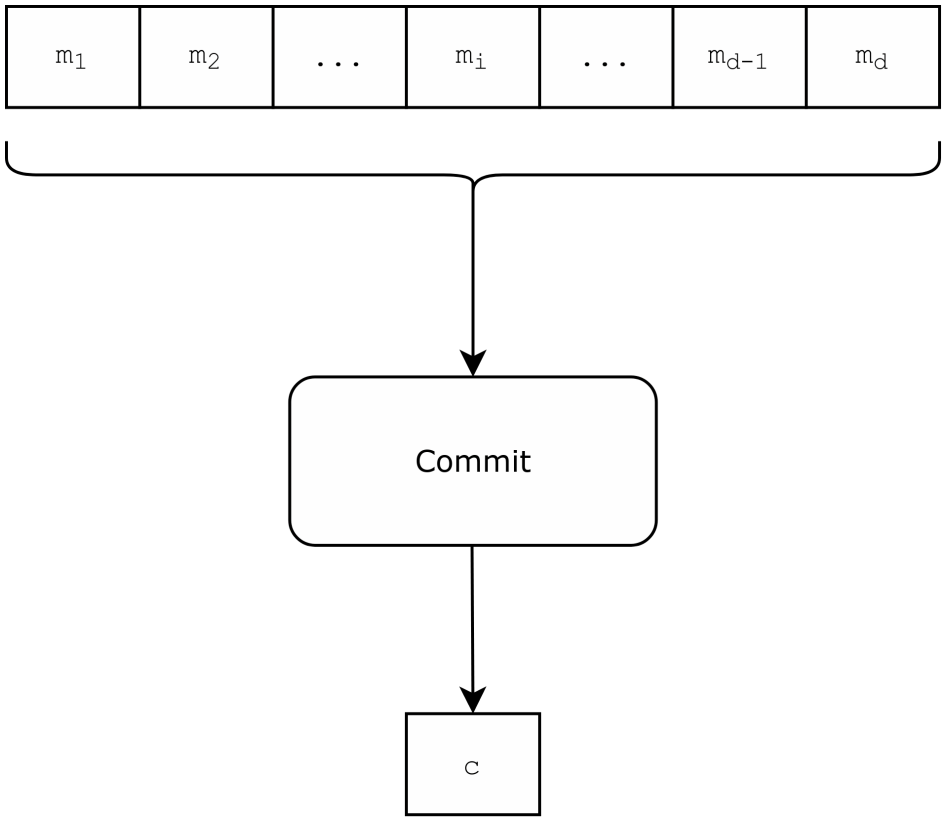
Vector and Functional Commitments from Lattices

Chris Peikert, Zachary Pepin, Chad Sharp

Vector Commitments [LY10, CF13]



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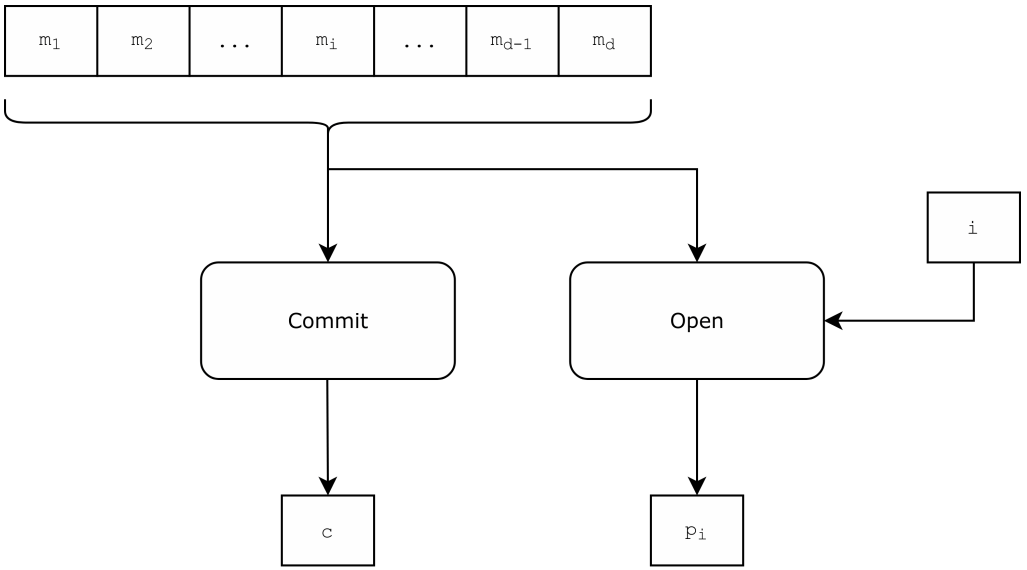

