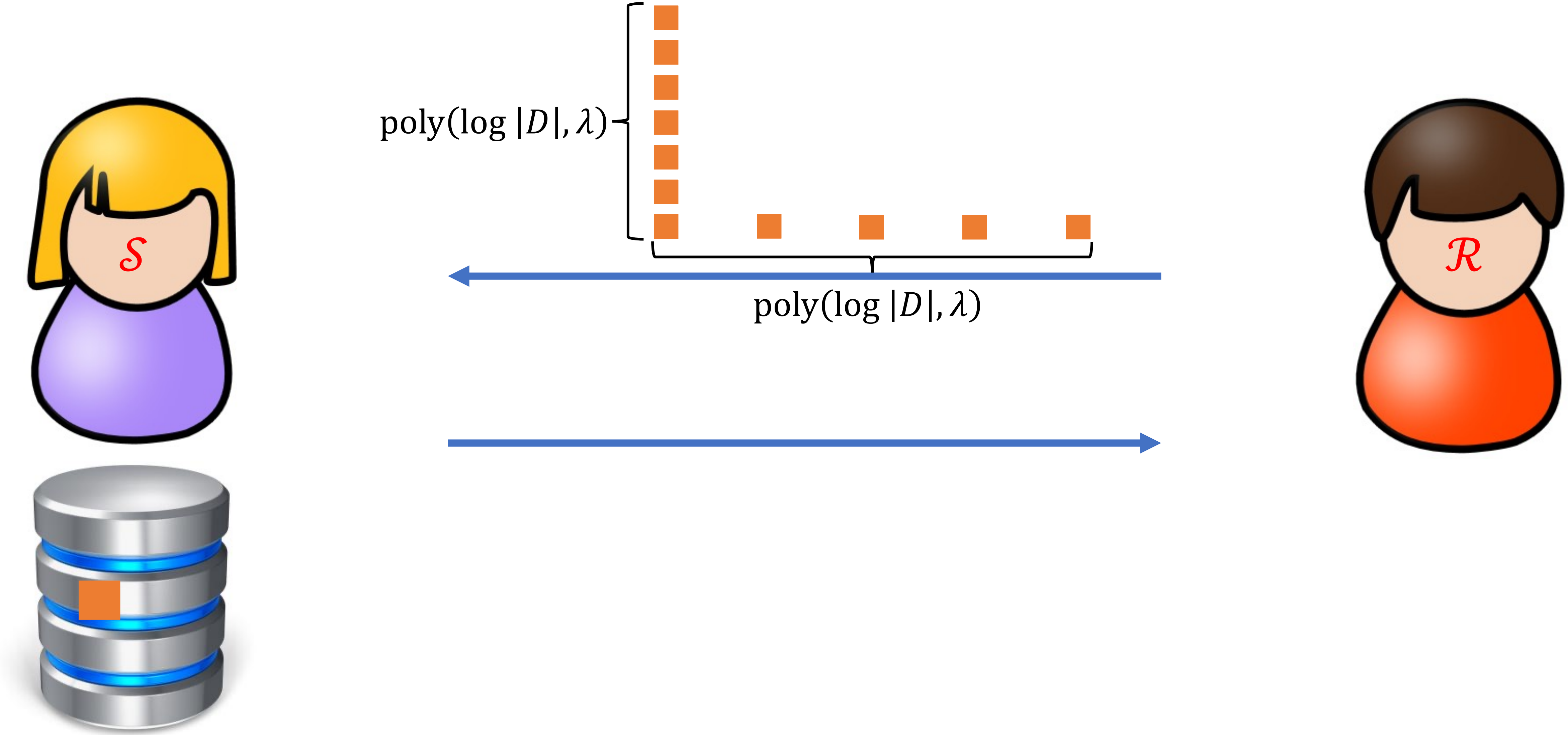
Bilinear Power DDH Assumption:

Bilinear map $e\colon \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$

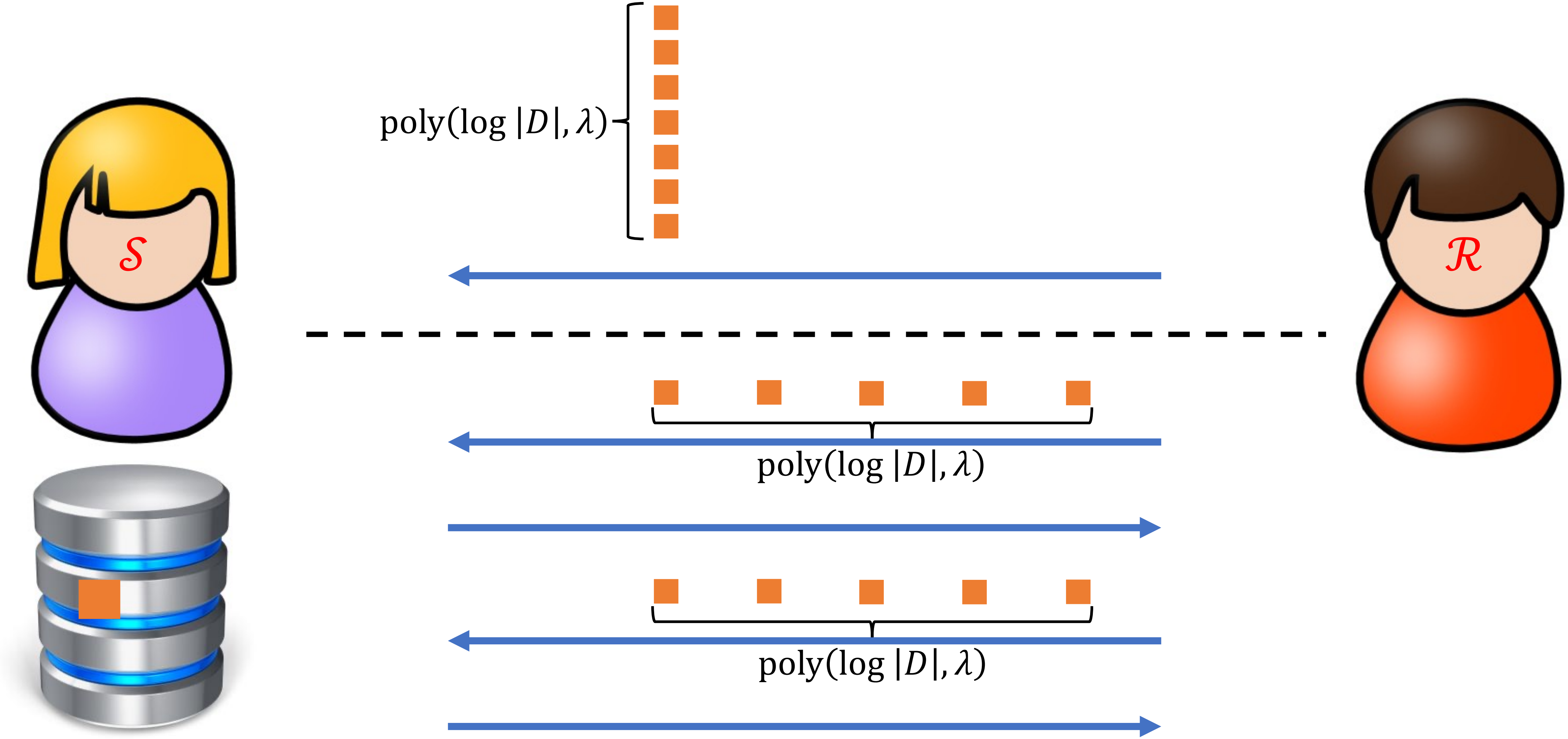
$\mathbb{G}_1$ is Power DDH hard

$\mathbb{G}_2$ is DDH hard

Our Results: Applications from Bilinear Power DDH



Our Results: Applications from Bilinear Power DDH


$\text{poly}(\log|D|,\lambda)$
$\mathcal{S}$
$\mathcal{R}$
$\text{poly}(\log|D|,\lambda)$
$\text{poly}(\log|D|,\lambda)$