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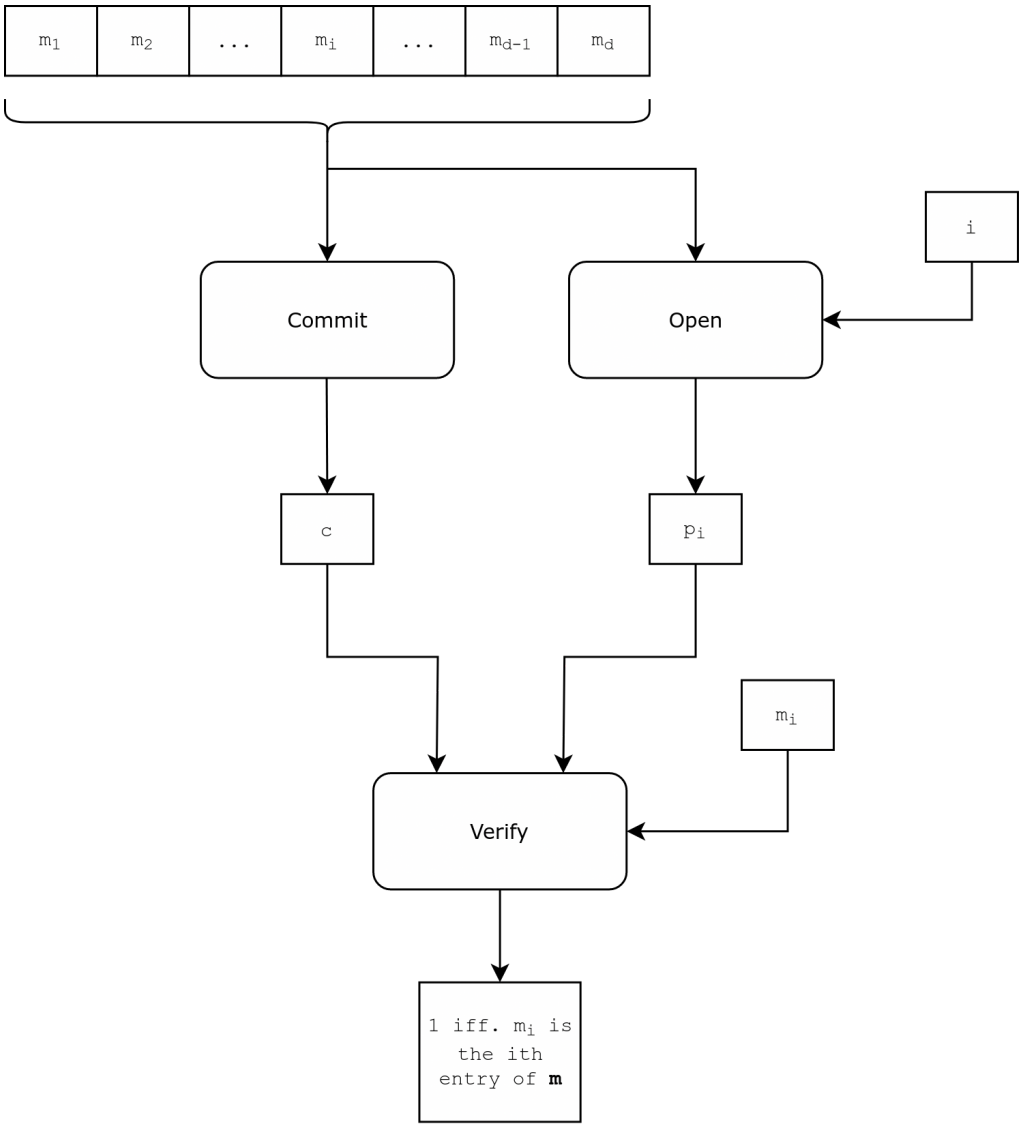