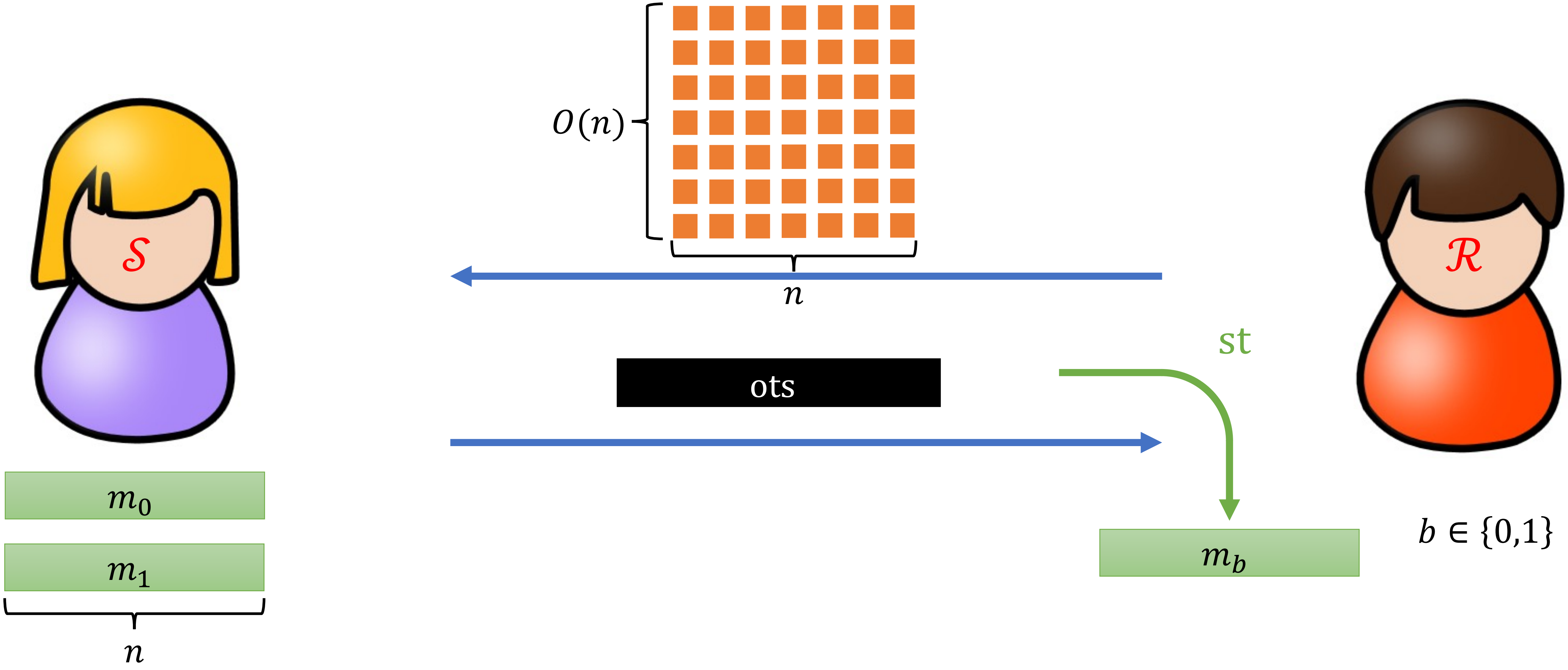
Summary

Problem	Work	Receiver Offline	Receiver Online	Assumption
Rate-1 OT	[DGIMMO19]	N/A	$O(n^2)$	DDH
Amortized Rate-1 OT	Ours	$O(n^2)$	$O(1)$	Bilinear SXDH
Rate-1 OT	[GHO20]	N/A	$O(n)$	Power DDH
Amortized Rate-1 OT	Ours	$O(n)$	$O(1)$	Bilinear Power DDH
Single-Server PIR	[GHO20]	N/A	$O(\lambda \cdot \log^2 N)$	Power DDH
Single-Server PIR	Ours	$O(\lambda \cdot \log N)$	$O(\log N)$	Bilinear Power DDH
Unbalanced PSI	[GHO20]	N/A	$O(\lambda \cdot \log^2 N \cdot m)$	Power DDH
Unbalanced PSI	Ours	$O(\lambda \cdot \log N)$	$O(\log N \cdot m)$	Bilinear Power DDH

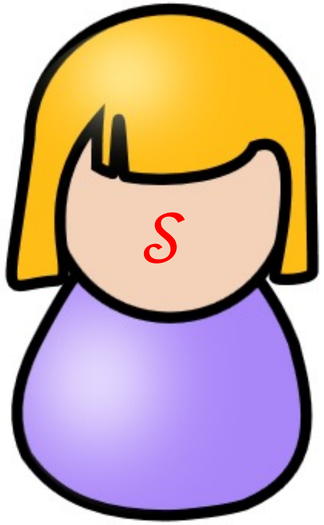
Outline

- Rate-1 OT from DDH [DGIMMO19]
- Amortized Rate-1 OT from Bilinear SXDH
- Optimizations

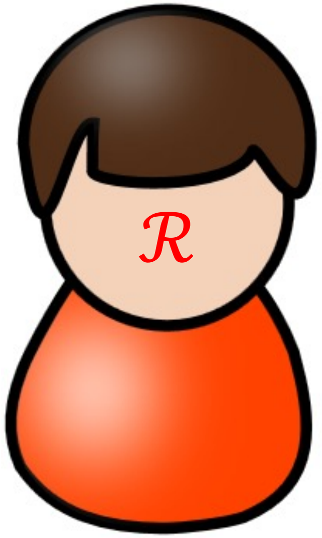
Rate-1 OT from DDH [DGIMMO19]



Rate-1 OT from DDH [DGIMMO19]



$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek =$	$g_{0,1}^{\rho} \cdot g^{1-b}$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho} \cdot g^{b}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$
$b = 0$:	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$
$b = 1$:	$g_{0,1}^{\rho}$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho} \cdot g$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$



m_0

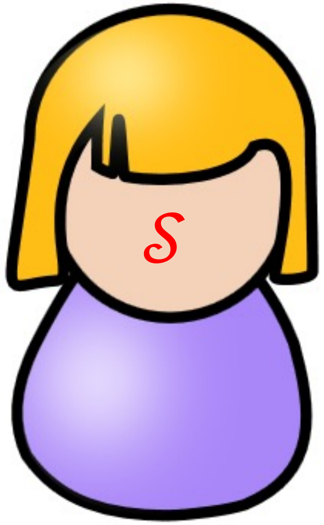
m_1

$\underbrace{\qquad\qquad}_{n}$

$b \in \{0,1\}$

Rate-1 OT from DDH [DGIMMO19]

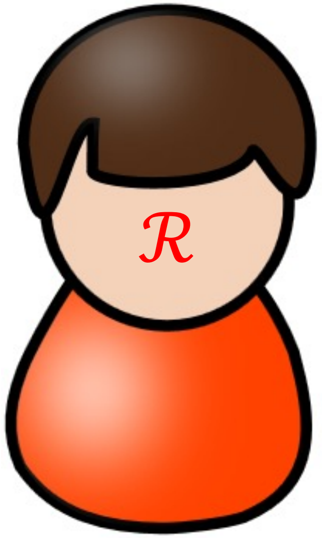
$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$



m_0

m_1

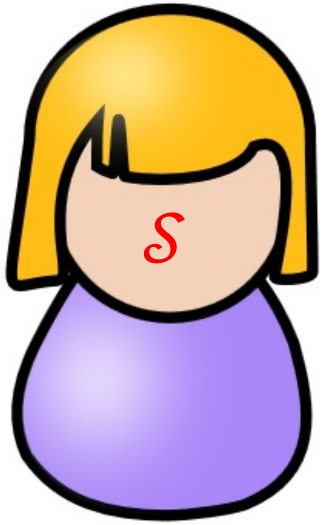
n

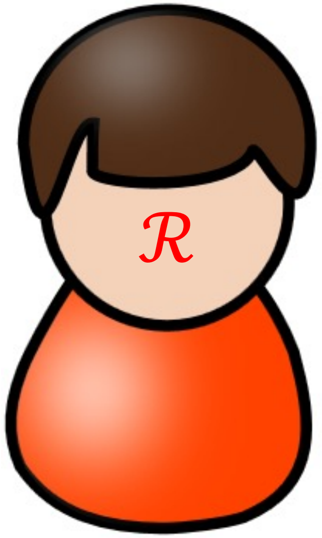


$b = 0$

Rate-1 OT from DDH [DGIMMO19]

$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$





$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$m =$		m_0						m_1				
$ek =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$

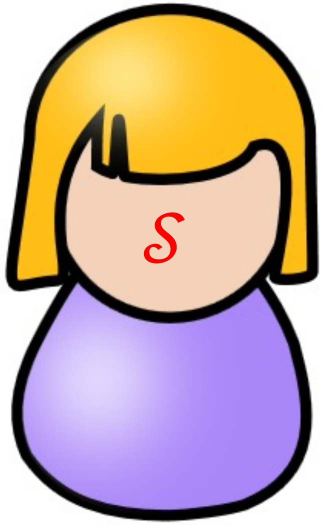
$$h = \langle hk, m \rangle = \prod_{i \in [2n]} hk[i]^{m[i]}$$

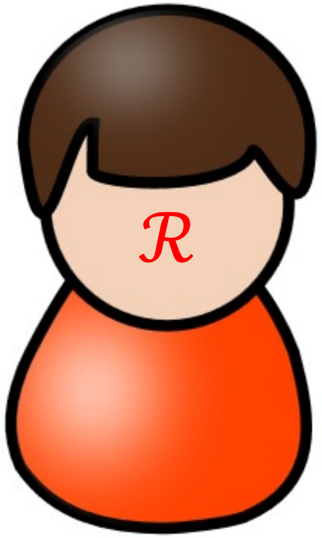
$$e = \langle ek, m \rangle = \prod_{i \in [2n]} ek[i]^{m[i]}$$

$b = 0$

Rate-1 OT from DDH [DGIMMO19]

$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$





$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$m =$	0	0	1	0	1	1	1	0	1	1	0	1
$ek =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$

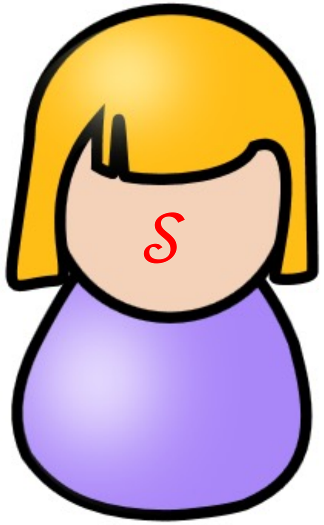
$$h = \langle hk, m \rangle = \prod_{i \in [2n]} hk[i]^{m[i]}$$

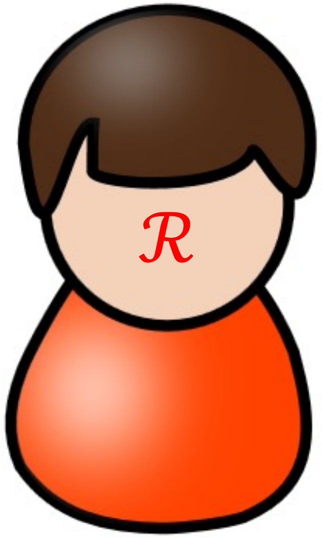
$$e = \langle ek, m \rangle = \prod_{i \in [2n]} ek[i]^{m[i]}$$

$b = 0$

Rate-1 OT from DDH [DGIMMO19]

$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$





$b = 0$

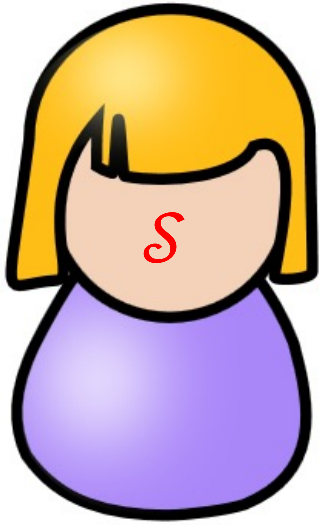
$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$m =$	0	0	1	0	1	1	1	0	1	1	0	1
$ek =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$

$$h = \langle hk, m \rangle = \prod_{i \in [2n]} hk[i]^{m[i]}$$

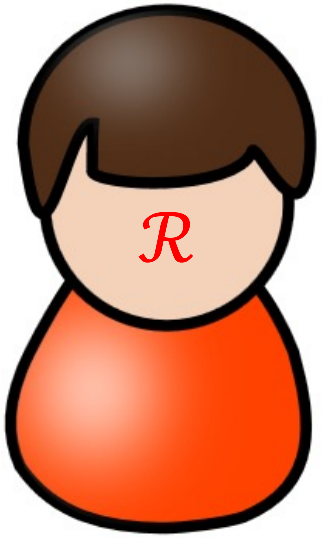
$$e = \langle ek, m \rangle = \prod_{i \in [2n]} ek[i]^{m[i]}$$

$m_0[1] = 0$: $e = h^{\rho}$

Rate-1 OT from DDH [DGIMMO19]



$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$



$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$m =$	1	0	1	0	1	1	1	0	1	1	0	1
$ek =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$

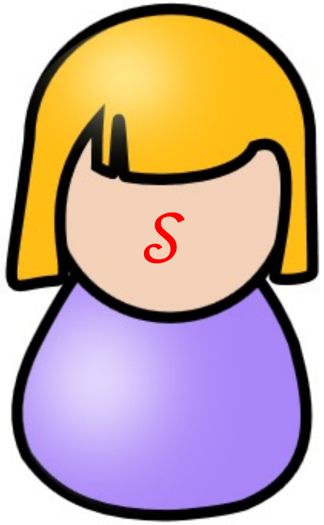
$$h = \langle hk, m \rangle = \prod_{i \in [2n]} hk[i]^{m[i]}$$

$$e = \langle ek, m \rangle = \prod_{i \in [2n]} ek[i]^{m[i]}$$

$b = 0$

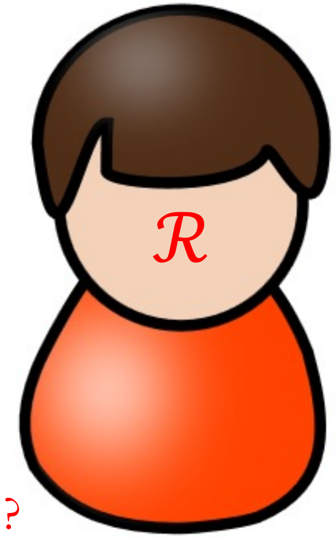
$m_0[1] = 1$: $e = h^{\rho} \cdot g$

Rate-1 OT from DDH [DGIMMO19]



$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek =$	$g_{0,1}^{\rho}$ $\cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$

h, e



$e = h^{\rho}$ or $h^{\rho} \cdot g$?

$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$m =$	1	0	1	0	1	1	1	0	1	1	0	1
$ek =$	$g_{0,1}^{\rho}$ $\cdot g$	$g_{0,2}^{\rho}$	.	.	.	$g_{0,n}^{\rho}$	$g_{1,1}^{\rho}$	$g_{1,2}^{\rho}$	.	.	.	$g_{1,n}^{\rho}$

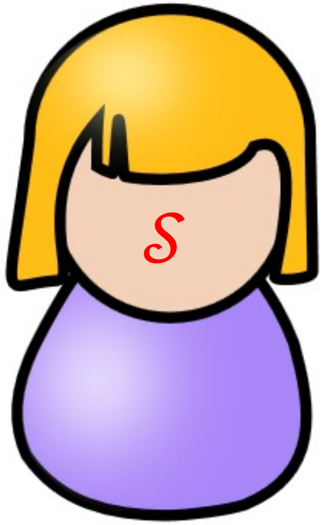
$$h = \langle hk, m \rangle = \prod_{i \in [2n]} hk[i]^{m[i]}$$

$$e = \langle ek, m \rangle = \prod_{i \in [2n]} ek[i]^{m[i]}$$

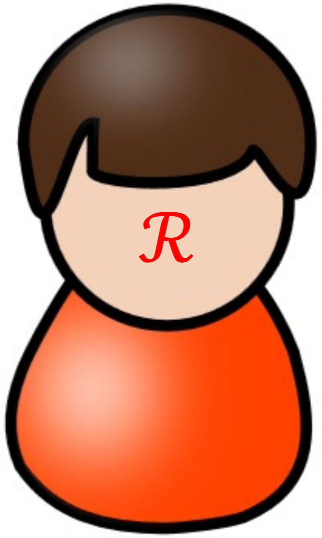
$b = 0$

$e = h^{\rho}$ or $h^{\rho} \cdot g$

Rate-1 OT from DDH [DGIMMO19]



$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek_1 =$	$g_{0,1}^{\rho_1} \cdot g$	$g_{0,2}^{\rho_1}$	.	.	.	$g_{0,n}^{\rho_1}$	$g_{1,1}^{\rho_1}$	$g_{1,2}^{\rho_1}$	.	.	.	$g_{1,n}^{\rho_1}$
$ek_2 =$	$g_{0,1}^{\rho_2}$	$g_{0,2}^{\rho_2} \cdot g$	.	.	.	$g_{0,n}^{\rho_2}$	$g_{1,1}^{\rho_2}$	$g_{1,2}^{\rho_2}$	.	.	.	$g_{1,n}^{\rho_2}$
							⋮					
$ek_n =$	$g_{0,1}^{\rho_n}$	$g_{0,2}^{\rho_n}$	.	.	.	$g_{0,n}^{\rho_n} \cdot g$	$g_{1,1}^{\rho_n}$	$g_{1,2}^{\rho_n}$	.	.	.	$g_{1,n}^{\rho_n}$



$m =$	m_0	m_1

$b = 0$

$$h = \langle hk, m \rangle$$
$$e_1 = \langle ek_1, m \rangle$$
$$e_2 = \langle ek_2, m \rangle$$
$$\vdots$$
$$e_n = \langle ek_n, m \rangle$$

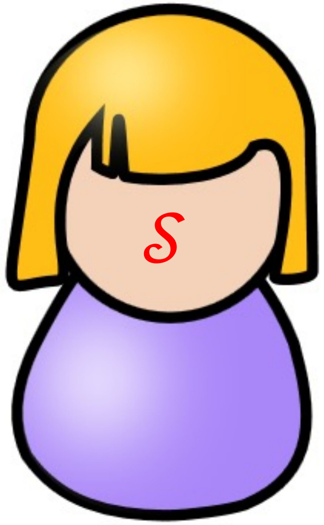
$h, e_1, e_2, \ldots, e_n$

$(n+1)$ group elements!

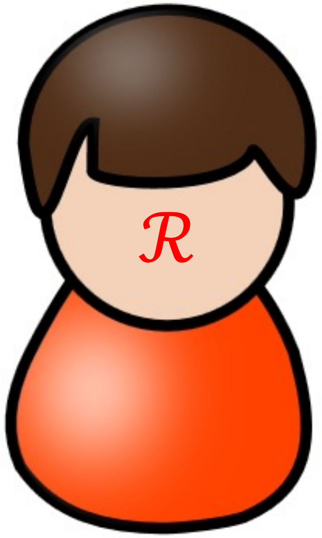
Goal: 1 group element + n bits

$e_1 = h^{\rho_1}$ or $h^{\rho_1} \cdot g$?
$e_2 = h^{\rho_2}$ or $h^{\rho_2} \cdot g$?
$\vdots$
$e_n = h^{\rho_n}$ or $h^{\rho_n} \cdot g$?