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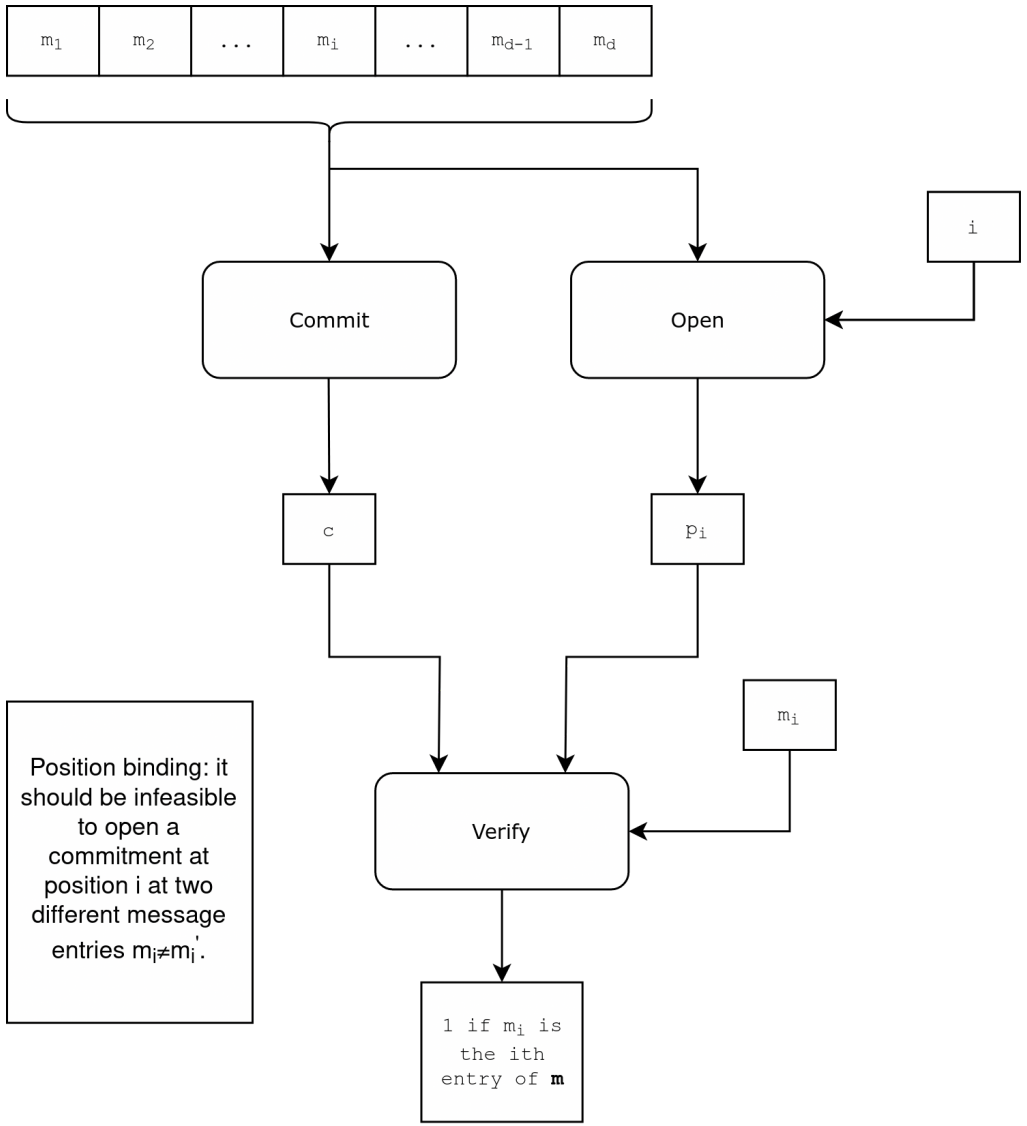
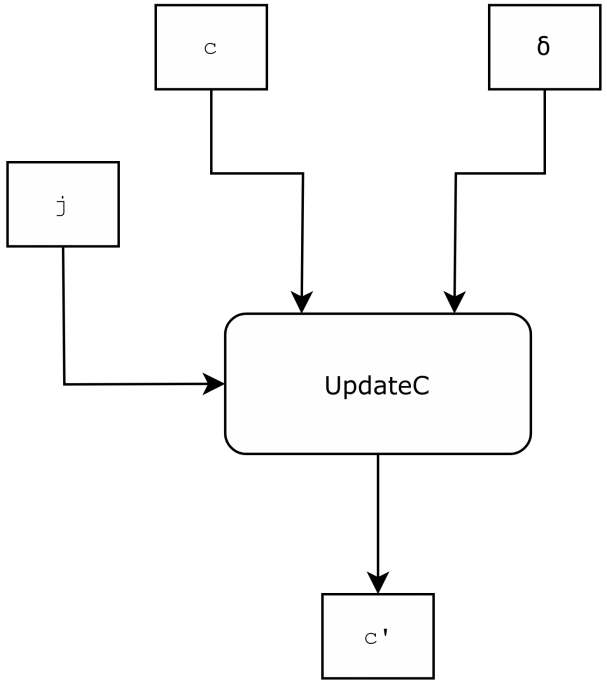
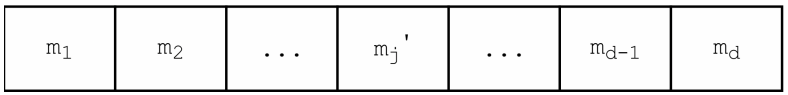


Vector Commitments: (Stateless) Updates





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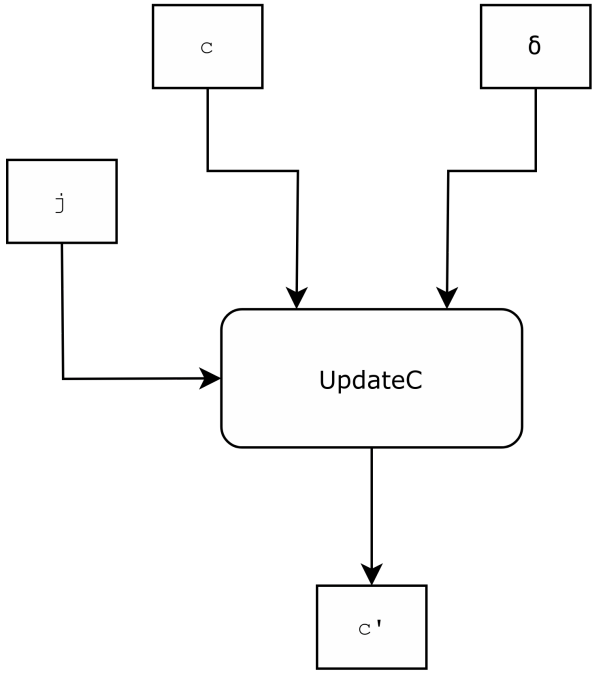