Rate-1 OT from DDH [DGIMMO19]



$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek_1 =$	$g_{0,1}^{\rho_1} \cdot g$	$g_{0,2}^{\rho_1}$	.	.	.	$g_{0,n}^{\rho_1}$	$g_{1,1}^{\rho_1}$	$g_{1,2}^{\rho_1}$	.	.	.	$g_{1,n}^{\rho_1}$
$ek_2 =$	$g_{0,1}^{\rho_2}$	$g_{0,2}^{\rho_2} \cdot g$	.	.	.	$g_{0,n}^{\rho_2}$	$g_{1,1}^{\rho_2}$	$g_{1,2}^{\rho_2}$	.	.	.	$g_{1,n}^{\rho_2}$
							$\vdots$					
$ek_n =$	$g_{0,1}^{\rho_n}$	$g_{0,2}^{\rho_n}$	.	.	.	$g_{0,n}^{\rho_n} \cdot g$	$g_{1,1}^{\rho_n}$	$g_{1,2}^{\rho_n}$	.	.	.	$g_{1,n}^{\rho_n}$



$m =$	m_0	m_1

$b = 0$

$h = \langle hk, m \rangle$

$e_1 = \langle ek_1, m \rangle \rightarrow b_1$

$e_2 = \langle ek_2, m \rangle \rightarrow b_2$

$\vdots$

$e_n = \langle ek_n, m \rangle \rightarrow b_n$

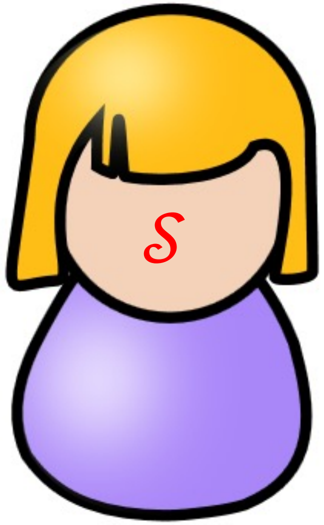
$h, b_1, b_2, \dots, b_n$

Goal: 1 group element + n bits

Rate-1 OT from DDH [DGIMMO19]

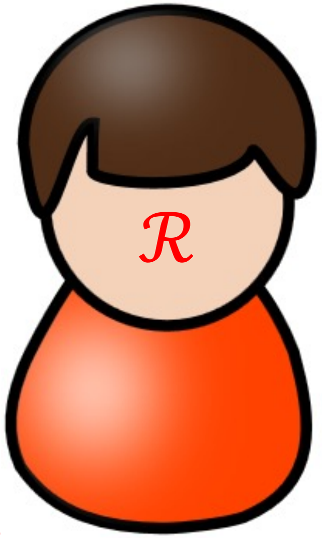
$$hk = \begin{array}{|c|c|c|c|c|c|}\hline g_{0,1} & g_{0,2} & . & . & . & g_{0,n} \\ \hline \end{array}\ \begin{array}{|c|c|c|c|c|c|}\hline g_{1,1} & g_{1,2} & . & . & . & g_{1,n} \\ \hline \end{array}$$

$$ek = \begin{array}{|c|c|c|c|c|c|}\hline g_{0,1}^{\rho} \cdot g & g_{0,2}^{\rho} & . & . & . & g_{0,n}^{\rho} \\ \hline \end{array}\ \begin{array}{|c|c|c|c|c|c|}\hline g_{1,1}^{\rho} & g_{1,2}^{\rho} & . & . & . & g_{1,n}^{\rho} \\ \hline \end{array}$$



$$\xleftarrow{\qquad\qquad\qquad\qquad}$$

$$\xrightarrow{h,\ \Phi(e)}$$



$\Phi(e) = \Phi(h^{\rho})$ or $\Phi(h^{\rho} \cdot g)$?

$b = 0$

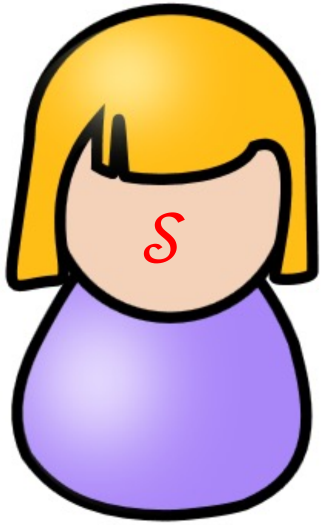
$$m = \begin{array}{|c|c|}\hline m_0 & m_1 \\ \hline \end{array}$$

$h = \langle hk, m \rangle$
$e = \langle ek, m \rangle$

$e = h^{\rho}$ or $h^{\rho} \cdot g$

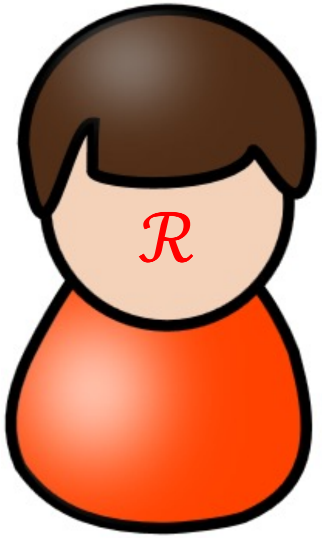
$\Phi\colon \mathbb{G} \to \{0,1\}$ such that
$\forall v \in \mathbb{G},\ \ \Phi(v) \neq \Phi(v \cdot g)$
(can be achieved by [BGI16])

Rate-1 OT from DDH [DGIMMO19]



$hk =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$ek_1 =$	$g_{0,1}^{\rho_1} \cdot g$	$g_{0,2}^{\rho_1}$	.	.	.	$g_{0,n}^{\rho_1}$	$g_{1,1}^{\rho_1}$	$g_{1,2}^{\rho_1}$	.	.	.	$g_{1,n}^{\rho_1}$
$ek_2 =$	$g_{0,1}^{\rho_2}$	$g_{0,2}^{\rho_2} \cdot g$	.	.	.	$g_{0,n}^{\rho_2}$	$g_{1,1}^{\rho_2}$	$g_{1,2}^{\rho_2}$	.	.	.	$g_{1,n}^{\rho_2}$
						⋮						
$ek_n =$	$g_{0,1}^{\rho_n}$	$g_{0,2}^{\rho_n}$	.	.	.	$g_{0,n}^{\rho_n} \cdot g$	$g_{1,1}^{\rho_n}$	$g_{1,2}^{\rho_n}$	.	.	.	$g_{1,n}^{\rho_n}$

←



$m =$	m_0	m_1

$b = 0$

$$h = \langle hk, m \rangle$$
$$e_1 = \langle ek_1, m \rangle \xrightarrow{\Phi} b_1$$
$$e_2 = \langle ek_2, m \rangle \longrightarrow b_2$$
$$\vdots$$
$$e_n = \langle ek_n, m \rangle \longrightarrow b_n$$

$h, b_1, b_2, \ldots, b_n$ →

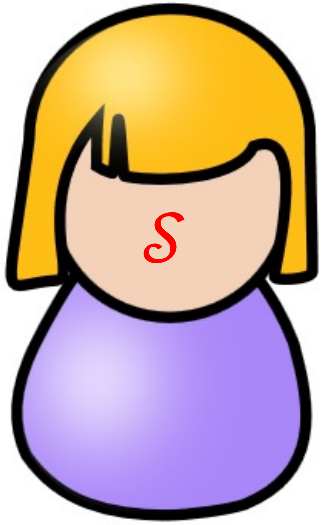
Outline

- Rate-1 OT from DDH [DGIMMO19]

- Amortized Rate-1 OT from Bilinear SXDH

- Optimizations

Amortized Rate-1 OT from Bilinear SXDH



$M =$	$g_{0,1}$	$g_{0,2}$	.	.	.	$g_{0,n}$	$g_{1,1}$	$g_{1,2}$	.	.	.	$g_{1,n}$
$vk_1 =$	$g_{0,1}^{\rho} \cdot g$	$g_{0,2}^{\rho_1}$	.	.	.	$g_{0,n}^{\rho_1}$	$g_{1,1}^{\rho_1} \cdot g$	$g_{1,2}^{\rho_1}$	.	.	.	$g_{1,n}^{\rho_1}$
$vk_2 =$	$g_{0,1}^{\rho_2}$	$g_{0,2}^{\rho_2} \cdot g$	.	.	.	$g_{0,n}^{\rho_2}$	$g_{1,1}^{\rho_2}$	$g_{1,2}^{\rho_2} \cdot g$	.	.	.	$g_{1,n}^{\rho_2}$
						$\vdots$						
$vk_n =$	$g_{0,1}^{\rho_n}$	$g_{0,2}^{\rho_n}$	.	.	.	$g_{0,n}^{\rho_n} \cdot g$	$g_{1,1}^{\rho_n}$	$g_{1,2}^{\rho_n}$	.	.	.	$g_{1,n}^{\rho_n} \cdot g$

$\in \mathbb{G}_1$

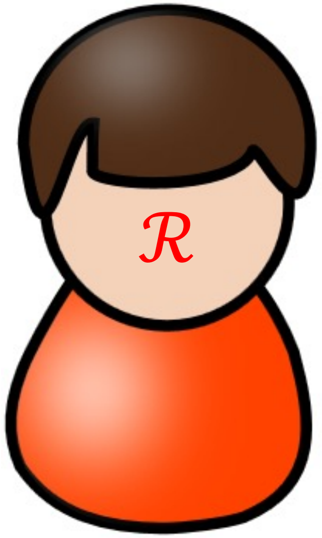
e

u $\in \mathbb{G}_2$

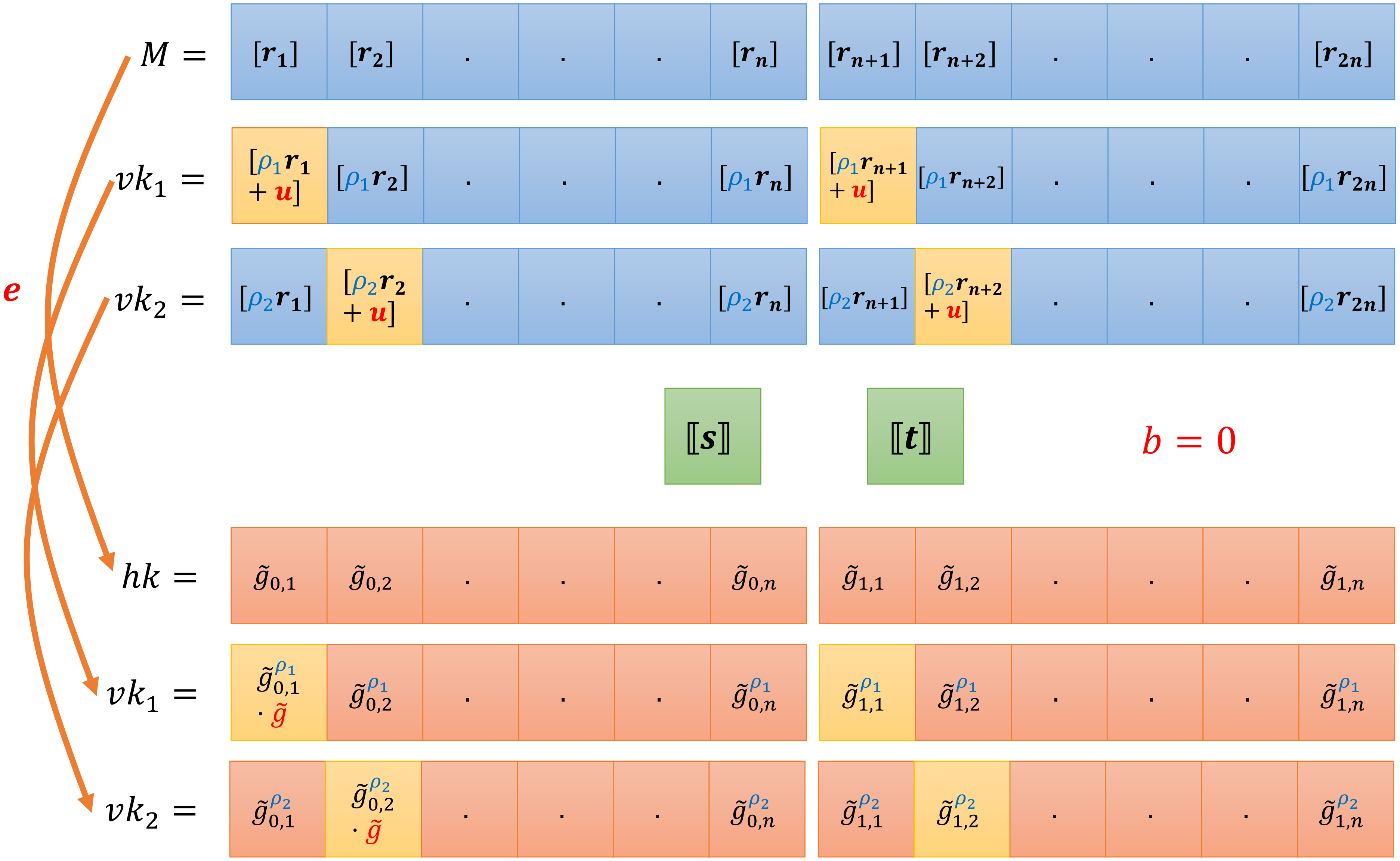
$b = 0$

$hk =$	$\tilde{g}_{0,1}$	$\tilde{g}_{0,2}$	.	.	.	$\tilde{g}_{0,n}$	$\tilde{g}_{1,1}$	$\tilde{g}_{1,2}$	.	.	.	$\tilde{g}_{1,n}$
$ek_1 =$	$\tilde{g}_{0,1}^{\rho_1} \cdot \tilde{g}$	$\tilde{g}_{0,2}^{\rho_1}$	.	.	.	$\tilde{g}_{0,n}^{\rho_1}$	$\tilde{g}_{1,1}^{\rho_1}$	$\tilde{g}_{1,2}^{\rho_1}$	.	.	.	$\tilde{g}_{1,n}^{\rho_1}$
$ek_2 =$	$\tilde{g}_{0,1}^{\rho_2}$	$\tilde{g}_{0,2}^{\rho_2} \cdot \tilde{g}$	.	.	.	$\tilde{g}_{0,n}^{\rho_2}$	$\tilde{g}_{1,1}^{\rho_2}$	$\tilde{g}_{1,2}^{\rho_2}$	.	.	.	$\tilde{g}_{1,n}^{\rho_2}$

$\in \mathbb{G}_T$



Amortized Rate-1 OT from Bilinear SXDH



$M =$	$[\boldsymbol{r}_1]$	$[\boldsymbol{r}_2]$	.	.	.	$[\boldsymbol{r}_n]$	$[\boldsymbol{r}_{n+1}]$	$[\boldsymbol{r}_{n+2}]$	.	.	.	$[\boldsymbol{r}_{2n}]$
$vk_1 =$	$[\rho_1\boldsymbol{r}_1 + \boldsymbol{u}]$	$[\rho_1\boldsymbol{r}_2]$	.	.	.	$[\rho_1\boldsymbol{r}_n]$	$[\rho_1\boldsymbol{r}_{n+1} + \boldsymbol{u}]$	$[\rho_1\boldsymbol{r}_{n+2}]$	.	.	.	$[\rho_1\boldsymbol{r}_{2n}]$
$vk_2 =$	$[\rho_2\boldsymbol{r}_1]$	$[\rho_2\boldsymbol{r}_2 + \boldsymbol{u}]$	.	.	.	$[\rho_2\boldsymbol{r}_n]$	$[\rho_2\boldsymbol{r}_{n+1}]$	$[\rho_2\boldsymbol{r}_{n+2} + \boldsymbol{u}]$	.	.	.	$[\rho_2\boldsymbol{r}_{2n}]$

$[\![\boldsymbol{s}]\!]$ $[\![\boldsymbol{t}]\!]$ $b = 0$

$hk =$	$\tilde{g}_{0,1}$	$\tilde{g}_{0,2}$	.	.	.	$\tilde{g}_{0,n}$	$\tilde{g}_{1,1}$	$\tilde{g}_{1,2}$	.	.	.	$\tilde{g}_{1,n}$
$vk_1 =$	$\tilde{g}_{0,1}^{\rho_1} \cdot \tilde{g}$	$\tilde{g}_{0,2}^{\rho_1}$	.	.	.	$\tilde{g}_{0,n}^{\rho_1}$	$\tilde{g}_{1,1}^{\rho_1}$	$\tilde{g}_{1,2}^{\rho_1}$	.	.	.	$\tilde{g}_{1,n}^{\rho_1}$
$vk_2 =$	$\tilde{g}_{0,1}^{\rho_2}$	$\tilde{g}_{0,2}^{\rho_2} \cdot \tilde{g}$	.	.	.	$\tilde{g}_{0,n}^{\rho_2}$	$\tilde{g}_{1,1}^{\rho_2}$	$\tilde{g}_{1,2}^{\rho_2}$	.	.	.	$\tilde{g}_{1,n}^{\rho_2}$

$\boldsymbol{r}_i, \boldsymbol{u} \xleftarrow{\$} \binom{\mathbb{Z}_p}{\mathbb{Z}_p}$

$[\boldsymbol{r}] := \binom{g^{r[0]}}{g^{r[1]}}$

$\rho_i \xleftarrow{\$} \mathbb{Z}_p$

$\boldsymbol{s}, \boldsymbol{t} \xleftarrow{\$} \binom{\mathbb{Z}_p}{\mathbb{Z}_p}$ s.t.