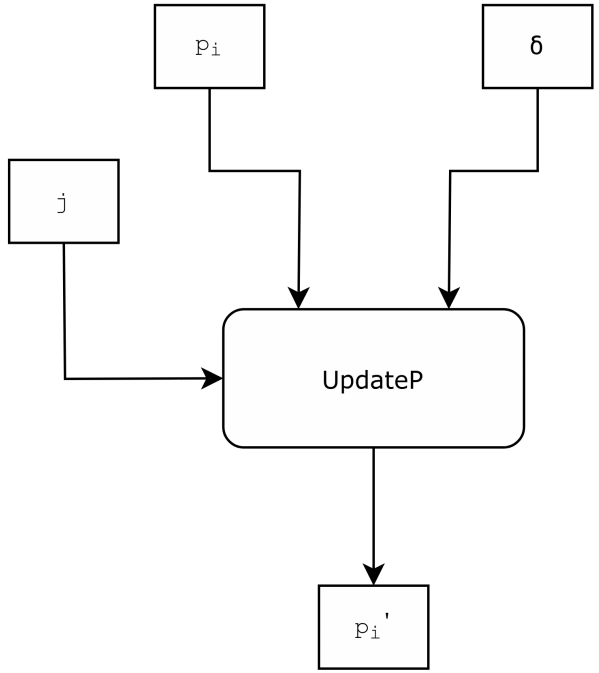

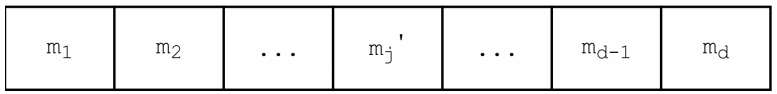

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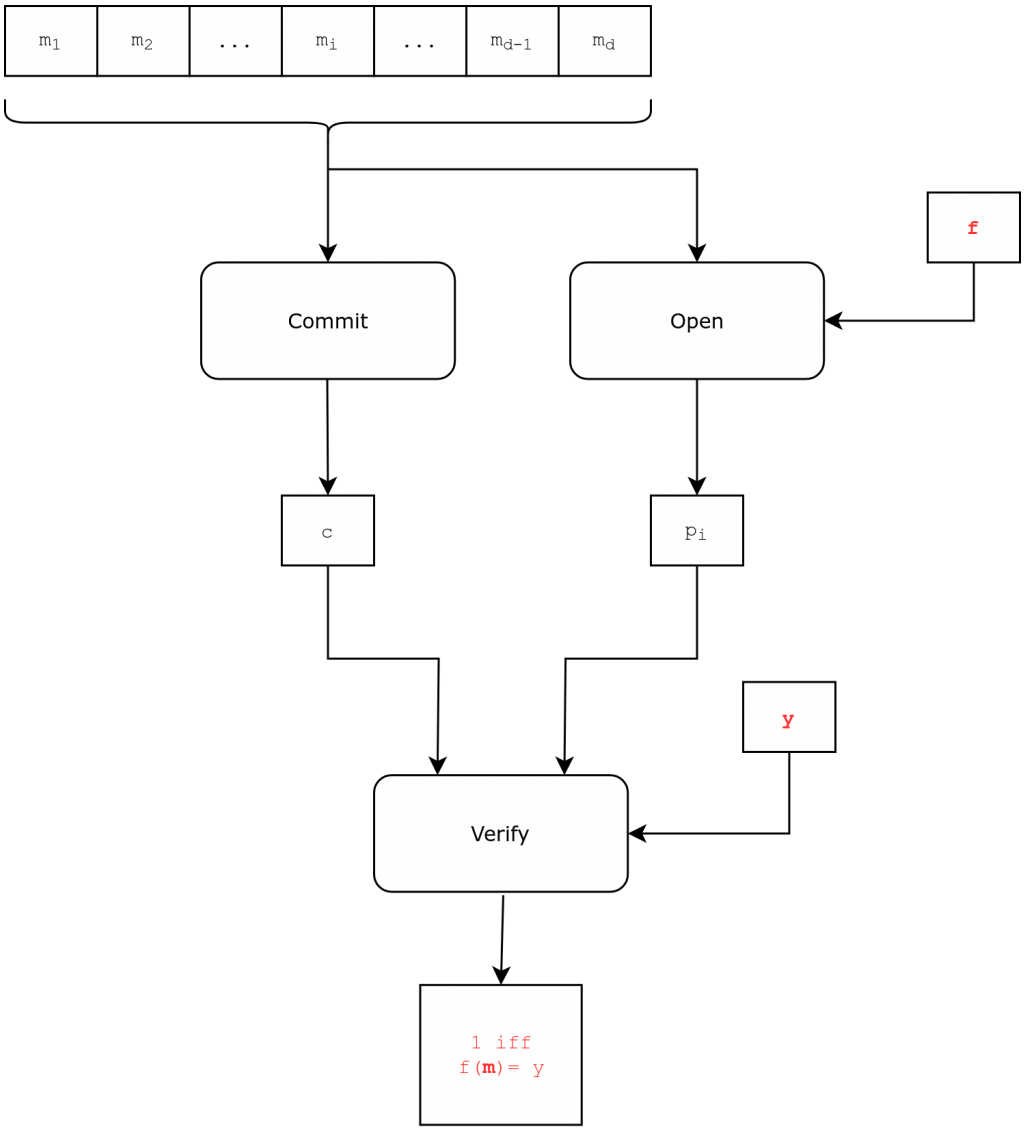

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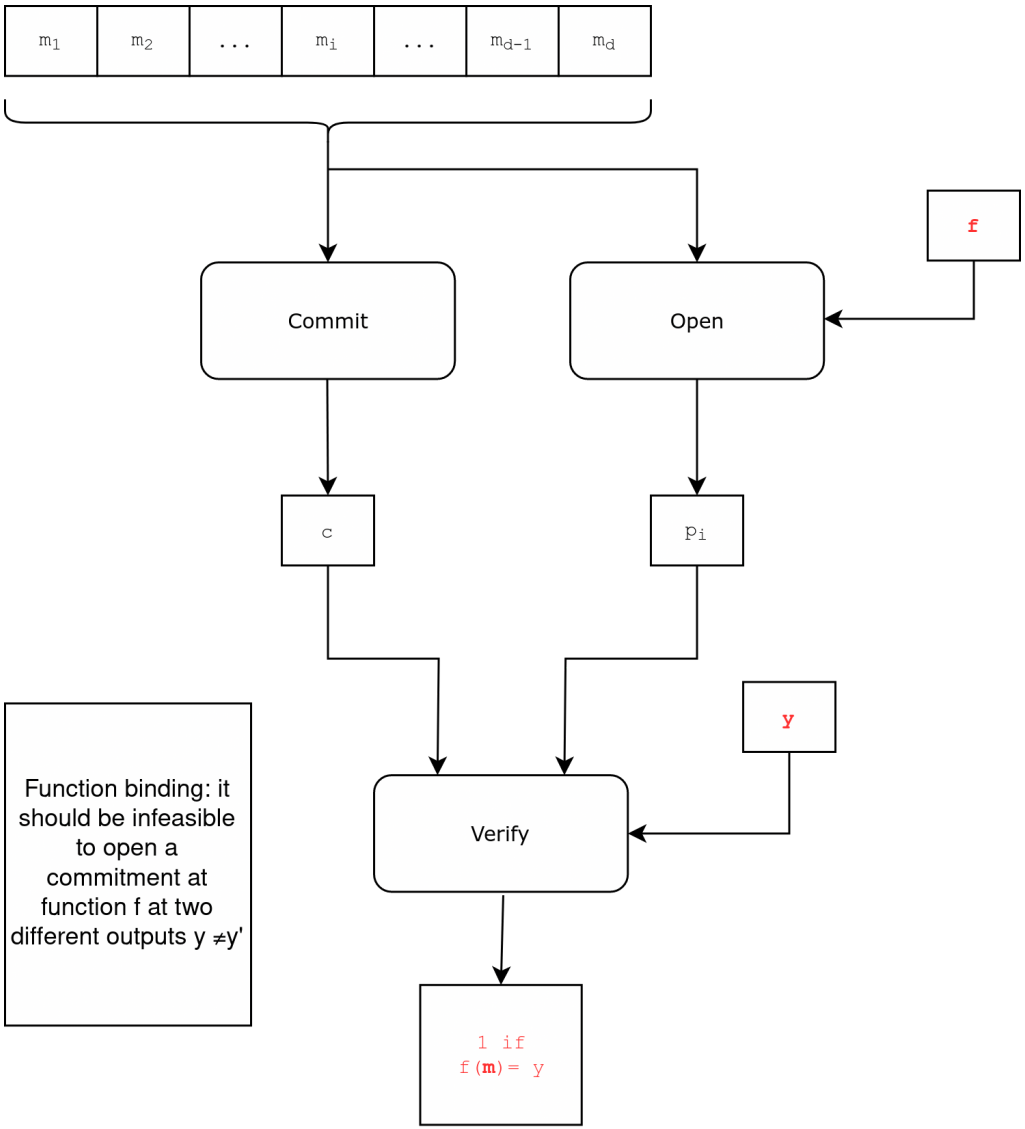

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- Applications: verifiable oursourcing of storage [CF13, BGV11], verifiable zero knowledge sets [MRK03], cryptographic accumulators [BdM93], pseudononymous credentials [KZG10], cryptocurrencies [CPSZ18].

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- Applications: verifiable secret sharing [LRY10], content extraction signatures [SBZ01], and zero-knowledge SNARKS [BFS20, BDFG20].