$\boldsymbol{s} \cdot \boldsymbol{u} = 1$

$\boldsymbol{t} \cdot \boldsymbol{u} = 0$

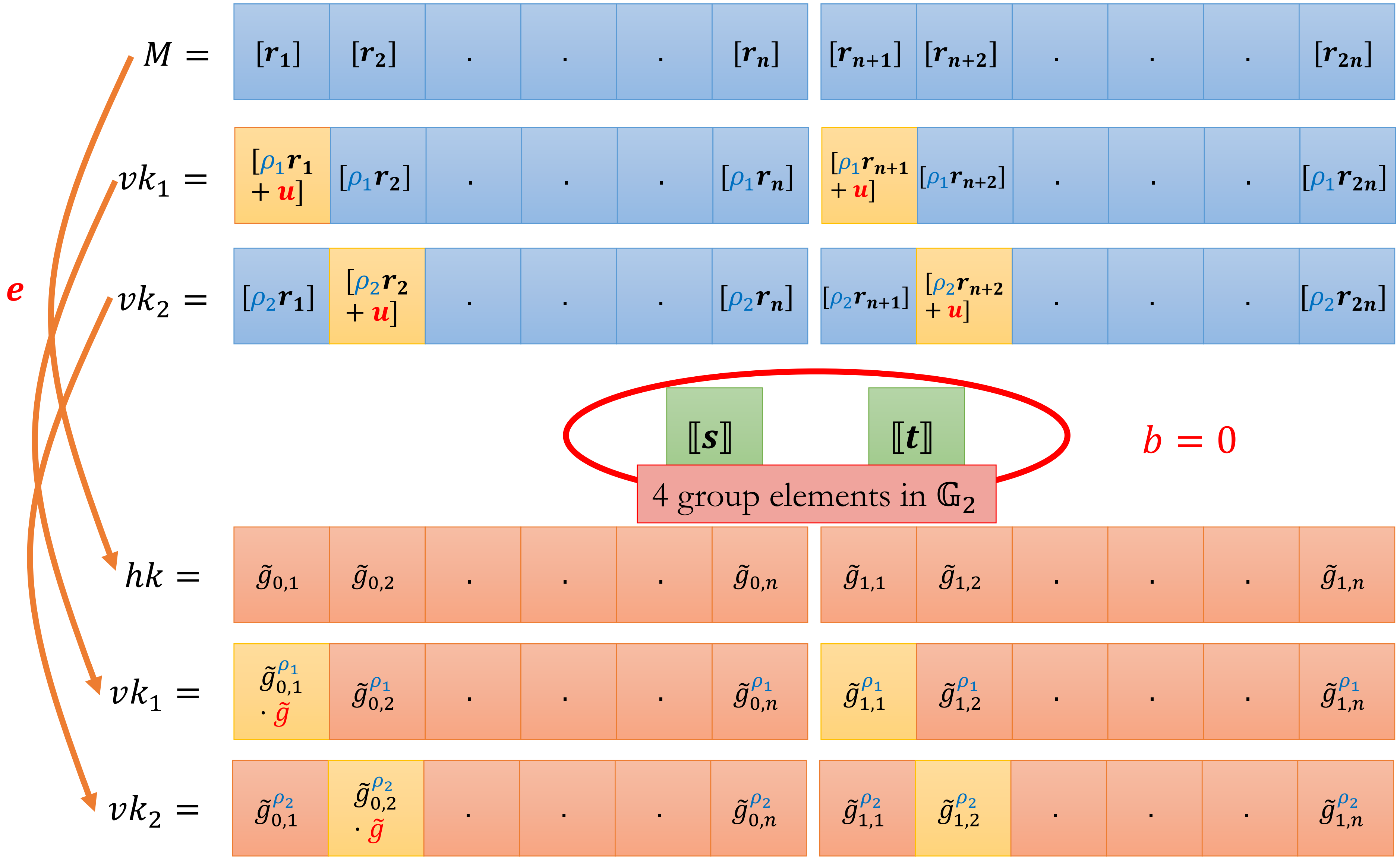
$[\![\boldsymbol{s}]\!] := \binom{h^{s[0]}}{h^{s[1]}}$

$$\boldsymbol{e}([\boldsymbol{r}], [\![\boldsymbol{s}]\!]) \coloneqq \boldsymbol{e}\left(g^{r[0]}, h^{s[0]}\right) \cdot \boldsymbol{e}\left(g^{r[1]}, h^{s[1]}\right)$$

Outline

- Rate-1 OT from DDH [DGIMMO19]
- Amortized Rate-1 OT from Bilinear SXDH
- Optimizations

Amortized Rate-1 OT from Bilinear SXDH



$M =$	$[\boldsymbol{r}_1]$	$[\boldsymbol{r}_2]$	.	.	.	$[\boldsymbol{r}_n]$	$[\boldsymbol{r}_{n+1}]$	$[\boldsymbol{r}_{n+2}]$	.	.	.	$[\boldsymbol{r}_{2n}]$
$vk_1 =$	$[\rho_1 \boldsymbol{r}_1 + \boldsymbol{u}]$	$[\rho_1 \boldsymbol{r}_2]$	.	.	.	$[\rho_1 \boldsymbol{r}_n]$	$[\rho_1 \boldsymbol{r}_{n+1} + \boldsymbol{u}]$	$[\rho_1 \boldsymbol{r}_{n+2}]$	.	.	.	$[\rho_1 \boldsymbol{r}_{2n}]$
$vk_2 =$	$[\rho_2 \boldsymbol{r}_1]$	$[\rho_2 \boldsymbol{r}_2 + \boldsymbol{u}]$	.	.	.	$[\rho_2 \boldsymbol{r}_n]$	$[\rho_2 \boldsymbol{r}_{n+1}]$	$[\rho_2 \boldsymbol{r}_{n+2} + \boldsymbol{u}]$	.	.	.	$[\rho_2 \boldsymbol{r}_{2n}]$

$[\![\boldsymbol{s}]\!]$ $[\![\boldsymbol{t}]\!]$

4 group elements in $\mathbb{G}_2$

$b = 0$

$hk =$	$\tilde{g}_{0,1}$	$\tilde{g}_{0,2}$	.	.	.	$\tilde{g}_{0,n}$	$\tilde{g}_{1,1}$	$\tilde{g}_{1,2}$	.	.	.	$\tilde{g}_{1,n}$
$vk_1 =$	$\tilde{g}_{0,1}^{\rho_1} \cdot \tilde{g}$	$\tilde{g}_{0,2}^{\rho_1}$	.	.	.	$\tilde{g}_{0,n}^{\rho_1}$	$\tilde{g}_{1,1}^{\rho_1}$	$\tilde{g}_{1,2}^{\rho_1}$	.	.	.	$\tilde{g}_{1,n}^{\rho_1}$
$vk_2 =$	$\tilde{g}_{0,1}^{\rho_2}$	$\tilde{g}_{0,2}^{\rho_2} \cdot \tilde{g}$	.	.	.	$\tilde{g}_{0,n}^{\rho_2}$	$\tilde{g}_{1,1}^{\rho_2}$	$\tilde{g}_{1,2}^{\rho_2}$	.	.	.	$\tilde{g}_{1,n}^{\rho_2}$

$\boldsymbol{r}_i, \boldsymbol{u} \overset{\$}{\leftarrow} \begin{pmatrix} \mathbb{Z}_p \\ \mathbb{Z}_p \end{pmatrix}$

$[\boldsymbol{r}] := \begin{pmatrix} g^{r[0]} \\ g^{r[1]} \end{pmatrix}$

$\rho_i \overset{\$}{\leftarrow} \mathbb{Z}_p$

$\boldsymbol{s}, \boldsymbol{t} \overset{\$}{\leftarrow} \begin{pmatrix} \mathbb{Z}_p \\ \mathbb{Z}_p \end{pmatrix}$ s.t.