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Scheme	$\lvert pp\rvert$	$\lvert c\rvert$	$\lvert \pi\rvert$	Setup	PQ
[PSTY13] (SIS)	$h^2 d$	$h \log d$	$h^3 d \log^2 d$	Public	✓
Tree construction (SIS)	$h^2 d^2$	$h \log d$	$h^3 \log^2 d$	Private	✓

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Secondary contributions:

1. Formal definition and generic construction of a zero-knowledge vector commitment scheme.
2. Formal analysis of a (folklore) Merkle-like tree transformation for VC schemes that makes them suitable for vectors of large arity.

Short Integer Solution ($\mathsf{SIS}_{n,m,q,\beta}$)

Given uniformly random $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$, find nonzero $\mathbf{x} \in \mathbb{Z}^m$ such that

$$\mathbf{A}\mathbf{x} = \mathbf{0} \bmod q$$

and

$$\|\mathbf{x}\| \leq \beta.$$

SIS-based VC scheme

Setup$(1^n, 1^d)$:

- Sample $\mathbf{U} = [\mathbf{u}_0 \mid \cdots \mid \mathbf{u}_{d-1}] \in \mathbb{Z}_q^{n \times d}$ u.a.r.

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- For all distinct $i, j \in [d]$, sample $\mathbf{A}_i \in \mathbb{Z}_q^{n \times m}$ and (short) $\mathbf{r}_{i,j} \in \mathbb{Z}^m$ (via trapdoor pre-image sampling) such that:

$$\mathbf{A}_i \mathbf{r}_{i,j} = \mathbf{u}_j.$$

- Output $\mathbf{A}_i$s, $\mathbf{r}_{i,j}$s, and $\mathbf{U}$ as public parameters.

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$$\begin{bmatrix} \mathbf{A}_0 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{d-1} \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{0} & \mathbf{r}_{0,1} & \cdots & \mathbf{r}_{0,d-1} \\ \mathbf{r}_{1,0} & \mathbf{0} & \cdots & \mathbf{r}_{1,d-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{r}_{d-1,0} & \mathbf{r}_{d-1,1} & \cdots & \mathbf{0} \end{bmatrix}}_{\tilde{\mathbf{R}}} = \begin{bmatrix} \mathbf{0} & \mathbf{u}_1 & \cdots & \mathbf{u}_{d-1} \\ \mathbf{u}_0 & \mathbf{0} & \cdots & \mathbf{u}_{d-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{u}_0 & \mathbf{u}_1 & \cdots & \mathbf{0} \end{bmatrix}$$

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$$\mathbf{A}_i(\tilde{\mathbf{R}}_{i,*}\mathbf{m}) + m_i\mathbf{u}_i = \sum_{j\neq i} m_j\mathbf{A}_i\mathbf{r}_{i,j} + m_i\mathbf{u}_i = \sum_{j\neq i} m_j\mathbf{u}_j + m_i\mathbf{u}_i = \mathbf{U}\mathbf{m} = \mathbf{c}$$

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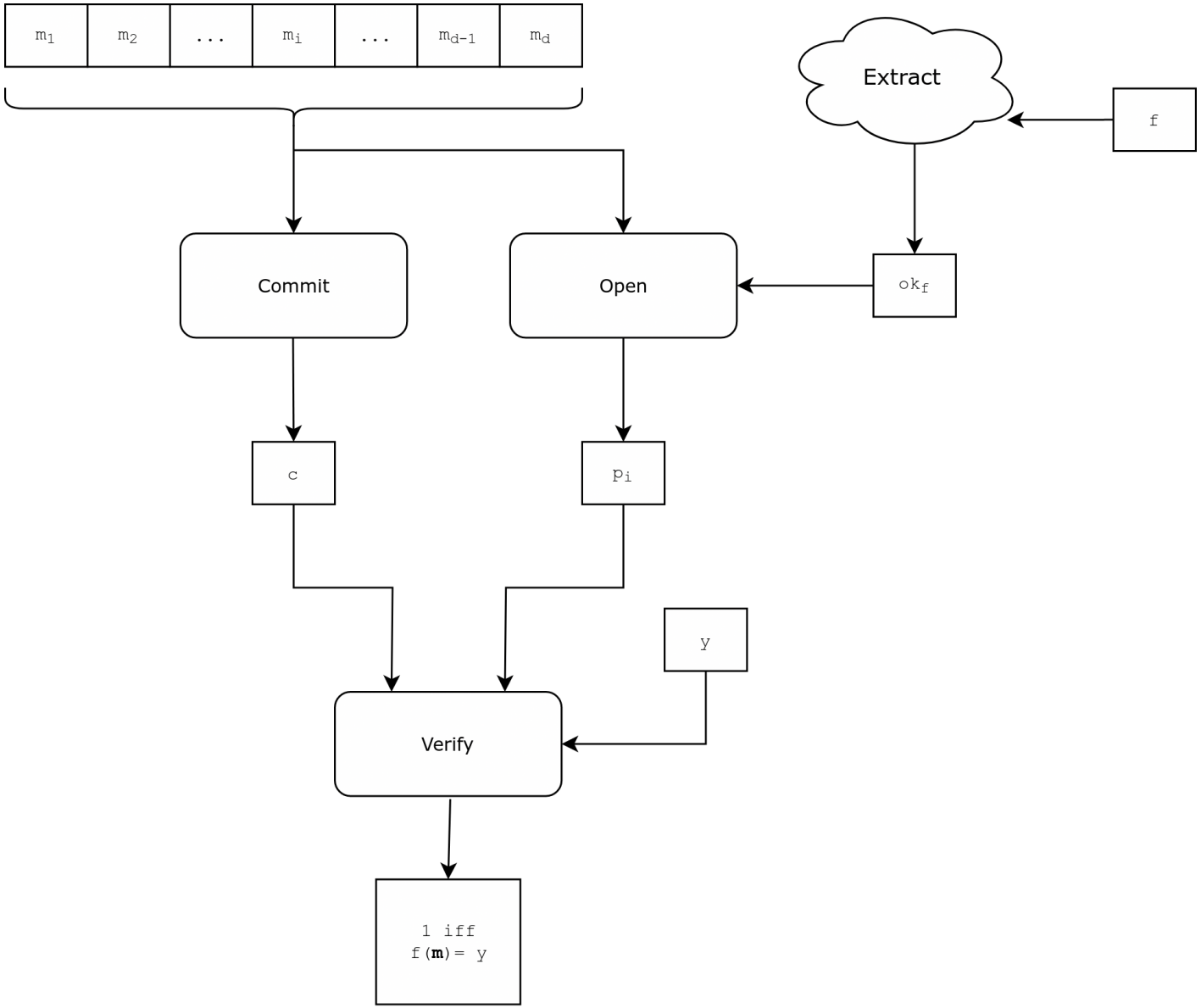
Functional Commitments with Authority



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- Additive and multiplicative homomorphisms.

Homomorphic commitments [GSW13]

$$\mathsf{Eval}(f, \mathbf{C}_x[, \mathbf{R}_x]) \text{ outputs } (\mathbf{C}_{x,f}[, \mathbf{R}_{x,f}])$$

such that $\mathbf{R}_{x,f}$ is still short and if

$$\mathbf{C}_x = \mathsf{Com}_{\mathbf{A}}(x; \mathbf{R}_x),$$

then

$$\mathbf{C}_{x,f} = \mathsf{Com}_{\mathbf{A}}(f(x), \mathbf{R}_{x,f}).$$

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- Think of $\mathbf{C}$ as a superposition of commitments to all $f \in \mathcal{F}$.

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$\mathsf{Extract}_{ek}(f)$:

- Use $\mathbf{T}$ to generate a witness $\mathbf{R}_f$ that $\mathbf{C}$ is a commitment to f wrt $\mathbf{A}_f$.
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$\mathsf{Open}_{pp}(\mathbf{m}, f, ok_f = \mathbf{R}_f)$:

- compute $(\mathbf{C_m}; \mathbf{R}_{\mathbf{m},f}) = \mathsf{Eval}(U_{\mathbf{m}}, \mathbf{C}; \mathbf{R}_f)$ and output $\mathbf{R}_{\mathbf{m},f}$.

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