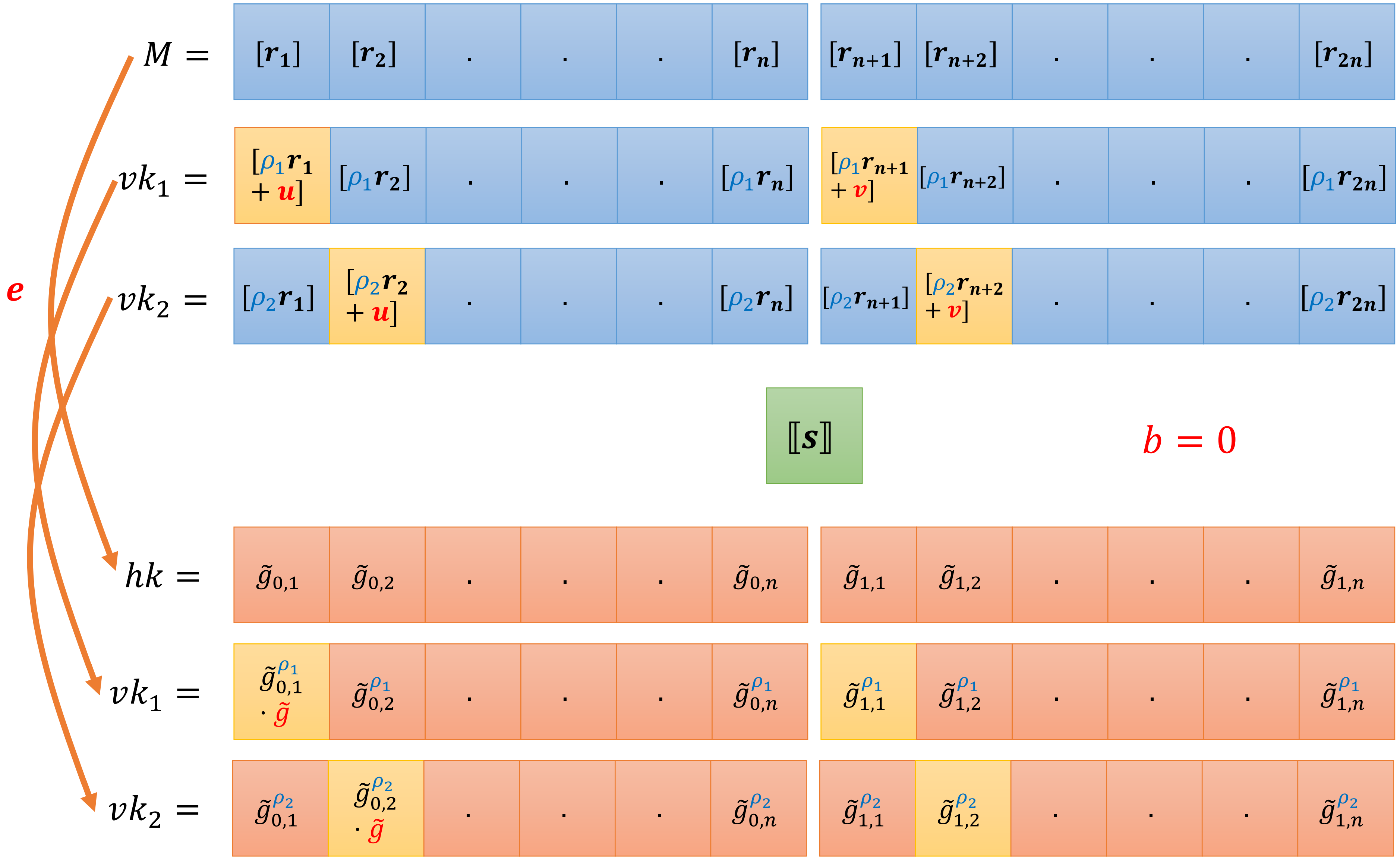
$\boldsymbol{s} \cdot \boldsymbol{u} = 1$

$\boldsymbol{t} \cdot \boldsymbol{u} = 0$

$[\![\boldsymbol{s}]\!] := \begin{pmatrix} h^{s[0]} \\ h^{s[1]} \end{pmatrix}$

$$\boldsymbol{e}([\boldsymbol{r}], [\![\boldsymbol{s}]\!]) \coloneqq \boldsymbol{e}\left(g^{r[0]}, h^{s[0]}\right) \cdot \boldsymbol{e}\left(g^{r[1]}, h^{s[1]}\right)$$

From 4 to 3 Group Elements in $\mathbb{G}_2$



$M =$	$[\boldsymbol{r}_1]$	$[\boldsymbol{r}_2]$	.	.	.	$[\boldsymbol{r}_n]$	$[\boldsymbol{r}_{n+1}]$	$[\boldsymbol{r}_{n+2}]$	.	.	.	$[\boldsymbol{r}_{2n}]$
$vk_1 =$	$[\rho_1 \boldsymbol{r}_1 + \boldsymbol{u}]$	$[\rho_1 \boldsymbol{r}_2]$	.	.	.	$[\rho_1 \boldsymbol{r}_n]$	$[\rho_1 \boldsymbol{r}_{n+1} + \boldsymbol{v}]$	$[\rho_1 \boldsymbol{r}_{n+2}]$	.	.	.	$[\rho_1 \boldsymbol{r}_{2n}]$
$vk_2 =$	$[\rho_2 \boldsymbol{r}_1]$	$[\rho_2 \boldsymbol{r}_2 + \boldsymbol{u}]$	.	.	.	$[\rho_2 \boldsymbol{r}_n]$	$[\rho_2 \boldsymbol{r}_{n+1}]$	$[\rho_2 \boldsymbol{r}_{n+2} + \boldsymbol{v}]$	.	.	.	$[\rho_2 \boldsymbol{r}_{2n}]$

$\llbracket \boldsymbol{s} \rrbracket$

$b = 0$

$hk =$	$\tilde{g}_{0,1}$	$\tilde{g}_{0,2}$	.	.	.	$\tilde{g}_{0,n}$	$\tilde{g}_{1,1}$	$\tilde{g}_{1,2}$	.	.	.	$\tilde{g}_{1,n}$
$vk_1 =$	$\tilde{g}_{0,1}^{\rho_1} \cdot \tilde{g}$	$\tilde{g}_{0,2}^{\rho_1}$	.	.	.	$\tilde{g}_{0,n}^{\rho_1}$	$\tilde{g}_{1,1}^{\rho_1}$	$\tilde{g}_{1,2}^{\rho_1}$	.	.	.	$\tilde{g}_{1,n}^{\rho_1}$
$vk_2 =$	$\tilde{g}_{0,1}^{\rho_2}$	$\tilde{g}_{0,2}^{\rho_2} \cdot \tilde{g}$	.	.	.	$\tilde{g}_{0,n}^{\rho_2}$	$\tilde{g}_{1,1}^{\rho_2}$	$\tilde{g}_{1,2}^{\rho_2}$	.	.	.	$\tilde{g}_{1,n}^{\rho_2}$

$$\boldsymbol{r}_i, \boldsymbol{u}, \boldsymbol{v} \overset{\$}{\leftarrow} \begin{pmatrix}\mathbb{Z}_p\\ \mathbb{Z}_p\\ \mathbb{Z}_p\end{pmatrix}$$

$$[\boldsymbol{r}] := \begin{pmatrix} g^{r[0]}\\ g^{r[1]}\\ g^{r[2]}\end{pmatrix}$$

$$\rho_i \overset{\$}{\leftarrow} \mathbb{Z}_p$$

$$\boldsymbol{s} \overset{\$}{\leftarrow} \begin{pmatrix}\mathbb{Z}_p\\ \mathbb{Z}_p\\ \mathbb{Z}_p\end{pmatrix} \text{ s.t.}$$

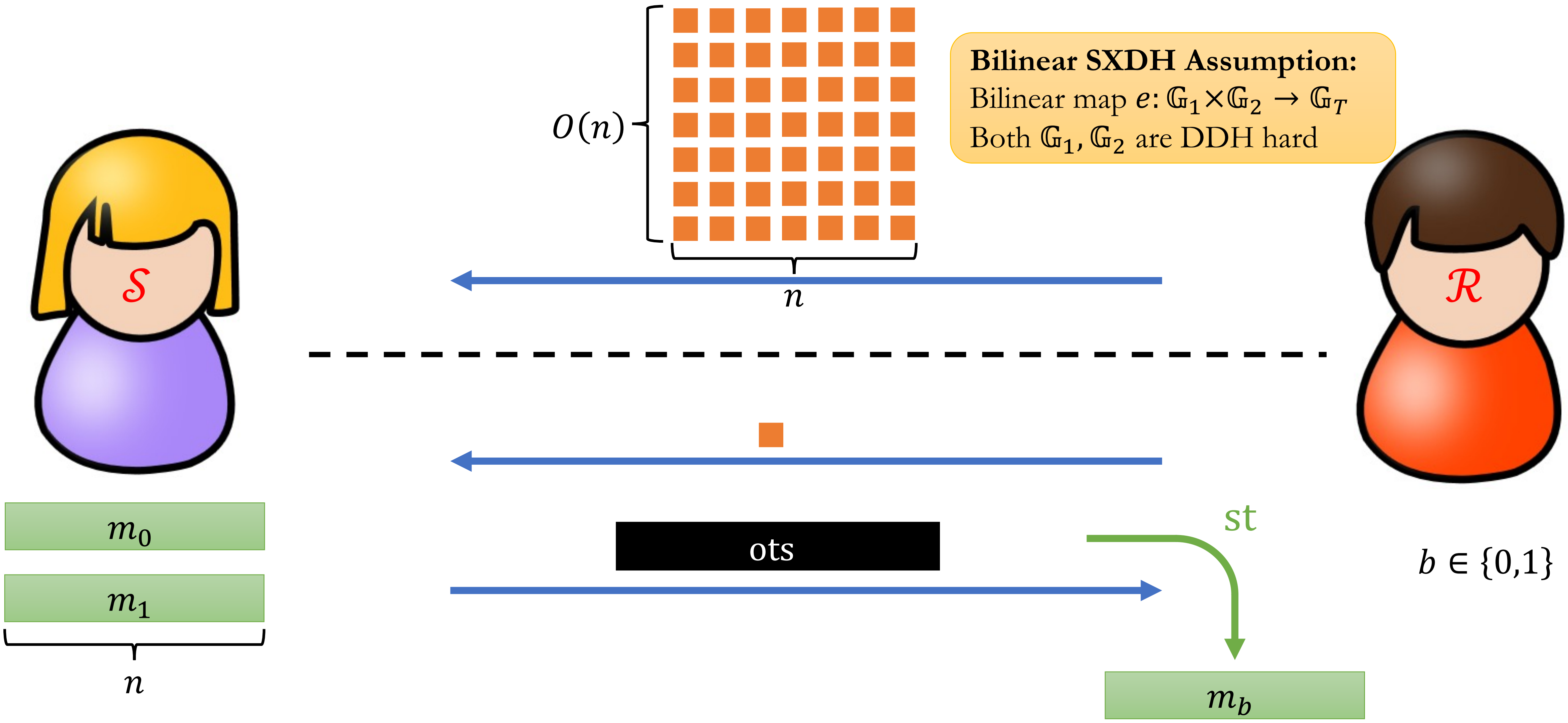
$$\boldsymbol{s} \cdot \boldsymbol{u} = 1$$

$$\boldsymbol{s} \cdot \boldsymbol{v} = 0$$

$$\llbracket \boldsymbol{s} \rrbracket := \begin{pmatrix} h^{s[0]}\\ h^{s[1]}\\ h^{s[2]}\end{pmatrix}$$

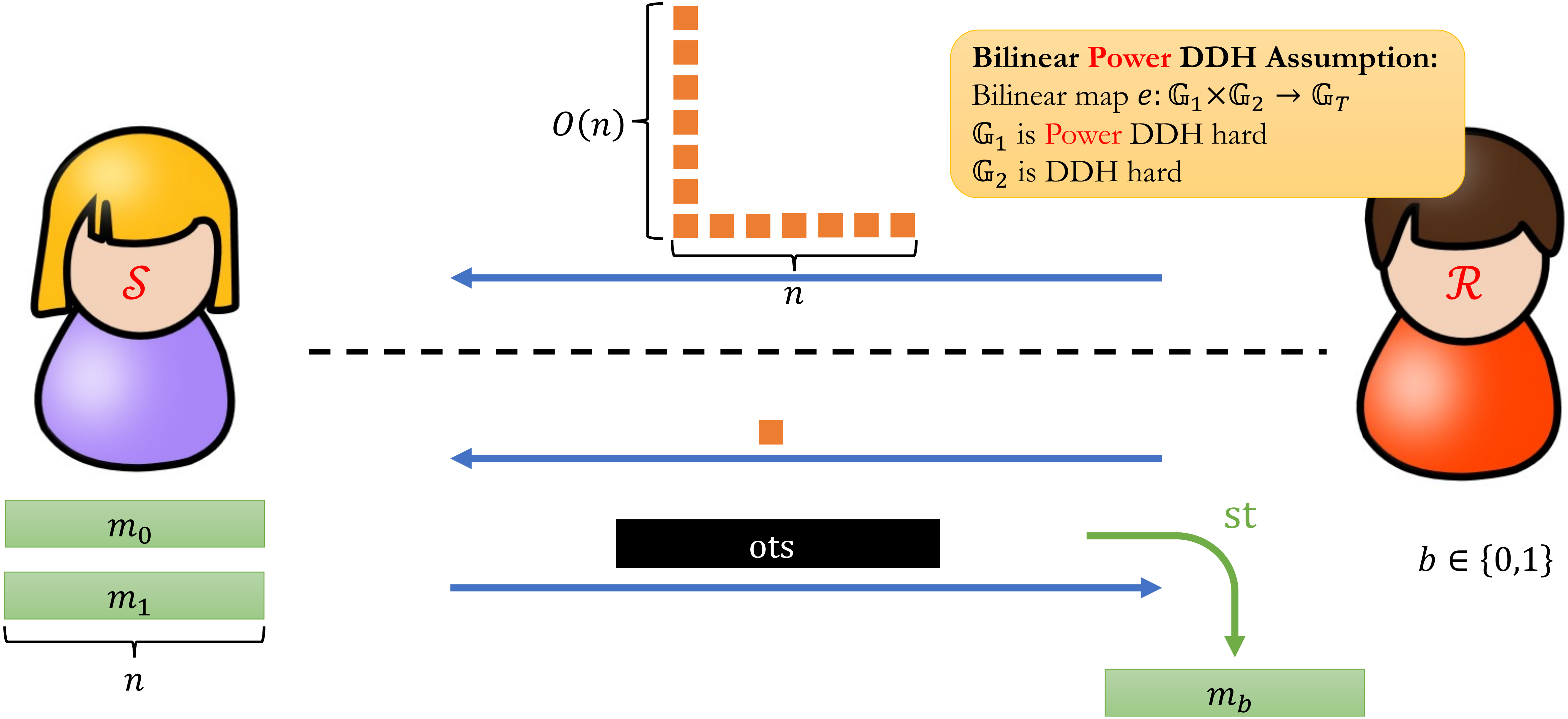
$$\boldsymbol{e}([\boldsymbol{r}], \llbracket \boldsymbol{s} \rrbracket) := \boldsymbol{e}\left(g^{r[0]}, h^{s[0]}\right) \cdot \boldsymbol{e}\left(g^{r[1]}, h^{s[1]}\right) \cdot \boldsymbol{e}\left(g^{r[2]}, h^{s[2]}\right)$$

Summary: Amortized Rate-1 OT



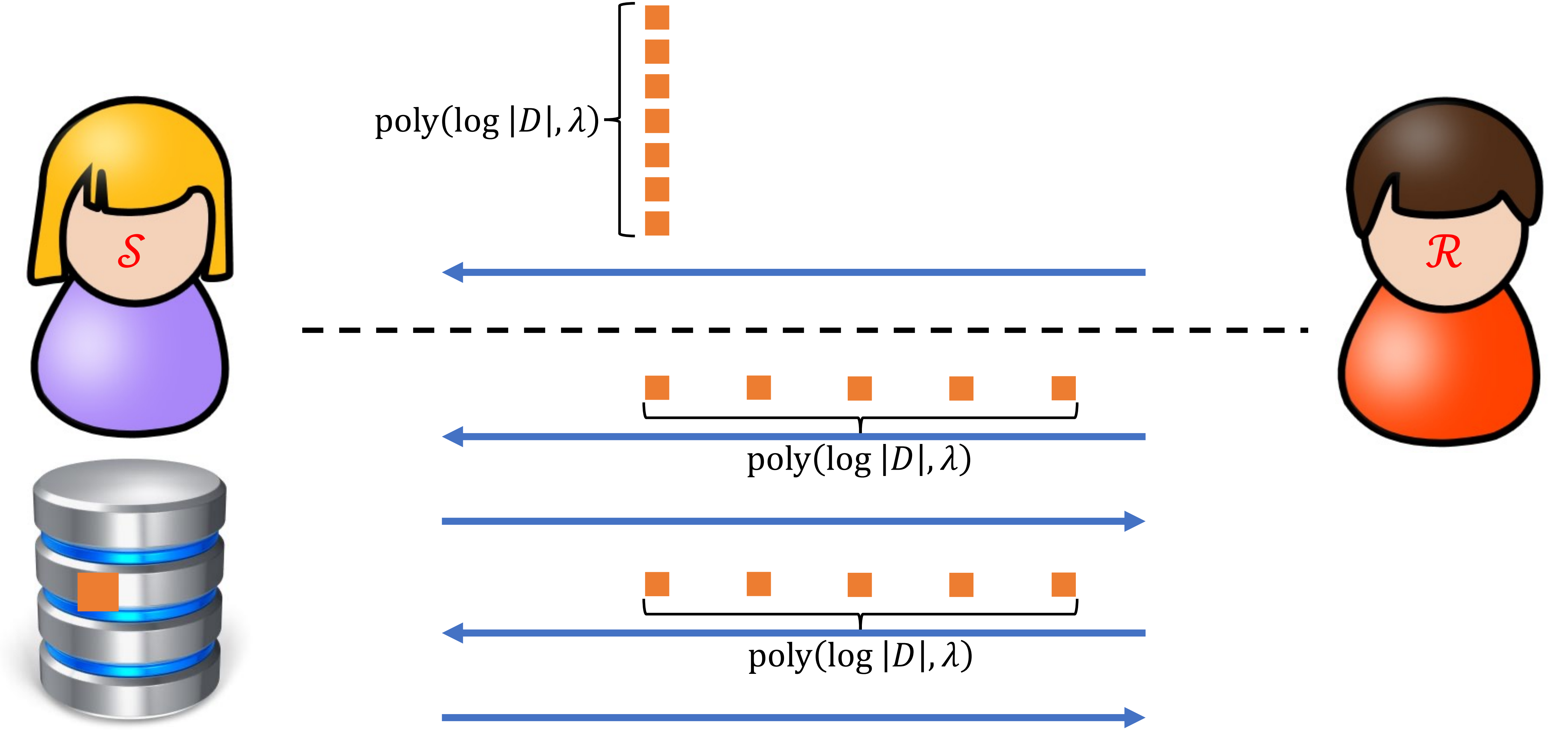
Bilinear SXDH Assumption:
Bilinear map $e: \mathbb{G}_1 \times \mathbb{G}_2 \rightarrow \mathbb{G}_T$
Both $\mathbb{G}_1, \mathbb{G}_2$ are DDH hard

Summary: Amortized Rate-1 OT



Bilinear Power DDH Assumption:
Bilinear map $e: \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$
$\mathbb{G}_1$ is Power DDH hard
$\mathbb{G}_2$ is DDH hard

Summary: Applications (PIR & PSI)



Open Problems

- Amortized Rate-1 OT from other assumptions

- Amortized Rate-1 OT extension

- Applications
    - More applications of amortized Rate-1 OT
    - Concretely efficient implementation of the applications

Thank you!